Module 1: Design for Static Strength

<u>July/August 2021</u>

1.a) Explain stress tensor and stress concentration factor. (05 Marks)

Stress is defined as force per unit area. If we take a cube of material and subject it to an arbitrary load we can measure the stress on it in various directions as shown in the figure.These measurements will form a second rank tensor; the stress tensor.

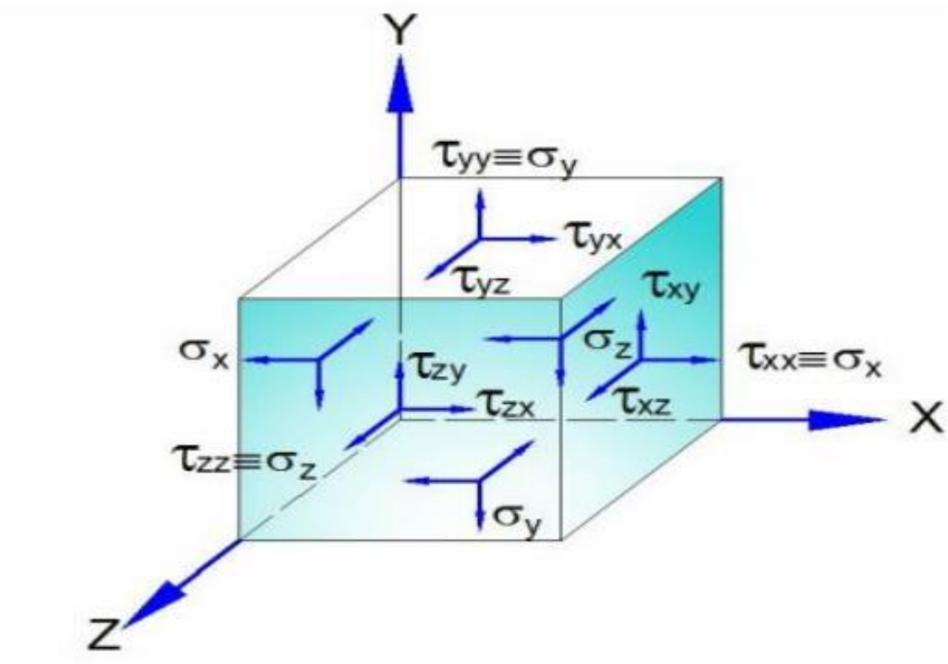


Figure 2.4 Stress components acting on parallelopiped

Figure 2.4 depicts three-orthogonal co-ordinate planes representing a parallelopiped on which are nine components of stress. Of these three are direct stresses and six are shear stresses. In tensor notation, these can be expressed by the tensor τ_{ij}, where $i = x, y, z$ and $j = x, y, z$.

Stress Tensor is expressed as,

$$\tau_{ij} = \begin{bmatrix} \sigma_x & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \sigma_y & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \sigma_z \end{bmatrix} \text{MPa}$$

Stress concentration factor (K_t)

It is defined as the ratio of maximum or significant stress at the discontinuity to the nominal Stress at minimum cross section.

$$K_t = \frac{\text{Highest value of actual stress at fillet, notch, hole etc.}}{\text{Nominal stress as given by elementary equation at minium cross section}}$$

Theoretical stress concentration factor is based on the geometrical shape of the discontinuity and the material is assumed to be elastic, isotropic and homogenous. Values of stress concentration factor can be found experimentally by Photo elastic analysis, Lasers, Holography, direct measurement using strain gauges.

1.b) Body subjected to tensile stress of 100MPa and 70MPa along two mutually perpendicular directions. The point is also subjected to shear stress of 50MPa. Determine:

- **i) Normal and shear stress at plane inclined at 120° with reference to 100MPa stress plane.**
- **ii) Principal and maximum, minimum shear stress**
- **iii) Principal plane orientation**
- **iv) Normal stress on planes of maximum and minimum shear stress.**

(15 Marks)

Data:

σ_x = 100 Mpa, σ_y = 70 Mpa, τ_{xy} = 50 Mpa, θ = 120°

Soln:

(i) Normal and shear stress at plane inclined at 120° with reference to 100MPa stress plane

