Module 2 → Design for Impact and fatigue loads.

1). A steel rod of 1.5m long has to resist longitudinally an impact 2.5 kN falling under gravity at a velocity of 0.99 m/s. Maximum compressive stress is limited to 150 MPa. Determine i). Diameter of rod required, ii). Impact factor. Use, $E = 206.8\times10^3\ N/mm^2$.

→ $l = 1.5m = 1.5\times10^3\ mm$.

$W = 2.5\ kN$.

$v = 0.99\ m/s$.

$\boxed{\sigma' = 150\ MPa}$.

$E = 206.8\times10^3\ N/mm^2$.

$L = 1.5m$

$W = 2.5kN$

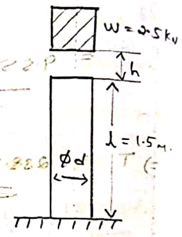
$v = 0.99 m/s$

$\sigma_{max} = 150 MPa$.

d

IF

$E = 206.8\times10^3 N/mm^2$



$$\sigma' = \frac{W}{A}\left(1+\sqrt{1+\frac{2hEA}{WL}}\right)$$

$h = v^2/2g$.

i) Diameter of the Rod →

$$\sigma' = \frac{W}{A}\left(1+\sqrt{1+2h/\delta}\right) = \frac{W}{A}\left(1+\sqrt{1+2hEA/WL}\right).$$

$\delta = \frac{WL}{AE}$ eqn. 1.2(b) Pg 3.

eqn. 2.26(c) Pg. 27

$$150 = \frac{2.5\times10^3}{\frac{\pi(d)^2}{4}}\left(1+\sqrt{1+\frac{2\times49.95\times206.8\times10^3\times\frac{\pi d^2}{4}}{2.5\times10^3\times1.5\times10^3}}\right)$$

3.75×10^6

$h = \frac{v^2}{2g}$

$= \frac{0.99^2}{2\times9.81}$

$h = 0.049 m \times10^3$

$h = 49.95 mm$.

$d = 44.61 mm$.

Impact factor $= 1+\sqrt{1+\frac{2hEA}{WL}}$.

$$= 1+\sqrt{1+\frac{2\times49.95\times206.8\times10^3\times\frac{\pi\times44.61^2}{4}}{2.5\times10^3\times1.5\times10^3}}$$

$= 93.8$.

9). A cantilever beam of span 800 mm has a rectangular c/s. of depth 200 mm. The free end of the beam is subjected to a transverse load of 1 kN that drops on to it from a height of 40 mm. Selecting 40C8 steel ($\sigma_y = 328.6$ MPa) & FOS = 3. Determine the width of rectangular cross section.

2012-13

$L = 800$ mm

reqd.

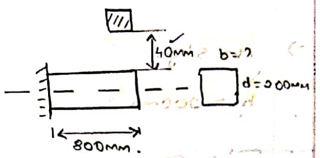
$d = 200$ mm

$W = 1$ kN

$h = 40$ mm

$\sigma_y, 1$

$b = ?$



→ $l = 800$ mm.

$d = 200$ mm.

$W = 1$ kN.

$h = 40$ mm.

$\sigma_y = 328.6$ MPa.

FOS $= 3$.

$b = ?$

$\sigma_b' = \sigma_b\left(1 + \sqrt{\frac{1+2h}{\delta}}\right)$ eq. ①

$\frac{\sigma_y}{FOS}$ $\quad \frac{M}{I} = \frac{\sigma}{c} \rightarrow \frac{h}{2} \rightarrow 200$

$M \rightarrow FL$

$I = \frac{bh^3}{12}$ 200

$\delta = \frac{1}{3}\frac{FL^3}{EI}$

MQ2 8M similar

To find width →

Impact stress due to Bending.

$\sigma_b' = \sigma_b\left(1 + \sqrt{1+2h/\delta}\right)$. eqn. 2.26(c) Pg. 27.

(Table 1.4. Pg. 15.)

Cantilever, end load.

$\delta = \frac{1}{3} \times \frac{WL^3}{EI}$

$E = 207$ GPa Table I.18. Pg. 473 (8)

$= 207 \times 10^3$ N/mm²

$\delta = \frac{1}{3} \times \frac{1 \times 10^3 \times 800^3}{207 \times 10^3 \times \frac{b \times 200^3}{12}}$

$\delta = \frac{1.2367}{b}$

General Bending eqn.

$\frac{M}{I} = \frac{E}{R} = \frac{\sigma}{C}$ eqn 1.3(b) Pg. 3

$\sigma_b = \frac{MC}{I}$.

$= \frac{1 \times 10^3 \times 800}{\frac{bd^3}{12}} \times \frac{d}{2}$. Pg. 12.

$\sigma_b = \frac{120.12}{b}$

FOS $= \frac{\sigma_y}{\sigma_{working}}$.

FOS $= \frac{\sigma_y}{\sigma_b'}$

$\sigma_b' = \frac{\sigma_y}{FOS}$

$= \frac{328.6}{3}$.

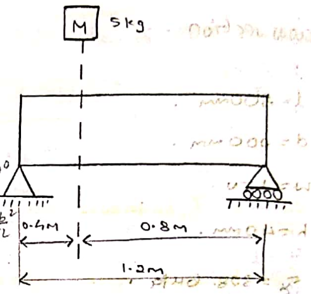
$\sigma_b' = 109.53$ N/mm²

$\therefore \sigma_b' = \sigma_b\left(1 + \sqrt{1+2h/\delta}\right)$. ⇒ $109.53 = \frac{120.12}{b} \times \left(1 + \sqrt{\frac{1+2\times40}{\frac{1.2367}{b}}}\right)$.

