# Module - 1.

## Static longitudinal stability and control - stick fixed.

## I. Historical perspective:

* By the start of the 20th century, the aeronautical community had solved many of the technical problems necessary for achieving powered flight of a heavier-than-air aircraft.
* One problem still beyond the grasp of these early investigators was a lack of understanding of the relationship between stability and control. as well as the influence of the pilot on the pilot-machine system.
* Most of the ideas regarding stability and control came from experiments with uncontrolled hand-launched gliders.
* Earlier aviation pioneers such as Albert Zahm in the United States, Alphonse Penaud in France, and Frederick Lanchester in England contributed to the notion of stability.
* Zahm, however, was the first to correctly outline the requirements for static stability in a paper he presented in 1893. In his paper, he analyzed the conditions necessary for obtaining a stable Equilibrium for an airplane descending at a constant speed.

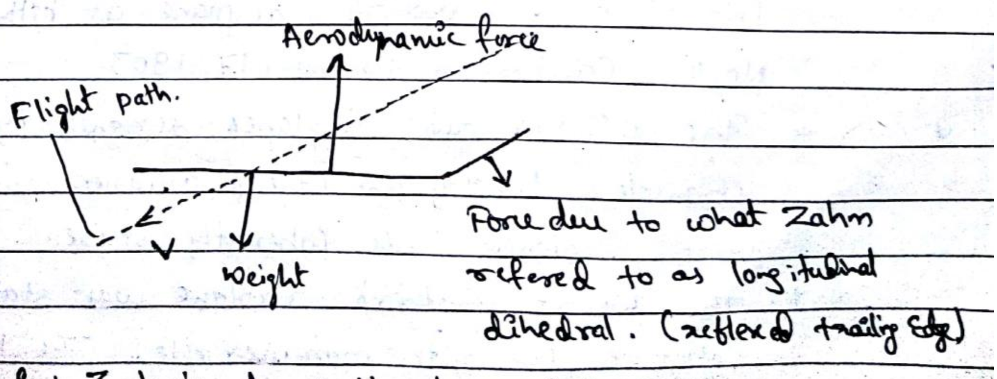



fig: Zahm's description of longitudinal stability.

* Zahm concluded that the center of gravity had to be in front of the aerodynamic force and the vehicle would require what he referred as "longitudinal dihedral". to have a stable Equilibrium point.
*. From the United states and Europe, the scientists were working with gliders and have given contribution to the stability. In that mainly Otto Lilienthal of Germany and Octave Chanute and Samuel Pierpont Langley of the United States.
* Otto Lilienthal's experiments includes the human carrying gliders and the determination of the properties of curved or cambered wings.
*. Octave Chanute, for his gliders, Incorporated biplane and multiplane wings, controls to adjust the wings to maintain Equilibrium, and vertical tail for steering.
*. Around 1890, Samuel Pierpont Langley, become interested in problems of flight. Initially his work consisted of collecting and examining all the available aerodynamic data. and in 1896 he built heavier than air powered flight.
* The Wright brothers made their historic first flight on a powered airplane at Kitty Hawk, North Carolina, on December 17, 1903.
* The gliders and airplanes designed by Lilenthal, Chanute, Langley, and others aviation pioneers were designed to be inherently stable.
*. The Wright brothers airplane was statically unstable but quite maneuverable. The lack

of stability made their work as pilots very difficult. However, through their glider experiments they were able to teach themselves to fly their unstable airplane.

## II. Aerodynamic Nomenclature:

For the aircraft motion, two co-ordinate systems are used. One co-ordinate system is fixed to the Earth and may be considered for the purpose of aircraft motion analysis to be an inertial coordinate system. The other coordinate system is fixed to the airplane and is referred to as a body co-ordinate system.

Below figure shows the body coordinate system (right handed coordinate system) of aircraft.

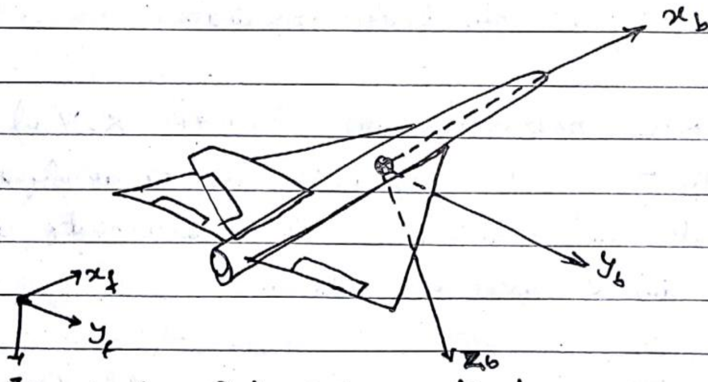



fig: Body axis coordinate system.

The forces acting on an airplane in flight consist of aerodynamic, thrust, and gravitational forces. These forces can be resolved along an axis system fixed to the airplane's center of gravity, as illustrated in below figure.

| | Roll axis $x_b$ | pitch axis $y_b$ | Yaw axis $z_b$ |
|---|---|---|---|
| Angular rates | p | q | r |
| velocity components | u | v | w |
| Aerodynamic force components | X | Y | Z |
| Aerodynamic moment components | L | M | N |

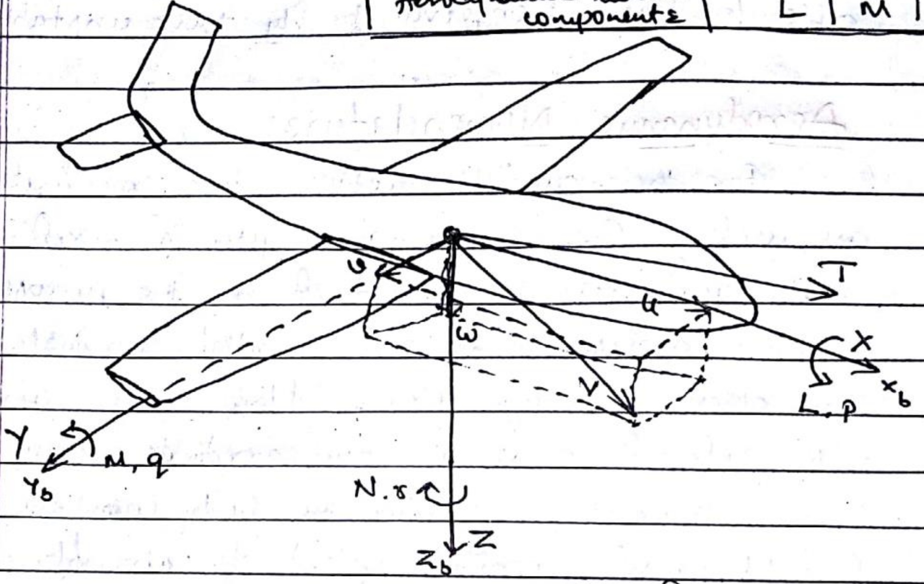



fig: Definition of forces, moments and velocity components in a body fixed coordinate.

The force components are denoted X, Y and Z, $T_x$, $T_y$, $T_z$, $W_x$, $W_y$, and $W_z$ for the aerodynamic, thrust, and gravitational force components along x, y, and z axes respectively.

The aerodynamic forces are defined in terms of dimensionless coefficient, the flight dynamic pressure Q, and a reference area S as follows:

$X = C_x Q S$ Axial force --- (1)

$Y = C_y Q S$ Side force --- (2)

$Z = C_z Q S$ Normal force --- (3)

In the similar manner, the moments on the airplane can be divided into moments created by the aerodynamic load distribution and the thrust force not acting through the center of gravity. The components of the aerodynamic moment also are expressed in terms of dimensionless coefficients, flight dynamic pressure, reference area and a characteristic length as follows:

$$L = C_l Q S l \quad \text{Rolling moment} \quad \text{---(4)}$$

$$M = C_m Q S l \quad \text{pitching moment} \quad \text{---(5)}$$

$$N = C_n Q S l \quad \text{Yawing moment} \quad \text{---(6)}$$

For airplanes the reference area $S$ is taken as the wing platform area and the characteristic length $l$ is taken as the wing span for the rolling and yawing moment and the mean chord for the pitching moment.

The aerodynamic coefficients $C_x, C_y, C_z, C_l, C_m$ and $C_n$ primarily are a function of the mach number, Reynolds number, angle of attack and sideslip angle: they are secondary functions of the time rate of change of angle of attack and sideslip, and the angular velocity of the airplane.

The aerodynamic force and moment acting on the airplane and its angular and translational velocity are illustrated in above figure. The $x$ and $z$ axes are in the plane of symmetry, with the $x$ axis pointing along the fuselage and the positive $y$ axis along the right wing. The resulting force and moment, as well as the airplane's velocity can be resolved along these axes.

The angle of attack and sideslip can be defined in terms of the velocity components as illustrated in below figure.

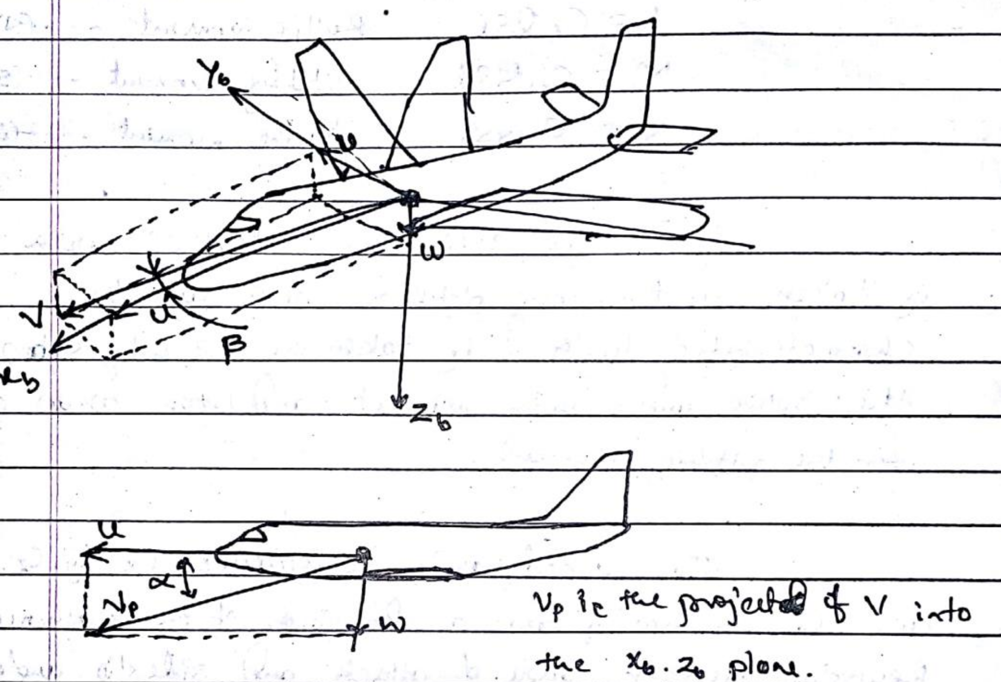



fig: Definition of angle of attack and sideslip.

The angle of attack and sideslip can be defined in terms of the velocity components as illustrated in above figure. The equations for $\alpha$ and $\beta$:

$$\alpha = \tan^{-1}\frac{w}{u} \quad ----(7)$$

$$\beta = \sin^{-1}\frac{v}{V} \quad ----(8)$$

where $V = (u^2+v^2+w^2)^{1/2}$ ----(9)

If the angle of attack and sideslip are small, that is $<15°$, then Eqn (7) & (8) can be approximated by

$$\alpha = \frac{w}{u} \quad \text{------ (10)}$$

$$\beta = \frac{v}{u} \quad \text{---- (11)}$$

where $\alpha$ & $\beta$ are in radians

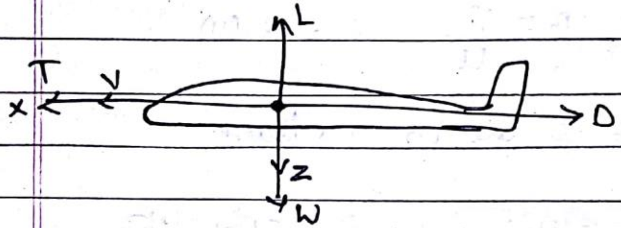
## III. Airplane Equilibrium and Stability :

* "By stability we mean the tendency of the airplane to return to its equilibrium position after it has been disturbed."
* "stability is a property of any equilibrium state of a system". (System = Airplane).
* The disturbance may be generated by the pilot's actions or atmospheric phenomena. The atmospheric disturbances can be wind gusts, wind gradients, or turbulent air.
* To discuss stability we must first define what is meant by equilibrium.
* "Equilibrium state is nothing but system's position of rest or A dynamic condition with uniform velocities, no acceleration involved."
* If an airplane is to remain in steady uniform flight, the resultant force as well as the resultant moment about the center of gravity must both be equal to 0.