Normal Stress on inclined plane,

$$\sigma_\theta = \frac{\sigma_x + \sigma_y}{2} + \left(\frac{\sigma_x - \sigma_y}{2}\right) \cos 2\theta + \tau_{xy} \sin 2\theta$$

$$\sigma_\theta = \frac{100 + 70}{2} + \left(\frac{100 - 70}{2}\right) \cos 2 \times 120 + 50 \sin 2 \times 120$$

$$\boldsymbol{\sigma_\theta = 34.2 \text{ MPa}}$$

Shear Stress on inclined plane,

$$\tau_\theta = -\left(\frac{\sigma_x - \sigma_y}{2}\right) \sin 2\theta + \tau_{xy} \cos 2\theta$$

$$\tau_\theta = -\left(\frac{100 - 70}{2}\right) \sin 2 \times 120 + 50 \cos 2 \times 120$$

$$\boldsymbol{\tau_\theta = -12 \text{ MPa}}$$

(ii) Principal and maximum, minimum shear stress

Maximum and Minimum Principal Stresses,

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$\sigma_{1,2} = \frac{100 + 70}{2} \pm \sqrt{\left(\frac{100 - 70}{2}\right)^2 + 50^2}$$

$$\sigma_{1,2} = 85 \pm 52.2$$

Maximum Principal Stress, $\boldsymbol{\sigma_1 = 137.2\ MPa}$

Minimum Principal Stress, $\boldsymbol{\sigma_2 = 32.8\ MPa}$

Maximum and Minimum Shear Stress,

$$\tau_{max,min} = \pm\sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + {\tau_{xy}}^2}$$

$$\tau_{max,min} = \pm\sqrt{\left(\frac{100 - 70}{2}\right)^2 + 50^2}$$

Maximum Shear Stress,

$$\boldsymbol{\tau_{max} = 52.2\ MPa}$$

Minimum Shear Stress,

$$\boldsymbol{\tau_{min} = -52.2\ MPa}$$

(iii) Principle Plane Orientation

Plane of Maximum Principal Stress,

$$\tan 2\theta_{p1} = \frac{2\tau_{xy}}{\sigma_x - \sigma_y}$$

$$\tan 2\theta_{p1} = \frac{2 * 50}{100 - 70}$$

$$\boldsymbol{\theta_{p1} = 36.65^o}$$

Plane of Minimum Principal Stress, $\theta_{p2} = \theta_{p1} + 90$

$$\boldsymbol{\theta_{p2} = 126.65^o}$$

(iv) Normal stress on planes of maximum and minimum shear stress.

Plane of Maximum Shear Stress, $\tan 2\theta_{s1} = -\left(\frac{\sigma_x - \sigma_y}{2\tau_{xy}}\right)$

$$\tan 2\theta_{s1} = -\left(\frac{100 - 70}{2 * 50}\right)$$

$$\theta_{s1} = -8.35^o$$

Plane of Minimum Shear Stress, $\theta_{s2} = \theta_{s1} + 90$

$$\theta_{s2} = 81.65^o$$

Normal Stress on plane of Maximum Shear Stress,

$$\sigma_{\theta_{s1}} = \frac{\sigma_x + \sigma_y}{2} + \left(\frac{\sigma_x - \sigma_y}{2}\right)\cos 2\theta_{s1} + \tau_{xy}\sin 2\theta_{s1}$$

$$\sigma_{\theta_{s1}} = \frac{100 + 70}{2} + \left(\frac{100 - 70}{2}\right)\cos 2(-8.35) + 50\sin 2(-8.35)$$

$$\boldsymbol{\sigma_{\theta_{s1}} = 85\ MPa}$$

Normal Stress on plane of Minimum Shear Stress,

$$\sigma_{\theta_{s2}} = \frac{\sigma_x + \sigma_y}{2} + \left(\frac{\sigma_x - \sigma_y}{2}\right) \cos 2\theta_{s2} + \tau_{xy} \sin 2\theta_{s2}$$

$$\sigma_{\theta_{s2}} = \frac{100 + 70}{2} + \left(\frac{100 - 70}{2}\right) \cos 2(81.65) + 50 \sin 2(81.65)$$

$$\boldsymbol{\sigma_{\theta_{s2}} = 85\ MPa}$$

2.a) Explain various methods by which stress concentration can be reduced with sketches. (05 Marks)