$b = 80.95$ mm

3. A 5kg block is dropped from a height of 200mm onto a beam as shown in fig. The material has an allowable stress of 50MPa. Determine the dimensions of rectangular c/s whose depth is 1.5 times the width. Take E = 70 GPa.

Sem/Apr 2017-18 8M.



$\rightarrow$ $M = 5kg \rightarrow N$

$h = 200mm$

$\sigma_b' = 50MPa$

$E = 70 GPa = 70\times10^3 \; N/mm^2$.

$d = 1.5b$.

$M, h, \sigma_{max} = 50$

$d = 1.5b \quad d \,\&\, b = ?$

$\sigma_b' = \sigma_b\left(1+\sqrt{1+\frac{2h}{\delta}}\right) \rightarrow 200$, $\delta \rightarrow \frac{Wa^2b^2}{3EIL}$

$\frac{M}{I} = \frac{\sigma}{c} \rightarrow \frac{h}{2} \rightarrow \frac{1.5b}{2}$

$M \rightarrow F\frac{ab}{L}$

$I = \frac{bh^3}{12}(1.5b)$.

or $\frac{I}{c} = Z = \frac{bh^2}{6}$

Bending.

$\sigma_b' = \sigma_b\left(1+\sqrt{1+2h/\delta}\right)$ eqn. 2.06(c) Pg. 27.

Max. $M = F\frac{ab}{L}$ [Table 7.4 fig 5 Pg. 16]

$M = \frac{Mg \times a \times b}{L}$

$= \frac{5\times9.81\times0.4\times10^3\times0.8\times10^3}{1.2\times10^3}$.

$M = 13080$ N-mm

$I = \frac{bd^3}{12} = \frac{b\times(1.5b)^3}{12}$.

$= 0.28125b^4$.

$c = \frac{d}{2} = \frac{1.5b}{2} = 0.75b$.

$\sigma_b = \frac{13080 \times 0.75b}{0.28125b^4}$

$\sigma_b = \frac{34880}{b^3}$.

$$\boxed{\delta = \frac{Wa^2b^2}{3EIL}}$$

remember in DHB.

$\delta = \frac{5\times9.81\times(0.4\times10^3)^2\times(0.8\times10^3)^2}{3\times70\times10^3\times0.28125b^4\times1.2\times10^3}$

$\delta = \frac{70867.3}{b^4}$ //

$$50 = \frac{34880}{b^3}\left(1 + \sqrt{1 + \frac{2(200)}{\frac{70867.3}{b^4}}}\right)$$

$\Rightarrow b = 52.7$ mm.

$\therefore d = 1.5b = 79.08$ mm.

• **NOTE →** Instantaneous max. deflection, $\delta' = \delta \times$ Impact factor.

Instantaneous Max. stress, $\sigma_b' = \sigma_b \times$ Impact factor.

Instantaneous max. load, $W' = W \times$ Impact factor.

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5). A Beam of 300 mm depth I-section is resting on two supports 5m apart. It is loaded by a weight of 5000N falling through a height 'h' and striking the Beam at midpoint. Moment of Inertia of the section is 9.6×10^7 mm^4; Modulus of elasticity is 21×10^4 N/mm^2. Determine the Permissible of height 'h'. if the stress is limited to 130 N/mm^2.

→ $d = 300$ mm.

$L = 5$m $= 5 \times 10^3$ mm.

$W = 5000$N.

$I = 9.6 \times 10^7$ mm^4.

$E = 21 \times 10^4$ N/mm^2.

$\sigma_b' = 130$ N/mm^2

determine $h = ?$

$d, L, W, h = ?$

$I, E, \sigma_{max} = 130$

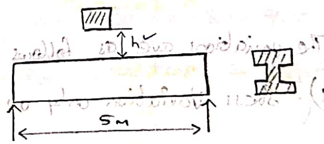
$\sigma_b' = \sigma_b\left(1 + \sqrt{\frac{1 + 2h}{\delta}}\right)$

$\delta \to \frac{1}{48}\frac{FL^3}{EI}$

$\frac{M}{I} = \frac{\sigma}{c}$

$M \to \frac{1}{4}FL$

$c \to d/2 \to \frac{300}{2}$



Bending,

$$\sigma_b' = \sigma_b\left(1 + \sqrt{1 + 2h/\delta}\right).$$

$\sigma_b = \frac{MC}{I}$.

$\sigma_b = 9.7656$ N/mm^2.

$I = 9.6 \times 10^7$ mm^4

$c = \frac{d}{2} = \frac{300}{2} = 150$.

(Pg. 15 (4)).

Max. $M = \frac{1}{4}FL$.

$= \frac{5000 \times 5 \times 10^3}{4}$

$M = 6.25 \times 10^6$ N-mm

$$\text{Max. } y = \frac{1}{48}\frac{FL^3}{EI}.$$

$$y = \frac{1}{48} \times \frac{5000 \times (5\times10^3)^3}{21\times10^4 \times 9.6\times10^7}$$

$$y = \underline{\underline{0.65}}.$$

Ans. $h = 49$

$$\therefore\ 130 = 9.7656\left(1+\sqrt{1+\frac{2\times h}{0.65}}\right).$$

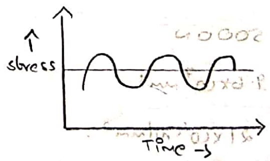
$h = 48.76 \approx \underline{\underline{49}}$ mm //

- Fatigue →

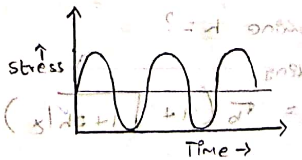
Fatigue or variable load → when a load or stress on a machine part changes in magnitude or direction or both, the load is known as variable load or fatigue load.

