Module 4

GAS TURBINE ENGINE COMPRESSORS

Syllabus:

Centrifugal compressors: Principle of operation of centrifugal compressors. Work done and pressure rise -Velocity diagrams, Diffuser vane design considerations. Performance characteristics. Concept of Pre-whirl, Rotating stall.

Axial flow compressors: Elementary theory of axial flow compressor, Velocity triangles, Degree of reaction, three dimensional flow. Air angle distribution for free vortex and constant reaction designs, Compressor blade design. Axial compressor performance characteristics.

4.1 Principle of operation of centrifugal compressor

Essential parts of a centrifugal compressor:

- The **inlet casing** with converging nozzle, whose function is to accelerate the fluid to the impeller inlet. The outlet of the inlet casing is known as the eye.
- **The impeller**, in which the energy transfer takes place, resulting in a rise of fluid kinetic energy and static pressure.

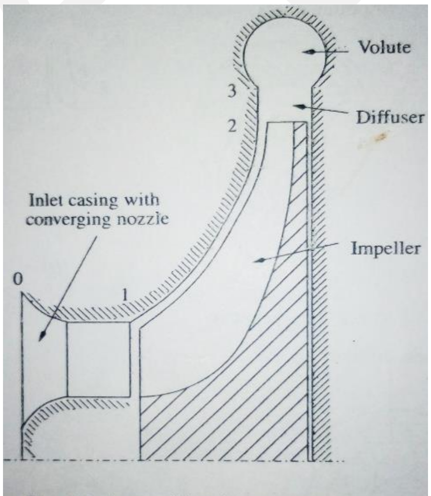


Fig. 8.1 A centrifugal compressor stage

- **The diffuser**, whose function is to transform the high kinetic energy of the fluid at the impeller outlet into static pressure.
- The **outlet casing**, which comprises a fluid collector known as a volute or scroll.
- **The impeller vanes** help to transfer the energy from the impeller to the fluid.

- The hub, which is surface AB.
- The **shroud,** which is surface CD. Impellers enclosed on the surface CD are known as shrouded impellers, but the surface CD is referred to as the shroud in descriptions of impeller geometry whether the impeller is enclosed or not.
- The **inducer**, the section EF. whose function is to increase the angular momentum of the fluid without increasing its radius of rotation.

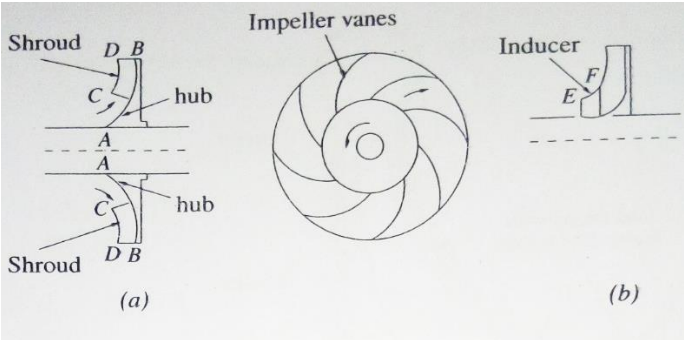
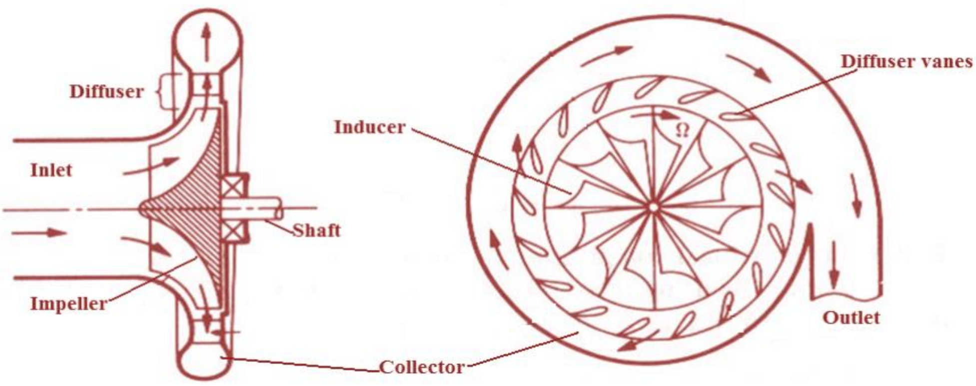


Fig. 8.2 The impeller



Schematic of a typical centrifugal compressor

Principle of Operation:

- Air is sucked into the impeller eye through an accelerating nozzle and whirled round at high speed by the vanes on the impeller disc.
- At any point, in the impeller, the flow experiences a centripetal acceleration due to a pressure head.
- Hence, the static pressure of the air increases from the eye to the tip of the impeller. The remainder of the static pressure rise is obtained in the diffuser.
- It may be noted that air enters the impeller eye with a very high velocity.
- The friction in the diffuser will cause some loss in stagnation pressure.
- It is a normal practice to design the compressor such that 50% pressure rise occurs in the impeller and another 50% in the diffuser.

- It is to be understood that owing to the action of the vanes in carrying the air around, with the impeller, there will be a slightly higher static pressure on the forward face of a vane than the trailing face.
- This pressure differential will make the air to flow around the space between the impeller and the casing. This naturally results is a loss of efficiency.
- Therefore, the clearance must be kept as small as possible to reduce such a loss.
- The manufacturing difficulties are greatly increased to achieve this. Further there would be a disc friction or windage loss associated with the shroud.

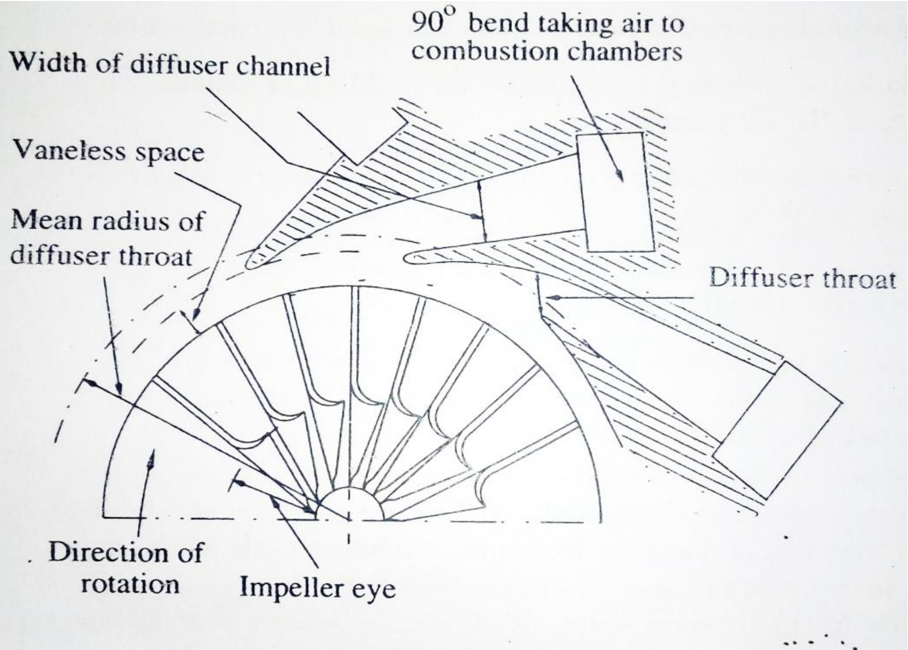


Fig. 8.4 Schematic diagram of a centrifugal compressor

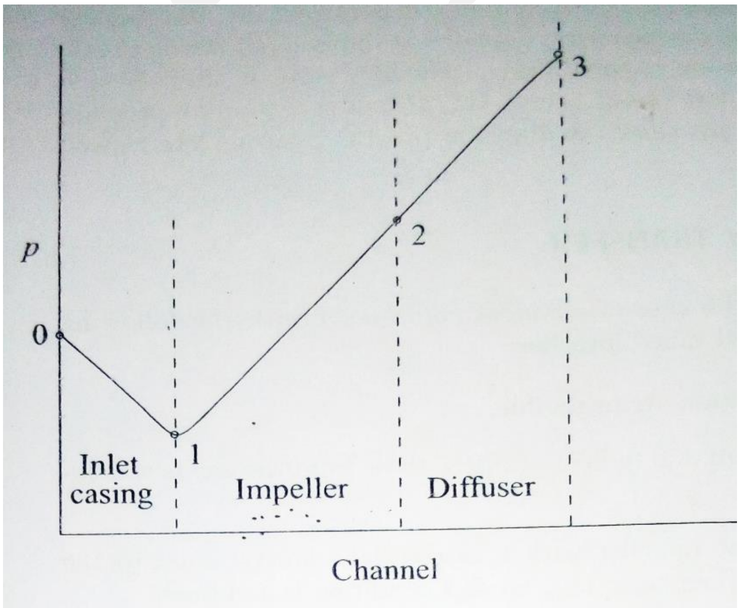


Fig. 8.6 Pressure rise across centrifugal compressor

4.2 Work done and pressure rise of centrifugal compressor

Since no work is done on the air in the diffuser, the energy absorbed by the compressor will be determined by the conditions of the air at the inlet and outlet of the impeller. Figure 4.2 shows the nomenclature employed.