∴ Sum of External forces $\sum F = 0 = m\frac{dV}{dt}$ | $\bar{V}$ = const.

Sum of External moments $\sum M = 0 = I\frac{d\omega}{dt}$ | ∵ $\omega$ = const.

Ex 1: Cruise level flight.




consider a flight that is in Equilibrium condition,

Velocity of aircraft = $V = [u.\ v.\ w] = [u.\ 0.\ 0]$

for Equilibrium state,

$$\sum F = 0.$$

Eqn of motion along x-axis ⇒ ∴ $m\frac{du}{dt} = T - D = 0.$

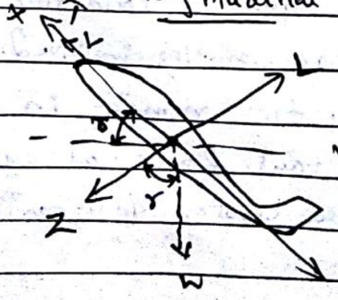
& Eqn of motion along z-axis ⇒ $m\frac{dw}{dt} = W - L = 0.$

∴ $T = D$

$W = L$

$\sum M_{cg} = 0$ = longitudinal pitching moment.

Ex 2: longitudinal flight condition [Climbing]




$V = [u.v.w] = [u.0.0]$

Equation of motion along x-axis,

$$m\frac{du}{dt} = T - W\sin\gamma - D = 0$$

equation of motion along z axis.

$$m\frac{dw}{dt} = W\cos\gamma - L = 0$$

$$\therefore \Sigma M_{cg} = 0.$$

Ex : 3: Non longitudinal equilibrium state:

at Steady level turn state.

bank angle $\phi$ = constant

Flight path angle $\gamma = 0$

sideslip angle $\beta = 0$.

IV. Stability:

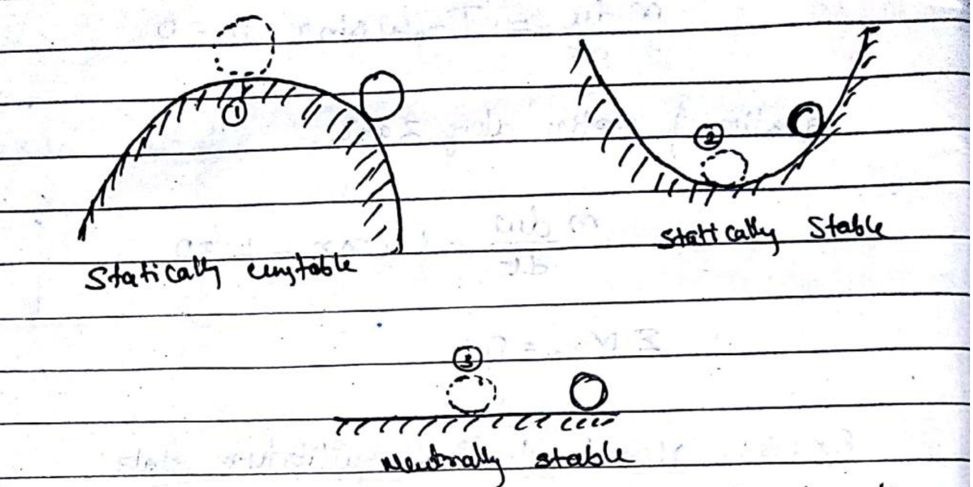
"Stability refers to system's property to comeback its original position when its disturbed. [any physical System => Airplane]."

stability → Static Stability
stability → Dynamic Stability

Static Stability:

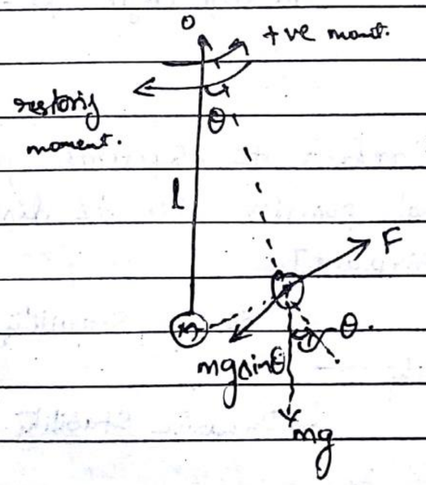
"static stability refers to system's tendency to return to its original Equilibrium state".

Ex 1:




Ball is in position of rest at ① ② & ③. how the system is responding to the disturbance comes under static stability.

Ex 2:




if $\theta = 0$ in Equilibrium state.

∴ initially Equilibrium state.

$$F = -mg \sin\theta.$$

moment around suspension poin O.

$$\therefore \quad I \frac{d^2\theta}{dt^2} = -mg \sin\theta \cdot l. \qquad \text{----(p)}$$

we know that $I = ml^2$

$\therefore$ Equation (12) becomes,

$$ml^2 \frac{d^2\theta}{dt^2} = -mg\sin\theta . l$$

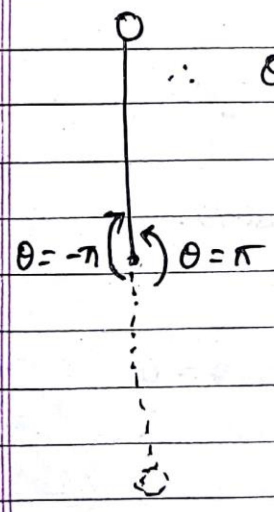
$$\therefore \boxed{\frac{d^2\theta}{dt^2} + \frac{g}{l}\sin\theta = 0} \Rightarrow \text{Eqn of motion for pendulum.}$$

if $\dfrac{d^2\theta}{dt^2} = 0$ $\Rightarrow$ Equilibrium state

$\Rightarrow$ Time rate of change of any derivative should be zero.

$\therefore$ If $\sin\theta = 0$.

then $\theta = n\pi = -1, -2, -3, 0, 1, 2, 3 \ldots$



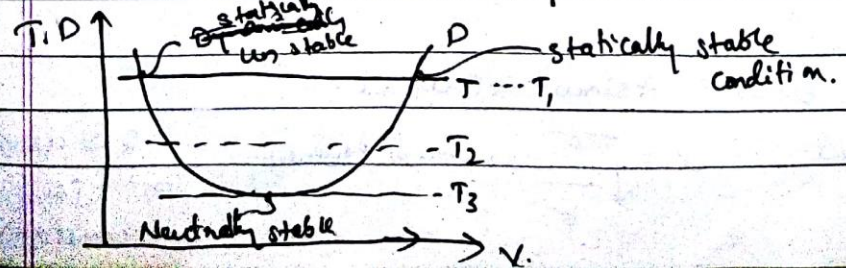

$\therefore \theta = -\pi, 0, \pi$ = Equilibrium position.

$\theta = 0 \Rightarrow$ statically stable.

$\theta = -\pi, \pi \Rightarrow$ statically unstable.

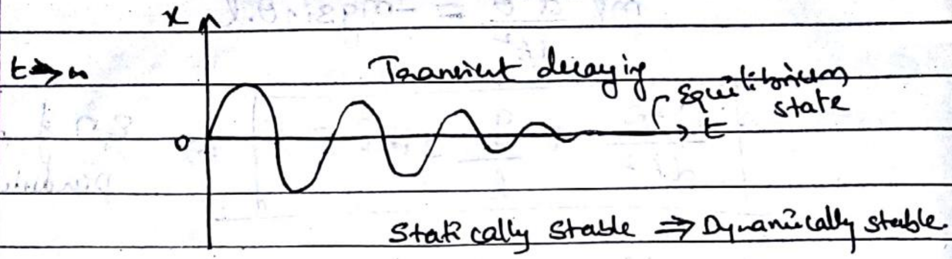
Ex 3: Aircraft dynamics:

Take cruise level flight condition, $T = D$ & $L = W$. $\Sigma M_{cg} = 0$.



## Dynamic Stability :

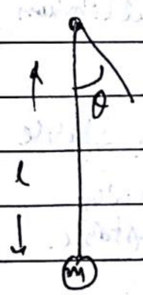
* "It is related to the actual time history of system's state."




Statically stable ⇒ Dynamically stable

* " system (Airplane) is dynamically stable at an equilibrium state, when it eventually ($t \to \infty$) aquires its equilibrium state from where it was disturbed".

Ex 1:




The equation of motion,

$$\frac{d^2\theta}{dt^2} = -\frac{g}{l}\sin\theta.$$

$$\therefore \frac{d^2\theta}{dt^2} + \frac{g}{l}\sin\theta = 0$$

$\theta = 0$ Equilibrium state.

$\because \theta$ is small,

$$\sin\theta \simeq \theta.$$

$$\therefore \frac{d^2\theta}{dt^2} + \frac{g}{l}\theta = 0$$

$$\Rightarrow \theta(t) = A\sin\omega t + B\cos\omega t.$$

where, $\omega = \sqrt{\frac{g}{l}}$ = natural frequency

A & B depends on initial condition.

when $\theta(0) = 0, \Rightarrow \dot{\theta}(0) \neq 0$

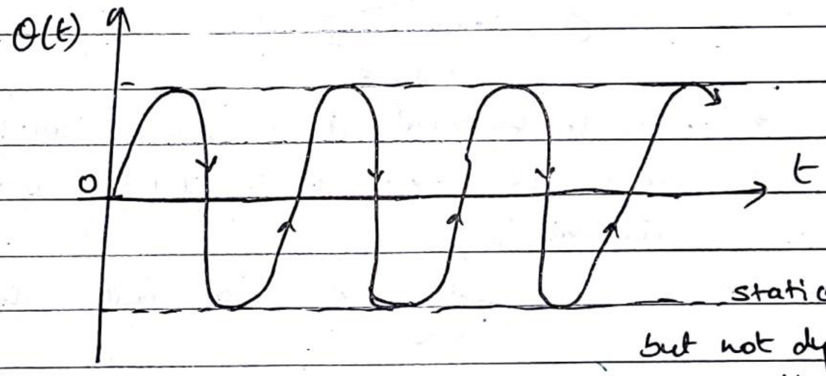
$\therefore \theta(0) = A(0) + B(1) \Rightarrow B = 0.$

$\therefore \boxed{\theta(t) = A \sin \omega t}.$

$\therefore \dot{\theta}(t) = A\omega \cos \omega t$

$\dot{\theta}(0) = A\omega(1) \Rightarrow A = \frac{\dot{\theta}(0)}{\omega}$

$\therefore \boxed{\theta(t) = \frac{\dot{\theta}(0)}{\omega} \sin \omega t}$




statically stable but not dynamically stable.

$\therefore$ Statically stable $\xRightarrow{\text{Does not always imply}}$ Dynamically stable.

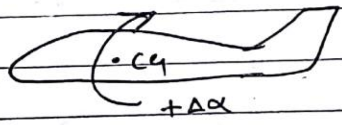
but Dynamically stable $\Longrightarrow$ Statically stable

## V Stability Criteria / Static Stability criteria / guidelance

### (i) Longitudinal static stability / Pitch stability :

* $\frac{\partial M}{\partial \alpha}$ pitching moment.

* This refers to rotational motion about CG of aircraft in their plane of axis.




* " Due to external disturbances causing pitching up motion will have to be counter acted by the aircraft".

$$\frac{\partial M}{\partial \alpha} < 0 \quad \text{for static stability in pitch.}$$

here, $M = QS\bar{c}C_m$

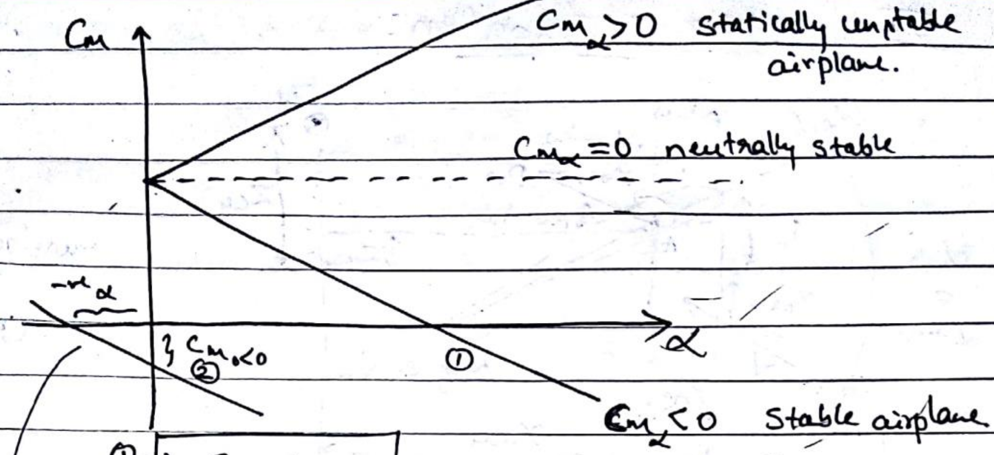
$$\frac{\partial M}{\partial \alpha} \propto \frac{\partial C_m}{\partial \alpha}$$

$$\therefore \boxed{C_{m_\alpha} < 0}$$

$$\therefore \boxed{\frac{dC_m}{dC_L} < 0} \quad \text{for pitch stability.}$$

$$\frac{dC_m}{d\alpha} = \frac{dC_m}{dC_L}\underbrace{\frac{dC_L}{d\alpha}}_{+ve}$$

$$\therefore \frac{dC_m}{d\alpha} < 0 \Rightarrow \frac{dC_m}{dC_L} < 0$$




$\therefore$ ① $\rightarrow$ $\boxed{C_{m_0} > 0}$ $\rightarrow$ to trim at +ve $\alpha$

$\boxed{C_{m_\alpha} < 0}$ $\rightarrow$ to have static stability pitch.