The variations are as follows,

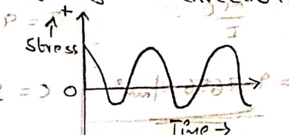
1). Stress Variation only on positive side.



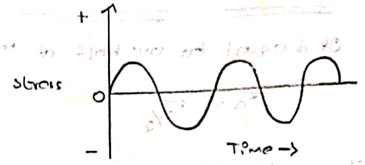
2). Stress variation only on positive side with zero minimum stress.



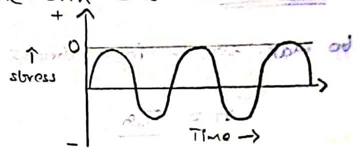
3). Stress variation in both positive & negative direction.



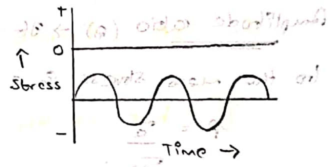
4). Equal stress variation both in positive & negative direction (Completely reversed load).



5). Stress variation on negative side with zero max. stress



6). stress variation only on negative side.

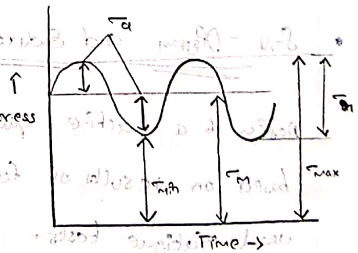


Stress Cycle → It is the smallest portion of the stress - Time plot which is repeated periodically & identically.

Fluctuating stress → The stresses which vary from a min. to max. value of same nature (tensile or compressive) are called fluctuating stresses.

Maximum stress → (σ_{max}) It is the largest algebraic stress in a stress cycle. Tensile stress is taken as positive & Compressive stress taken as negative.

Minimum stress → (σ_{min}).
It is the smallest algebraic stress in a stress cycle.



Mean stress → (σ_m)

It is the algebraic mean of the max. & min stresses in one cycle.

$$\sigma_m = \frac{\sigma_{max} + \sigma_{min}}{2}$$

Range of stress → (σ_R)

It is the algebraic difference b/w the max & min stresses in a stress cycle. $\sigma_R = \sigma_{max} - \sigma_{min}$

Stress amplitude or variable stress (σ_a) →

It is equal to one half of the range of stress.

$$\sigma_a = \sigma_r/2 .$$

Stress ratio → (r) It is defined as the ratio of min. stress to max. stress in a stress cycle.

$$r = \frac{\sigma_{min}}{\sigma_{max}}$$

Amplitude ratio (A) → It is defined as the ratio of variable stress to the mean stress in a stress cycle.

$$A = \frac{\sigma_a}{\sigma_m} .$$

Repeated stresses → The stresses which vary from zero to a certain max. value are called repeated stresses.

← [fig. 2 & fig 5].

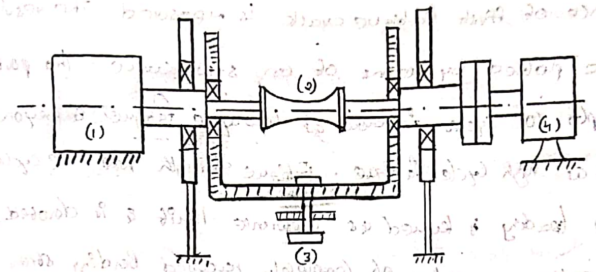
Alternating or Completely reversed stress → The stress which vary from a minimum value to a max. value of the opposite nature are called completely reversed or Alternating or cyclic stresses.

- S-N - Diagram and Endurance limit →

Design of a machine part that is subjected to a fatigue load is based on results of fatigue tests. The following are the usual fatigue tests.

1). Completely Reversed Bending stress. (Rotating Beam Test).
2). Completely Reversed axial stress (Push - Pull Test).
3). Repeated Torsional stress.
4). Oscillating stress & superimposed static stress.

1). Completely Reversed Bending stress (Rotating Beam Test):

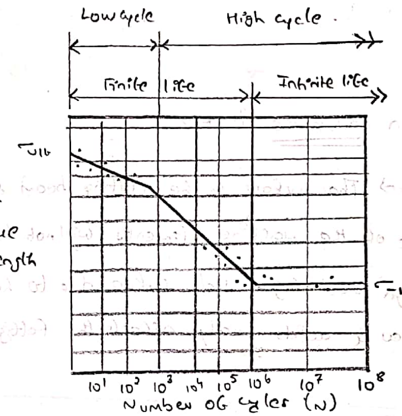


1). Revolution Counter.

2). Specimen.

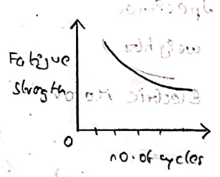
3). weights.

4). Electric Motor.



The mostly widely used fatigue testing device is the R.R. Moore high speed Rotating Beam machine shown in fig. The specimen is very carefully machined & polished with a final polishing in the axial direction. to avoid circumferential scratches. Self alligning ball bearings are used to ensure that only radial loads are applied to the specimen. The test specimen is subjected to pure Bending moment & the magnitude of Bending stress is adjusted by means of weights. As the Beam rotates the fibres of the surface of the specimen undergo a reversal of stress.