In the first instance it will be assumed that the air enters the impeller eye in the axial direction, so that the initial angular momentum of the air is zero. The axial portion of the vanes must be curved so that the air can pass smoothly into the eye. The angle which the leading edge of a vane makes with the tangential direction α will be given by the direction of the relative velocity of the air at inlet, V_1, as shown in fig. 4.2.

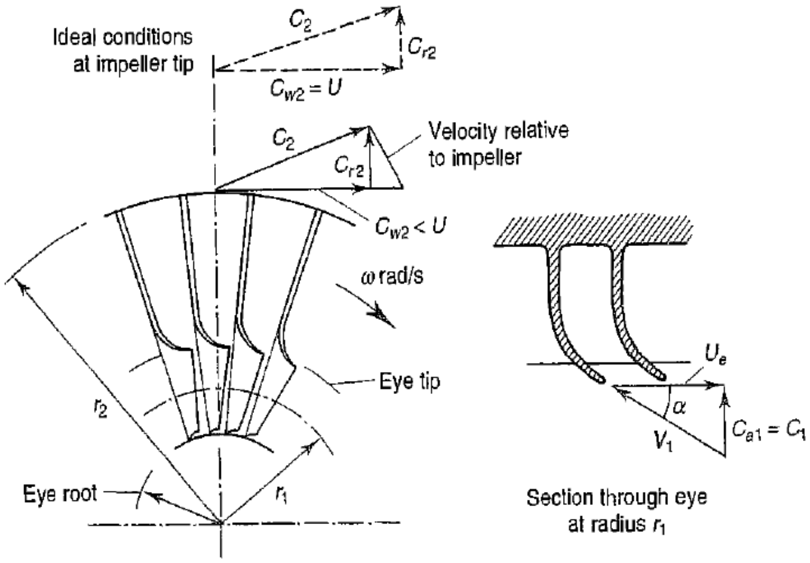


FIG. 4.2 Nomenclature

If the air leaves the impeller tip with an absolute velocity C_2, it will have a tangential or whirl component C_{w2}, and a comparatively small radial component C_{r2}. Under ideal conditions C_2 would be such that the whirl component is equal to the impeller tip speed U.

Due to its inertia, the air trapped between the impeller vanes is reluctant to move round with the impeller, and we have already noted that this results in a higher static pressure on the leading face of the vane than on the trailing face. It also prevents the air from acquiring a whirl velocity equal to the impeller speed. This effect is known as slip. How far the whirl velocity t the impeller tip falls short of the tip speed depends largely upon the number if vanes on the impeller.

The greater the number of vanes, the smaller the slip, i.e the more nearly C_{w2} approaches U. It is necessary in design to assume value for the slip factor σ, where σ is defined as the ratio of C_{w2}/U.

$$\sigma = 1 - \frac{0{\cdot}63\pi}{n}$$

where n is the number of vanes.

As explained in any elementary text on applied thermodynamics, the theoretical torque which must be applied to the impeller will be equal to the rate of change of angular momentum experienced by the air. Considering unit mass flow of air, this torque is given by

$$\text{theoretical torque} = C_{w2} r_2 \tag{4.1}$$

If ω is the angular velocity, the work done on the air will be

$$\text{theoretical work done} = C_{w2} r_2 \omega = C_{w2} U$$

Or, introducing the slip factor,

$$\text{theoretical work done} = \sigma U^2 \tag{4.2}$$

For convenience, in both the chapters on compressors we shall treat the work done on the air as a positive quantity.

Due to friction between the casing and the air carried round by the vanes, and other losses which have a braking effect such as disc friction or 'windage', the applied torque and therefore the actual work input is greater than this theoretical value. A *power input factor* ψ can be introduced to take account of this, so that the actual work done on the air becomes

$$\text{work done} = \psi \sigma U^2 \tag{4.3}$$

If $(T_{03} - T_{01})$ is the stagnation temperature rise across the whole compressor then, since no energy is added in the diffuser, this must be equal to the stagnation temperature rise $(T_{02} - T_{01})$ across the impeller alone. It will therefore be equal to the temperature equivalent of the work done on the air given by equation (4.3), namely

$$T_{03} - T_{01} = \frac{\psi \sigma U^2}{c_p} \tag{4.4}$$

where c_p is the mean specific heat over this temperature range. Typical values for the power input factor lie in the region of 1·035–1·04.

So far we have merely considered the work which must be put into the compressor. If a value for the *overall* isentropic efficiency η_c be assumed, then it is known how much of the work is usefully employed in raising the pressure of the air. The *overall* stagnation pressure ratio follows as

$$\frac{p_{03}}{p_{01}} = \left(\frac{T'_{03}}{T_{01}}\right)^{\gamma/(\gamma-1)} = \left[1 + \frac{\eta_c (T_{03} - T_{01})}{T_{01}}\right]^{\gamma/(\gamma-1)} = \left[1 + \frac{\eta_c \psi \sigma U^2}{c_p T_{01}}\right]^{\gamma/(\gamma-1)}$$

The distinction between the power input factor and the slip factor should be clearly understood: they are neither independent of one another nor of η_c. The power input factor represents an increase in the work input, the whole of which is absorbed in overcoming frictional loss and therefore degraded into thermal energy.

4.3 Diffuser vane design consideration

the problem of designing an efficient combustion system is eased if the velocity of the air entering the combustion chamber is as low as possible. It is necessary, therefore, to design the diffuser so that only a small part of the stagnation temperature at the compressor outlet corresponds to kinetic energy. Usually the velocity of the air at the compressor outlet is in the region of 90 m/s.

it is much more difficult to arrange for an efficient deceleration of flow than it is to obtain an efficient acceleration. There is a natural tendency in a diffusing process for the air to break away from the walls of the diverging passage, reverse its direction, and flow back in the direction of the pressure gradient—see Fig. 4.5(a). If the divergence is too rapid, this may result in the formation of eddies with consequent transfer of some kinetic energy into internal energy and a reduction in useful pressure rise. A small angle of divergence, however, implies a long diffuser and a high value of skin friction loss. Experiments have shown that the optimum included angle of divergence is about 7 degrees, although angles of up to twice this value can be used with diffusers of low length/width (or radius) ratio without incurring a serious increase in stagnation pressure loss. During acceleration in a converging passage, on the other hand, the gas naturally tends to fill the passage and follow the boundary walls closely, as in Fig. 4.5(b), however

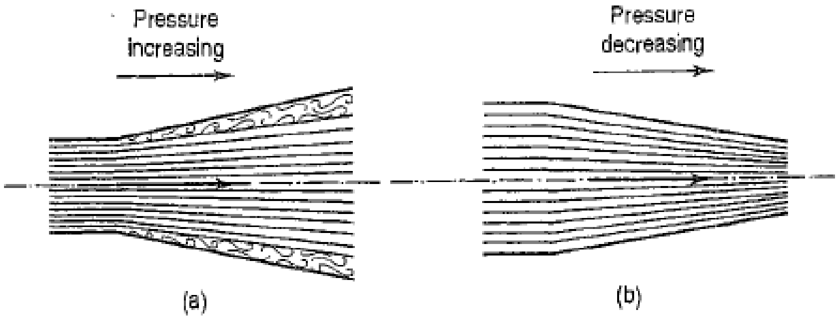


FIG. 4.5 Diffusing and accelerating flow

rapid the rate of convergence. Only the normal frictional losses will be incurred in this case.

In order to control the flow of air effectively and carry out the diffusion process in as short a length as possible, the air leaving the impeller is divided into a number of separate streams by fixed diffuser vanes. Usually the passages formed by the vanes are of constant depth, the width diverging in accordance with the shape of the vanes, as shown in Fig. 4.1. The angle of the diffuser vanes at the leading edge must be designed to suit the direction of the absolute velocity of the air at the radius of the leading edges, so that the air will flow smoothly over the vanes. As there is always a radial gap between the impeller tip and the leading edges of the vanes, this direction will not be that with which the air leaves the impeller tip. , when the effects of compressibility in this region are discussed.

