Module - 2

Static Longitudinal Stability & Static Directional Stability and Control - Stick free

I. Elevator Hinge moment :

It is important to know the moment acting at the hinge line of the elevator (or other type of control surface). The hinge moment, of course, is the moment the pilot must overcome by exerting a force on the control stick. Therefore to design the control system properly we must know the hinge moment characteristics. The hinge moment is defined as shown in below figure.

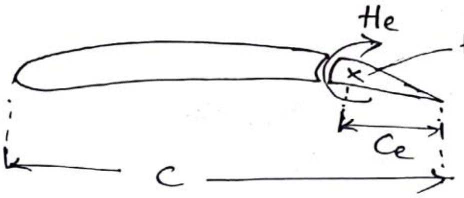


fig: Definition of hinge moment.

$H_e = C_{h_e} \frac{1}{2} \rho V^2 S_e C_e$

S_e = Area aft of the hinge line

C_e = Chord measured from hinge line to trailing edge of the flap

If we assume that the hinge moment can be expressed as the addition of the effects of angle of attack, Elevator deflection angle, and tab angle taken separately, then we can express the hinge moment coefficient in the following manner,

$$C_{h_e} = C_{h_0} + C_{h_{\alpha_t}} \cdot \alpha_t + C_{h_{\delta_e}} \cdot \delta_e + C_{h_{\delta_t}} \cdot \delta_t$$

where C_{h_0} is the residual moment and

$$C_{h_{\alpha_t}} = \frac{dC_h}{d\alpha_t} \quad , \quad C_{h_{\delta_e}} = \frac{dC_h}{d\delta_e} \quad , \quad C_{h_{\delta_t}} = \frac{dC_h}{d\delta_{e_t}}$$

The hinge moment parameters just defined are very difficult to predict analytically with great precision. Wind-tunnel tests usually are required to provide the control system designer with the information needed to design the control system properly.

When elevator is set free, that is, the control stick is released, the stability and control characteristics of the airplane are affected. For simplicity, we shall assume that both $\boxed{\delta_t \text{ and } C_{h_0} \text{ are Equal to } 0.}$ Then, for the case when the Elevator is allowed to be free,

$$C_{h_e} = 0 = C_{h_{\alpha_t}} \cdot \alpha_t + C_{h_{\delta_e}} \cdot \delta_e.$$

$\therefore$ Solving for δ_e yields,

$$(\delta_e)_{free} = -\frac{C_{h_{\alpha_t}}}{C_{h_{\delta_e}}} \cdot \alpha_t.$$

Usually, the coefficients $C_{h_{\alpha_t}}$ and $C_{h_{\delta_e}}$ are negative. If this indeed is the case, then above Equation for $(\delta_e)_{free}$ tells us that the elevator will float upwards as the angle of attack is increased.

The lift coefficient for tail with a free elevator is given by

$$C_{L_t} = C_{L_{\alpha_t}} \cdot \alpha_t + C_{L_{\delta_e}} \cdot \delta_e.$$

$$\therefore \quad C_{L_t} = C_{L_{\alpha_t}} \cdot \alpha_t - C_{L_{\delta_e}} \cdot \frac{C_{h_{\alpha_t}}}{C_{h_{\delta_e}}} \cdot \alpha_t.$$

which simplifies to,

$$C_{L_t} = C_{L_{\alpha_t}} \alpha_t \left(1 - \frac{C_{L_{\delta_e}}}{C_{L_{\alpha_t}}} \cdot \frac{C_{h_{\alpha_t}}}{C_{h_{\delta_e}}}\right)$$

$$\therefore \quad C_{L_t} = C'_{L_{\alpha_t}} \cdot \alpha_t$$

where, $C'_{L_{\alpha_t}} = C_{L_{\alpha_t}}\left(1 - \frac{C_{L_{\delta_e}}}{C_{L_{\alpha_t}}}\frac{C_{h_{\alpha_t}}}{C_{h_{\delta_e}}}\right)$

$$\therefore \quad C'_{L_{\alpha_t}} = C_{L_{\alpha_t}} f .$$

The slope of the tail lift curve is modified by the term in the ~~paran~~ parentheses. The factor 'f' can be greater or less than unity, depending on the sign of the hinge parameters ($C_{h_{\alpha_t}}$ and $C_{h_{\delta_e}}$).

Now, if we were to develop the equations for the total pitching moment for the freeelevator case, by taking into account of wing, fuselage & tail planes contribution. The only difference would be that the term $C_{L_{\alpha_t}}$ would be replaced by $C'_{L_{\alpha_t}}$.

$$\therefore \quad C'_{m_0} = C_{m_{0_w}} + C_{m_{0_f}} + C'_{L_{\alpha_t}} \eta . V_H (\varepsilon_0 + i_w - i_t)$$

$$C'_{m_\alpha} = C_{L_{\alpha_w}}\left(\frac{X_{cg} - X_{ac}}{\bar{c}}\right) + C_{m_{\alpha_f}} - C'_{L_{\alpha_t}} \eta V_H \left(1 - \frac{d\varepsilon}{d\alpha}\right)$$

where the prime indicates elevator-free values. To determine the influence of a free elevator on the static longitudinal stability, we again examine the condition in which $C_{m_\alpha} = 0$. Setting $\boxed{C'_{m_\alpha} \text{ Equal to } 0}$ in above Equation and solving for ~~X_{NP}~~ $\frac{X_{NP}}{\bar{c}}$ yields to the stick-free neutral point.