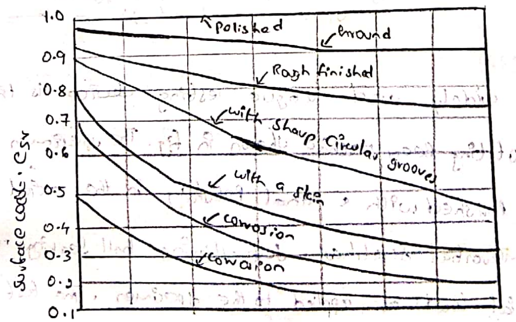
To determine the endurance limit stress of a material, a large number of tests are carried out. Each test consists of applying a constant bending load & the number of revolutions before the appearance of first fatigue crack is measured. The results of these tests are plotted by means of an S-N-curve. The portion of the curve upto 10^3 cycles is known as low cycle fatigue & beyond 10^3 cycles is known as high cycle fatigue. Fatigue strength at 10^6 cycles of reversible loading is termed as endurance limit & is denoted by σ_{-1}. defined as the max. value of completely reversed bending stress which a polished standard specimen can withstand without failure for infinite no. of cycles.



for non-ferrous

- Effect of factors on Endurance limit →

1). Surface condition → The surface of the rotating beam specimen is highly polished. But most of the machine elements will not have such a high quality finish. Rough machining, scaly surface due to hot rolling and corrosion from water & acids badly effect the fatigue strength.



(fig. 4.6 2.33)

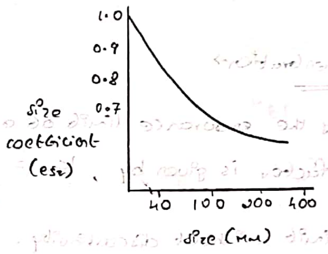
For a mirror polished specimen, the surface correction coeff. $C_{sr} = 1$. but for a specimen other than mirror polished surface condition, $C_{sr} < 1$

d). **Size effect** → The standard test specimen used in rotating beam method has a nominal diameter of about 7.5 mm. Therefore, if the nominal diameter is not equal to 7.5 mm, a size correction coeff. should be taken into consideration. Fatigue strength has been observed to decrease with increase in the machine part size because of the following reasons.

a). Probability of increase in internal defects because of greater volume.

b). Higher probability of initiation of fatigue crack because of larger area.

c). Difficulties in obtaining uniform strength & heat treatment throughout the whole crossection of large parts.



Diameter (mm).	Size coefficient (e_{sz}).
$d \leq 7.5$	1
$7.5 < d < 50$	0.85
$d > 50$	0.75

e). **Effect of load** → The endurance limit of a material as determined by the rotating beam method is for reversed Bending load. There are many machine members which are subjected to loads other than reversed Bending loads. Therefore, the endurance limit will be different for different type of loads.

Denoted by, $e_L = 1$ for reversed Bending load.

$e_L = 0.7$ for axial load.

$e_L = 0.6$ for torsional load.

4). Effect of reliability → The coeff. of reliability (e_r) depends on the reliability i.e. considered in the design of component.

Reliability (%)	Reliability coeff (e_r)
50	1
90	0.897
95	0.868
99	0.814
99.9	0.753

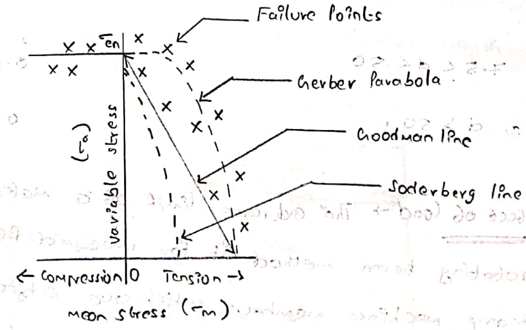
5). Effect of stress concentration →

All discontinuities effects the endurance limit of a machine Part. The endurance stress conc. factor is given by, $K_{en} = \frac{\sigma_{en}}{\sigma_{en}'}$

where, σ_{en} = endurance limit without discontinuity.

σ_{en}' = endurance limit with discontinuity.

Fatigue strength under fluctuating stresses →



1). A steel rod (SAE 9260) oil quenched $\sigma_u = 1089.5$ MPa, $\sigma_y = 689.4$ MPa, $\sigma_{en} = 427.6$ MPa is subjected to a tensile load which varies from 120 kN to 40 kN. Design a safe diameter of the rod using Soderberg diagram. Adopt FOS as 2, stress conc. factor as unity and correction factors for load, size & surface as 0.75, 0.85 & 0.91 respectively.

→ Given → (SAE 9260).

F_{max}, F_{min}, $d = ?$

Soderberg.

$\sigma_u = 1089.5$ MPa. $\quad e_l = 0.75$

$\sigma_y = 689.4$ MPa. $\quad e_{sz} = 0.85$

$\sigma_{en} = 427.6$ MPa. $\quad e_s = 0.91$

$F_{max} = 120$ kN. $\quad K_{tf} = 1$

$F_{min} = 40$ kN.

FOS = 2. $\quad d = ?$

$\sigma_a = \frac{F_a}{A} \Rightarrow F_a = \frac{F_{max} - F_{min}}{2}$, $A \to \frac{\pi d^2}{4}$

The Soderberg relationship,

for ductile materials, $\frac{K_{tf}\,\sigma_a}{ABC\,\sigma_{en}} + \frac{\sigma_m}{\sigma_{yp}} = \frac{1}{n}$ (2.21(c)) Pg. 25

$\sigma_m = \frac{F_m}{A}$; $F_m = \frac{F_{max} + F_{min}}{2}$, $A \to \frac{\pi d^2}{4}$

Ultimate stress, $\sigma_a = \frac{\sigma_{max} - \sigma_{min}}{2}$

$\sigma = F/A$, $F = \frac{F_{max} - F_{min}}{2}$

Average or mean normal stress, $\sigma_m = \frac{\sigma_{max} + \sigma_{min}}{2}$

$$F_a = \frac{F_{max} - F_{min}}{2} = \frac{120 - 40}{2} = 40\text{ kN}.$$

$$\therefore \sigma_a = \frac{F_a}{A} = \frac{40 \times 10^3}{\frac{\pi d^2}{4}}.$$

$\sigma_a = \frac{F_a}{A}$.