② $\rightarrow$ $\boxed{C_{m_0} < 0}$ Statically stable in pitch but can

$\boxed{C_{m_\alpha} < 0}$ trim ($C_{m_{cg}} = 0$) only at negative angle of attack.

(ii) Roll static stability:

$$\boxed{\frac{dC_l}{d\beta} < 0} \quad \text{dihedral effect} \quad \boxed{C_{l_\beta} < 0}$$

"change in rolling moment coefficient respect to change in sideslip angle must be negative."

here, $C_l$ = rolling coefficient.

$\beta$ = side slip angle.

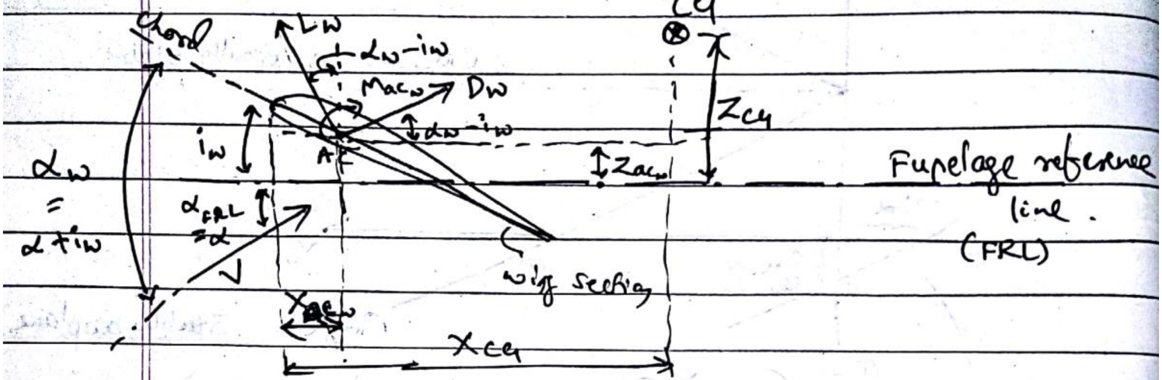
(iii) Yaw stability / Weathercock stability:

$$\boxed{\frac{dC_n}{d\beta} > 0} \Rightarrow \boxed{C_{n_\beta} > 0}$$

$C_n$ = Yawing moment coefficient.

# Longitudinal Stick fixed Stability of aircraft

## VI. Wing contribution:




here, $i_w$ = wing incidence angle
$\alpha$ = angle of attack of aircraft which is measured with respect to FRL.
$L_w$ = Lift created at wing.
$D_w$ = Drag created at wing.

$\therefore M_{cg} = \sum$ pitching moment about the CG.

$$\therefore M_{cg} = Mac_w + L_w \cos(\alpha_w - i_w)(x_{cg} - x_{ac_w}) + L_w \sin(\alpha_w - i_w)(Z_{cg} - Z_{ac_w}) - D_w \cos(\alpha_w - i_w)(Z_{cg} - Z_{ac_w}) + D_w \sin(\alpha_w - i_w)(x_{cg} - x_{ac_w}) \quad \text{---(1)}$$

we know that
$$M_{cg} = \tfrac{1}{2}\rho V^2 S \bar{c} C_{m_{cg}}$$
$$Mac_w = \tfrac{1}{2}\rho V^2 S \bar{c} C_{mac_w}$$
$$L_w = \tfrac{1}{2}\rho V^2 S C_{L_w}$$
$$D_w = \tfrac{1}{2}\rho V^2 S C_{D_w}.$$

$\therefore$ eqn (1) becomes,

$$\frac{1}{2}\rho V^2 S\bar{c}\, C_{m_{cg}} = \frac{1}{2}\rho V^2 S\bar{c}\, C_{m_{ac_w}} + \frac{1}{2}\rho V^2 S C_{L_w} \cos(\alpha_w - i_w)(X_{cg} - X_{ac_w}) + \frac{1}{2}\rho V^2 S C_{L_w} \sin(\alpha_w - i_w)(Z_{cg} - Z_{ac_w}) - \frac{1}{2}\rho V^2 S C_{D_w} \cos(\alpha_w - i_w)(Z_{cg} - Z_{ac_w}) + \frac{1}{2}\rho V^2 S C_{D_w} \sin(\alpha_w - i_w)(X_{cg} - X_{ac_w})$$

∴ dividing $\frac{1}{2}\rho V^2 S\bar{c}$ on above equation,

$$C_{m_{cg}} = C_{m_{ac_w}} + C_{L_w} \cos(\alpha_w - i_w)(X_{cg} - X_{ac_w}) + C_{L_w} \sin(\alpha_w - i_w)(Z_{cg} - Z_{ac_w}) - C_{D_w} \cos(\alpha_w - i_w)(Z_{cg} - Z_{ac_w}) + C_{D_w} \sin(\alpha_w - i_w)(X_{cg} - X_{ac_w}) \quad ---(2)$$

Assuming that $(\alpha_w - i_w)$ is small (angle is small).
and also $C_{L_w} >> C_{D_w}$ lift is larger than drag.
and $(Z_{cg} - Z_{ac_w})$ is small.

∴ $\cos(\alpha_w - i_w) \simeq 1$

& $\sin(\alpha_w - i_w) = (\alpha_w - i_w)$

further vertical components are neglected.

∴ Eqn (2) becomes,

∴

$$\boxed{C_{m_{cg}} = C_{m_{ac_w}} + C_{L_w}\left(\frac{X_{cg} - X_{ac_w}}{\bar{c}}\right)} \quad ---(3)$$

if we include camber effect, then $C_{L_w}$ becomes,

$$C_{L_w} = C_{L_{0_w}} + C_{L_{\alpha_w}} \cdot \alpha_w$$

∴ Eqn (3) becomes,

$$\boxed{C_{m_{cg}} = C_{m_{ac_w}} + \left(C_{L_{0_w}} + C_{L_{\alpha_w}} \alpha_w\right)\left(\frac{X_{cg} - X_{ac}}{\bar{c}}\right)} \quad ---(4)$$

Separating eqn (4) becomes,

$$C_{m_{cg}} = \left[C_{m_{ac_w}} + C_{L_{0w}}\left(\frac{X_{cg}-X_{ac_w}}{\bar{c}}\right)\right] + \left[a_w \alpha_w \left(\frac{X_{cg}-X_{ac}}{\bar{c}}\right)\right] \quad \text{---(5)}$$

where. $a_w = C_{L_{\alpha_w}}$ = lift curve slop of the wing.

we know that

$$C_{m_{cg}} = C_{m_0} + C_{m_\alpha}$$

∴ Eqn (5) becomes.

$$C_{m_0} = C_{m_{ac_w}} + C_{L_{0w}}\left(\frac{X_{cg}-X_{ac_w}}{\bar{c}}\right)$$

$$C_{m_\alpha} = a_w\left(\frac{X_{cg}-X_{ac_w}}{\bar{c}}\right)$$

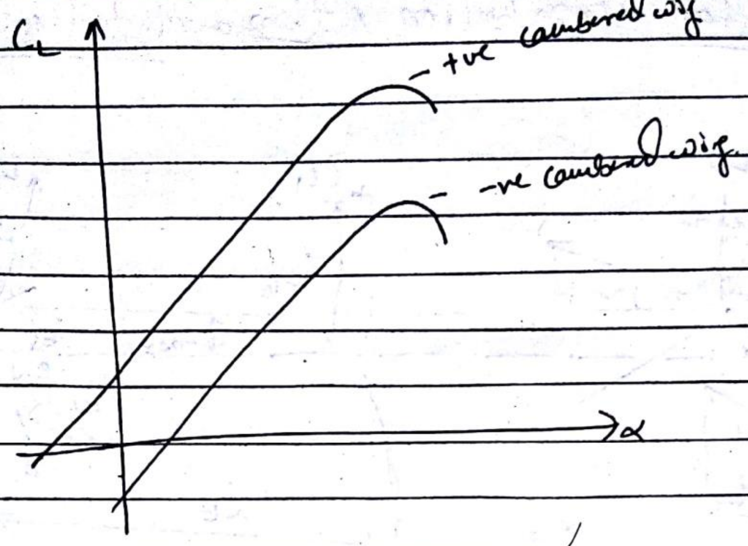
∵ here $C_{m_\alpha} < 0$,

means $X_{cg} < X_{ac_w}$

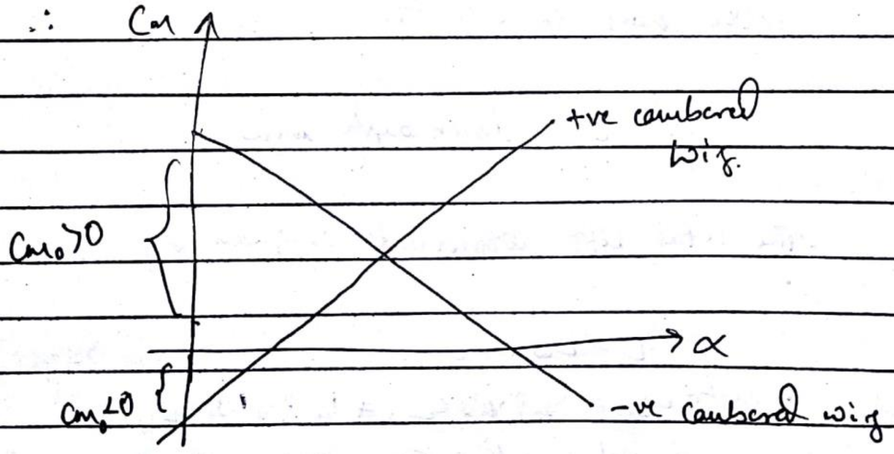
∴ " for static stability in pitch for a wing alone configuration, cg must lie ahead of aerodynamic center."

for positive cambered wing, $\boxed{C_{m_0} < 0}$

for negative cambered wing, $\boxed{C_{m_0} > 0}$



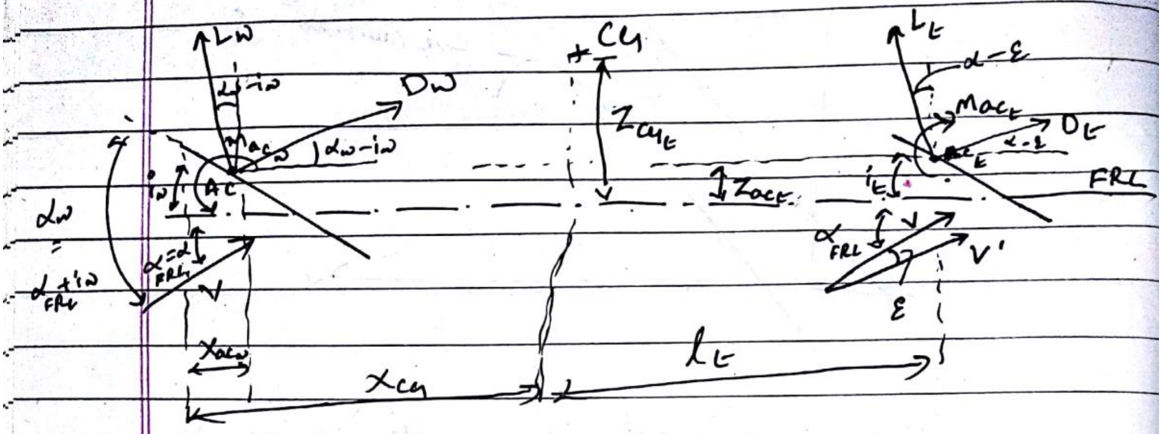

"Negative cambered wing results in loss in $C_{L_{max}}$, which will affect the performance of the aircraft. So to minimize this, there will be a ~~ex~~ usage of reflexed trailing edge wing.