$$\therefore \quad 0 = C_{L_{\alpha_w}}\left(\frac{X'_{NP}}{\bar{c}} - \frac{X_{ac}}{\bar{c}}\right) + C_{m_{\alpha_f}} - C'_{L_{\alpha_t}} \eta V_H \left(1 - \frac{d\varepsilon}{d\alpha}\right)$$

here $X'_{NP} = X_{cg}$ because $C_{m_\alpha} = 0$

$$\therefore \quad \frac{X'_{NP}}{\bar{c}} = \frac{X_{ac}}{\bar{c}} + V_H \eta \frac{C'_{L_{\alpha_t}}}{C_{L_{\alpha_w}}}\left(1 - \frac{d\varepsilon}{d\alpha}\right) - \frac{C_{m_{\alpha_f}}}{C_{L_{\alpha_w}}}$$

The difference between the stick fixed neutral point and the stick free neutral point can be expressed as follows:

$$\frac{X_{NP}}{\bar{c}} - \frac{X'_{NP}}{\bar{c}} = (1-f)\, V_H \eta \left(\frac{C_{L_{\alpha t}}}{C_{L_{\alpha w}}}\right)\left(1-\frac{d\varepsilon}{d\alpha}\right)$$

The factor determines whether the stick free neutral point lies forward or aft of the stick fixed neutral point.

Static margin is a term that appears frequently in the literature. The static margin is simply the distance between the neutral point and the actual center of gravity position.

$$\text{stick fixed static margin} = \frac{X_{NP}}{\bar{c}} - \frac{X_{cg}}{\bar{c}}$$

$$\text{stick free static margin} = \frac{X'_{NP}}{\bar{c}} - \frac{X_{cg}}{\bar{c}}$$

For most aircraft designs, it is desirable to have a stick fixed static margin of approximately 5 percent of the mean chord. The stick fixed or stick-free static neutral points represent an aft limit on the center of gravity travel for the airplane.

II Stick forces

To deflect a control surface the pilot must move the control stick or rudder pedals. The forces exerted by the pilot to move the control surface is called the stick force or pedal force, depending which control is being used.

The stick force is proportional to the hinge moment acting on the control surface:

$$F = f_n(H_e)$$

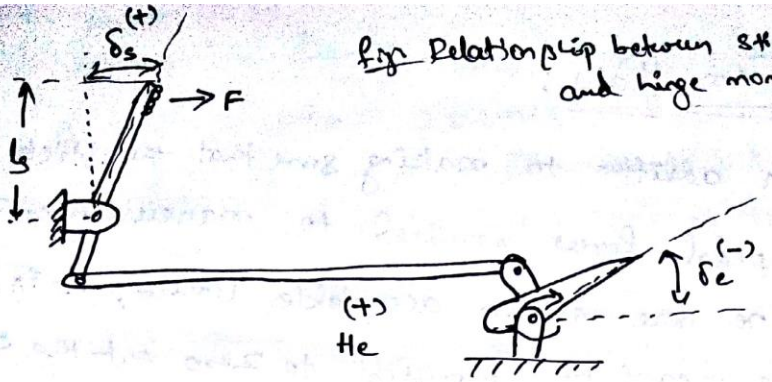


fig: Relationship between stick force and hinge moment.

above figure is a sketch of a simple ~~mech~~ mechanical system used for deflecting the elevator. ~~The work of~~ displacing which control ~~is being used~~. ~~The stick force~~.

The work of displacing the control stick is equal to the work in moving the control surface of the desired deflection angle. From above figure we can write the expression for the work performed at the stick and elevator:

$$F l_s \delta_s = H_e \delta_e$$

$$\Rightarrow F = \frac{\delta_e}{l_s \delta_s} . H_e.$$

$$\Rightarrow F = G H_e.$$

where $G = \frac{\delta_e}{l_s \cdot \delta_s}$ called the gearing ratio, is a measure of the mechanical advantage provided by the control system.

Substituting the expression for the hinge moment defined earlier into the stick force equation yields,

$$F = G C_{h_e} \tfrac{1}{2} \rho V^2 S_e \bar{c}_e$$

From this expression we see that the magnitude of the stick force increases with the size of the airplane and the square of the airplane's speed. Similar expressions can be obtained for the rudder peldal force and aileron stick force.

The convention for longitudinal control is that a pull force should always rotate the nose upward, which causes the airplane to slow down. A push down will have ~~speed~~ up the opposite effect. i.e, nose down leads to speed up.

(i) Trim Tabs:

In addition to making sure that the stick and rudder pedal forces required to maneuver or trim the airplane are within acceptable limits, it is important that some means be provided to zero out the stick force at the trimming flight speed. If such a provision is not made, the pilot will become fatigued by trying to maintain the necessary stick force.

The stick force at trim can be made zero by incorporating a tab on either the elevator or the rudder. The tab is a small flap located at the trailing edge of the control surface. The trim tab can be used to zero out the hinge moment and thereby eliminate the stick or pedal forces.

Below figure illustrates the concept of a trim tab. Although the trim tab has a great influence over the hinge moment, it has only a slight effect on the lift produced by the control surface.