$F_a = \frac{F_{max} - F_{min}}{2}$

$$F_m = \frac{F_{max} + F_{min}}{2} = \frac{120 + 40}{2} = 80\text{ kN}.$$

$$\therefore \sigma_m = \frac{F_m}{A} = \frac{80 \times 10^3}{\frac{\pi d^2}{4}}.$$

$$\therefore \frac{1 \times \dfrac{40\times10^3}{\frac{\pi d^2}{4}}}{0.75\times0.85\times0.91\times 407.6} + \frac{\dfrac{20\times10^3}{\frac{\pi d^2}{4}}}{689.4} = \frac{1}{2}$$

236.06145

(19.57)

$d = 26.57\ mm \approx 28\ mm.$

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2). A Piston rod is subjected to a max. reversed axial load of 110 kN. It is made of steel having an ultimate stress of $900 N/mm^2$ & the surface is machined. The average endurance limit is 50% of the ultimate strength. Take the size correction coeff as 0.85 & FOS = 1.75. Determine the diameter of the rod.

→ $F_{max} = 110kN$.

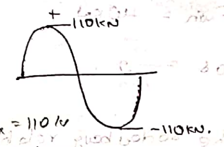
$F_{min} = -110kN$.

$\sigma_u = 900 N/mm^2$.

$\sigma_{en} = 0.5\sigma_u = 450\ N/mm^2$.

$C_{sz} = 0.85$, FOS = 1.75.

M22 shallow 10M



$F_{max} = 110 kN$
$F_{min} = -110 kN$.

$\sigma_u = 900$, $\sigma_{en} = 50\% \sigma_u$

B, n, d=?

Goodman

$\sigma_a = \frac{F_a}{A \to \frac{\pi d^2}{4}}$; $F_a = \frac{F_{max} - F_{min}}{2}$.

The Goodman's relationship,

for ductile materials, $\frac{K_{tf}\,\sigma_a}{ABC\,\sigma_{en}} + \frac{\sigma_m}{\sigma_u} = \frac{1}{n}$ (eq. 2.21(a) pg.25)

Assume K_{tf} as 1. 0.7

$\sigma_m = \frac{F_m}{A}$; $F_m = \frac{F_{max}+F_{min}}{2}$; $\sigma_a = \frac{F_a}{A}$. $F_a = \frac{F_{max} - F_{min}}{2}$.

A = 0.7 for reversed axial loading.

C = (Table.(2.2) pg.32) (when not mention go to cold drawn)

Interpolate,

820	0.82
900	?
1030	0.78

$\Rightarrow \frac{900-820}{1030-820} = \frac{x-0.82}{0.78-0.82}$

$x = 0.8$.

$\sigma_a = \frac{\sigma_{max} - \sigma_{min}}{2}$.

$F_a = \frac{F_{max} - F_{min}}{2} = 110\ kN.$

$\therefore \sigma_a = \frac{F_a}{A} = \frac{110\times10^3}{\frac{\pi d^2}{4}}$.

$\sigma_m = \dfrac{\sigma_{max} + \sigma_{min}}{2}$ $\qquad$ $F_m = \dfrac{F_{max} + F_{min}}{2} = \underline{\underline{0}}$.

$\therefore \sigma_m = 0$.

$$\therefore \frac{1 \times \dfrac{110\times10^3}{\frac{\pi d^2}{4}}}{0.7\times0.85\times0.8\times450} + \frac{0}{900} = \frac{1}{1.75}$$

$$d = \underline{\underline{33.82 \text{ mm}}}$$

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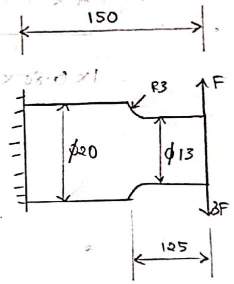
3. A cantilever beam made of cold drawn carbon steel ($\sigma_u = 550$ MPa) $\sigma_y = 470$ MPa, $\sigma_{en} = 275$ MPa) of circular c/s. shown in fig. is subjected to load which varies from $(-F)$ to $3F$. Determine the max. load the cantilever beam can withstand for an infinite life, using FOS. of 2.

$\sigma_u, \sigma_y, \sigma_{en}$

cr/w. $(-F)$ to $(3F)$

$F = ?$ $\quad$ $n = 2$.

Soderberg.



using Soderberg relationship. (Pg. 25) (2.21(c)).

For ductile materials, $\dfrac{K_{tf}\,\sigma_a}{ABC\,\sigma_{en}} + \dfrac{\sigma_m}{\sigma_{yp}} = \dfrac{1}{n}$.

$A = 1.0$ for reversed bending.

$B = 0.85$ (size effect). $(7.5 < d < 50)$.

$C =$ (Table (2.2) Pg. 32) $=$ for $\sigma_u = 550$, $C = 0.88$.

$K_{tf} = 1 + q(K_t - 1)$.

$= 1 + 1(K_t - 1) = \underline{\underline{1.4}}$.

$\boxed{M = F \times L}$

$\dfrac{M_{ba}}{I} = \dfrac{\sigma_{ba}}{2}$ $\quad$ $\dfrac{3F\times125 - (-F\times125)}{2} = \dfrac{M_{b\,max} - M_{b\,min}}{2}$

$\sigma_{ba} = \dfrac{M_{ba}}{Z}$ $\quad$ $Z \to \dfrac{\pi d^3}{32}$

$\dfrac{M_{bm}}{I} = \dfrac{\sigma_{bm}}{2} \Rightarrow \sigma_{bm} = \dfrac{M_{bm}}{Z}$ ✓

$M_{bm} = \dfrac{M_{bmax} + M_{bmin}}{2}$

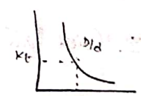
Fig. 2.25 K_t

$K_{tf} = 1 + q(K_t - 1)$

$Z = \dfrac{I}{c} = \dfrac{\pi d^3}{32}$

(Fig. 2.25 Pg. 43)

$\dfrac{r}{d} = \dfrac{3}{13} = 0.2307$.