A number of methods are available to reduce stress concentration in machine parts. Some of them are as follows:

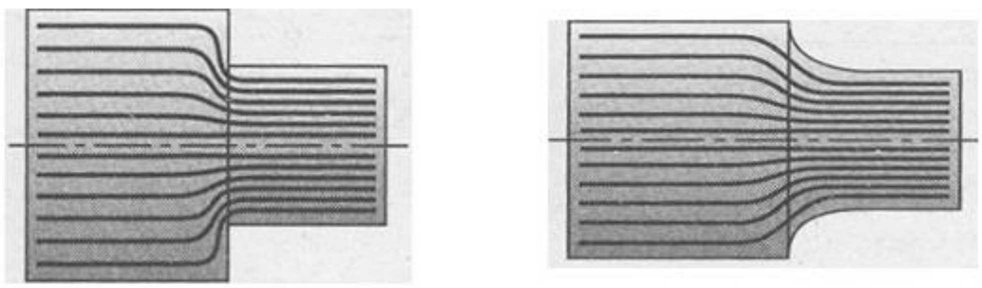
1. Provide a fillet radius so that the cross-section may change gradually.

2. Sometimes an elliptical fillet is also used.

3. If a notch is unavoidable it is better to provide a number of small notches rather than a long one. This reduces the stress concentration to a large extent.

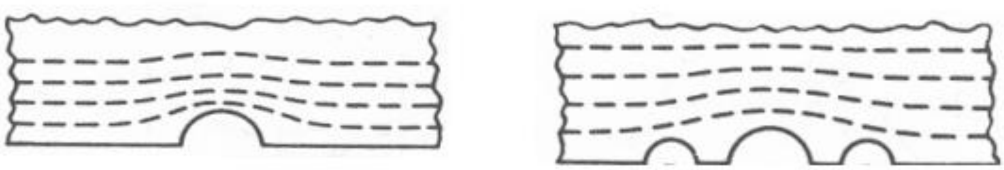
4. If a projection is unavoidable from design considerations it is preferable to provide a narrow notch than a wide notch.

5. Stress relieving groove are sometimes provided. These are demonstrated in the following figures.



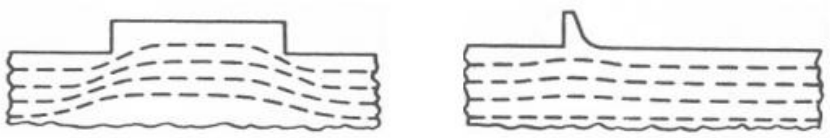
(a) Force flow around a sharp corner

Force flow around a corner with fillet: Low stress concentration.

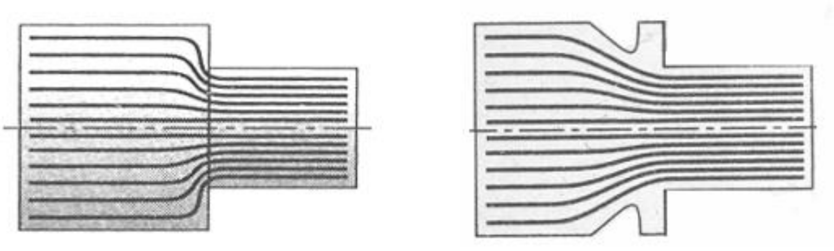


(b) Force flow around a large notch

Force flow around a number of small notches: Low stress concentration.



(c) Force flow around a wide projection Force flow around a narrow projection: Low stress concentration.



(d) Force flow around a sudden change in diameter in a shaft Force flow around a stress relieving groove.