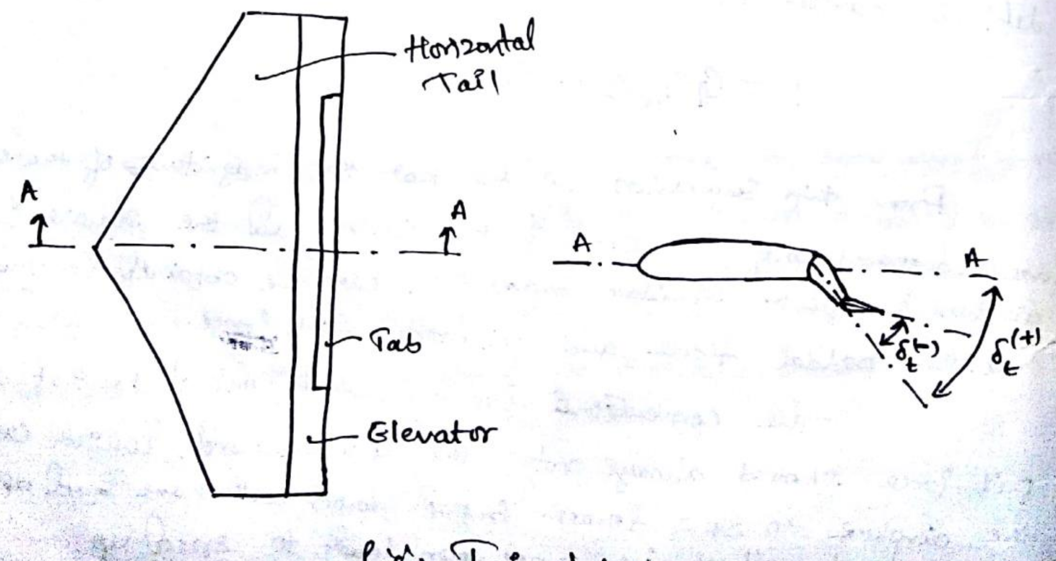


Fig. Trim tabs.

(ii) Stick force gradients:

Another important parameter in the design of a control system is the stick force gradient. Below figure shows the variation of the stick force with speed.

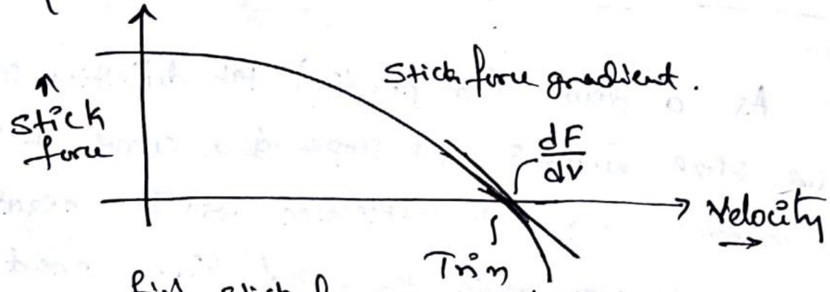


fig. stick force versus velocity.

The stick force gradient is a measure of the change in stick force needed to change the speed of the airplane. To provide the airplane with speed stability, the stick force gradient must be negative; that is $\boxed{\frac{dF}{dV} < 0}$

The need for a negative stick-force gradient can be appreciated by examining the trim point in above figure. If the airplane slows down, a positive stick force occurs that rotates the nose of the airplane downward, which causes the airplane to increase its speed back toward the trim velocity.

If the airplane exceeds the trim velocity, a negative (pull) stick force causes the airplane's nose to pitch up, which causes the airplane to slow down. The negative stick force gradient provides the pilot and airplane with speed stability.

The larger the gradient, the more resistant the airplane will be to disturbances in the flight speed. If an airplane did not have speed stability the pilot would have to continuously monitor and control the airplane's speed. This would be highly ~~undesi~~ undesirable from the pilot's point of view.

III. Coefficient of hinge moment due to tab deflection

As a down (or positive) tab deflection creates a negative hinge moment, the slope of a curve of control hinge moment versus tab deflection will be negative. Assuming linearity again, the control hinge moment coefficient can be written as,

$$C_{h_e} = C_{h_0} + \left(\frac{dC_h}{d\alpha_t}\right) \alpha_t + \left(\frac{dC_h}{d\delta_e}\right) \cdot \delta_e + \left(\frac{dC_h}{d\delta_t}\right) \cdot \delta_t$$

or, in short hand notation,

$$C_{h_e} = C_{h_0} + C_{h_{\alpha_t}} \cdot \alpha_t + C_{h_{\delta_e}} \delta_e + C_{h_{\delta_t}} \cdot \delta_t .$$

The control floating angle will now become,

$$\delta_e = \delta_{Flat} = -\frac{C_{h_{\alpha_t}}}{C_{h_{\delta_e}}} \cdot \alpha_t - \frac{C_{h_{\delta_t}}}{C_{h_{\delta_e}}} \cdot \delta_t .$$

here $C_{h_0} = C_{h_i} = 0$ = trim condition.

From δ_{float} indicates again that deflecting the tab can change the control surface floating angle.

In estimating the parameter $C_{h_{\delta_t}}$ tab hinge moment, it is convenient to obtain the two dimensional slope parameter $C_{h_{\delta_t}}$ and then correct for the three dimensional case. The section parameter $C_{h_{\delta_t}}$ depends on the airfoil section, the flap to airfoil chord ratio, c_f/c and the tab to airfoil chord ratio c_t/c

The three dimensional hinge moment slope. $C_{h_{\delta_t}}$ can be estimated, first assuming the tab is a full span flap from the following formula,

$$C_{h_{\delta_t}} = c_{h_{\delta_t}} - c_{h_\alpha} \tau_t \left(1 - \frac{a}{a_0}\right)$$

$\because C_{h_{\delta_t}} \Rightarrow$ for 3D
$c_{h_{\delta_t}} \Rightarrow$ 2D
$c_{h_\alpha} \Rightarrow$ 2D

where τ_t is the effectiveness of the tab. i.e.
a & $a_0 \Rightarrow$ lift variations with α_t & δ_t.

As in most cases, the tab is only a partial span flap; a correction to above equation is required to allow for this. The simplest way to do this is to assume that the correction will be in proportion to the area moment ratios of the actual tab to the full span tab of equal chord ratio.
If the area moment ratio is given as K,

$$K = \frac{(\text{Area moment}) \text{ Partial span tab}}{(\text{Area moment}) \text{ Full span tab.}}$$

$$\therefore \boxed{C_{h_{\delta_t}} = c_{h_{\delta_t}} \cdot K - \tau_t c_{h_\alpha}\left(1 - \frac{a}{a_0}\right)}$$

IV. Restriction on aft C.G.

The concept of the stick-free neutral point, X'_{NP}, as that center of gravity location where the stability criterion $(dCm/dC_L)_{free}$ vanishes or where stick force versus velocity gradient is zero through a trim speed, brings new restriction on the aft limit to the allowable C.G range.