$\dfrac{D}{d} = \dfrac{20}{13} = 1.5384$.

$\boxed{Z = \dfrac{\pi d^3}{32}}$

$\underline{K_t = 1.4}$.

$\left[\dfrac{M}{I} = \dfrac{\sigma}{c}\right]$

Max. bending moment $= 3F \times 125$.

$= \underline{\underline{375F}}$.

Min. bending moment $= \underline{\underline{-125F}}$.

$\boxed{\dfrac{M_{ba}}{I} = \dfrac{\sigma_{ba}}{C}}$, $\quad$ $\boxed{\dfrac{M_{b\,max} - M_{b\,min}}{2} = M_{ba}}$ $\Rightarrow M_{ba} = \underline{\underline{250F}}$.

$$\sigma_{ba} = \frac{M_{ba}}{I} \times c = \frac{M_{ba}}{Z} = \frac{250F}{\frac{\pi d^3}{32}}$$

$$\sigma_{ba} = 1.159F$$

$$\boxed{\frac{M_{bm}}{I} = \frac{\sigma_{bm}}{c}}, \quad \frac{M_{bmax} + M_{bmin}}{2} = M_{bm} = 125F$$

$$\sigma_{bm} = \frac{M_{bm}}{Z} = \frac{125F}{\frac{\pi d^3}{32}} \Rightarrow \sigma_{bm} = 0.5795F$$

$$\therefore \frac{1.4 \times 1.159F}{1 \times 0.85 \times 0.88 \times 275} + \frac{0.5795F}{470} = \frac{1}{2}$$

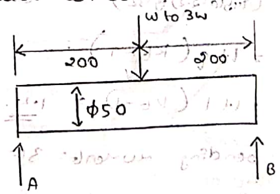
205.7

$$F = 54.8175 \simeq 55N \; (\uparrow)$$

$$3F = 165N \; (\downarrow)$$

---

7). Determine the max. load for the simply supported Beam cyclically loaded as shown in fig. The ultimate strength is 700MPa, the yield point in tension is 520 MPa & endurance limit in reversed bending is 320 MPa. use FOS as 1.25. The load, size & surface correction factors are 1, 0.75, & 0.9, resp.



Ans (yield given so ductile).

Given → $\sigma_u = 700MPa$. $A = 1$; $B = 0.75$; $C = 0.9$

$\sigma_y = 520MPa$.

$\sigma_{en} = 320MPa$.

$n = 1.25$.

Soderberg relationship, (Fig 25)

For ductile, $\frac{K_{fa}\,\tau_a}{ABC\,\tau_{en}} + \frac{\tau_m}{\tau_{yp}} = \frac{1}{n}$ since $K_{ff} = 1$. (static) (since no fig. is given K_f)

(Eq. 15) $\boxed{M_b = \frac{1}{4} W \times l}$

$M_{b\,max} = \frac{1}{4} \times 3W \times l = 300\,W$ ($l = 400$)

$M_{b\,min} = \frac{1}{4} \times W \times l = 100\,W$

$\boxed{M = \frac{1}{4} FL}$

$\tau_{ba} = \frac{M_{ba}}{I} \times C$

$\boxed{\frac{M_{ba}}{I} = \frac{\tau_{ba}}{C}}$

$\frac{I}{C} = Z = \frac{\pi D^3}{32}$

ϕ given so calculate

$\boxed{\frac{M_{bmax} - M_{bmin}}{2} = M_{ba}}$

$Z = \frac{I}{C} = \frac{\pi D^3}{32}$ Fig. 13

$\Rightarrow M_{ba} = 100\,W$

$\therefore \tau_{ba} = \frac{100\,W}{\frac{\pi D^3}{32}} = 8.1487 \times 10^{-3}\,W$

$\frac{M_{bm}}{I} = \frac{\tau_{bm}}{C}$, $\frac{M_{bmax} + M_{bmin}}{2} = M_{bm} = 200\,W$

$\therefore \tau_{bm} = \frac{200\,W}{\frac{\pi D^3}{32}} = 16.2974 \times 10^{-3}\,W$

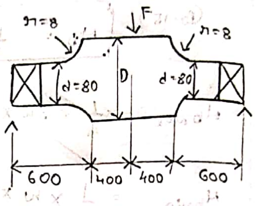
$\tau_y, \tau_e, \tau_{en}$, $n = 1.25$, A, B, C, $W = ?$, $M_{max} \to \frac{1}{4} FL$ (400), $M_{min} \to \frac{1}{4} FL$ Fig (4)

Soderberg, $K_{ff} = 1$.

$\frac{M_{ba}}{I} = \frac{\tau_{ba}}{c} \Rightarrow \tau_{ba} = \frac{M_{ba}}{Z}$, $M_{ba} \to \frac{M_{bmax} - M_{bmin}}{2}$, $Z \to \frac{\pi D^3}{32}$.

7). A non rotating shaft shown in fig. is subjected to a load F, varying from 4000N to 12000N. The shaft material has σ_u = 600 MPa & σ_{en} = 300 MPa. The correction coeff. for surface, size & load are 0.8, 0.85 & 0.9 respectively. Find the dimension 'D' for a FOS of 3.5 & notch sensitivity factor 0.9.