To find the correct inlet angle for the diffuser vanes, the flow in the vaneless space must be considered. No further energy is supplied to the air after it leaves the impeller so that, neglecting the effect of friction, the angular momentum $C_w r$ must be constant. Hence C_w decreases from impeller tip to diffuser vane ideally in inverse proportion to the radius. For a channel of constant depth, the area of flow in the radial direction is directly proportional to the radius. The radial velocity C_r will therefore also decrease from impeller tip to diffuser vane, in accordance with the equation of continuity. If both C_r and C_w decrease, then the resultant velocity C will decrease from the impeller tip, and some diffusion evidently takes place in the vaneless space. The consequent increase in density means that C_r will not decrease in inverse proportion to the radius as does C_w, and the way in which C_r varies must be found from the continuity equation. An example at the end of this section will show how this may be done. When C_r and C_w have been calculated at the radius of the leading edges of the diffuser vanes, then the direction of the resultant velocity can be found and hence the inlet angle of the vanes.

It will be apparent that the direction of the air flow in the vaneless space will vary with mass flow and pressure ratio, so that when the compressor is operating under conditions other than those of the design point the air may not flow smoothly into the diffuser passages in which event some loss of efficiency will result. In gas turbines where weight and complexity are not so important as high part-load efficiency, it is possible to incorporate adjustable diffuser vanes so that the inlet angle is correct over a wide range of operating conditions.

For a given pressure and temperature at the leading edge of the diffuser vanes, the mass flow passed will depend upon the total throat area of the diffuser passages (see Fig. 4.1). Once the number of vanes and the depth of passage have been decided upon, the throat width can be calculated to suit the mass flow required under given conditions of temperature and pressure. For reasons given later in section 4.6, the number of diffuser vanes is appreciably less than the number of impeller vanes. The length of the diffuser passages will of course be determined by the maximum angle of divergence permissible, and the amount of diffusion required, and use would be made of the data in Ref. (8) to arrive at an optimum design. Although up to the throat the vanes must be curved to suit the changing direction of flow, after the throat the air flow is fully controlled and the walls of the passage may be straight. Note that diffusion can be carried out in a much shorter flow path once the air is controlled than it can be in a vaneless space where the air follows an approximately logarithmic spiral path (for an incompressible fluid $\tan^{-1}(C_r/C_w) = \text{constant}$).

After leaving the diffuser vanes, the air may be passed into a volute (or scroll) and thence to a single combustion chamber (via a heat-exchanger if one is employed). This would be done only in an industrial gas turbine unit: indeed, in some small industrial units the diffuser vanes are omitted and the volute alone is used. For aircraft gas turbines, where volume and frontal area are important, the individual streams of air may be retained, each diffuser passage being connected to a separate combustion chamber. Alternatively the streams can be fed into an annular combustion chamber surrounding the shaft connecting the compressor and turbine.

4.4 Performance parameters/characteristics of centrifugal compressor

8.8.1 Power Input Factor

The ideal energy transfer is given by Eq. 8.2, which is $E = u_2^2$. In practice, the actual energy transfer to the air from the impeller is lower than that given by Eq. 8.34, which is μu_2^2. Further, some energy is lost in friction between the casing and the air carried round by the vanes, and in disc friction or windage. In order to take this into account, *power input factor*, P_{if} is introduced, so that the work input to the compressor becomes

$$W_c = P_{if}\,\mu\,u_2^2 \qquad (8.43)$$

The normal value used for P_{if} is between 1.035 to 1.04.

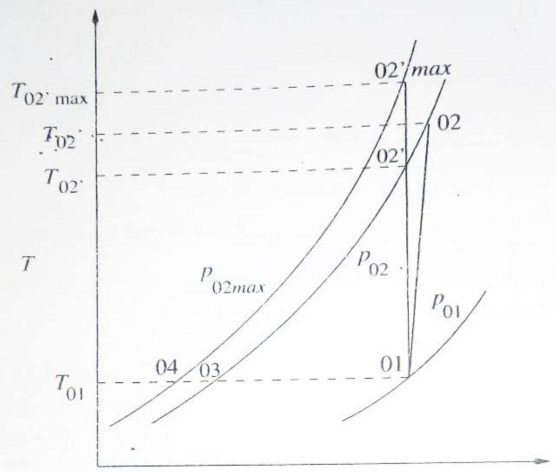
Let $\Delta T_c = T_{02} - T_{01}$ be the total head temperature rise across the compressor. As there is no energy addition in the diffuser this temperature rise is also the temperature rise across the impeller. Thus,

$$\Delta T_c = T_{02} - T_{01} = \frac{P_{if}\mu u_2^2}{C_p} \qquad (8.44)$$

8.8.2 Pressure Coefficients

This is a performance parameter which is useful in comparing various centrifugal compressors.

Each impeller has a definite maximum work capacity limited by the maximum tangential velocity at the exit. If this maximum work is utilized to the maximum advantage, an isentropic compressor will result, and a delivery pressure of $p_{02\ max}$ will be obtained. Now referring to Fig. 8.30 we have,



$$T_{02'max} = T_{01}\left(\frac{p_{02max}}{p_{01}}\right)^{\frac{\gamma-1}{\gamma}} \qquad (8.45)$$

$$W_{isen} = W_{max} = C_pT_{01}\left[\left(\frac{p_{02max}}{p_{01}}\right)^{\frac{\gamma-1}{\gamma}} - 1\right] \qquad (8.46)$$

The actual compressor produces only p_{02} pressure after an expenditure of adiabatic work given by

$$W_{adia} = C_pT_{01}\left[\left(\frac{p_{02}}{p_{01}}\right)^{\frac{\gamma-1}{\gamma}} - 1\right] \qquad (8.47)$$

Now pressure coefficient ψ_p is defined as

$$\psi_p = \frac{W_{adia}}{W_{isen}} = \frac{C_p T_{01}\left[\left(\frac{p_{02}}{p_{01}}\right)^{\frac{\gamma-1}{\gamma}} - 1\right]}{C_p T_{01}\left[\left(\frac{p_{02max}}{p_{01}}\right)^{\frac{\gamma-1}{\gamma}} - 1\right]} \quad (8.48)$$

and from Eq. 8.7, the denominator is u_2^2 and therefore

$$\psi_p = \frac{C_p T_{01}\left[(r_c)^{\frac{\gamma-1}{\gamma}} - 1\right]}{u_2^2} \quad (8.49)$$

8.8.3 Compressor Efficiency

We have already dealt with the compressor efficiency. However, it is better to recall it once again here. The overall stagnation isentropic efficiency (Fig. 8.31) in terms of pressure ratio can be written as

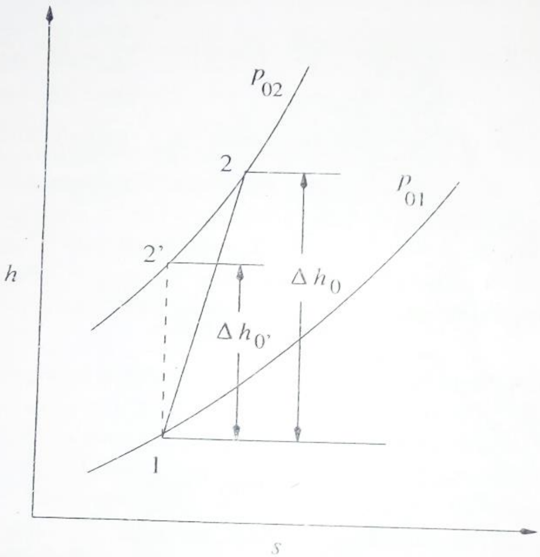


Fig. 8.31 h–s diagram of the compressor

$$\eta_c = \frac{\Delta h_{0'}}{\Delta h_0}$$

In terms of temperature and pressure ratios

$$\eta_c = \frac{T_{01}\left(r_c^{\frac{\gamma-1}{\gamma}} - 1\right)}{(T_{02} - T_{01})} \quad (8.50)$$

From Eq. 8.44,

$$T_{02} - T_{01} = \frac{P_{if}\mu u_2^2}{C_p} \quad (8.51)$$

From Eq. 8.49,

$$T_{01}\left(r_c^{\frac{\gamma-1}{\gamma}} - 1\right) = \frac{\psi_p u_2^2}{C_p} \quad (8.52)$$

Substituting in the Eq.8.50

$$\eta_c = \frac{\psi_p}{P_{if}\mu} \quad (8.53)$$

8.9 LOSSES IN CENTRIFUGAL COMPRESSORS

Total losses in a centrifugal compressor may be divided into two groups:

(i) *Frictional losses* These are proportional to c^2 and hence proportional to $\dot{m}^2$.