At the present writing the Army and Navy specifications call only for the most aft CG to be ahead of the stick free neutral points. It is obvious, however, that zero stick force gradients are undesirable, and any well-designed airplane should always try to maintain a stable stick force gradient even at the most aft C.G location. This is difficult to do on modern high-speed airplanes with large variations in C.G; however, by proper design and making use of aids such as the downspring, bobweight or Vee tab, adequate gradients can be obtained. No desirable minimum gradient is suggested herein, but it should be tied up rather closely with airplane type, control friction, etc.,

To point up the new aft limit and its further restriction on the C.G range, the CG limitations on a typical mean chord are shown in below figure

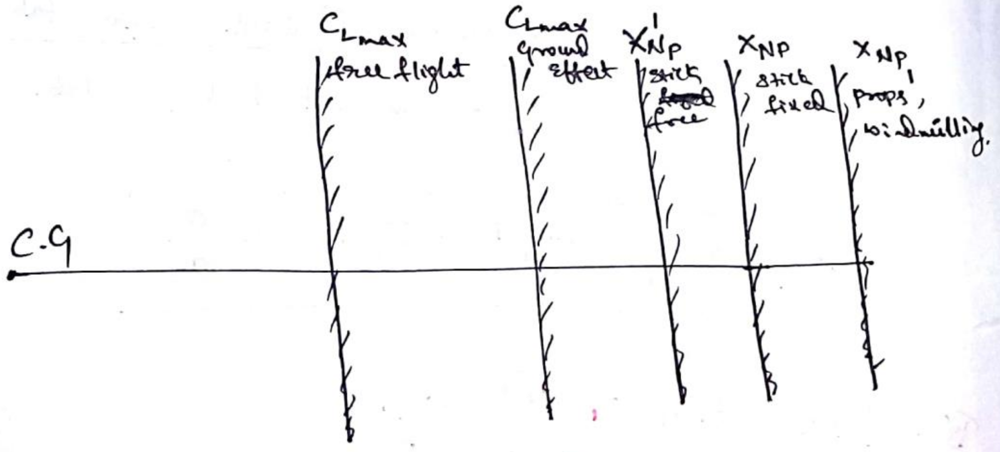


fig: Typical center of gravity limits.

IV. Defination of directional stability

Directional, or weathercock, stability is concerned with the static stability of the airplane about z axis. Just as in the case of longitudinal static stability, it is desirable that the airplane should tend to return to an equilibrium condition when subjected to some form of yawing disturbance

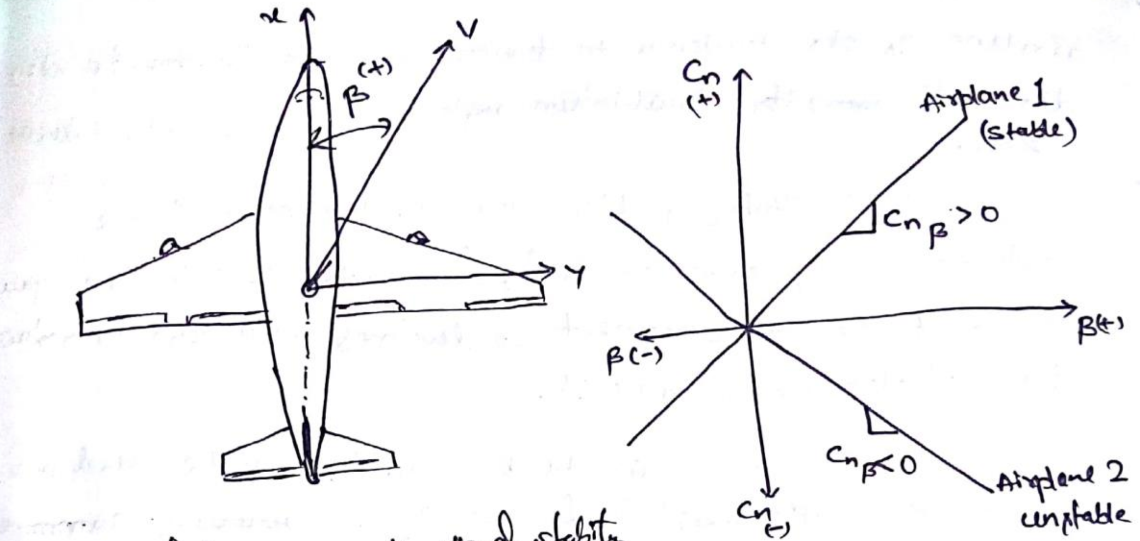


Fig: Static directional stability

from above figure, the yawing moment coefficient versus sideslip angle β for two airplane configurations.

To have static directional stability, the airplane must develop a yawing moment that will restore the airplane to its equilibrium state. Assume that both airplanes are disturbed from their equilibrium condition, so that the airplanes are flying with positive sideslip angle β. Airplane 1 will develop a restoring moment that will tend to rotate the airplane back to its equilibrium condition that is, a zero side slip angle. Airplane 2 will develop a yawing moment that will tend to increase the sideslip angle. Examining these curves, we see that to have static directional stability the slope of the yawing moment curve must be positive ($C_{n_\beta} > 0$). Note that an airplane possessing static directional stability will always point into the relative wind, hence the name weathercock stability.

VII Static Directional Stability - Rudder fixed, Contribution of aircraft components.

Just as in the longitudinal case, where the static stability is defined as the tendency of the airplane to return to a given equilibrium angle of attack or lift coefficient when disturbed, the static directional stability of the airplane is its tendency to develop restoring moments when disturbed from its equilibrium angle of sideslip, usually taken as zero.