2016



[Since yield stress is not given & material is not specified, we] Goodman's eqn,

4000, 12000
σ_u σ_{en} ABC
D=? n=3.5 q=0.9

for brittle materials,

$$\frac{K_{tf}\,\sigma_a}{ABC\,\sigma_{en}} + K_f \frac{\sigma_m}{\sigma_u} = \frac{1}{n}$$

$\frac{M_{ba}}{I} = \frac{\sigma_{ba}}{c} \Rightarrow \sigma_a = \frac{M_{ba}}{Z}$

$M_{ba} = \frac{M_{bmax} - M_{bmin}}{2}$

static, F/2.

σ_u = 600MPa.

σ_{en} = 300MPa.

F_{max} = 12000N.

F_{min} = 4000N.

A = 0.9

B = 0.85

C = 0.8

q = 0.9

FOS = 3.5.

Since given fig. is symmetric, the reaction $z = \frac{\pi d^3}{32}$

M_{bm}

$$\boxed{\text{force} = \frac{F}{2}}$$

K_{tf}

$K_{tf} = 1 + q(K_t - 1)$

K_t =

r/d

D/d

$M_{bmax} = \frac{F_{max}}{2} \times 600$ (where bending occurs) = 3600 kN-mm

$M_{bmin} = \frac{F_{min}}{2} \times 600$ = 1200 kN-mm.

$M_{bmax} = \frac{F}{2} \times 600$ ($F \to F_{max}$)

$M_{bmin} = \frac{F}{2} \times 600$ ($F \to F_{min}$)

$$\boxed{\frac{M_{ba}}{I} = \frac{\sigma_{ba}}{c}}$$

$$\boxed{\frac{M_{bmax} - M_{bmin}}{2} = M_{ba}}$$

$\Rightarrow \sigma_{ba} = \frac{M_{ba}}{I} \times c$.

$\Rightarrow M_{ba}$ = 1200 kN-mm.

Fig. 2.25, $z = \frac{I}{c} = \frac{\pi d^3}{32}$.

$$\sigma_{ba} = \frac{1200 \times 10^3}{\frac{\pi d^3}{32}} = 23.87 \text{ N/mm}^2.$$

d = 80

$\frac{M_{bm}}{I} = \frac{\sigma_{bm}}{c}$

$\frac{M_{bmax} + M_{bmin}}{2} = M_{bm}$.

$\Rightarrow \sigma_{bm} = \frac{M_{bm}}{I} \times c$

M_{bm} = 2400 kN-mm.

$\sigma_{bm} = \frac{2400 \times 10^3}{\pi d^3/\ldots}$ = 47.74 N/mm².

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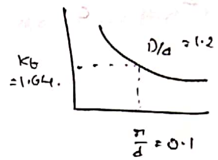
$$\Rightarrow \frac{k_{tf} \times 23.87}{0.9 \times 0.85 \times 0.8 \times 300} + \overset{\text{Assume}}{(1)} \times \frac{47.74}{600} = \frac{1}{3.5}.$$

$$k_{tf} = 1.58.$$

$$k_{tf} = 1 + q(k_t - 1).$$



$$\Rightarrow k_t = 1.64.$$

$$\frac{r}{d} = \frac{8}{80} = 0.1.$$

$$\Rightarrow D/d = 1.2.$$

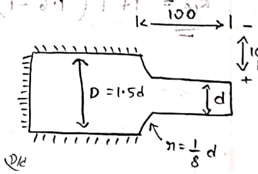
$$\therefore \frac{D}{d} = 1.2.$$

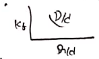
$$\Rightarrow D = 1.2 \times 80 = 96\text{ mm}.$$

---

5). A stepped shaft of a circular c/s shown in fig. is subjected to variable load which is completely reversed with a value = 100 kN. It is made of SAE 1045 steel annealed. Determine the diameter d & radius 'r' so that the max. stress will be limited to a value corresponding to a FOS 2. notch sensitivity index = 1.

circular.
w = 100 kN.
SAE 1045
d & r.
n, q.
soderberg.





k_{tt}.

$$\frac{M_{ba}}{I} = \frac{\sigma_{ba}}{c} \Rightarrow \sigma_{ba} = \frac{M_{ba}}{Z}$$

$100 \times 10^3 \times 100$

$-100k \times 100$

$$M_{ba} = \frac{M_{max} - M_{bm}}{2}$$

$$Z \rightarrow \pi d^3/32.$$

$+ M_{bm}$

$\rightarrow$ SAE 1045, annealed. (Pg. 473).

$\sigma_u = 586\ MN/m^2 = 586\ N/mm^2.$

$\sigma_y = 310\ N/mm^2.$

$\sigma_{en} = 289\ N/mm^2.$ $q = 1$, FOS = 2.

$F_{max} = 100$ kN. $F_{min} = -100$ kN.

Soderberg. doctrine.

$$\frac{k_{tf}\,\sigma_a}{ABC\,\sigma_{en}} + \frac{\sigma_m}{\sigma_{yp}} = \frac{1}{n}.$$

A = 1.

B = 0.85 (assume).

C = (table. 0.2 Pg. 32.)