(ii) *Incidence losses* These loses in terms of drag coefficient C_D are proportional to $C_D c^2$.

Figure 8.32 shows the variation of the above two losses with respect to the mass flow rate.

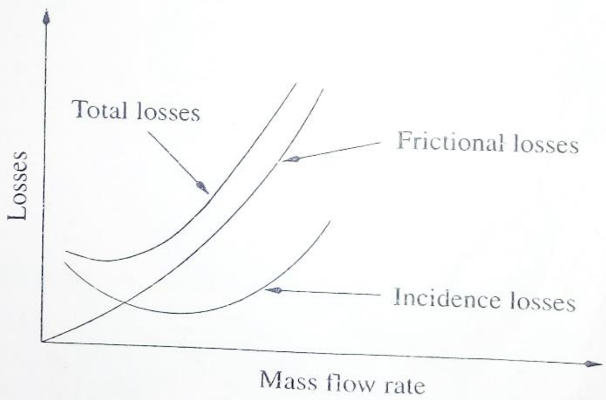


Fig. 8.32 Variation of losses with respect to the mass flow rate

8.10 COMPRESSOR CHARACTERISTICS

The ideal performance characteristics as shown in Fig. 8.15 for a radial-vane compressor are to be modified because of the losses mentioned above. If these losses are subtracted from the ideal energy transfer for a radial-vaned impeller, then the constant-pressure ratio straight line characteristics becomes curved, with a maximum value of energy at some particular value of the mass flow rate as shown in Fig. 8.33. Radial-vaned impellers are mostly used with gas turbine compressors. Therefore, a characteristic which will have a point of maximum pressure ratio with a positive and a negative slope is shown in Fig. 8.33. This particular variation decides upon the range of compressor operation.

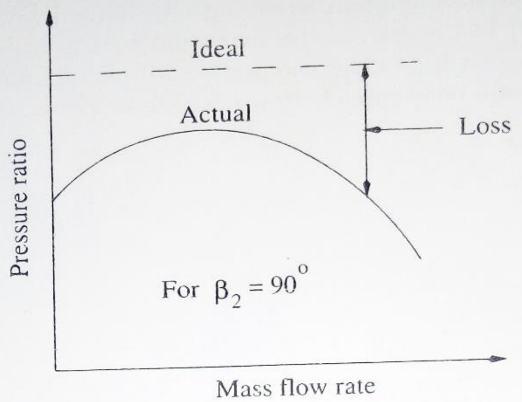


Fig. 8.33 Typical compressor characteristics at ideal and actual conditions

8.11 SURGING AND CHOKING

A typical characteristic of the compressor at one particular speed has been shown in Fig. 8.34 whereas Figs. 8.35 and 8.36 give the details for various speeds. Let us assume that the compressor is running at point C. If the resistance to flow is increased, say, by closing the valve provided at the delivery line of the compressor, the equilibrium point will shift to B.

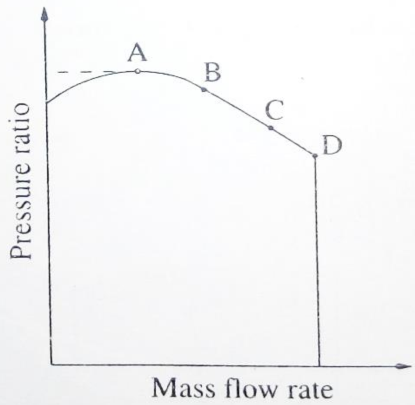


Fig. 8.34 Compressor characteristic at a constant speed

Any further restriction to the flow will cause the operating point to shift to the left, ultimately reach the point A. At this point maximum pressure ratio is obtained. If the flow is still reduced from this point then the compressor cannot increase the pressure ratio any further and therefore, the pressure ratio starts reducing. At this moment, there is a higher pressure in the downstream of the system (near exit) than at compressor delivery and the flow stops momentarily and may even reverse its direction. This reduces the pressure downstream. After a short interval of time, compressor again starts to deliver fluid and the operating point shifts to C again. Again the pressure starts increasing and the operating point moves from right to left. If the downstream conditions are unchanged then once again the flow will break down after point A and the cycle will be repeated with a high frequency. This phenomenon is known as *surging* or pumping. This will be again explained in greater detail with respect to axial flow compressor in the next chapter.

This instability will be severe in compressors producing high pressure ratios, which may ultimately lead to physical damage due to impact loads and high-frequency vibration. Because of this particular phenomenon of surging or pumping at low-mass flow rates, the compressor cannot be operated at any point to the left of the maximum pressure ratio point A, i.e., it cannot be operated on the positive slope of the characteristic.

At higher mass flow rate points on the characteristic a different situation occurs. At a constant rotor speed the tangential velocity component at the impeller tip remains constant. As can be seen from Fig. 8.34, with the increase in mass flow rate the pressure ratio decreases ($ABCD$) and hence the density also decreases. These effects result in a considerably increased velocity which increases the absolute velocity and the incidence angle at the diffuser vane top. Thus there is a rapid progress towards a choking state. The slope of the characteristic therefore steepens and finally after point D mass flows cannot be increased any further. The characteristic finally becomes vertical. The point D on the characteristic curve is called a *choking point*.

Rotating stall:

The important cause of instability and poor performance, which may contribute to surge but can exist in the nominally stable operating range: this is the rotating stall. When there is any no uniformity in the flow or geometry of the channels between vanes or blades, breakdown in the flow in one channel, say B as shown in figure causes the air to be deflected in such a way that channel C receives.

Fluid at a reduced angle of incidence and channel A at an increased incidence. Channel A then stalls, resulting in a reduction of incidence to channel B enabling the flow in that channel to recover. Thus the stall passes from channel to channel: at the impeller eye it would rotate. In a direction opposite to the direction of rotation of the impeller. Rotating stall may lead to aerodynamically induced vibrations resulting in fatigue failures in other parts of the gas turbine.

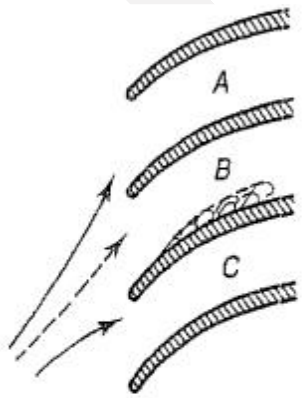


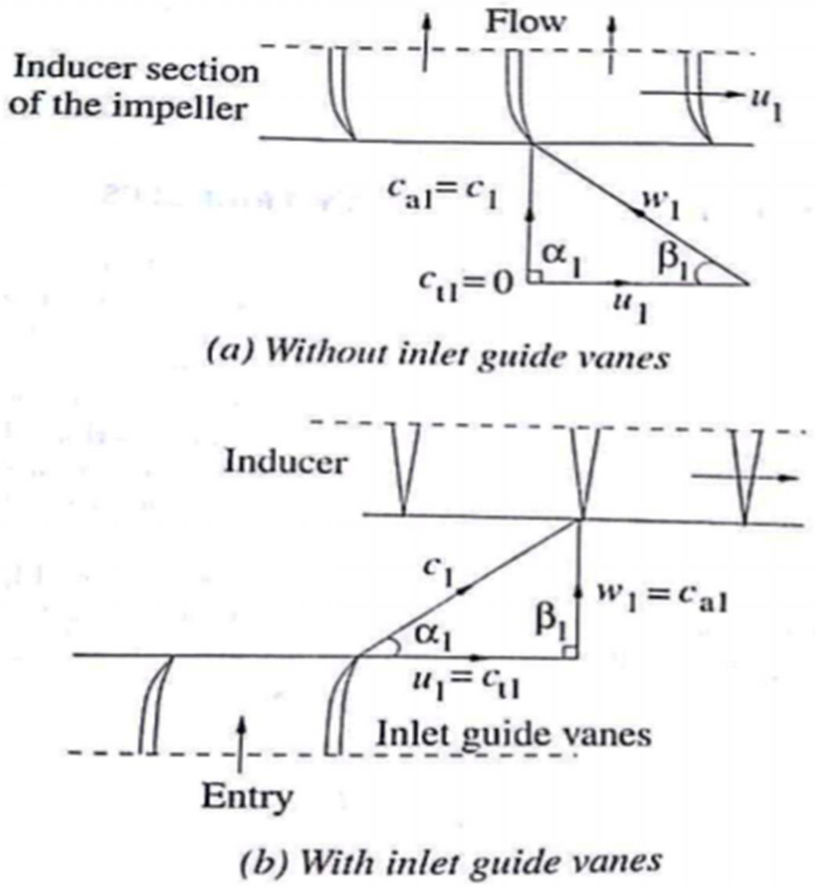
FIG. 4.9 Rotating stall

4.5 Concept of Prewhirl

The tangential component of velocity at the inlet to the impeller is usually zero as the flow enters the impeller axially. If prewhirl (or inlet guide) vanes are installed to the inlet duct before the impeller, then the incoming air has a tangential component of velocity. This velocity component depends on the vane outlet angle. Prewhirl may be positive or Positive prewhirl reduces the inlet relative velocity, while negative prewhirl increases the inlet relative velocity. In aero engines, positive prewhirl is frequently used to reduce the inlet relative speed.