∴



Airplane to have in stability in pitch/ fly at +ve AOA, has to satisfy $\boxed{\begin{array}{l} C_{m_0} > 0 \\ C_{m_\alpha} < 0 \end{array}}$

Horizontal tail going to be used to augment the wing alone's +ve cambered and it going to make $C_{m_0} < 0$ to $C_{m_0} > 0$.

## VII Tail contribution / Horizontal tail contribution




for +ve cambered wing, $C_{m_0}$ or $C_{m_{ac_w}}$ should be '-ve'

Because of tail location, downwash angle (tip & trailing vortex) ⇒ change of AOA at FRL associated with change in velocity.

∴ $\varepsilon$ = downwash angle

∴ The total lift coefficient is given by,

$$L = L_w + L_t$$

$$\therefore \tfrac{1}{2}\rho V^2 S C_L = \tfrac{1}{2}\rho V^2 S C_{L_w} + \tfrac{1}{2}\rho V'^2 S C_{L_t}$$

divided by $\frac{1}{2}\rho V^2 S$ on above equation,

$$C_L = C_{L_w} + \frac{\frac{1}{2}\rho V'^2}{\frac{1}{2}\rho V^2}\frac{S_t}{S_w} C_{L_t} \quad \Big| \; S_w = S$$

$$C_L = C_{L_w} + \frac{Q_t}{Q_w}\frac{S_t}{S_w} C_{L_t} \quad \Big| \; \eta = \frac{Q_t}{Q_w} \text{ = tail efficiency}$$

$$\therefore C_L = C_{L_w} + \eta \frac{S_t}{S_w} C_{L_t}.$$

Angle of attack to tail part,

$$\alpha_{FRL_t} = \alpha_{FRL_w} - \varepsilon = (\alpha - \varepsilon)$$

Effective angle of attack at tail,

$$\alpha_{eff} \text{ at tail} = \alpha - \varepsilon + i_t$$
$$\alpha_t = \alpha - \varepsilon + i_t$$
$$\alpha_t = \alpha_w - i_w + i_t - \varepsilon$$

From finite wing theory,

$$\varepsilon = \frac{2 C_{Lw}}{\pi AR_w}$$

$$\therefore \varepsilon = \varepsilon_0 + \frac{d\varepsilon}{d\alpha} \cdot \alpha_w \quad \text{at zero AOA.}$$

$$\therefore \alpha_t = \alpha_w - i_w + i_t - \varepsilon_0 - \frac{d\varepsilon}{d\alpha} \alpha_w.$$

need to find out the moment of the force at tail about CG of Aircraft.

$$M_{CG_t} = -L_t \cos(\alpha - \varepsilon) l_t + L_t \sin(\alpha - \varepsilon)(Z_{CG} - Z_{ac}) - D_t \cos(\alpha - \varepsilon)(Z_{CG} - Z_{ac_t}) - D_t \sin(\alpha - \varepsilon) l_t + M_{ac_t} \quad \text{---- (6)}$$

here. $M_{ac_t} = 0$ because tail is symmetric airfoil.

Assuming $L_t \gg D_t$.

$(Z_{CG} - Z_{ac_t})$ is very small,

angles are small, $\cos(\alpha - \varepsilon) = 1$ & $\sin(\alpha - \varepsilon) = (\alpha - \varepsilon)$

$\therefore$ Eqn (6) leads to,

$$\boxed{M_{cg_t} = -L_t l_t}$$

Elaborating above eqn,

$$\frac{1}{2}\rho V^2 S_w \bar{c}\, C_{m_{cg_t}} = -\frac{1}{2}\rho V^2 S_t C_{L_t} \cdot l_t$$

$$\therefore \quad C_{m_{cg_t}} = \frac{-\frac{1}{2}\rho V'^2}{\frac{1}{2}\rho V^2} \frac{S_t l_t}{S_w \bar{c}} \cdot C_{L_t}$$

$$\therefore \quad C_{m_{cg_t}} = -\eta V_H C_{L_t}$$

$V_H$ = tail volume ratio $= \frac{S_t l_t}{S_w \bar{c}}$

$$\therefore \quad C_{m_{cg_t}} = -\eta V_H C_{L_{\alpha_t}} \cdot \alpha_t$$

$$C_{m_{cg_t}} = -\eta V_H a_t \left(\alpha_w - i_w + i_t - \varepsilon_0 - \frac{d\varepsilon}{d\alpha} \cdot \alpha_w\right) \quad \text{------ (7)}$$

$C_{L_{\alpha_t}} = a_w$

We know that $C_{m_{cg}} = C_{m_0} + C_{m_\alpha}$

$\therefore$ Eqn (7) becomes,

$$C_{m_{cg_t}} = -\eta V_H a_t \left(-i_w + i_w - \varepsilon_0\right) - \eta V_H a_t \left(1 - \frac{d\varepsilon}{d\alpha}\right)\alpha_w$$

$$\therefore \quad C_{m_{0_t}} = \eta V_H a_t \left(i_w - i_t + \varepsilon_0\right)$$

$$C_{m_\alpha} = -\eta V_H a_t \left(1 - \frac{d\varepsilon}{d\alpha}\right)\alpha_w$$

from above eqns we can seen as $C_{m_0} > 0$ and $C_{m_\alpha} < 0$.

for wing-tail contribution,

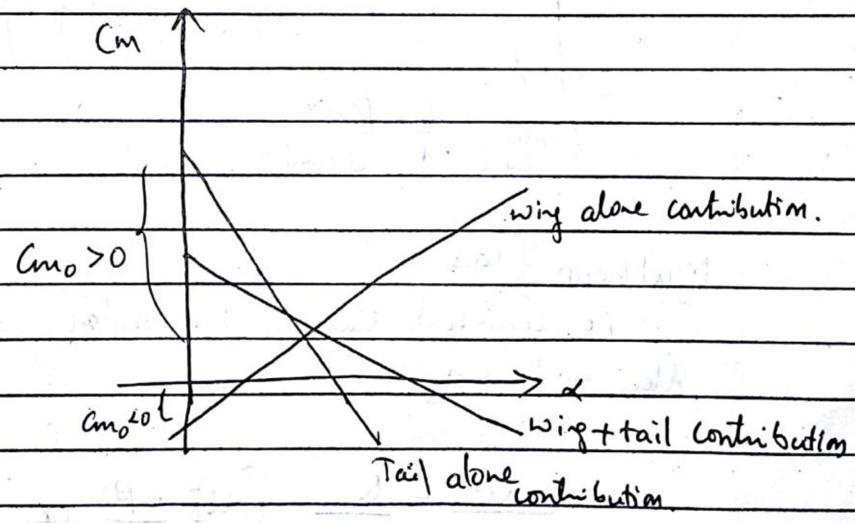
$$C_{m_{0(w+t)}} = C_{m_{0_w}} + C_{m_{0_t}}$$

$$C_{m_{\alpha(w+t)}} = C_{m_{\alpha_w}} + C_{m_{\alpha_t}}.$$

VIII. Wing plus tail contribution:

$$C_{m_{0(w+t)}} = C_{m_{0_t}} + C_{m_{0_w}}$$

$$C_{m_{\alpha(w+t)}} = C_{m_{\alpha_t}} + C_{m_{\alpha_w}}$$




IX Fuselage Contribution:

Max Munk: (1923)

Fuselage on pitching stability for slender fuselage contribution,

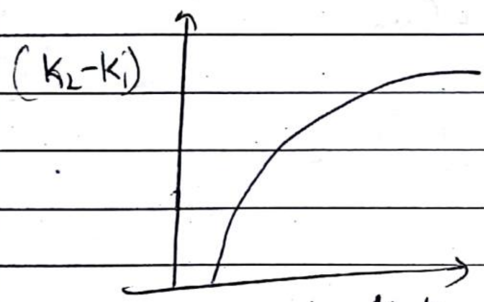
$$\frac{dM}{d\alpha} = fn(\text{Volume}. \tfrac{1}{2}\rho V^2)$$

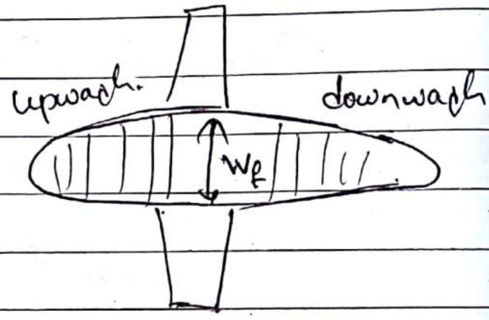
$$\frac{dM}{d\alpha} = \frac{\text{Volume}}{28.7} Q \text{ per degree}$$

$$\frac{dM}{d\alpha} = \frac{\text{Volume}}{28.7} (K_2 - K_1) Q$$

where $(K_2 - K_1)$ = Account for Slenderness ratio

$$\frac{dM}{d\alpha} \frac{Q(K_2 - K_1)}{36.5} \int_0^l w_f^2 \, dx.$$




Multhopp (1941):

he considered that for the fuselage, wing disturbs the flow to fuselage,

$$\frac{dM}{d\alpha} = \frac{Q}{36.5} \int_0^l w_f^2 \frac{d\beta}{dx} \, dx.$$

here, $\beta$ = local angle of attack

$$\beta = \alpha + \alpha_i$$

$$\frac{d\beta}{d\alpha} = 1 + \frac{d\varepsilon}{d\alpha}, \qquad \frac{d\beta}{d\alpha} = 1 - \frac{d\varepsilon}{d\alpha}$$

For fuselage portion ahead of the wing. — for the fuselage portion rear of the wing

$$\frac{dM}{d\alpha} = \frac{Q}{36.5} \sum_{i=1}^{n} W_f^2 \left(\frac{d\beta}{dx}\right) \Delta x$$

Fuselage effect on wing pitching moment,
fuselage affect to local flow,

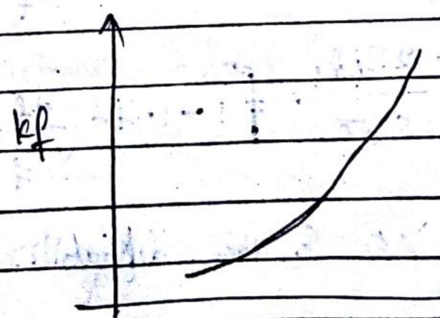
$$\frac{dM}{d\alpha} = \frac{Q\,\bar{c}^2}{290}\left(W_{LE} + 2W_{mid} + 3W_{TE}\right)$$

where $W$ = width of the fuselage at different section along the length.

$$\left(\frac{dC_m}{dC_L}\right)_{fuselage} = \frac{\left(\frac{dM}{d\alpha}\right)_{fuselage}}{S_w \bar{c} Q C_{L_{\alpha w}}}$$

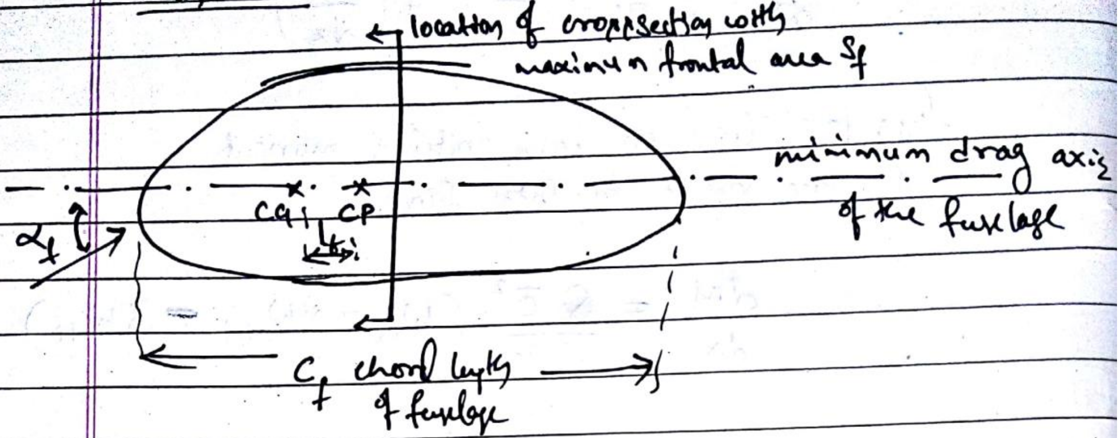
$$\left(\frac{dC_m}{dC_L}\right)_{fuselage} = \frac{k_f W_f^2 L_f}{S_w \bar{c} C_{L_{\alpha w}}}$$

where $k_f$ = an Empirical factor determined from Experiments
$W_f$ = max. width of fuselage.
$L_f$ = fuselage length.




Position of 1/4 root chord on body in % of body length.