Fig. Illustrations of different methods to reduce stress concentration

2.b) A point in a plate is subjected to a horizontal tensile stress of 100 N/mm^2 and vertical stress of 60 N/mm^2 . Find the magnitude of principal stresses. (15 Marks)

Data: σ_x = 100 N/mm^2, σ_y = 60 N/mm^2, τ_{xy} = 0

Soln:

Maximum Principal Stress,

$$\sigma_1 = \frac{\sigma_x + \sigma_y}{2} + \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}{}^2}$$

$$\sigma_1 = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}$$

$$\boldsymbol{\sigma_1 = \sigma_x = 100\ N/mm^2}$$

Minimum Principal Stress,

$$\sigma_2 = \frac{\sigma_x + \sigma_y}{2} - \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}{}^2}$$

$$\sigma_2 = \frac{\sigma_x + \sigma_y}{2} - \frac{\sigma_x - \sigma_y}{2}$$

$$\boldsymbol{\sigma_2 = \sigma_y = 60\ N/mm^2}$$

Maximum Shear Stress,

$$\tau_{max} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + {\tau_{xy}}^2}$$

$$\tau_{max} = \frac{\sigma_x - \sigma_y}{2}$$

$$\tau_{max} = \frac{100 - 60}{2}$$

$$\tau_{max} = 20\ N/mm^2$$

Jan./Feb. 2021

1.a) Explain the following: i) Principal stress ii) Plane stress iii) Stress Tensor (06 Marks)

(i) Principal Stress: Principal stresses are those stresses which are acting on the principal planes. Principal planes are these planes within the material such that the resultant stresses across them are wholly normal stresses or planes across which no shearing stresses occur. The plane carrying the maximum normal stress is called the major principal plane and the stress acting on it is called major principal stress. The plane carrying minimum normal stress is known as minor principal plane and the stress acting on it is called as minor principal stress.

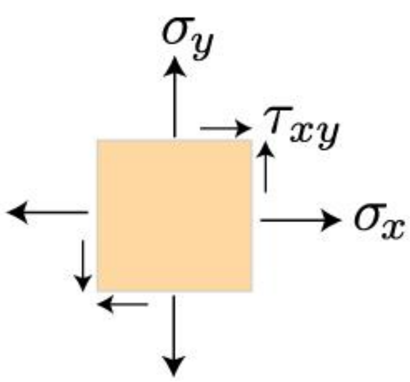
Magnitude of Maximum and Minimum Principal Stress is given by,

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + {\tau_{xy}}^2}$$

(ii) Plane Stress: Plane stress is defined to be a state of stress in which the normal stress and shear stress directed perpendicular to the z plane are assumed to be zero.

Thus for plane stress condition, $\sigma_z = \tau_{xz} = \tau_{yz} = 0$

Ex: A thin planar body subjected to in-plane loading as shown in the figure:



1.b) A point in a structural member subjected to a plane stress is shown in Fig.Q.1(b). Determine the following: i) Normal and Tangential stress intensities on plane MN inclined at an angle 45° ii) Principal stresses and their directions. iii) Maximum shear stress and direction of planes on which it occurs. (14 Marks)

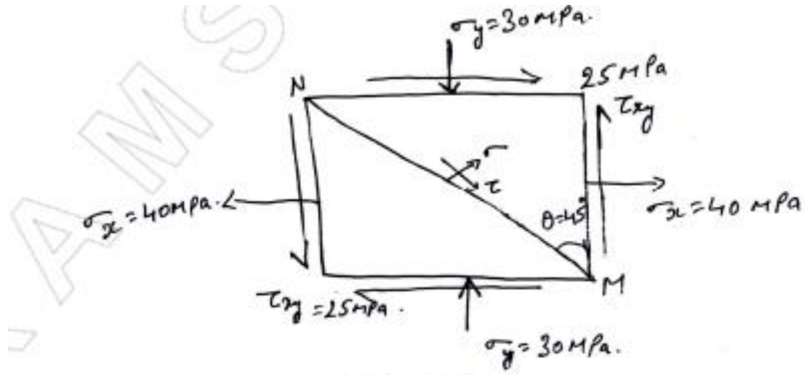


Fig.Q.1(b)

Data: $\sigma_x = 40\ MPa, \sigma_y = -30\ MPa, \tau_{xy} = 25\ MPa, \theta = 45°$

Soln:

(i) ***Normal and Tangential stress intensities on plane MN inclined at an angle 45°***