The static problem, then, is the study of the airplane's yawing moments developed because of sideslip or yaw, to see if the yawing moment so developed will tend to reduce the sideslip or increase it.

The static directional stability can be developed as was the static longitudinal stability, by ~~assuming~~ summing up the stability contributions of the component parts of the airplane. Each of these components produces yawing moments when flying at angles of sideslip, and the study of the variation of the total yawing moment with angle of sideslip or angle of yaw gives the magnitude of the directional stability.

The contribution of the wing to directional stability usually is quite small in comparison to the fuselage, provided the angle of attack is not large. The fuselage and engine nacelles, in general, create a destabilizing contribution to directional stability. The wing-fuselage contribution can be calculated from the following empirical expression,

$$C_{n_{\beta_{wf}}} = -k_n k_{Rl} \frac{S_{fs} l_f}{S_w b} \quad \text{(per degree)}$$

where k_n = An empirical wing-body interference factor that is a function of the fuselage geometry.

k_{Rl} = An empirical correction factor that is a function of the fuselage Reynolds number.

S_{fs} = the projected side area of the fuselage.

l_f = the length of the fuselage.

b = the projected side span of the wing.

S_w = the projected side area of the wing.

Since the wing-fuselage contribution to directional stability is destabilizing, the vertical tail must be properly sized to ensure that the airplane has directional stability.

The mechanism by which the vertical tail produces directional stability is shown in below figure.

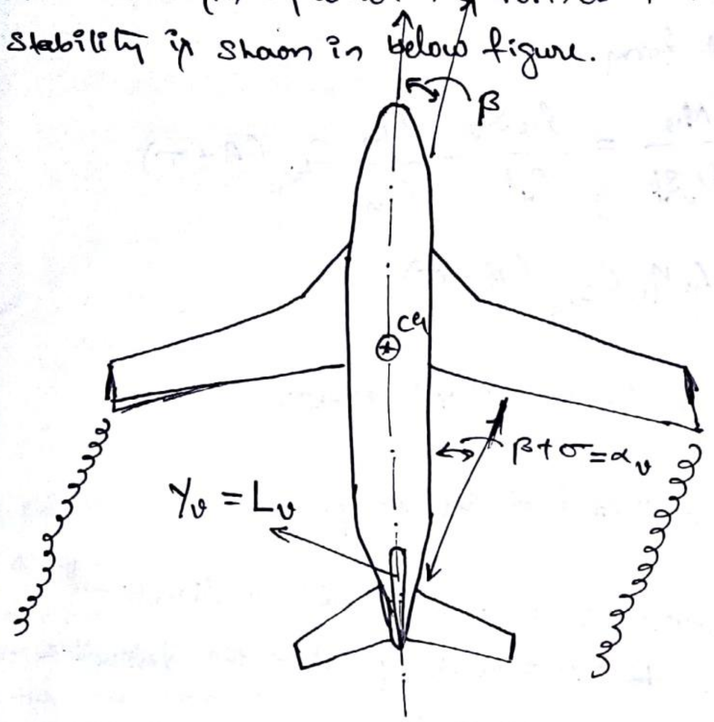


fig: Vertical tail contribution to directional stability

σ = side wash due to wing vortices

If we consider the vertical tail surface in above figure, we see that when the aircraft is flying at a positive sideslip angle the vertical tail produces a sideforce (lift force in xy plane) that tends to rotate the airplane about its center of gravity. The moment produced is restoring moment. The sideforce acting on the vertical tail can be expressed as

$$Y_v = -C_{L_{\alpha_v}} \alpha_v Q_v S_v$$

where the subscript v refers to properties of the vertical tail. The angle of attack α_v that the vertical tail plane will experience can be written as

$$\alpha_v = \beta + \sigma$$

where σ is the sidewash angle. The sidewash angle is analogous to the downwash angle ε for the horizontal tail plane. The sidewash caused by the flow field distortion due to the wing and fuselage.

The moment produced by the vertical tail can be written as a function of the side force acting on it:

$$N_v = l_v Y_v = l_v C_{L_{\alpha_v}} (\beta + \sigma) Q_v S_v$$

or in coefficient form

$$C_n = \frac{N_v}{Q_w S_w b} = \frac{l_v S_v}{S_w b} \frac{Q_v}{Q_w} C_{L_{\alpha_v}} (\beta + \sigma)$$

$$C_n = V_v \eta_v C_{L_{\alpha_v}} (\beta + \sigma)$$

where $V_v = \dfrac{l_v S_v}{S_w b}$ = Vertical tail volume ratio

$\eta_v = \dfrac{Q_v}{Q_w}$ = Vertical tail efficiency.

The contribution of the vertical tail to directional stability now can be ~~obtaind~~ obtained by taking the derivative of above equation with respect to β:

$$\boxed{C_{n_{\beta_v}} = V_v \eta_v C_{L_{\alpha_v}} \left(1 + \frac{d\sigma}{d\beta}\right)}$$

A simple algebraic equation for estimating the combined sidewash and tail efficiency factor η_v is reproduced here:

$$\eta_v \left(1 + \frac{d\sigma}{d\beta}\right) = 0.724 + 3.06 \frac{S_v/S_w}{1 + \cos \Lambda_{c/4w}} + 0.4 \frac{z_w}{d} + 0.009 AR_w$$

where,

S_w = the wing area

S_v = the vertical tail area, including submerged area to the fuselage centerline.

z_w = the distance, parallel to z axis, from wing root quarter chord point to fuselage centerline

d = The maximum fuselage length

AR_w = the aspect ratio of the wing

$\Lambda_{c/4w}$ = Sweep of wing quarter chord.

VI. Directional control

Directional control is achieved by a control surface called a rudder, located on the vertical tail, as shown in below figure.