Hoak (1960) :




$$C_m = \frac{M_{cg_f}}{\frac{1}{2}\rho V_\infty^2 c_f S_f} = \frac{-2 l_f}{c_f}\left[1 - 1.76\left(\frac{d_f}{c_f}\right)^{3/2}\right]\alpha_f$$

$(C_p)_f$ can be assumed to be halfway between the nose and the point of maximum frontal area.

$$(\Delta C_m)_f = \frac{S_f l_f}{S_w \bar{c}} C_{m_f} = \frac{-2 S_f l_f}{S_w \bar{c}}\left[1 - 1.76\left(\frac{d_f}{c_f}\right)^{3/2}\right]\alpha_f$$

$$(\Delta C_m)_f = (\Delta C_{m_0})_f + (\Delta C_{m_\alpha})_f .$$

$$\therefore (\Delta C_{m_0})_f = \frac{-2 S_f l_f}{S_w \bar{c}}\left[1 - 1.76\left(\frac{d_f}{c_f}\right)^{3/2}\right]\alpha_{0_f}$$

$$(\Delta C_{m_\alpha})_f = \frac{-2 S_f l_f}{S_w \bar{c}}\left[1 - 1.76\left(\frac{d_f}{c_f}\right)^{3/2}\right]$$

$\therefore (C_{m_\alpha})_f > 0$ & its destabilizing.

## Problems:

1. The wing - fuselage pitching moment characteristics of a high-wing, single-engine general aviation airplane follow along with pertinent geometric data:

$$C_{m_{cg_{wf}}} = -0.05 - 0.035\alpha$$

where $\alpha$ is the fuselage reference line angle of attack in degrees and wf means wing-fuselage

$S_w = 178\ ft^2$ $\quad x_{cg}/c = 0.1$

$b_w = 35.9\ ft$ $\quad AR_w = 7.3$

$\bar{c}_w = 5.0\ ft$ $\quad C_{L_{\alpha_{wf}}} = 0.07/deg.$

$i_w = 2^{\circ}$ $\quad C_{L_{\alpha=0}} = 0.26$

Estimate the horizontal tail area and tail incidence angle, $i_t$, so that the complete airplane has the following pitching moment characteristics.

$$C_{m_{cg_{wft}}} = 0.15 - 0.025\alpha$$

where $\alpha$ is in degrees and wft is the wing fuselage - tail contribution. Assume the following with regard to the horizontal tail.

$l_t = 14.75\ ft$ $\quad \eta = 1$

$AR_t = 4.85$ $\quad C_{L_{\alpha_t}} = 0.073/deg.$

→ Need to find the horizontal tail contribution.

Subtracting the wing-fuselage contribution from the wing-fuselage-tail contribution

$$C_{m_{cg_t}} = C_{m_{cg_{wft}}} - C_{m_{cg_{wf}}}$$

$$C_{m_{0_t}} = 0.15 - (-0.05) = 0.2$$

$$C_{m_{\alpha_t}} = -0.025 - (-0.0035) = -0.0215/\text{deg}.$$

we know that

$$C_{m_{\alpha_t}} = -\eta V_H a_t \left(1 - \frac{d\varepsilon}{d\alpha}\right) = -0.0215/\text{deg}.$$

$$C_{m_{0_t}} = \eta V_H a_t (i_w + \varepsilon_0 - i_t) = 0.2$$

$$V_H = \frac{S_t l_t}{S_w \bar{c}} \qquad a_t = C_{L_{\alpha_t}}$$

from above equation unknown terms are $\varepsilon_0$, $\frac{d\varepsilon}{d\alpha}$, $S_t$ & $i_t$

we know that,

$$\varepsilon = \text{downwash angle} = \frac{2 C_{L_w}}{\pi AR_w}$$

$$\therefore \varepsilon_0 = \frac{2 C_{L_0}}{\pi AR_w} = \frac{2 \times 0.26}{\pi \times 7.3} = \underline{\underline{0.0226 \text{ rad}}}$$

$$\frac{d\varepsilon}{d\alpha} = \frac{2 C_{L_{\alpha_w}}}{\pi AR_w} = \frac{2 \times 0.07 \times 180}{\pi \times 7.3 \times \pi} = \underline{\underline{0.35 \text{ rad}}}.$$

$| \; C_{L_{\alpha_{wf}}} = C_{L_{\alpha_w}}$

Note: If in the problem directly length & other parameters given in "meter" then "don't convert into radian" (angles).

$$\therefore \eta V_H a_t \left(1-\frac{d\varepsilon}{d\alpha}\right) = \frac{0.0215\times180}{\pi}$$

$$\eta \frac{S_t l_t}{S_w \bar{c}} C_{L\alpha_t}\left(1-\frac{d\varepsilon}{d\alpha}\right) = \frac{0.0215\times180}{\pi}$$

$$(1)\frac{S_t(14.75)}{(178\times5)}\left(\frac{0.073\times180}{\pi}\right)(1-0.35) = \frac{0.0215\times180}{\pi}.$$

$$\therefore \boxed{S_t = 27.3\ ft^2}$$

$$\eta V_H a_t(i_w+\varepsilon_0-i_t) = 0.2$$

$$\eta \frac{S_t l_t}{S_w \bar{c}} C_{L\alpha_t}(i_w+\varepsilon_0-i_t) = 0.2.$$

$$(1)\frac{(27.3\times14.75)}{(178\times5)}\left(\frac{0.073\times180}{\pi}\right)\left(\frac{2\times180}{\pi}+0.0226-i_t\right) = 0.2.$$

$$i_t = 0.50\ rad$$

$$\therefore \boxed{i_t = -2.7\ deg}$$

The horizontal tail is mounted to the fuselage at a negative 2.7°.

2. For a given wing body combination, the aerodynamic center lies 0.05 chord length ahead of the center of gravity. The moment coefficient about the aerodynamic center is −0.016. If the lift coefficient is 0.45, calculate the moment coefficient about the center of gravity.

→ From wing contribution,

$$C_{m_{cg}} = C_{m_{ac_w}} + C_{L_w}\left[\frac{x_{cg} - x_{ac_w}}{\bar{c}}\right]$$

from given data,

$$\frac{x_{cg} - x_{ac_w}}{\bar{c}} = 0.05$$

$C_{m_{cg}} = ?$

$$C_{m_{ac_w}} = -0.016$$

$$C_{L_w} = 0.45$$

$$\therefore C_{m_{cg}} = -0.016 + 0.45\,(0.05)$$

$$\boxed{C_{m_{cg}} = 0.0065}$$

3. For a given wing body combination, the aerodynamic center lies 0.03 chord length ahead of the center of gravity. The moment coefficient about the center of gravity is 0.005 and the lift coefficient is 0.5. calculate the moment coefficient about the aerodynamic center.

→

$$\left(\frac{x_{cg} - x_{ac_w}}{\bar{c}}\right) = 0.03$$

$C_{L_w} = 0.5$

$C_{m_{ac_w}} = ?$

$C_{m_{cg}} = 0.005$

$$C_{m_{cg}} = C_{m_{ac_w}} + C_{L_w}\left(\frac{x_{cg} - x_{ac_w}}{\bar{c}}\right)$$

$$0.005 = C_{m_{ac_w}} + 0.5\,(0.03)$$

$$\boxed{C_{m_{ac_w}} = -0.01}$$

4. Consider a model of wing-body shape mounted in a wind tunnel. The flow conditions in the test section are standard sea level properties with a velocity of $100\,ms^{-1}$. The wing area and chord are $1.5\,m^2$ & $0.45\,m$. respectively using the wind tunnel force & moment measuring balance. the moment about the CG when the lift is zero found to be $-12.4$ Nm. when the model is pitched to another angle of attack. the lift and moment about the CG are measured to be 3675 N and 20.67 Nm. respectively. calculate the value of the moment coefficient about the aerodynamic center and the location of the aerodynamic center.

→ $V_\infty = 100\,m/s$

$S_w = 1.5\,m^2$

$\bar{c} = 0.45\,m$

$M_{cg_w} = -12.4\,Nm$ | when lift is zero.

$L_w = 3675\,N$

$M_{cg_w} = 20.67\,Nm$ (pitched to another angle of attack)

$C_{m_{ac_w}} = ?$

we know that,

$$M_{cg_w} = \frac{1}{2}\rho_\infty V_\infty^2 S \bar{c}\, C_{m_{cg_w}}.$$

$$C_{m_{cg_w}} = C_{m_{ac_w}} = \frac{-12.4}{\frac{1}{2} \times 1.225 \times 100^2 \times 1.5 \times 0.45}$$

$$\boxed{\text{at zero lift} = C_{m_{cg_w}} = C_{m_{ac_w}} = -0.003}$$

moment coefficient about aerodynamic center.

and also we know wing contribution,

$$C_{m_{cg_w}} = C_{m_{ac_w}} + C_{L_w}\left(\frac{x_{cg} - x_{ac_w}}{\bar{c}}\right)$$

$$C_{L_w} = \frac{L}{q_\infty S_w} = \frac{L}{\frac{1}{2}\rho V_\infty^2 S_w} = \frac{3675}{\frac{1}{2} \times 1.225 \times 100^2 \times 1.5}$$

$$\underline{\underline{C_{L_w} = 0.4}}$$

$$C_{m_{cg_w}} = \frac{M_{cg_w}}{q_\infty S \bar{c}} = \frac{M_{cg_w}}{\frac{1}{2}\rho_\infty V_\infty^2 S \bar{c}}$$

$$C_{m_{cg_w}} = \frac{20.67}{\frac{1}{2} \times 1.225 \times 100^2 \times 1.5 \times 0.45}$$

$$C_{m_{cg_w}} = \underline{\underline{0.005}}$$

$$\therefore C_{m_{cg_w}} = C_{m_{ac_w}} + C_{L_w}\left(\frac{x_{cg} - x_{ac_w}}{\bar{c}}\right)$$

$$0.005 = -0.003 + 0.4\left(\frac{x_{cg} - x_{ac_w}}{\bar{c}}\right)$$

$$\therefore \boxed{\left(\frac{x_{cg} - x_{ac_w}}{\bar{c}}\right) = 0.02}$$

The location of aerodynamic center. $0.02\bar{c}$.

## XI Stick fixed Neutral point.

The total pitching moment for the airplane can now obtained by summing the wing, fuselage & tail contributions:

$$C_{m_{cg}} = C_{m_0} + C_{m_\alpha} \cdot \alpha$$

where

$$C_{m_0} = C_{m_{0_w}} + C_{m_{0_f}} + C_{m_{0_t}}$$

$$C_{m_\alpha} = C_{m_{\alpha_w}} + C_{m_{\alpha_f}} + C_{m_{\alpha_t}}.$$

$$\therefore \cancel{C_{m_\alpha} = a_w\left(\frac{x_{cg} - x_{ac}}{\bar{c}}\right) + C_{m_{\alpha_f}} - \eta V_H a_t\left(1 - \frac{d\varepsilon}{}\right)}$$

$$\therefore C_{m_0} = C_{m_{0_w}} + C_{m_{0_f}} + \eta V_H a_t(\varepsilon_0 + i_w - i_t)$$

$$C_{m_\alpha} = a_t\left(\frac{x_{cg} - x_{ac}}{\bar{c}}\right) + C_{m_{\alpha_f}} - \eta V_H a_t\left(1 - \frac{d\varepsilon}{d\alpha}\right) \quad \text{---(1)}$$

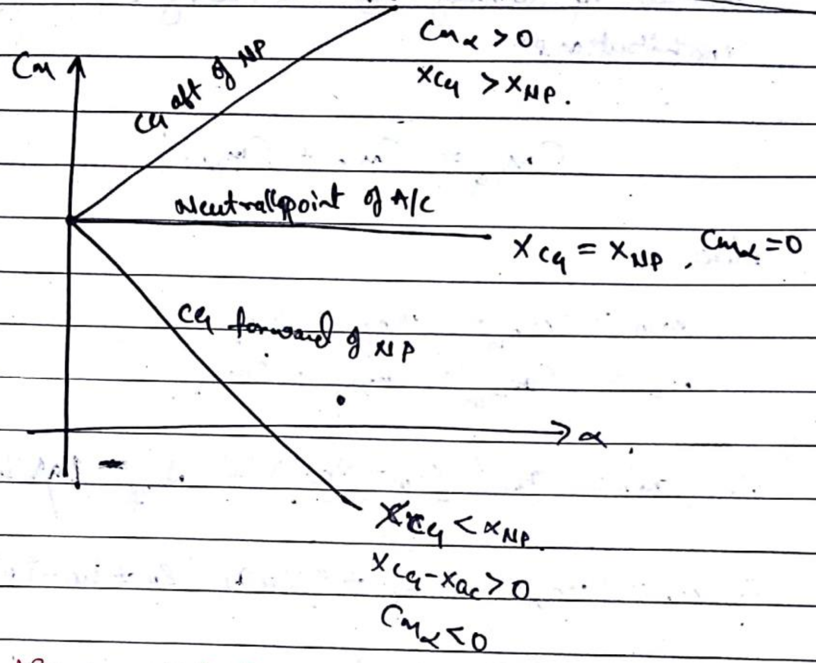
Notice that the expression $C_{m_\alpha}$ depends upon the center of gravity as well as the aerodynamic characteristics of the airplane. The center of gravity of an airplane varies during the course of its operation; therefore it is important to know if there are any limits to the center of gravity level. To ensure that the airplane possesses static longitudinal stability, we should like to know at what point. $\boxed{C_{m_\alpha} = 0}$. Setting $C_{m_\alpha}$ equal to 0 and solving for the center of gravity position yields, eqn (1) becomes,

$$0 = a_w\left(\frac{x_{NP} - x_{ac}}{\bar{c}}\right) + C_{m_{\alpha_f}} - \eta V_H a_t\left(1 - \frac{d\varepsilon}{d\alpha}\right)$$