Normal Stress on plane MN,

$$\sigma_\theta = \frac{\sigma_x + \sigma_y}{2} + \left(\frac{\sigma_x - \sigma_y}{2}\right)\cos 2\theta + \tau_{xy}\sin 2\theta$$

$$\sigma_\theta = \frac{40 - 30}{2} + \left(\frac{40 + 30}{2}\right)\cos 2 \times 45 + 25\sin 2 \times 45$$

$$\boldsymbol{\sigma_\theta = 30\ \text{MPa}}$$

Tangential Stress on plane MN,

$$\tau_\theta = -\left(\frac{\sigma_x - \sigma_y}{2}\right)\sin 2\theta + \tau_{xy}\cos 2\theta$$

$$\tau_\theta = -\left(\frac{40 + 30}{2}\right)\sin 2 \times 45 + 25\cos 2 \times 45$$

$$\boldsymbol{\tau_\theta = -35\ \text{MPa}}$$

ii) Principal stresses and their directions

Maximum and Minimum Principal Stresses,

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$\sigma_{1,2} = \frac{40 - 30}{2} \pm \sqrt{\left(\frac{40+30}{2}\right)^2 + 25^2}$$

$$\sigma_{1,2} = 5 \pm 43.01$$

Maximum Principal Stress, $\boldsymbol{\sigma_1 = 48.01\ MPa}$

Minimum Principal Stress, $\boldsymbol{\sigma_2 = -38.01\ MPa}$

Plane of Maximum Principal Stress,

$$\tan 2\theta_{p1} = \frac{2\tau_{xy}}{\sigma_x - \sigma_y}$$

$$\tan 2\theta_{p1} = \frac{2 \times 25}{40 + 30}$$

$$\boldsymbol{\theta_{p1} = 17.77°}$$

Plane of Minimum Principal Stress, $\theta_{p2} = \theta_{p1} + 90$

$$\boldsymbol{\theta_{p2} = 107.77°}$$

iii) Maximum shear stress and direction of planes on which it occurs

Maximum Shear Stress,

$$\tau_{max} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$\tau_{max} = \sqrt{\left(\frac{40+30}{2}\right)^2 + 25^2}$$

$$\boldsymbol{\tau_{max} = 43.01\ MPa}$$

Plane of Maximum Shear Stress, $\tan 2\theta_{s1} = -\left(\frac{\sigma_x - \sigma_y}{2\tau_{xy}}\right)$

$$\tan 2\theta_{s1} = -\left(\frac{40+30}{2 \times 25}\right)$$

$$\boldsymbol{\theta_{s1} = -27.23°}$$

2.a) Explain the following failure theories: i) Maximum normal stress theory ii) Maximum shear stress theory iii) Maximum strain theory iv) Distortion energy theory (16 Marks)

i) Maximum Normal Stress Theory

- This theory was proposed by Rankine.
- It states that failure will occur when the maximum principal stress (σ_1) in the complex system reaches the value of maximum stress (σ_{yt}) at the elastic limit simple tension or the minimum principal stress (i.e. maximum principal compression stress) reaches the elastic limit (σ_{yc}) in simple compression.

$\sigma_1 = \sigma_{yt}$ in simple tension

$|\sigma_3| = \sigma_{yc}$ in simple compression

- For the design, the maximum principal stress should not exceed the working stress σ for the material. $\sigma_1 \le \sigma$,
- Working stress, $\sigma = \dfrac{\sigma_y}{F}$

F : Factor of safety

- This theory is valid for brittle metals such as cast iron.

ii) Maximum Shear Stress Theory

- This theory is also called Coulomb Guest's or Treasca's theory
- It states that the material will fail when the maximum shear stress (τ_{max}) in the complex system reaches the value of maximum shear stress in simple tension at the elastic limit.

$$\tau_{max} = \frac{\sigma_1 - \sigma_3}{2} = \frac{\sigma_{yt}}{2}$$

$$\sigma_1 - \sigma_3 = \sigma_{yt}$$

$$\sigma_1 - \sigma_3 = \sigma_t$$

- This theory gives good correlation with the results of experiments on ductile materials.
- It gives satisfactory results for ductile materials particularly in case of shafts.
- The theory does not give accurate results for the state of stress of pure shear.
- The theory is not applicable in the case where the state of stress consists of triaxial tensile stresses of nearly equal magnitude.