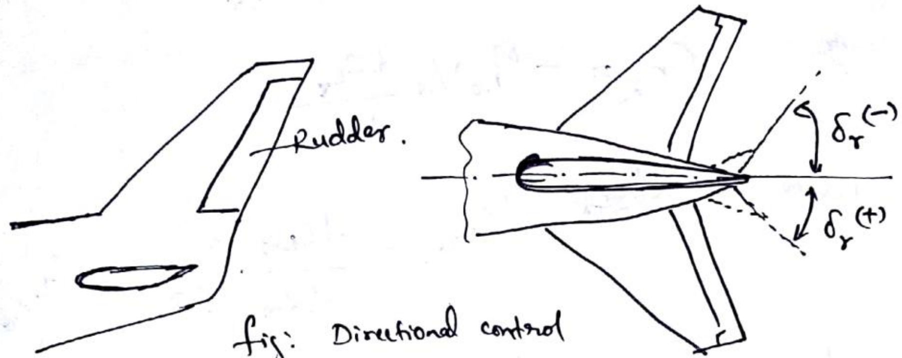


fig: Directional control by means of rudder.

The rudder is a hinged flap, the lift force (side force) on the fixed vertical surface can be varied to create a yawing moment about center of gravity. The size of the rudder is determined by the directional control requirements. The rudder control power must be sufficient to accomplish the requirements like adverse yaw, crosswind ~~long~~ landings, asymmetric ~~asymmetric~~ power condition and spin recovery.

The yawing moment produced by the rudder depends on the change in lift on the vertical tail due to the deflection on the rudder times its distance from the center of gravity. For a positive rudder deflection, a positive side force is created on the vertical tail. A positive side force will produce a negative yawing moment:

$$N_v = -l_v Y_v$$

where side force is given by

$$Y_v = C_{L_v} Q_v S_v$$

Rewriting this equation in terms of yawing moment coefficient yields

$$C_n = \frac{N}{Q_w S b} = -\frac{Q_v}{Q_w}\,\frac{l_v S_v}{S_w b}\,\frac{dC_{L_v}}{d\delta_r}\cdot\delta_r$$

$$\boxed{C_n = -\eta_v V_v \frac{dC_{L_v}}{d\delta_r}\cdot\delta_r}$$

The rudder control effectiveness is the rate of change of yawing moment with rudder deflection angle:

$$C_n = C_{n_{\delta_r}} \cdot \delta_r = -\eta_v V_v \frac{dC_{L_v}}{d\delta_r} \delta_r$$

or

$$C_{n_{\delta_r}} = -\eta_v V_v \frac{dC_{L_v}}{d\delta_r}$$

where

$$\frac{dC_{L_v}}{d\delta_r} = \frac{dC_{L_v}}{d\alpha_v} \cdot \frac{d\alpha_v}{d\delta_r} = C_{L_{\alpha_v}} \tau$$

here τ is rudder control effectiveness.

$$\therefore \boxed{C_{n_{\delta_r}} = -\eta_v V_v C_{L_{\alpha_v}} \tau}$$

Requirements for directional control:

(i) **Adverse Yaw:**

When an airplane is banked to execute a ~~banking~~ turning maneuver the ailerons may create a yawing moment that opposes the turn (i.e., adverse yaw). The rudder must be able to overcome the adverse yaw so that a coordinated turn can be achieved. The critical condition for a adverse yaw occurs when the airplane is flying slow. i.e., high C_L.

(ii) **Crosswind Landing:**

To maintain alignment with the runway during a cross wind landing requires the pilot to fly the airplane at a sideslip angle. The rudder must be powerful enough to permit the pilot to trim the airplane for the specified crosswinds. For transport airplanes, landing may be carried out for crosswind upto 15.5 m/s or 51 ft/s.

(iii) Asymmetric power condition:

The critical asymmetric power condition occurs for a multiengine airplane when one engine fails at low flight speeds. The rudder must be able to overcome the yawing moment produced by the asymmetric thrust arrangements.

(iv) Spin recovery:

The primary control for spin recovery in many airplanes is a powerful rudder. The rudder must be powerful enough to oppose the spin rotation.

(v) One Engine inoperative condition:

A more sever requirement on the rudder, for single-engine high performance airplanes, with single rotation propellers is the need for overcoming the effects of slipstream rotation. This condition is critical at very low airspeeds with high power, being usually more critical for Navy airplanes than for Air force airplanes because the carrier landing technique for Navy airplanes requires full stall landings with full flaps at high power.

The critical design condition for the rudder on multiengined airplanes is the low speed flight condition with full antisymmetric power. The Air force and Navy requirement is that the rudder should be powerful enough to hold zero sideslip with the most critical engine windmilling and all other engines delivering full power down to 1.2 times the airplane's stalling speed in the take off configuration. The yawing moment coefficient due to the asymmetric thrust can be given as follows:

$$C_{n_T} = \frac{375\, BHP\, \eta_p\, l_y}{V_{mph}\, q_w\, S_w\, b}$$

This is a cubic in V_{mph} with the yawing moment coefficient increasing inversely with V^3. A typical curve of C_{n_T} versus V_{mph} is shown in below figure

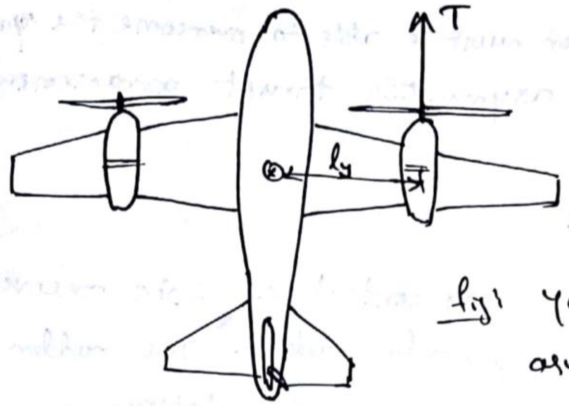


fig: yawing moment due to asymmetric power.