The objective here is to avoid the formation of shock waves on the blade suction side. The designer of compressor seeks for a small inlet area of the engine to reduce the drag force. At the same time, the air mass flow rate is chosen as maximum as possible to maximize the thrust force. Both factors led to an increase in the axial absolute velocity at inlet, and this in turn increases the relative velocity. Since the relative velocity is maximum at the tip radius of the inlet than when accelerated, there is always a tendency for the air to break away from the convex face of the curved part of the impeller vane. Here a shock wave might occur, which upon interaction with the boundary layer over the convex surface of blades, causes a large increase in boundary layer thickness. Thus, excessive pressure loss occurs.

The value of the inlet relative Mach number must be in the range from 0.8 to 0.85 to avoid the shock wave losses described previously, or where T_1 is the static inlet temperature. Though this Mach number may be satisfactory on ground operation, it may be too high at altitude as the ambient temperature decreases with altitude. For this reason, IGVs are added to decrease the Mach number. These IGVs are attached to the compressor casing to provide a positive prewhirl that decreases the magnitude of the maximum relative velocity (W_1) at the eye tip.



4.6 The effect of impeller blade shape on performance

As already mentioned in section 8.4 the various blade shapes utilized in impellers of centrifugal compressors can be classified as

(i) forward-curved blades ($\beta_2 > 90°$),
(ii) radial-curved blades ($\beta_2 = 90°$),
(iii) backward-curved blades ($\beta_2 < 90°$).

Figure 8.15 represents the variation of pressure-ratio that can be obtained with respect to mass flow rate for the above mentioned various shapes for operation at a given rpm. The basic velocity diagram at exit is shown for each type of blade is Fig. 8.16. For example, the Euler energy equation without inlet swirl ($c_{t1} = 0$) is written as

$$E = u_2 c_{t2} = u_2(u_2 - c_{r2} \cot \beta_2) \tag{8.22}$$

For any particular impeller running at a constant speed, energy equation can be written as

$$E = K_1 - K_2 Q \tag{8.23}$$

where Q is the volume flow rate which is proportional to flow velocity for given impeller, and K_1 and K_2 are constants as given by

$$K_1 = u_2^2 \tag{8.24}$$

and

$$K_2 = \frac{u_2 \cot \beta_2}{\pi d_2 b_2} \tag{8.25}$$

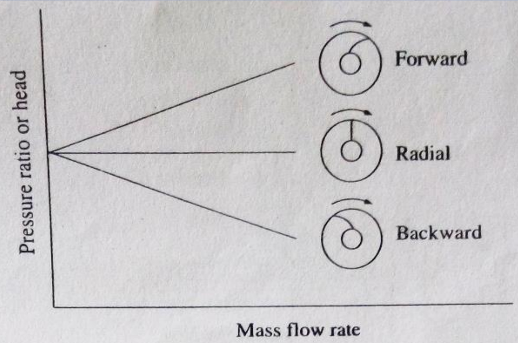


Fig. 8.15 The effect of impeller blade shape on performance

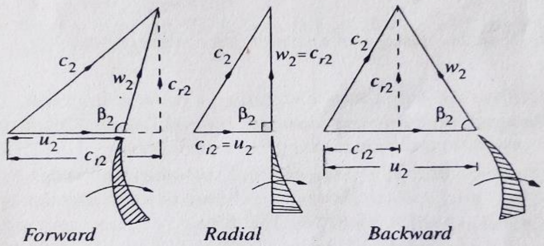


Fig. 8.16 Exit velocity diagrams for different shapes of blades

The comparison of performance for the three types of vanes are made for the same volume flow rate, each blade having unit depth, and for the same vector value of c_{r2}. Figure 8.16 shows the exit velocity triangles for three types of impellers from which the relative performance of the blades can be evaluated.

Centrifugal effects of the curved blades create a bending moment and produce increased stresses which reduce the maximum speed at which the impeller can be run. Good performance can be obtained with radial impeller blades. Backward-curved blades are slightly better in efficiency and are stable over a wider range of flows than either radial or forward-curved blades. The forward-curved impeller can produce the highest pressure ratio for a given blade tip speed; but is inherently less stable and has a narrow operating range. Its efficiencies are lower than that are possible with the backward-curved or radial-curved blades.

Although all the three types can be used in compressors, the radial blade is used almost exclusively in turbojet engine applications.

4.7 Problems on centrifugal compressor: (study all problems from V Ganesan, page no. 313 to 336)

8.1 A centrifugal compressor under test gave the following data:

Speed	:	11,500 rev/min
Inlet total head temperature	:	21°C
Outlet and inlet total head pressure	:	4 bar, 1 bar
Impeller dia	:	75 cm

If the slip factor is 0.92, what is the compressor efficiency?

Solution

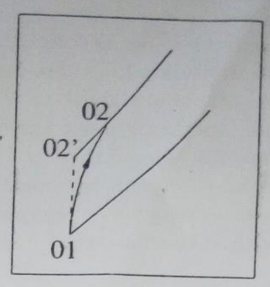


Fig. 8.37

$$u = \frac{\pi D n}{60}$$

$$u = \pi \times 0.75 \times \frac{11500}{60} = 451.6 \text{ m/s}$$

$$W_C = \mu u^2 = \frac{0.92 \times 451.6^2}{1000} = 187.63 \text{ kJ/kg}$$

$$W = C_p(T_{02} - T_{01})$$

$$T_{02} = \frac{187.63}{1.005} + 294 = 480.7 \text{ K}$$

$$p_{01} = 1 \text{ bar}$$

$$T_{02'} = T_{01}\left(\frac{p_{02}}{p_{01}}\right)^{\frac{\gamma-1}{\gamma}} = 294 \times \left(\frac{4}{1}\right)^{0.286} = 437 \text{ K}$$

$$\eta_c = \frac{437 - 294}{480.7 - 294} \times 100 = 76.6\% \quad \Leftarrow \text{Ans}$$

8.2 A centrifugal compressor has to deliver 35 kg of air per sec. The impeller is 76 cm diameter revolving at 11,500 rpm with an adiabatic efficiency of 80%. If the pressure ratio is 4.2:1, estimate the probable axial width of the impeller at the impeller tip if the radial velocity is 120 m/s. The inlet conditions are 1 bar and 47°C.

Solution

$$T_{02}' = T_{01} (r_c)^{\frac{\gamma-1}{\gamma}}$$

$$T_{02'} = 320 \times 4.2^{0.286} = 482.4 \text{ K}$$

$$\eta_c = \frac{T_{02}' - T_{01}}{T_{02} - T_{01}} \Rightarrow$$

$$T_{02} = T_{01} + \frac{T_{02'} - T_{01}}{\eta_c}$$

$$= 320 + \frac{482.4 - 320}{0.8} = 523 \text{ K}$$

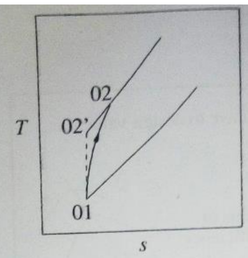


Fig. 8.38

$A_{tip} = \frac{\dot{m}_a}{\rho_2 \times \text{radial velocity}}$

Ignoring the effects of velocity of flow

$$\rho_2 = \frac{p_{02}}{RT_{02}} = \frac{4.2 \times 10^5}{287 \times 523} = 2.8 \text{ kg/m}^3$$

$$A_{tip} = \frac{35}{2.8 \times 120} = 0.1042 \text{ m}^2$$

Axial width $= \frac{A_{tip}}{\pi D}$

$$\text{Axial width} = \frac{0.1042}{\pi \times 0.76} = 0.0436 \text{ m} = \mathbf{4.36 \text{ cm}} \quad \text{Ans}$$

8.4 A centrifugal compressor takes in gas at 0°C and 0.7 bar and delivers at 1.05 bar. The efficiency of the process compared with the adiabatic compression is 83%. The specific heat of the gas at constant-pressure and constant-volume are 1.005 and 0.717 respectively. Calculate the final temperature of the gas and work done per kg of gas.