Iii) Maximum Strain Theory

- This theory was proposed by Saint Venant.
- It states that the failure of a material occurs when the major principal tensile strain reaches the strain at the elastic limit in simple tension or when the minor principal strain (i.e maximum principal compressive strain) reaches the strain at elastic limit in simple compression.
- This theory is more appropriate for ductile materials, brittle materials and materials under hydrostatic pressure.
- It does not fit well with the experimental results.

Principal strain in the direction of principal stress σ_1, $e_1 = \frac{1}{E}[\sigma_1 - \mu(\sigma_2 + \sigma_3)]$

Principal strain in the direction of principal stress σ_3, $e_3 = \frac{1}{E}[\sigma_3 - \mu(\sigma_1 + \sigma_2)]$

For the design purposes

$$\sigma_3 - \mu(\sigma_1 + \sigma_2) = \sigma_t$$

$$\sigma_3 - \mu(\sigma_1 + \sigma_2) = \sigma_c$$

Where σ_t and σ_c are the safe stresses.

iv) Distortion Energy Theory

- This theory was proposed by Von Mises-Henky
- It states that the elastic failure occurs when the shear strain energy per unit volume in the stressed material reaches a value equal to the shear strain energy per unit volume at the elastic limit point in tension.
- The theory gives best results for ductile material particularly in case of pure shear or $\sigma_{yc} = \sigma_{yt}$.

Shear strain energy per unit volume due to principal stresses σ_1, σ_2 and σ_3,

$$U_S = \frac{1+\mu}{3E}\left[\sigma_1^2 + \sigma_2^2 + \sigma_3^2 - (\sigma_1\sigma_2 + \sigma_3\sigma_2 + \sigma_1\sigma_3)\right]$$

According to this theory, the design condition is given by

$$\sigma_1^2 + \sigma_3^2 - \sigma_1\sigma_3 = \sigma_t^2$$

2.b) Explain the failure of brittle and ductile material. (04 Marks)

Failure involves the forced separation of a material into two or more parts. Brittle failure involves fracture without any appreciable plastic deformation (i.e. energy absorption). Ductile failure in the converse involves large plastic deformation before separation. The difference between brittle and ductile fracture is illustrated in Fig. 1. Remembering that the area under the Stress - Strain curve in Fig. 1 represents energy, we can see that much less energy is expended in brittle fracture than in ductile fracture.

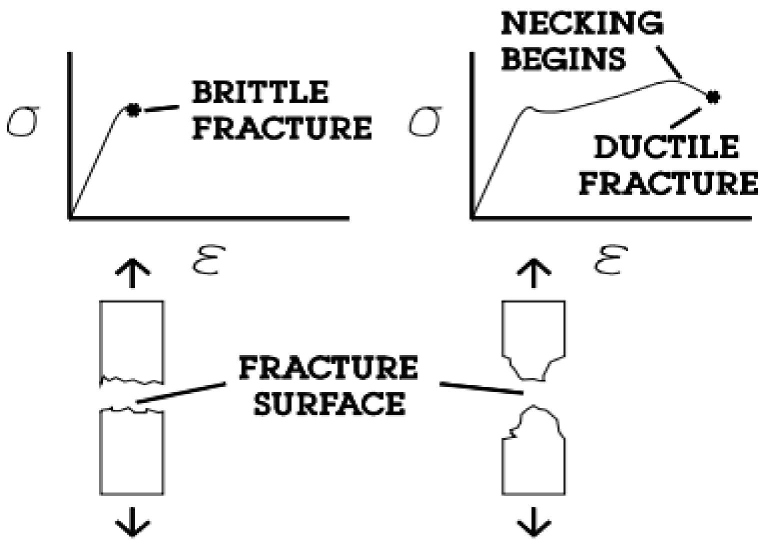


Figure 1: Brittle vs Ductile Fracture

Feb./Mar. 2022

1.a) Explain the following : i) Factor of safety ii) Uniaxial stress iii) True stress iv) Stress tensor v) Principal stress. (10 Marks)

i) ***Factor of Safety:***

It is defined as the ratio of maximum strength of the material to the working or design stress.