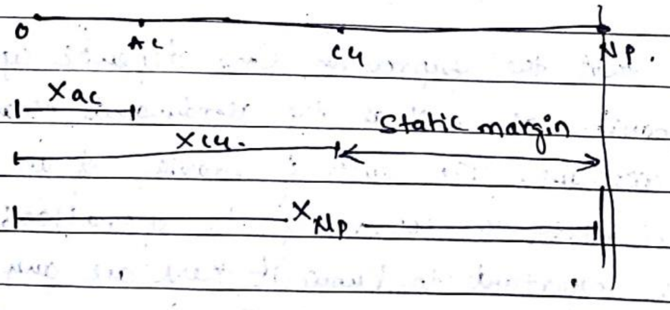
here $C_{m_\alpha} = 0$ & $x_{cg} = x_{NP}$

"Neutral point is the location of the center of gravity where $\frac{dC_m}{d\alpha} = 0$".

$$\therefore \boxed{\frac{X_{NP}}{\bar{c}} = \frac{X_{ac}}{\bar{c}} - C_{m_{0f}} + \eta V_H \frac{a_t}{a_w}\left(1 - \frac{d\varepsilon}{d\alpha}\right)}$$




## XII Static margin:

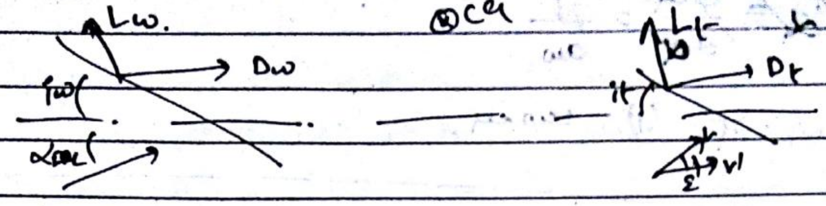



Static margin: the region between cg & NP.

$$\text{Stick fixed static margin} = \frac{X_{NP}}{\bar{c}} - \frac{X_{cg}}{\bar{c}}$$

Usually for most of aircrafts design; it is desirable to have a stick fixed static margin of approximately 5 percent of the mean chord.

wing + tail airplane configuration.




$$C_{m_{cg,\,w+t}} = C_{m_{ac_w}} + C_{L_w}\left(\frac{x_{cg}-x_{ac_w}}{\bar{c}}\right) - \cancel{\eta V_H C_{L_{\alpha_t}}(\ldots)}$$

$$- \eta V_H C_{L_{\alpha_t}}\left(-i_w - \varepsilon_0 + i_t\right)$$

$$- \eta V_H C_{L_{\alpha_t}}\left(1 - \frac{d\varepsilon}{d\alpha}\right)\alpha_w$$

$$\therefore\ C_{m_{0,\,w+t}} = C_{m_{ac_w}} + C_{L_{0_w}}\left(\frac{x_{cg}-x_{ac}}{\bar{c}}\right) + \eta V_H a_t\left(i_w + \varepsilon_0 - i_t\right)$$

$$C_{m_{\alpha,\,w+t}} = a_w\left(\frac{x_{cg}-x_{ac_w}}{\bar{c}}\right) - \eta V_H a_t\left(1 - \frac{d\varepsilon}{d\alpha}\right) \quad \text{---}(1)$$

here $a_t = C_{L_{\alpha_t}}$ $\quad a_w = C_{L_{\alpha_w}}$

$$C_{m_{\alpha,\,w+t}} = \frac{dC_m}{d\alpha}.$$

Considering Neutral point. $\frac{dC_m}{d\alpha} = 0$. $x_{cg} = x_{NP}$

$$\therefore\ 0 = a_w\left(\frac{x_{NP}}{\bar{c}} - \frac{x_{ac_w}}{\bar{c}}\right) - \eta V_H a_t\left(1 - \frac{d\varepsilon}{d\alpha}\right) \quad \text{---}$$

$$0 = \frac{x_{NP}}{\bar{c}} - \frac{x_{ac_w}}{\bar{c}} - \eta V_H \frac{a_t}{a_w}\left(1 - \frac{d\varepsilon}{d\alpha}\right) \quad \text{---}(2).$$

eqn (1) can be rewritten ~~as~~ by using below relation,

$$\frac{dC_m}{dC_L} = \frac{dC_m}{d\alpha}\cdot\frac{1}{\left(\frac{dC_L}{d\alpha}\right)} = \frac{1}{a_w}\,\frac{dC_m}{d\alpha}$$

∴

$$C_{m_{\alpha}} = \frac{dC_m}{d\alpha} = \frac{?}{a_w}$$

∴ Eqn (1) becomes,

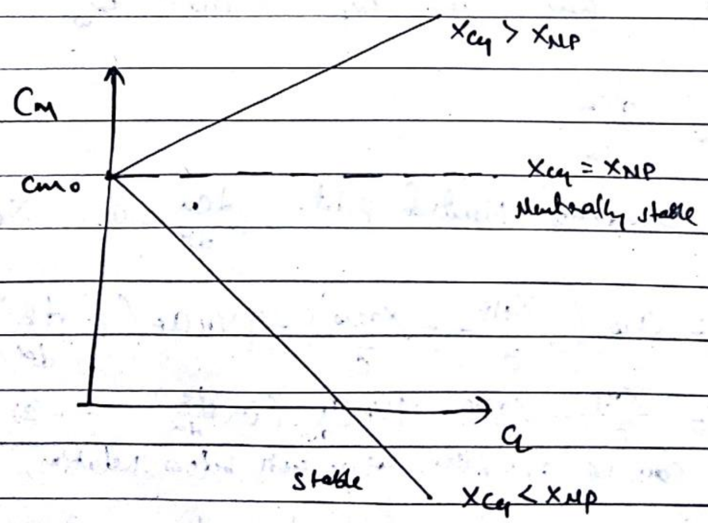
$$\boxed{\frac{dC_m}{dC_L} = \frac{X_{cg}}{\bar{c}} - \frac{X_{ac_w}}{\bar{c}} - \eta V_H \frac{a_t}{a_w}\left(1 - \frac{d\varepsilon}{d\alpha}\right)} \cdots (3)$$

Subtracting (2) from (3).

$$\therefore \quad 0 - \frac{dC_m}{dC_L} = \frac{X_{NP}}{\bar{c}} - \frac{X_{cg}}{\bar{c}}$$

$$\boxed{-\frac{dC_m}{dC_L} = \frac{X_{NP}}{\bar{c}} - \frac{X_{cg}}{\bar{c}}} = \text{Static margin} = H_n$$

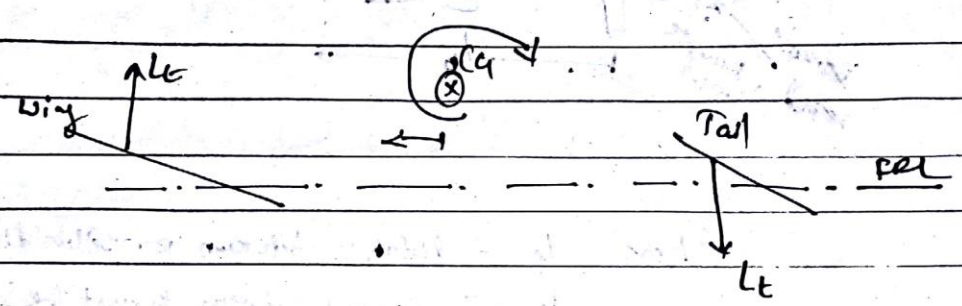
$X_{NP} - X_{cg} = 5\% \, \bar{c}$ for good handling qualities.




**Neutral point location is the most aft location of the center of gravity for a pitch stable airplane**

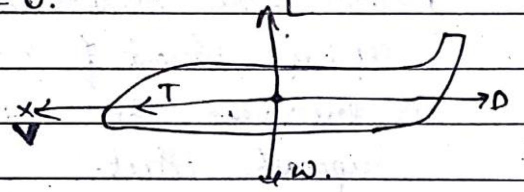
(i) Most forward location of the C.G / C.G limit:

when the lift producing at the tail become downward i.e. or '-ve' lift produced then at the C.G there will be a '+ve' moment.




We know that for equilibrium of the aircraft we should maintain trim condition, : $C_{m_{cg}} = 0$

or $\sum M_{cg} = 0$.



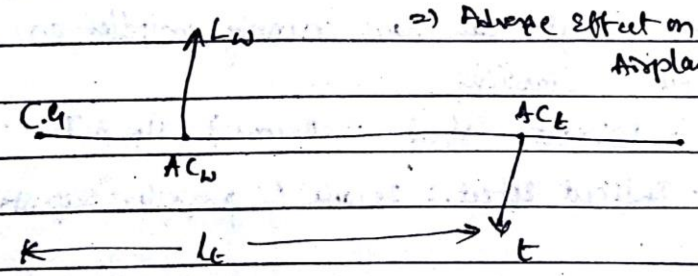

$\therefore$ $T = D$

$L = W = L_w + L_t$

if our tail plane is giving '-ve' lift then

$L = W = L_w - L_t$ ⇒ leads to high wing loading

⇒ Adverse effect on structural parts & Airplane performance

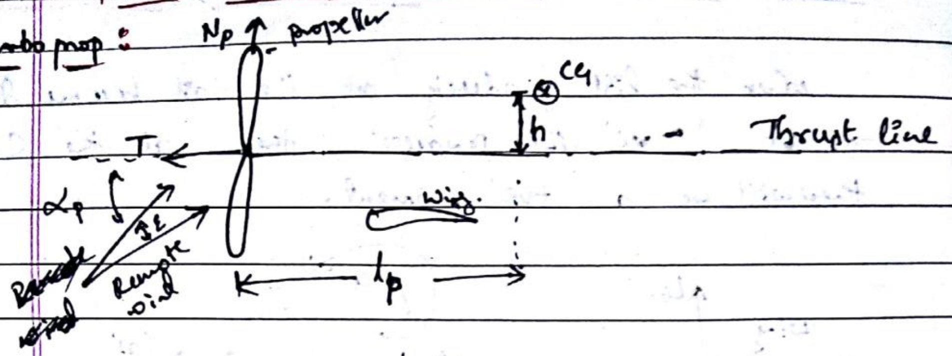



C.G of aircraft can lie forward of the aerodynamic center of wing. at that time the wing is having a stabilizing effect. while the tail is having a destabilizing effect. but we can get

$C_{m_{cg}} = 0$

## XII. Power Effects / Power plant Contribution:

(i) Turbo prop :




here $l_p$ = distance between propeller disc & C.G

$h$ = distance between thrust line & C.G

$N_p$ = Normal force acting on propeller.

Because of the wing placement, there will be a creation of upwash in front of propeller. So the angle of attack of freestream air ($\alpha_p$) will be acting based upon this upwash Effect.

$\varepsilon$ = upwash angle.

The two Effects can have running propeller on longitudinal static stability.

- Direct Effect : Because of $N_p$ & T.
- Indirect Effect : Because of propeller slip stream

Direct Effect:

$$M_{cg_p} = T(h) + N_p(l_p)$$

here T can be written as $T = C_T \cdot \rho V^2 D^2$

$$C_T = \frac{T}{\rho V^2 D^2} \quad ; \quad D = \text{diameter of the propeller disc}$$

and also $N_p = \frac{1}{2}\rho V^2 S_p C_{N_p}$ ; $S_p = \frac{\pi D^2}{4}$

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$\therefore \quad M_{cg_p} = Th + N_p l_p$

$\frac{1}{2}\rho V^2 S_w \bar{c}\, C_{m_{cg_p}} = C_T \rho V^2 D^2 . h + \frac{1}{2}\rho V^2 S_p . C_{N_p} . l_p$

$\therefore \quad C_{m_{cg_p}} = C_T \frac{2D^2}{S_w} . \frac{h}{\bar{c}} . N + C_{N_p} \frac{l_p}{S_w} . \frac{S_p}{\bar{c}} . N.$

here, N is number of blades

differentiating & dividing $C_L$,

$\therefore \left(\frac{dC_m}{dC_L}\right)_p = \underbrace{\left(\frac{dC_T}{dC_L}\right) \frac{2D^2}{S_w} \frac{h}{\bar{c}} . N}_{①} + \underbrace{\left(\frac{dC_{N_p}}{dC_L}\right) \frac{l_p}{S_w} . \frac{S_p}{\bar{c}} . N}_{②}$

here $C_L$ = lift coefficient. depends upon $\alpha_p$. (3)

If we look at Equation ① from above,

① $\therefore \quad C_T = K\eta_p C_L^{3/2}$

$K = f^n (BHP, \rho . W/S)$.