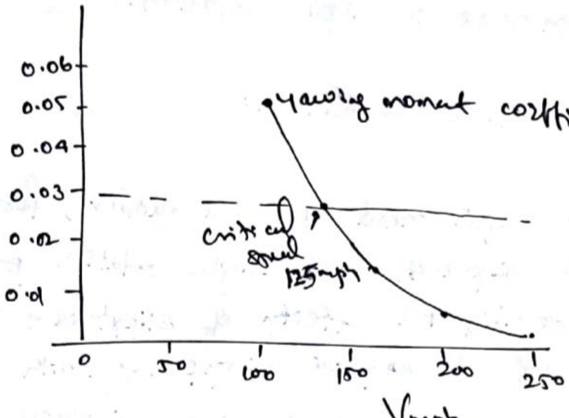


fig: Critical speed due to asymmetric power.

The rudder at full throw gives a constant yawing moment coefficient. The intersection of the two curves shown in above figure is the speed below which full rudder will not balance out the moment due to antisymmetric power.

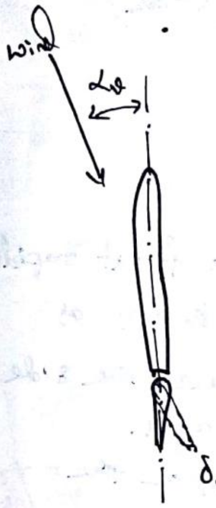
VIII. Stick free directional stability:

When the rudder ~~is held~~ is left free to float in response to its hinge moments, it can have large effects on the directional stability of the airplane, in the same manner that freeing the elevator was shown to have large effects on the longitudinal stability.

The floating angle of the rudder can be expressed analytically in terms of the two hinge moment coefficient parameters $C_{h_{\alpha v}}$ and $C_{h_{\delta r}}$

$$\therefore \delta_{r_{floating}} = -\frac{C_{h_{\alpha_v}}}{C_{h_{\delta_r}}} \cdot \alpha_v .$$

If the airplane sideslips, the restoring moment due to the tail will be decreased if the rudder floats with the wind and will be increased if the rudder floats against the wind. The floating rudder changes the effective angle of attack of the vertical tail,



$$\alpha_{v_{Effective}} = \beta + \tau \delta_{r_{float}} - \sigma$$

The restoring yawing moment coefficient developed because of the vertical tail will be,

$$C_n = -C_{L_{\alpha_v}}\left(\beta - \sigma + \tau \delta_{r_{float}}\right) \frac{S_v}{S_w} \frac{l_v}{b} \eta_v$$

∴ which upon substituting of equation of that,

fig. The floating rudder.

$$\therefore C_n = -C_{L_{\alpha_v}}\left(\beta - \sigma - \tau \frac{C_{h_{\alpha_v}}}{C_{h_{\delta_r}}} \beta\right) \frac{S_v}{S_w} \frac{l_v}{b} \eta_v$$

∴ The stability contribution of the vertical tail with a free rudder is therefore:

$$\left(C_{n_\beta}\right)_{v_{Free\ rudder}} = -C_{L_{\alpha_v}} \frac{S_v}{S_w} \frac{l_v}{b} \eta_v \left(1 - \frac{C_{h_{\alpha_v}}}{C_{h_{\delta_r}}} \tau\right) + \Delta_2 C_{n_\beta}$$

The Term $\Delta_2 C_{n_\beta}$ is that part of the contribution of the vertical tail surface to the airplane's directional stability that arises from the sidewash or interference flow from the wing fuselage combination.

(i) Rudder lock:

At high angles of side slip the well balanced rudder, which floats very slightly with the wing at low angles, will start to lose its aerodynamic balance and float over rapidly with increase in sideslip. This condition becomes accentuated at very high angles if the vertical tail commences to stall, which moves the center of pressure of the vertical tail well aft and causes the rudder to float well over. A typical curve of the floating angle of the rudder versus sideslip is shown in curve (a), for a closely balanced rudder.

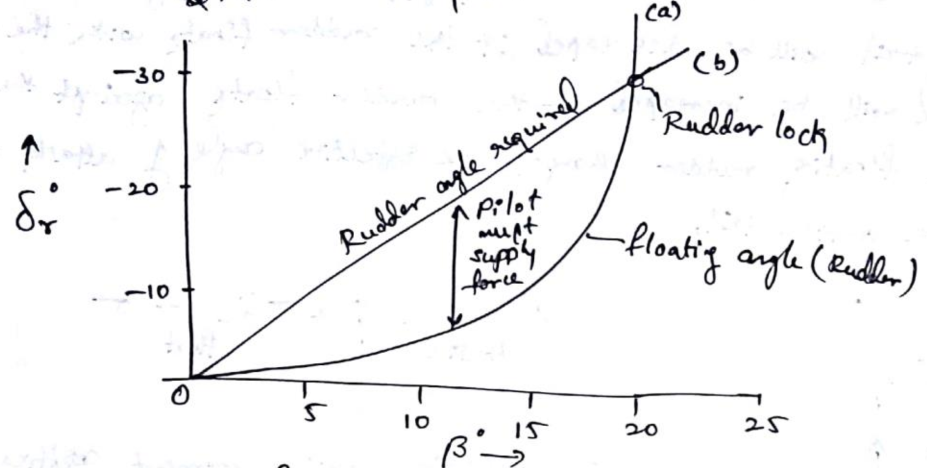
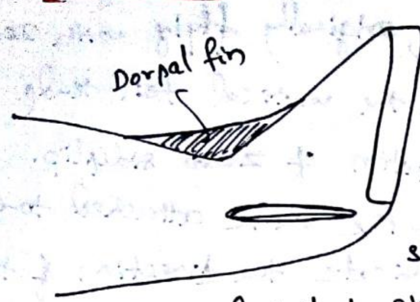


Fig: Rudder lock.