Mathematically,

$$\text{Factor of safety} = \frac{\text{Maximum stress}}{\text{Working or design stress}}$$

In case of ductile materials e.g. mild steel, where the yield point is clearly defined, the factor of safety is based upon the yield point stress. In such cases,

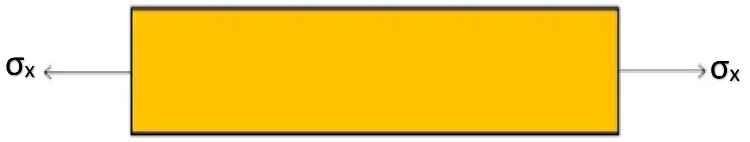
$$\text{Factor of safety} = \frac{\text{Yield stress}}{\text{Working or design stress}}$$

In case of brittle materials e.g. cast iron, the yield point is not well defined as for ductile materials. Therefore, the factor of safety for brittle materials is based on ultimate stress.

$$\text{Factor of safety} = \frac{\text{Ultimate stress}}{\text{Working or design stress}}$$

ii) Uniaxial Stress:

A one-dimensional state of stress in which normal stresses act along one direction only is called a uniaxial stress state. Figure shows a structural element subjected to uniaxial stress:



iii) True Stress

True Stress acting on the material is the applied load divided by instantaneous cross sectional area.

The relationship between true stress and engineering stress is given by,

$$\sigma_T = \sigma\,(1 + \varepsilon)$$

where σ_T is True Stress

σ is Engineering Stress

ε is Engineering Strain

iv) Stress Tensor:

Stress is defined as force per unit area. If we take a cube of material and subject it to an arbitrary load we can measure the stress on it in various directions as shown in the figure.These measurements will form a second rank tensor; the stress tensor.

Stress Tensor is expressed as,

$$\tau_{ij} = \begin{bmatrix} \sigma_x & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \sigma_y & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \sigma_z \end{bmatrix} \text{MPa}$$

v) Principal Stress:

Principal stresses are those stresses which are acting on the principal planes. Principal planes are these planes within the material such that the resultant stresses across them are wholly normal stresses or planes across which no shearing stresses occur. The plane carrying the maximum normal stress is called the major principal plane and the stress acting on it is called major principal stress. The plane carrying minimum normal stress is known as minor principal plane and the stress acting on it is called as minor principal stress.

Magnitude of Maximum and Minimum Principal Stress is given by,

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}{}^2}$$

1.b) A mild steel shaft of 50 mm diameter is subjected to a bending moment of 2000 N-m and a Torque 'T'. If the yield point of the steel in tension is 200 MPa, find the maximum value of this torque without causing yielding of the shaft according to, i) Maximum principal stress theory ii) Maximum shear stress theory iii) Maximum distortion strain energy theory. (10 Marks)

Data: $d = 50\ mm, M = 2000 \times 10^3\ N - mm, \sigma_y = 200\ MPa$

Soln:

Moment of Inertia of Shaft Section,

$$I = \frac{\pi d^4}{64}$$

$$I = \frac{\pi 50^4}{64}$$

$$I = 306.8 \times 10^3\ mm^4$$

Distance from neutral axis,

$$y = \frac{d}{2} = \frac{50}{2} = 25\ mm$$

Bending Stress,

$$\sigma = \frac{My}{I}$$

$$\sigma = \frac{2000 \times 10^3 \times 25}{306.8 \times 10^3}$$

$$\boldsymbol{\sigma = 163\ MPa}$$

For shaft subjected to torque T, Shear Stress

$$\tau = \frac{16T}{\pi d^3}$$

$$\tau = \frac{16T}{\pi 50^3}$$

$$\boldsymbol{\tau = 4.07 \times 10^{-5} T\ MPa}$$

Maximum and Minimum Principal Stress,

$$\sigma_{1,2} = \frac{\sigma}{2} \pm \sqrt{\left(\frac{\sigma}{2}\right)^2 + \tau^2}$$

$$\sigma_{1,2} = \frac{163}{2} \pm \sqrt{\left(\frac{163}{2}\right)^2 + (4.07 \times 10^{-5} T)^2}$$

$$\boldsymbol{\sigma_{1,2} = 81.5 \pm \sqrt{6642.25 + (1.66 \times 10^{-9} \times T^2)}}$$