$\therefore \frac{dC_T}{dC_L} = K\eta_p \frac{3}{2} C_L^{1/2}$

for $h>0$ the Effect of ① is destabilizing.

$\therefore \left(\frac{dC_m}{dC_L}\right)_T = 0.25 \frac{h}{\bar{c}}$ for a fighter A/C configuration

If we look at Equation ② from above,

② $\left(\frac{dC_{N_p}}{dC_L}\right) = \frac{dC_{N_p}}{d\alpha_p} \frac{d\alpha_p}{dC_L}$

$\alpha = \alpha_p + \varepsilon$

$\frac{d\alpha_p}{dC_L} = \frac{d\alpha_p}{d\alpha} . \frac{1}{\frac{dC_L}{d\alpha}} = \frac{1}{a_w}\left(1+\frac{d\varepsilon}{d\alpha}\right)$

$\therefore \frac{dC_{N_p}}{dC_L} = \left(\frac{dC_{N_p}}{d\alpha_p}\right) . \frac{1}{a_w}\left(1+\frac{d\varepsilon}{d\alpha}\right)$ --- (4)

Eqn (4) Substituting into eqn (2)

$$\therefore \left(\frac{dC_m}{dC_L}\right)_p = \left(\frac{dC_{N_p}}{d\alpha_p}\right) \cdot \frac{1}{a_w}\left(1 + \frac{d\varepsilon}{d\alpha}\right) \frac{l_p}{S_w} \frac{S_p}{\bar{c}} \cdot N.$$

$$\left(\frac{dC_{N_p}}{d\alpha}\right)_{p_{T=0}} =$$

| | |
|---|---|
| 0.00165 | 2 blade propeller. |
| 0.00235 | 3 blade propeller. |
| 0.00296 | 4 blade propeller. |
| 0.00510 | 6 blade propeller. |

$\left(\frac{dC_m}{dC_L}\right)_p$ is depends up on $\frac{dC_{N_p}}{d\alpha_p}$ and the mainly

depends up on $l_p$.

If $l_p$ is '+ve' ⇒ destability.

If $l_p$ is '−ve' ⇒ pusher type A/c = SARAS

⇒ will change the stability characteristics in large

**Indirect Effect:**

resolving all every component of A/c about C.G.

$$\therefore C_{m_{cg}} = C_{L_w}\left[\frac{X_{cg} - X_{ac_w}}{\bar{c}}\right] + C_{m_{ac_w}} + C_{m_{fus}} - \eta V_H C_{L_t}$$

$$+ C_T \frac{2D^2}{S_w} \frac{h}{\bar{c}} \cdot N + C_{N_p} \frac{l_p}{S_w} \frac{S_p}{\bar{c}} \cdot N.$$

(a). Effect of propeller slipstream on wing-fuselage pitching moment.
(b) Effect of propeller slipstream on wing lift coefficient.
(c) Effect of downstream (due to propeller slipstream) at the horizontal tail.
(d) Effect of increased dynamic pressure at the tail.

$$C_{m_t} = -V_H C_{L_t}\left(\frac{V_s}{V}\right)^2 \qquad \left(\frac{V_s}{V}\right)^2 = 1 + \frac{8C_T}{\pi}$$

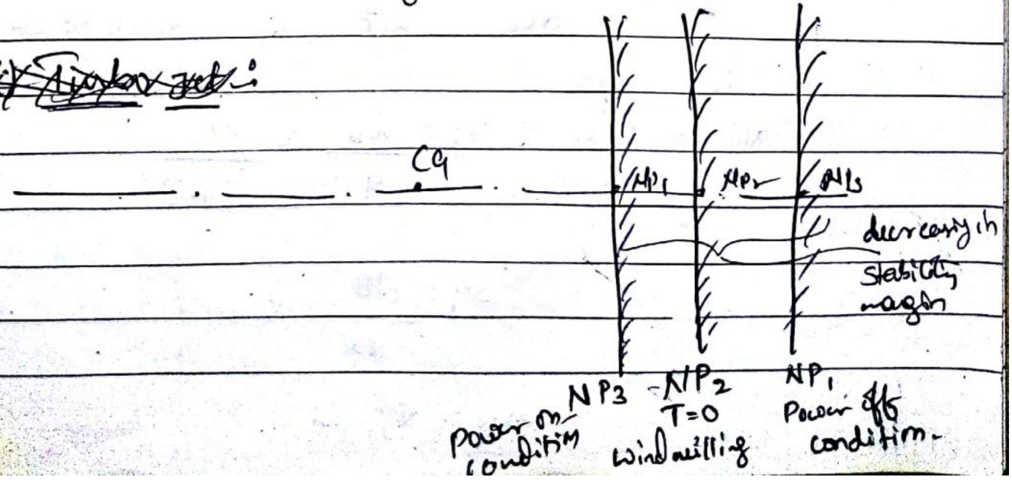
$$C_{m_t} = -V_H C_{L_{\alpha_t}}(\alpha_w - i_w + i_t - \varepsilon_w - \varepsilon_p)\left(\frac{V_s}{V}\right)^2$$

$$\therefore \left(\frac{dC_m}{dC_L}\right)_t = \underbrace{-\frac{a_t}{a_w}V_H\left(1 - \frac{d\varepsilon_w}{d\alpha} - \frac{d\varepsilon_p}{d\alpha}\right)\left(\frac{V_s}{V}\right)^2}_{①} \underbrace{- V_H C_{L_t}\frac{d\left(\frac{V_s}{V}\right)^2}{dC_L}}_{②}$$

here $\left(\frac{dC_m}{dC_L}\right)_t$ = Total contribution of tail to pitch stability including propeller indirect effect.

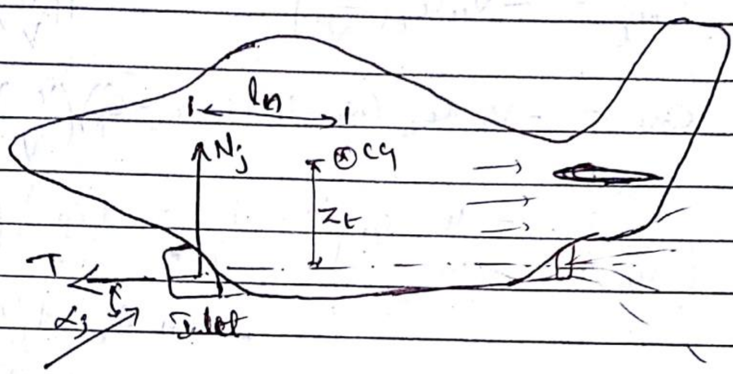
by looking at above equation ①, effect of downwash introduced by propeller slipstream at the tail is destabilizing.
and also equation ② is to make up for the stability change when C.G. having backward or forward.

(ii) ~~Jet engines~~:



(ii) Turbo jet : or power Effect on Jet Airplane pitch stability.

here considering Thrust constant with speed of Jet Airplane.




Direct Effects :

$$M_{cg} = T Z_t$$

$$C_{m_{cg}} = \frac{T Z_t}{S_w \bar{c} Q}$$

$L = W = Q S_w C_L$

$\therefore Q = (W/S_w) \, 1/C_L$

$$\therefore C_{m_{cg}} = \frac{T Z_t C_L}{W \bar{c}}$$

$$\therefore \boxed{\frac{dC_m}{dC_L} = \frac{T Z_t}{W \bar{c}}}$$

The sign of $Z_t$ will decide the stability in pitch.

Normal force $N_j = \frac{W_a}{g} V \frac{\alpha_j^{\circ}}{57.3}$

$W_a$ = weight of air mass flow rate lb/sec

$$\alpha_j = \frac{d\beta}{d\alpha} \cdot \alpha$$

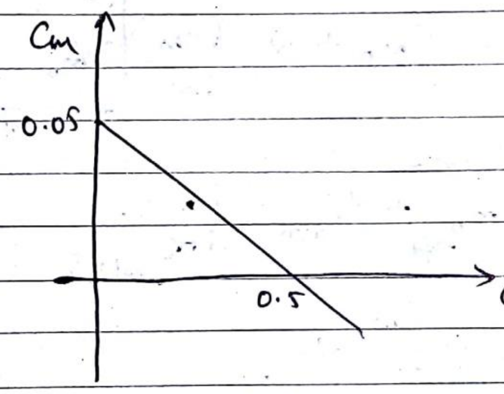
$$\therefore M_{cg} = N_j \cdot l_n$$

$$\therefore \quad C_{m_j} = \frac{2 W_a \left(\frac{d\beta}{d\alpha}\right) l_h \cdot \alpha}{57.3\, g\, \bar{c}\, \rho S_w V}$$

$$\left(\frac{dC_m}{dC_L}\right)_{N_j} = \frac{W_a}{g} \frac{(d\beta/d\alpha)}{a_{\alpha_w}} \frac{l_h}{\bar{c}} \frac{0.035}{\rho S_w V}$$

Sign of '$l_h$' will decide the stability.

## XIV. Longitudinal control:




consider a $C_m$ vs $C_L$ curve and for longitudinal stable, considering negative slope curve. and for reference we will give some values say $C_m = 0.05$ and $C_L = 0.5$.

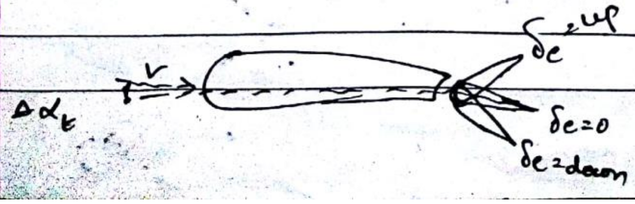
over the $C_L$ axis the airplane's curve is passing at 0.5 so. that point we can call it as trim $C_L$ value. because trim condition happen when $C_m = 0$.

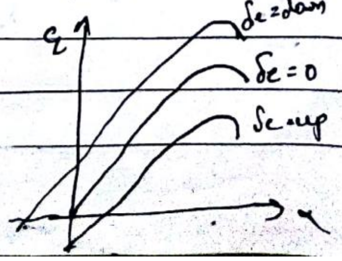
$\therefore C_{L_{trim}} = 0.5$ ~~here i.e.~~ where ~~slope~~ $C_m = 0$.

for here aircraft is flying at constant altitude with speed const.

$L = W = \frac{1}{2} \rho V_{trim}^2 C_{L_{trim}} S$

Now, if aircraft wants to change its velocity, then ultimately it has to make proper $C_{L_{trim}}$ value. this is done by flaps.



for the aircraft ~~Trim~~ Trim condition,

$L = W$

$C_{m_{cg}} = 0$

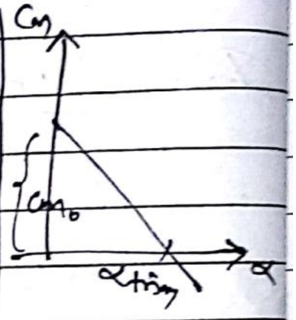
$T = D$.

$\therefore \Delta\alpha_t = \tau \delta_e$ here $\tau$ = Elevator / flap effectiveness parameter

for the longitudinal static stability, we know that

$$C_{m_{cg}} = C_{m_0} + C_{m_\alpha} \cdot \alpha$$

$$C_{m_{cg}} = C_{m_0} + \alpha \left(\frac{dC_m}{d\alpha}\right)$$



To change the $\alpha_{trim}$,

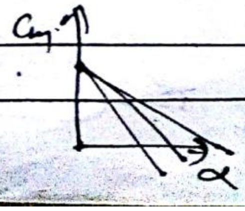
$$0 = C_{m_{0_{wft}}} + \alpha_{trim} \left(\frac{dC_m}{d\alpha}\right)_{wft}$$

$$\therefore \alpha_{trim} = -\frac{C_{m_{0_{wft}}}}{\left(\frac{dC_m}{d\alpha}\right)_{wft}}$$

"So, from above equation, by changing wing camber, we can change the $C_{m_{0_{wft}}}$, by that we can change the $\alpha_{trim}$".

$\Rightarrow$ stability change through the changed downwash at tail.

Again from above equation, change C.G in flight, i.e, $\left(\frac{dC_m}{d\alpha}\right)$ making variation.