If the gas were further compressed by passing through a second compressor having the same pressure ratio and efficiency and with no cooling between the two compressors, what would be the overall efficiency of the complete process?

Solution

$$T_{02'} = T_{01}\left(\frac{p_{02}}{p_{01}}\right)^{\frac{\gamma-1}{\gamma}} = 273 \times \left(\frac{1.05}{0.7}\right)^{0.286}$$

$$= 306.6 \text{ K}$$

$$T_{02} = T_{01} + \frac{T_{02'} - T_{01}}{\eta_c}$$

$$= 273 + \frac{306.6 - 273}{0.83} = \mathbf{313.5 \text{ K}} \quad \text{Ans}$$

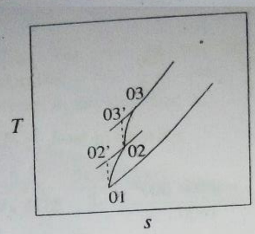


Fig. 8.40

$$W_c = 1.005 \times (313.5 - 273) = 40.7 \text{ kJ/kg}$$

With additional compressor

$$T_{03'} = 313.5 \times \left(\frac{1.05}{0.7}\right)^{0.286} = 352 \text{ K}$$

$$T_{03} = T_{02} + \frac{T_{03'} - T_{02}}{\eta_c}$$

$$= 313.5 + \frac{352.0 - 313.5}{0.83} = 359.9 \text{ K}$$

$$T_{03'} = 273 \times (1.5 \times 1.5)^{0.286} = 344.3 \text{ K}$$

$$\eta_{overall} = \frac{344.3 - 273}{359.9 - 273} \times 100 = 82\%$$

8.10 A centrifugal compressor compresses 30 kg of air per second at a rotational speed of 15000 rpm. The air enters the compressor axially, and the conditions at the exit sections are radius = 0.3 m, relative velocity of air at the tip = 100 m/s at an angle of 80°C with respect to plane of rotation. Take p_{01} = 1 bar and T_{01} = 300 K.

Find the torque and power required to drive the compressor and also the head developed.

Solution

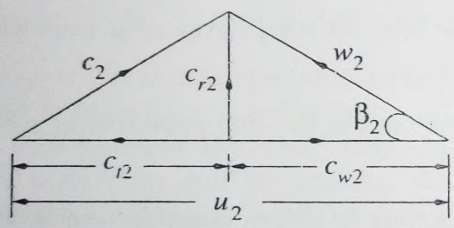


Fig. 8.45

$$u_2 = \pi D_2 \frac{N}{60} = \pi \times 0.6 \times \frac{15000}{60} = 471.24 \text{ m/s}$$

$$c_{t2} = u_2 - w_2 \cos\beta_1 = 471.24 - 100 \times \cos 80$$

$$= 453.88 \text{ m/s}$$

$$\text{Torque} = Fr = mc_{t2}r = 30 \times 453.88 \times 0.3$$

$$= \mathbf{4084.92\ Nm}$$

$$\text{Power} = T\omega = 4084.92 \times 2 \times \pi \times \frac{15000}{60}$$

$$= 6.417 \times 10^6 \text{ W}$$

$$= \mathbf{6.417 \times 10^3\ kW}$$

$$W = u_2 c_{t2} = 471.24 \times 453.88$$

$$= 213886.41$$

$$W = C_p T_{01}\left[\left(\frac{p_{02}}{p_{01}}\right)^{\frac{\gamma-1}{\gamma}} - 1\right]$$

$$213886.41 = 1005 \times 300 \times \left[\left(\frac{p_{02}}{p_{01}}\right)^{\frac{\gamma-1}{\gamma}} - 1\right]$$

$$\frac{p_{02}}{p_{01}} = 6.531$$

$$p_{02} = \mathbf{6.531\ bar}$$

4.8 Construction and working of Axial Compressors

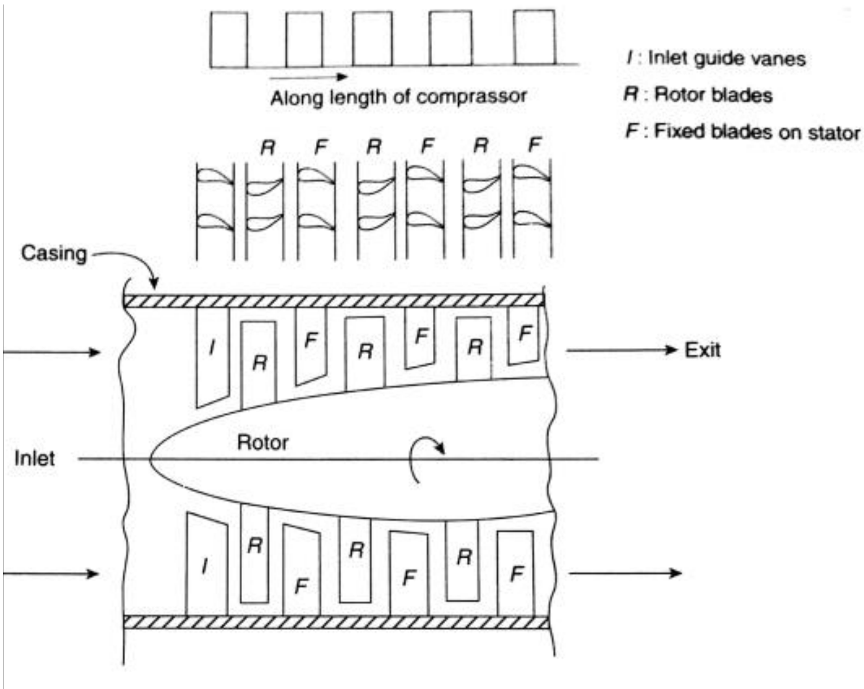


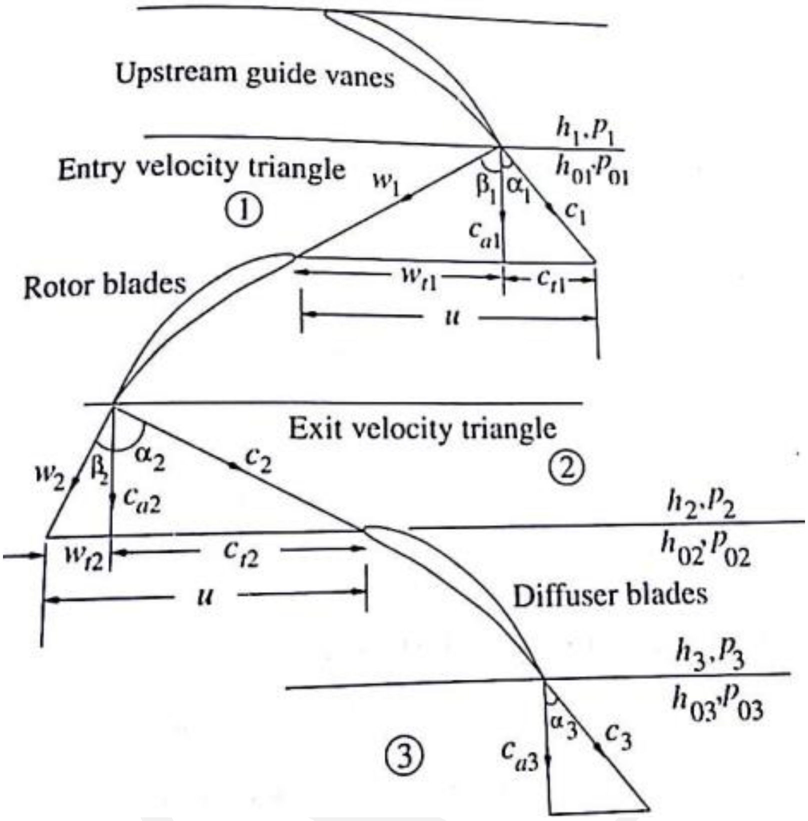
Figure 2 Schematic of axial flow compressor.