Maximum Shear Stress,

$$\tau_{max} = \sqrt{\left(\frac{\sigma}{2}\right)^2 + \tau^2}$$

$$\tau_{max} = \sqrt{\left(\frac{163}{2}\right)^2 + (4.07 \times 10^{-5}T)^2}$$

$$\boldsymbol{\tau_{max} = \sqrt{6642.25 + (1.66 \times 10^{-9} \times T^2)}}$$

i) Torque according to Maximum Principal Stress Theory

According to Maximum Principal Stress Theory, for safe design:

$$\sigma_1 = \sigma_y$$

$$81.5 + \sqrt{6642.25 + (1.66 \times 10^{-9} \times T^2)} = 200$$

$$\sqrt{6642.25 + (1.66 \times 10^{-9} \times T^2)} = 118.5$$

$$6642.25 + (1.66 \times 10^{-9} \times T^2) = 14042.25$$

$$(1.66 \times 10^{-9} \times T^2) = 7400$$

$$T^2 = 4.46 \times 10^{12}$$

$$\boldsymbol{T = 2112 \times 10^3\ N - mm = 2112\ N - m}$$

***ii) Torque according to* Maximum Shear Stress Theory**

According to Maximum Shear Stress Theory, for safe design:

$$\tau_{max} = \frac{\sigma_y}{2}$$

$$\sqrt{6642.25 + (1.66 \times 10^{-9} \times T^2)} = \frac{200}{2}$$

$$6642.25 + (1.66 \times 10^{-9} \times T^2) = 10000$$

$$(1.66 \times 10^{-9} \times T^2) = 3357.75$$

$$T^2 = 2022.74 \times 10^9$$

$$\boldsymbol{T = 1422 \times 10^3\ N - mm = 1422\ N - m}$$

iii) Torque according to Maximum Distortion Strain Energy Theory

According to Maximum Distortion Strain Energy Theory, for safe design:

$$\sigma_1{}^2 + \sigma_2{}^2 - \sigma_1\sigma_2 = \sigma_y{}^2$$

$$\left[81.5 + \sqrt{6642.25 + (1.66 \times 10^{-9} \times T^2)}\right]^2 + \left[81.5 - \sqrt{6642.25 + (1.66 \times 10^{-9} \times T^2)}\right]^2 - \left[81.5 + \sqrt{6642.25 + (1.66 \times 10^{-9} \times T^2)}\right]\left[81.5 + \sqrt{6642.25 - (1.66 \times 10^{-9} \times T^2)}\right] = 200^2$$

$$2[81.5^2 + 6642.5 + 1.66 \times 10^{-9}T^2] - [81.5^2 - 6642.5 + 1.66 \times 10^{-9}T^2] = 200^2$$

$$81.5^2 + 3(6642.5) + 3(1.66 \times 10^{-9}T^2) = 40000$$

$$\boldsymbol{T = 1642 \times 10^3\ N - mm = 1642\ N - m}$$

2.a) Why failure theory is important in aircraft structures? Explain the failure of brittle and ductile materials. (10 Marks)

Importance of failure theories in aircraft structures:

Aircraft structures must be designed to withstand a wide range of stresses and loads, including aerodynamic forces, weight, and impact loads. To ensure that the structures can withstand these loads without failing, engineers use failure theories to predict the behavior of materials and structures under different types of stress. There are several failure theories that are commonly used in aircraft design, including the maximum stress theory, maximum strain theory, and the maximum distortion energy theory. Each theory has its own assumptions and limitations, and engineers must carefully select the theory that best fits the specific application and material being used.

By using these failure theories, engineers can determine the maximum allowable stresses and loads that a material or structure can withstand before it fails. This information is critical for designing safe and reliable aircraft structures that can operate under a wide range of conditions and loads. In addition to helping prevent catastrophic failures, failure theories can also help reduce weight and material costs by allowing engineers to design structures that are optimized for the loads they will experience. This can help improve fuel efficiency and reduce operating costs for airlines and other aircraft operators.

Overall, failure theories are a critical tool for aircraft designers and engineers, helping to ensure the safety, reliability, and efficiency of aircraft structures.

Failure of Brittle and Ductile Materials: Refer 2b) of Jan/Feb 2021

2.b) Explain atleast four different theories of failure in detail. (10 Marks)

Refer solution of 2.a) Jan./Feb. 2021