This tendency of the most rudders to float rapidly at high angle of sideslip leads to the phenomenon known as rudder lock. The rudder angle required to produce the side-slip varies somewhat linearly upto rather high angle. The pedal force required of the pilot is a function of the difference between the required rudder angle and the floating angle. At high angles of sideslip the floating angle may catch up with the required angle, at which point the pedal force will be zero. If the sideslip is increased beyond this point, the pedal forces will reverse and the rudder will continue to deflect upto its stops. Considerable force is required of the pilot to break the lock and restore the airplane to zero sideslip.

The difficulty lies in the fact that the requirements on the rudder make it possible to develop large angles of sideslip with full rudder deflection. One way out of the rudder lock problem is to cut down the rudder effectiveness, thereby increasing the rudder deflection required at a given sideslip angle. This artifice can be used on most present-day airplanes, as they usually have more than enough rudder control. For more closely designed rudders on airplanes which encounter rudder lock, the addition of a dorsal fin helps the situation a great deal.

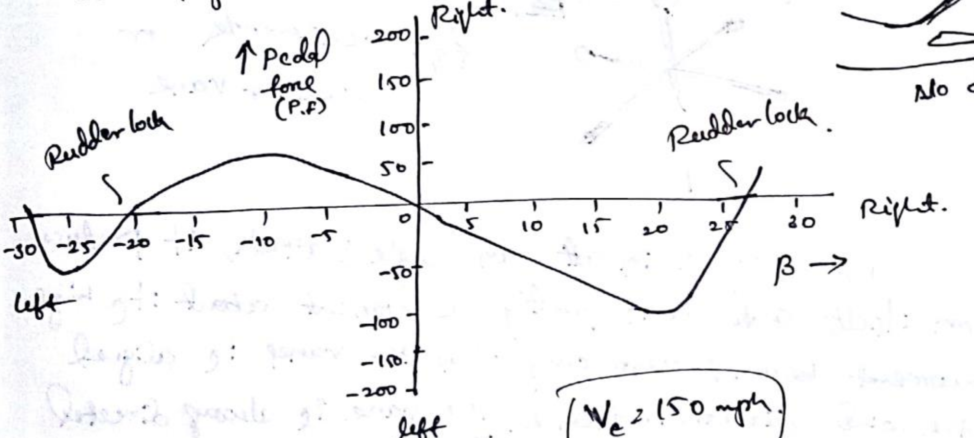
(ii) Dorsal fin:



The Dorsal fin is an auxiliary fin. The dorsal fin seems to do two things at once. One of these is to increase the fuselage stability at high angles of sideslip, and the second effect is to reduce the tendency of the vertical tail to stall.

The increase in directional stability at high sideslip angles will, of course, require more rudder angle for trim and thereby reduce the rudder lock possibilities.

A typical example of the effect of a dorsal fin on the pedal force versus side slip characteristics is shown in below figure.



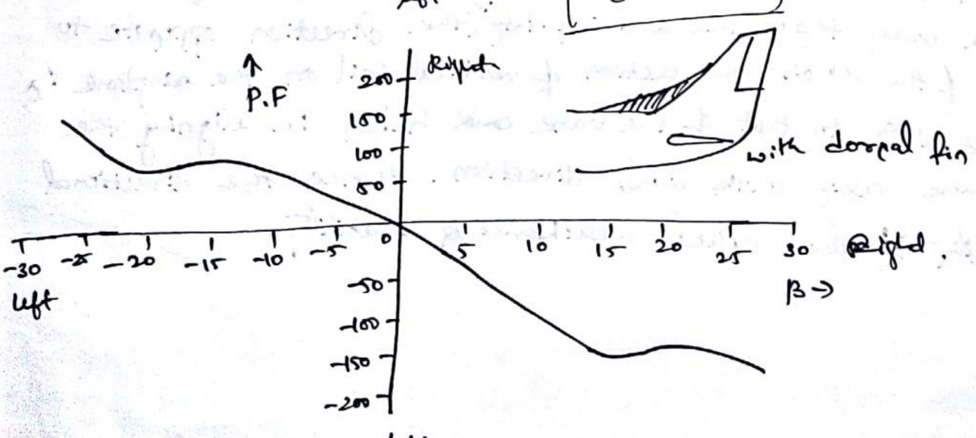


fig 1. Effect of dorsal fin on pedal force versus sideslip.

The dorsal fin has no effect on the force gradient through zero yaw, its effect being noticed only at large yaw angle.

IX. weather cocking Effect:

whenever the airplane, originally flying with zero sideslip, develops a sideslip (β), the vertical tail tends to bring it back to the original position of zero sideslip. This effect is similar to that of the vane attached to the weathercock which is used to indicate the direction of the wind and is located on top of buildings in ~~angle of attack,~~ ~~it produces lift on itself and consequently a moment about~~ meteorological departments and near airports.

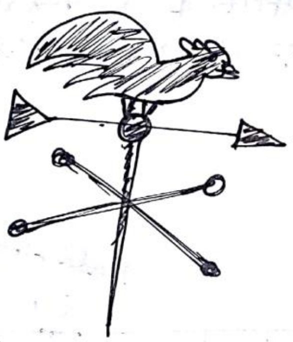


fig: weathercock or weather vane

when vane is at an angle of attack, it produces lift on itself and consequently a moment about its hinge. This moment becomes zero only when the vanes is aligned with the wind direction. Hence, the vane is always directed in a way that the arrow points in the direction opposite to that of the wind. The action of vertical tail on the airplane is also similar to that of the vane and helps in aligning the airplane axis with wind direction. Hence the directional stability ~~is~~ also called weathercock stability.