A axial Compressor consists of stator and rotor blades placed alternatively from the Inlet end to the exit end It has a moving inner core called rotor and static outer portion called stator or casing. The rotor has a set of blades mounted on it which rotate with the rotor. The casing or stator has static blades mounted on it. The fluid is admitted in the compressor from the inlet end through the inlet guide vanes smoothly impinge upon rotor blades. Flows over the rotor blades to the Stator blades and then again on rotor blade, and Stator blade

The working of axial flow compressors is based upon the addition of kinetic energy to the flowing fluid by the rotor blades and its subsequent conversion into the pressure rise. Here the fluid enters axially. Through the inlet guide Vanes, to the rotor blades at a suitable angle to ensure smooth flow. Then the fluid is rotated by the rotor blades and its kinetic energy gets increased. During this process, there occurs a very small rise in the pressure too. The rotor blades then discharge the fluid to the stator blades where the maximum rise in the pressure occurs due to the diffusion in the stator section.

The fluid subsequently enters into the rotor blades which are followed by the stator blades and the process continues so on till the exit end. The change in the total values of the pressure and temperature, and enthalpy occurs only in the rotor section. In an axial flow compressor, the fluid successively passes through the compressor stages causing slight rise in the pressure and temperature. This low pressure ratio of the order of 1.1 to 1.4 offers high efficiency and the high pressure ratio up to 40 may be obtained.

4.9 Velocity Triangles of axial compressors



The flow geometry at, the entry and exit of a turbomachine stage is described by the velocity triangles at these stations. As already mentioned earlier, the velocity triangles for a turbomachine contain the following three components:

- The peripheral velocity, (u), of the rotor blades,
- The absolute velocity, (c), of the fluid, and
- The relative Velocity, (w), of the fluid.

These velocities are related by the following well-known vector equation:

$$\vec{c} = \vec{u} + \vec{w}$$

This simple relation is frequently used and is very useful in drawing the velocity triangles for turbomachines. The notation used here to draw velocity triangles correspond to the x-y coordinates; the suffix (a) identifies components in the axial direction and the suffix (t) refers to the tangential direction. Air angles in the absolute system are denoted by alpha (α), where as those in the relative system are represented by beta (β).

From velocity triangles.

$$C_{a1} = C_1 \cos\alpha_1 = w_1 \cos\beta_1 \quad \text{— (1)}$$

$$C_{t1} = C_1 \sin\alpha_1 = C_{a1} \tan\alpha_1 \quad \text{— (2)}$$

$$w_{t1} = w_1 \sin\beta_1 = C_{a1} \tan\beta_1 \quad \text{— (3)}$$

$$u = C_{t1} + w_{t1} \quad \text{— (4)}$$

similarly

$$C_{a2} = C_2 \cos\alpha_2 = w_2 \cos\beta_2 \quad \text{— (5)}$$

$$C_{t2} = C_2 \sin\alpha_2 = C_{a2} \tan\alpha_2 \quad \text{— (6)}$$

$$w_{t2} = w_2 \sin\beta_2 = C_{a2} \tan\beta_2 \quad \text{— (7)}$$

$$u = C_{t2} + w_{t2} \quad \text{— (8)}$$

From eq^n 4 & 8 $\rightarrow$

$$C_{t1} + w_{t1} = C_{t2} + w_{t2}$$

useing eq^n 2, 3 6, 7

$$\cancel{C_{a2} \tan\alpha_2 + C_{a3} \tan\alpha_3 = C_{a2} \tan\beta_2}$$

$$C_{a1} \tan\alpha_1 + C_{a1} \tan\beta_1 = C_{a2} \tan\alpha_2 + C_{a2} \tan\beta_2$$

But $C_a = C_{a1} = C_{a2}$.

$$C_a [\tan\alpha_1 + \tan\beta_1] = [\tan\alpha_2 + \tan\beta_2] C_a$$

$$C_a [\tan\beta_1 - \tan\beta_2] = [\tan\alpha_2 - \tan\alpha_1] C_a$$

$$C_{t2} - C_{t1} = w_{t1} - w_{t2}$$

work done by the compressor

$$w = u(C_{t2} - C_{t1}) = C_a u [\tan\alpha_2 - \tan\alpha_1]$$

4.10 Degree of Reaction

A degree of Reaction for axial compressors can also be defined a ratio of actual change of the enthalpy in the rotor to actual change of enthalpy in the stage.

$$R = \frac{\text{actual change of enthalpy in rotor}}{\text{actual change of enthalpy in stage.}}$$

$$R = \frac{h_2 - h_1}{h_{03} - h_{01}} \quad \text{— (1)}$$

$$= \frac{1}{2u} \frac{w_1^2 - w_2^2}{(C_{t2} - C_{t1})} \quad \text{— (2)}$$

useing Phythagoras theorom

$$w_1^2 = w_{t1}^2 + C_{a1}^2$$

$$w_2^2 = w_{t2}^2 + C_{a2}^2$$

$$w_1^2 = w_2^2 = w_{t1}^2 - w_{t2}^2$$

$$R = \frac{w_{t1}^2 - w_{t2}^2}{2u(C_{t2} - C_{t1})} \quad \text{— (3)}$$

$$R = \frac{(W_{t1} + W_{t2})(W_{t1} - W_{t2})}{2u\,(C_{t2} - C_{t1})}$$

From velocity triangles

$$W_{t1} - W_{t2} = C_{t2} - C_{t1}$$

$$R = \frac{W_{t1} + W_{t2}}{2u} \quad \text{— (4)}$$

$$R = \frac{C_{a1}\tan\beta_1 + C_a\tan\beta_2}{2u}$$

$$R = \frac{C_a}{2u}\left[\tan\beta_1 + \tan\beta_2\right] \quad \text{— (5)}$$

But $W_{t2} = U - C_{t2}$ — (6)

6 in 4

$$R = \frac{C_{a1}\tan\beta_1 + U - C_{a2}\tan\alpha_2}{2u}$$

$$R = \frac{1}{2} + \frac{C_a}{2u}\left[\tan\beta_1 - \tan\alpha_2\right] \quad \text{— 6}$$

$W_{t1} = U - C_{t1}$ — (7)

using eqn 6 & 7 in 4 →

$$R = \frac{U - C_a\tan\alpha_1 + U - C_a\tan\alpha_2}{2u}$$

$$R = \frac{2u - C_a[\tan\alpha_1 + \tan\alpha_2]}{2u}$$

$$R = 1 - \frac{C_a}{2u}\left[\tan\alpha_1 - \tan\alpha_2\right] \quad \text{— (8)}$$

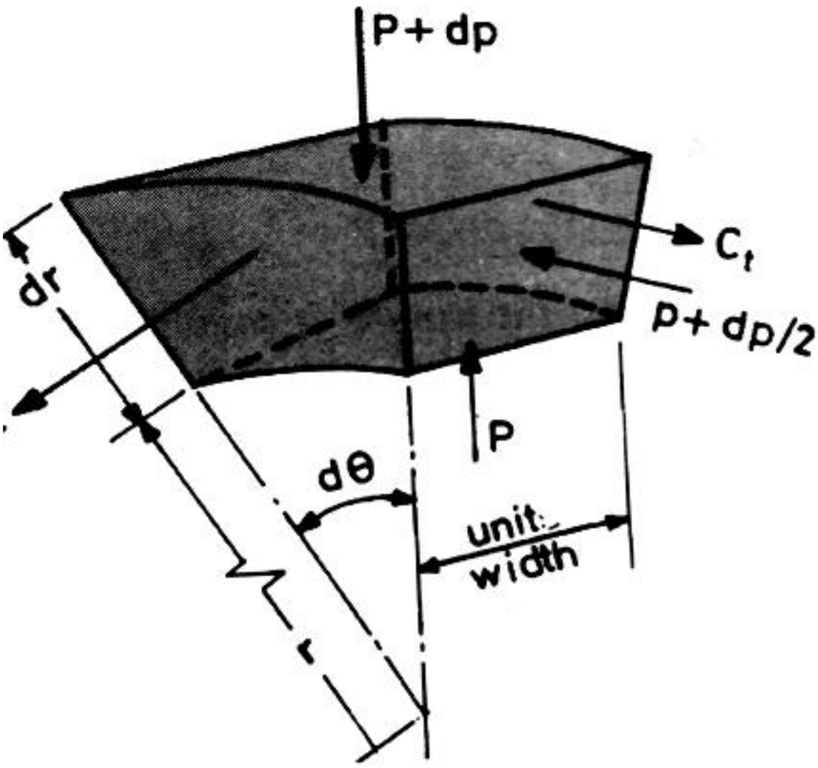
4.11 Blade Design Process

- Choice of rotational speed and annulus dimensions;
- Determination of number of stages, using an assumed efficiency;
- Calculation of the air angles for each stage at the mean radius;
- Determination of the variation of the air angles from root to tip;
- Investigation of compressibility effects;
- Selection of compressor blading, using experimentally obtained cascade data;

- Check on efficiency previously assumed, using the cascade data;
- Estimation of off-design performance;
- Rig testing

4.12 Free Vortex Theory (3D Flow)

Generally, the flow through compressor annulus is assumed to be two dimensiona1 considering that there is no effect due to the radial movement of the flow. This radial movement of the Fluid can be ignored in the later stages.