∴ control of an airplane can be achieved by providing an incremental lift force on one or more of the airplane's lifting surfaces. The lifting force can be produced by deflecting the entire lifting surface or by deflecting a flap incorporated in the lifting surface. Because the control flaps or movable lifting surfaces are located at some distance from the center of gravity, the incremental lifting force can be created a moment about the airplane's center of gravity.

Pitch control can be achieved by changing the lift on either a forward or aft control surface. If a flap is used; the flapped portion of the tail surface is called an elevator.

Factors affecting the design of a control surface are control effectiveness, hinge moments, and aerodynamic and mass balancing.

control effectiveness is a measure of how effective the control deflection is in producing the desired control moment. control effectiveness is a function of the size of the flap and tail volume ratio.

Hinge moments also are important because they are the aerodynamic moments that must be overcome to rotate the control surface. The hinge moment governs the magnitude of force required of the pilot to move the control surface.

Finally, aerodynamic and mass balancing deal with techniques to vary the hinge moments so that the control stick forces stay within an acceptable range.

## Elevator Effectiveness: (Elevator control power derivative)

The pitch attitude can be controlled by either an aft tail or forward tail (canard). Now, examine how an elevator on an aft tail provides the required control moments.

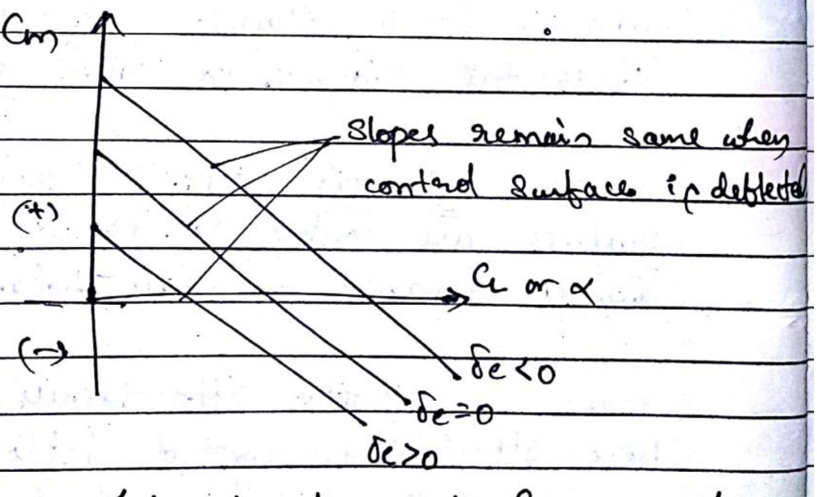



fig: Influence of the elevator on the $C_m$ versus $\alpha$ curve

from above figure, the elevator does not change the slope of the pitching moment curves but only shifts them so that different trim angles can be achieved.

when the elevator is deflected, it changes the lift and pitching moment of the airplane. The change in lift for the airplane can be expressed as follows:

$$\Delta C_L = C_{L_{\delta_e}} \cdot \delta_e$$

$$\text{where } C_{L_{\delta_e}} = \frac{dC_L}{d\delta_e}$$

$\therefore$ total lift: $C_L = C_{L_\alpha}\alpha + C_{L_{\delta_e}} \cdot \delta_e$

On the other hand, the change in pitching moment acting on the airplane can be written as,

$$\Delta C_m = C_{m_{\delta_e}} \cdot \delta_e \qquad \text{where, } C_{m_{\delta_e}} = \frac{dC_m}{d\delta_e}$$

The stability derivative $C_{m_{\delta_e}}$ is called the elevator control power. The larger the value of $C_{m_{\delta_e}}$, the more effective the control is in creating the control moment.

Adding $\Delta C_m$ to the pitching moment yields,

$$C_m = C_{m_0} + C_{m_\alpha}\alpha + C_{m_{\delta_e}} \cdot \delta_e .$$

The derivatives $C_{L_{\delta_e}}$ and $C_{m_{\delta_e}}$ can be related to the aerodynamic and geometric characteristics of the horizontal tail in the following manner.
The change in lift of the airplane due to deflecting the elevator is equal to the change in lift force acting on the tail:

$$\therefore \Delta L = \Delta L_t$$

$$\therefore \Delta Q S_w C_L = \Delta Q S_t C_{L_t}$$

$$\Delta C_L = \frac{S_t}{S_w} \eta \Delta C_{L_t}$$

$$\Delta C_L = \frac{S_t}{S_w} \eta \frac{dC_{L_t}}{d\delta_e} \cdot \delta_e$$

where $\frac{dC_{L_t}}{d\delta_e}$ is the elevator effectiveness. The elevator effectiveness is proportional to the size of the flap being used as an elevator and can be estimated from the equation,

$$\frac{dC_{L_t}}{d\delta_e} = \frac{dC_{L_t}}{d\alpha_t} \cdot \frac{d\alpha_t}{d\delta_e}$$

$$\frac{dC_{L_t}}{d\delta_e} = C_{L_{\alpha_t}} \cdot \tau$$

The parameter $\tau$ can be determined from below figure.

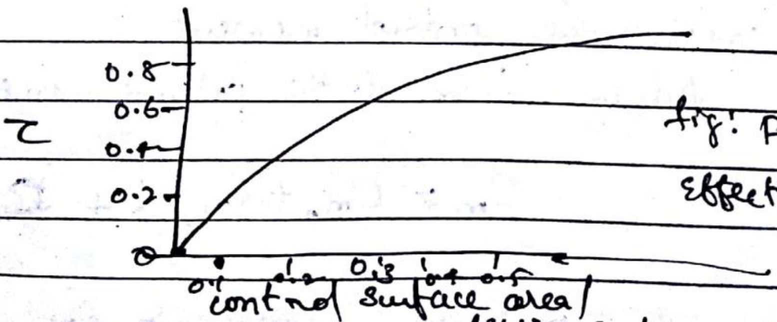



fig: Flap Effective parameter

$$\therefore C_{L_{\delta_e}} = \frac{S_t}{S_w} \eta \frac{dC_{L_t}}{d\delta_e}$$

The increment in airplane pitching moment is,

$$\Delta C_m = -V_H \eta \Delta C_{L_t}$$

$$\Delta C_m = -V_H \eta \frac{dC_{L_t}}{d\delta_e} \cdot \delta_e$$

$$\therefore \quad C_{m_{\delta_e}} \cdot \delta_e = -V_H \eta \frac{dC_{L_t}}{d\delta_e} \cdot \delta_e$$

$$C_{m_{\delta_e}} = -V_H \eta \frac{dC_{L_t}}{d\delta_e}$$

$$\therefore \quad \boxed{C_{m_{\delta_e}} = -V_H \eta C_{L_{\alpha_t}} \cdot \tau}$$

The designer can control the magnitude of the elevator control effectiveness by proper selection of the volume ratio and flap size.

## Elevator Angle to Trim:

An airplane is said to be trimmed if the forces and moments acting on the airplane are in equilibrium. Setting the pitching moment equation equal to 0 (the definition of trim) we can solve for the elevator angle required to trim the airplane:

$$C_m = 0 = C_{m_0} + C_{m_\alpha} \cdot \alpha + C_{m_{\delta_e}} \cdot \delta_e$$

$$\Rightarrow \quad \delta_{trim} = \frac{-(C_{m_0} + C_{m_\alpha} \alpha_{trim})}{C_{m_{\delta_e}}}$$

The lift coefficient to trim is,

$$C_{L_{trim}} = C_{L_\alpha}\alpha_{trim} + C_{L_{\delta_e}}\cdot\delta_{trim}$$

we can use this equation to obtain the trim angle of attack:

$$\alpha_{trim} = \frac{C_{L_{trim}} - C_{L_{\delta_e}}\cdot\delta_{trim}}{C_{L_\alpha}}$$

∴ If we substitute this equation back into $\delta_{trim}$ Equation, we get the following equation for the elevator angle to trim:

$$\delta_{trim} = -\frac{C_{m_0}C_{L_\alpha} + C_{m_\alpha}C_{L_{trim}}}{C_{m_{\delta_e}}C_{L_\alpha} - C_{m_\alpha}C_{L_{\delta_e}}}$$

~~Problems:~~

Elevator required for landing:

The longitudinal control Surface provides a moment that can be used to balance or trim the airplane at different operating angles of attack or lift coefficient.

The size of the control surface depends on the magnitude of the pitching moment that needs to be balanced by the control.

In general, the largest trim moments occur when an airplane is in the landing configuration.

(wing flaps and landing gear deployed) and the center of gravity is at its forwardmost location. This can be explained in the following manner.

In the landing configuration we fly the airplane at a high angle of attack or lift coefficient so that the airplane's approach speed can be kept as low as possible. Therefore the airplane must be trimmed at a high lift coefficient. Deployment of the wing flaps and landing gear create a nose-down pitching moment increment that must be added to the clean configuration pitching moment curve.

An additional nose-down or negative pitching moment increment due to the flaps and landing gear shifts the pitching moment curve.
As the center of gravity moves forward the slope of the pitching moment curve becomes more negative (the airplane is more stable). This results in a large trim moment at high lift coefficients. The largest pitching moment that must be balanced by ~~the center of gravity~~ The Elevator therefore occurs when the flaps and gear are deployed and the center of gravity is at its most forward position.

Problem;

1. Given general aviation airplane with the following configuration details:

Gross weight = 2750 kg,

Velocity = 170 m/sec.

$X_{cg} = 0.295\,\bar{c}$

Span = 33.4 m

$\bar{c}$ = 5.7 m

Tail area = 43 $m^2$

Tail chord = 16 m

$\eta = 0.8$

$S_e/S_t = 0.3$

$C_{L_{\alpha_t}} = 3.9$ /rad.

Assume the pitching moment curves for the landing configuration at the forward most C.G. position is given as $C_{m_{cg}} = -0.20 - 0.035\alpha$, where $\alpha$ is in degrees. Estimate the elevator effectiveness and the size of the elevator to trim the airplane at the landing angle of attack of 10°. Assume the elevator angle is constrained to +20° and −25°.

→ The increment in moment created by the control surface, $\Delta C_{m_{cg}}$, is both a function of the elevator control power $C_{m_{\delta_e}}$, and the elevator deflection angle $\delta_e$:

$$\Delta C_{m_{cg}} = C_{m_{\delta_e}} \delta_e.$$

for a 10° approach angle of attack, the pitching moment acting on the airplane can be estimated as follows:

$$\Delta C_{m_{cg}} = -0.20 - 0.035\,(10^\circ) = \underline{-0.55}$$

This moment must be balanced by an equal and opposite moment created by deflecting the elevator. The change in moment coefficient created by the elevator was shown to be

$$\Delta C_{m_{cg}} = C_{m_{\delta e}} \cdot \delta e$$

where $C_{m_{\delta e}}$ is referred as the elevator control power. The elevator control power is a function of the horizontal tail volume ratio, $V_H$, and the flap effectiveness factor $\tau$,

$$C_{m_{\delta e}} = -V_H \eta \tau C_{L_{\alpha t}}$$

The horizontal tail volume ratio $V_H$ is set by the static longitudinal stability requirements. Therefore the designer can change only the flap effectiveness parameter, $\tau$ to achieve the appropriate control effectiveness $C_{m_{\delta e}}$. The flap effectiveness factor is a function of the area of the control flap to the total area ~~of~~ of the lift surface on which it is attached. By proper selection of the elevator area the ~~ness~~ necessary control power can be achieved.

For positive moment, the control deflection angle must be negative; that is trailing edge of the elevator is up.

$$\Delta C^{(+)}_{m_{trim}} = C^{(-)}_{m_{\delta e}} \delta e^{(-)}$$

$$\therefore \quad C_{m_{\delta e}} = \frac{\Delta C_{m_{trim}}}{\delta e} = \frac{0.55}{-25^\circ}$$

$$C_{m_{\delta e}} = -0.022/\deg$$

Solving for the flap effectiveness parameter $\tau$,

$$\tau = -\frac{C_{m_{\delta e}}}{V_H \eta C_{L_{\alpha t}}}$$

~~Using the values of $V_H$, $\eta$, $C_{L_{\alpha t}}$ from~~

$$\therefore \text{ here } V_H = \frac{S_t l_t}{S_w \bar{c}}$$

$$V_H = \frac{43 \times 16}{(33.4 \times 5.7)(5.7)} = 0.634$$

$$\therefore \tau = -\frac{0.022}{(0.634)(0.8)(3.9 \times \pi/180)}$$

~~$\tau = 0.000194$~~

$$\boxed{\tau = 0.6372}$$

$$\therefore \frac{S_e}{S_t} = 0.30$$

$$\therefore S_e = 0.30\, S_t$$

$$S_e = (0.3)(43)$$

$$\boxed{S_e = 13\ m^2}$$