Consider a fluid element of unit width with density p as shown in Figure. The fluid element shown here has whirl component of velocity Ct, and axial component of velocity Ca Small radial component of the velocity given by Cr. is there in the fluid element due to curvature (r) of the stream line. Here the element will be in equilibrium when the pressure forces and inertia forces are equal

The inertia force will be consisting of (a) the centripetal force due to the circumferential flow, (b) the radial component of the centripetal force due to the flow along the curved stream line and (c) the radial component of the force causing linear acceleration along the streamline.

Inertia Force due to centripetal force due to flow along circumference is given by

$$F_{Ia} = \frac{m C_t^2}{r}$$

Inertia Force due to radial component of the centripetal force due to flow along the curved streamline will be

$$F_{Ib} = \frac{m C_s^2}{r_s} \cos \alpha_s$$

Inertia Force due to radial component of the force causing linear acceleration along the streamline will be

$$F_{Ic} = m \frac{dC_s}{dt} \sin \alpha_s$$

Total inertia force will be

$$F_I = F_{Ia} + F_{Ib} + F_{Ic}$$

$$F_I = m \left[\left(\frac{C_t^2}{r} \right) + \left(\frac{C_s^2}{r_s} \cos \alpha_s \right) + \left(\frac{dC_s}{dt} \sin \alpha_s \right) \right]$$

$$F_I = \rho r . dr d\theta \left[\left(\frac{C_t^2}{r} \right) + \left(\frac{C_s^2}{r_s} \cos \alpha_s \right) + \left(\frac{dC_s}{dT} \sin \alpha_s \right) \right]$$

Similarly the pressure Force yielding the inertia Force in the radial direction will be

$$F_p = \left[(P + dP)(r + dr) d\theta - P r d\theta - 2 \left(P + \frac{dP}{2} \right) dr \frac{d\theta}{2} \right.$$

Neglecting higher order terms

$$F_p = r \, dp \, d\theta$$

$$F_p = F_I$$

$$r \, dp \, d\theta = \rho . r . dr . d\theta \left[\frac{C_t^2}{r} + \left(\frac{C_s^2 \cos \alpha_s}{r_s} \right) + \left(\frac{dC_s}{dt} \sin \alpha_s \right) \right]$$

$$\frac{1}{\rho} \frac{dP}{dr} = \frac{C_t^2}{r} + \frac{C_s^2}{r_s} \cos \alpha_s + \frac{dC_s}{dt} \sin \alpha_s$$

The radius of curvature can be neglected. and α_s is small

$$\therefore \quad \frac{1}{\rho} \frac{dP}{dr} = \frac{C_t^2}{r}$$

Stagnation enthalpy at any radius is given by

$$h_0 = h + \frac{C^2}{2}$$

$$h_0 = h + \frac{C_a^2 + C_t^2}{2}$$

Differentiating with respect to radius

$$\frac{dh_0}{dr} = \frac{dh}{dr} + C_a \frac{dC_a}{dr} + C_t \frac{dC_t}{dr}$$

From I and II law of thermodynamics

$$T ds = dh - \frac{dP}{\rho}$$

$$dh = T ds - \frac{dP}{\rho}$$

Differentiating with respect to radius

$$\frac{dh}{dr} = T\frac{ds}{dr} + ds\frac{dT}{dr} + \frac{1}{\rho}\frac{dP}{dr} - \frac{1}{\rho^2}\frac{d\rho}{dr}dP$$

Neglecting higher order terms

$$\frac{dh}{dr} = T\frac{ds}{dr} + \frac{1}{\rho}\frac{dP}{dr}$$

equating . in $\Longrightarrow$

$$\frac{dh_0}{dr} = T\frac{ds}{dr} + \frac{1}{\rho}\frac{dP}{dr} + C_a\frac{dC_a}{dr} + C_t\frac{dC_t}{dr}$$

equating in $\Longrightarrow$

$$\frac{dh_0}{dr} = T\frac{ds}{dr} + \frac{C_t^2}{r} + C_a\frac{dC_a}{dr} + C_t\frac{dC_t}{dr}$$

Ignoring entropy gradient

$$\boxed{\frac{dh_0}{dr} = C_a\frac{dC_a}{dr} + C_t\frac{dC_t}{dr} + \frac{C_t^2}{r}}$$ $\rightarrow$ vortex energy equation

Equating $\frac{dh_0}{dr} = 0$ and $\frac{dC_a}{dr} = 0$

as C_a remains constant

$$C_t\frac{dC_t}{dr} + \frac{C_t^2}{r} = 0$$

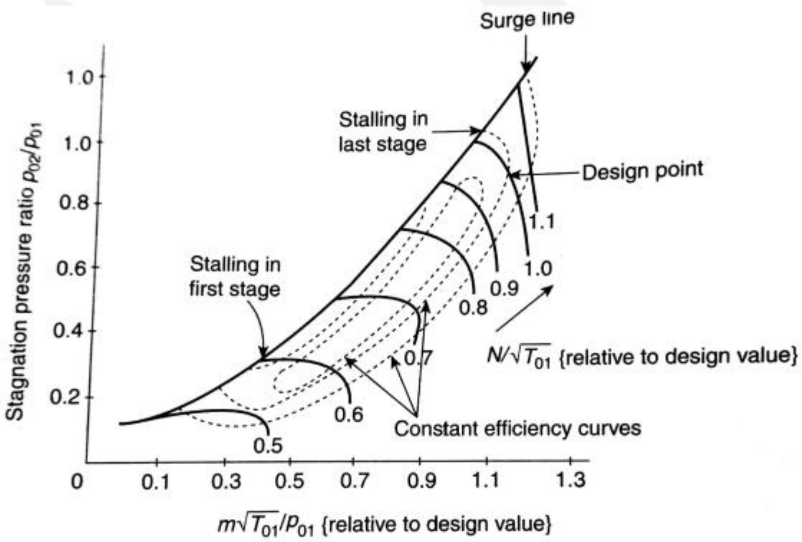
$$\boxed{C_t r = \text{constant}}$$

This is Free vortex condition.

4.13 Axial Compressor Characteristics

- The characteristic curves indicate that the pressure rise is quite steep at higher speeds of the rotation.

- Surging starts occurring even before the curves achieve the maximum value and due to this the design point is close to the peak of characteristic curve near the surge line.
- Even a small reduction in the mass flow will show the increase in the pressure ratio and density and this will cause reduction in the axial velocity causing stalling
- Even small increase in the mass flow will cause drop in the pressure ratio and so reduction in the density causing increase in the axial velocity causing stalling at later stages
- The reduction in the speed of rotation will cause reduction in the mass flow, axial velocity and blade, thus stalling in the initial stage
- The stable operation range is quite narrow and so the use of axial flow compressor needs high care while operating at off design conditions.
- Quite large speed of rotation leads to nearly vertical characteristic curve, thus increase in the density pressure ratio, speed U and so increased mass flow leading to choking.



Consequences of Surge:

- There is no steady operating point once surge occurs. Thus, it is impossible to achieve design pressure rise and mass flow and consequently the thrust of the engine.
- Transient consequences such as inlet overpressure can be severe.
- Surge may damage blades due to vibrational stresses (aero-elastic flutter and divergence).

- Internal component damage due to hot air passing upstream through the cold sections of the engine.
- Other consequences of surge include power loss and engine flame out

Consequences of rotating stall are:

- Reduced pressure rise in the stage affected.
- Vibrations: The stall cells rotate at a fraction of the rotor speed. As the blade passes in and out of these stalled regions, vibrations are induced. If the natural frequency of blades coincides with which stall cell crosses a blade, resonance occurs leading to possible fatigue failure.
- Hysteresis: Throttling is used to correct rotating stall. However, to recover from stall, the corrected mass flow (or flow coefficient) has to be much larger than that at which the compressor would stall. This phenomenon is known as hysteresis

4.14 Problems on axial compressor from V. Ganesan