550	0.88.
586	?
620	0.86.

$$\Rightarrow \frac{586 - 550}{620 - 550} = \frac{x - 0.88}{0.86 - 0.88}$$

$x = 0.86.$//

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$$M_{b_{max}} = 100 \times 100 = 10000 \ \text{kN-mm}$$

$$M_{b_{min}} = -100 \times 100 = -10000 \ \text{kN-mm}$$

$$\frac{M_{ba}}{I} = \frac{\sigma_{ba}}{c} \Rightarrow \sigma_{ba} = \frac{M_{ba}}{I} \times c$$

$$\frac{M_{b_{max}} - M_{b_{min}}}{2} = M_{ba}$$

$$\Rightarrow M_{ba} = 10000 \ \text{kN-m}$$

$$\sigma_{ba} = \frac{10000 \times 10^3}{\frac{\pi d^3}{32}} \rightarrow (\text{Pg } 4.3 \ \text{Pg } 2.25)$$

$$\frac{M_{bm}}{I} = \frac{\sigma_{bm}}{c} \Rightarrow \sigma_{bm} = \frac{M_{bm}}{I} \times c$$

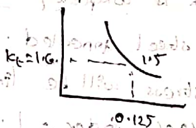
$$\frac{M_{b_{max}} + M_{b_{min}}}{2} = M_{bm}$$

$$\Rightarrow M_{bm} = 0$$

$$\sigma_{bm} = 0$$

$$K_{tf} = 1 + q(K_t - 1)$$

$$K_{tf} = 1 + 1(1.6 - 1) = 1.6$$



$$\frac{r}{d} = \frac{\frac{1}{8}d}{d} = \frac{1}{8} = 0.125$$

$$\frac{D}{d} = \frac{1.5d}{d} = 1.5$$

$$K_t = 1.6$$

$$\therefore \frac{1.6 \times \frac{10000 \times 10^3}{\frac{\pi d^3}{32}}}{1 \times 0.85 \times 0.86} + 0 = \frac{1}{2}$$

Problems on Combined loadings

1). A steel member of circular C/s is subjected to a torsional stress that varies from zero to 35 MPa. & At the same time it is subjected to an axial stress that varies from −14 MPa to +28 MPa. Neglecting stress concentration & column effect & assuming that the max. stresses in torsion & axial load occur at the same time. Determine
i). Maximum equivalent shear stress.
ii). FOS based upon yield in shear.

$\sigma_{en} = 206$ MPa, $\sigma_y = 480$ MPa, $d < 12$ mm, load correction factor $A = 1$
surface correction factor $C = 1$.

→ $\tau_{min} = 0$ $d < 12$, $B = 0.85$
$\tau_{max} = 35$ MPa

$\sigma_{min} = -14$ MPa
$\sigma_{max} = 28$ MPa
neglect stress conc.

$\sigma_a = \frac{\sigma_{max} - \sigma_{min}}{2}$ | $\sigma_m = \frac{\sigma_{max} + \sigma_{min}}{2}$
$\tau_a = \frac{\tau_{max} - \tau_{min}}{2}$ | $\tau_m = \frac{\tau_{max} + \tau_{min}}{2}$
$\tau_{yp} = 0.6\,\sigma_y$

equivalent max. shear stress

$$\tau_{eq(max)} = \sqrt{\left(\frac{1}{2}\sigma_{eq\text{-}n}\right)^2 + (\tau_{eq})^2} = \frac{\tau_{yp}}{n} \quad (eq.\ 2.23(c),\ pg.\ 26)$$

$$\sigma_{eq\text{-}n} = \sigma_m + \left(\frac{\sigma_{yp}}{\sigma_{en}}\right)\frac{k_{tf}\,\sigma_a}{ABC} \quad (eq.\ 2.22(a),\ pg.\ 26).$$

$\sigma_a = \frac{\sigma_{max} - \sigma_{min}}{2}$ ✓ $\sigma_m = \frac{\sigma_{max} + \sigma_{min}}{2}$

$\sigma_a = 21$ N/mm² $\sigma_m = 7$ N/mm².

$$\therefore \sigma_{eq\text{-}n} = 7 + \left(\frac{480}{206}\right)\frac{21}{1 \times 0.85 \times 1} = 64.56\ N/mm^2$$

$$\tau_{eq} = \tau_m + \left(\frac{\tau_{yp}}{\sigma_{en}}\right)\frac{k_{tf}\,\tau_a}{ABC} \quad (eq.\ 2.22(b)\ pg.\ 26).$$

$\tau_a = \frac{\tau_{max} - \tau_{min}}{2}$ $\tau_m = \frac{\tau_{max} + \tau_{min}}{2}$

$= 17.5$ N/mm² $= 17.5$ N/mm².

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$\boxed{\tau_{yp} = 0.6\,\sigma_y}$ $= 0.6 \times 480 = 288\ N/mm^2$

$\therefore \tau_{eq} = 17.5 + \left(\frac{288}{206}\right) \times \frac{17.5}{1 \times 0.85 \times 1} = 46.28\ N/mm^2$

$\therefore \tau_{eq(max)} = \sqrt{\left(\frac{1}{2} \times (64.56)\right)^2 + (46.28)^2}$

$\tau_{eq(max)} = 56.46 = \frac{\tau_{yp}}{n}$

$\therefore 56.46 = \frac{288}{n} \Rightarrow n = 5.1$

---

9). A hot rolled steel shaft is subjected to a torsional load that varies from 330 N-M clockwise to 110 N-M counterclockwise & an applied Bending Moment at the critical section varies from +440 N-M to -220 N-M. The shaft is of uniform c/s & no keyway is present at the critical section. Determine the required shaft diameter. The material has an ultimate strength of 550 MN/m² & yield strength of 410 MN/m². FOS = 1.5. Size & surface correction coefficients are 0.85 & 0.62 respectively. Take the endurance limit as half the ultimate strength.

→ $T_{max} = 330$ N-M.

$T_{min} = -110$ N-M

$M_{bmax} = 440$ N-M.

$M_{bmin} = -220$ N-M.

$\sigma_u = 550\ MN/m^2 = 550\ N/mm^2$.

$\sigma_y = 410\ MN/m^2 = 410\ N/mm^2$.

FOS = 1.5

$B = 0.85$, $C = 0.62$.

Given take endurance limit as half the ultimate strength.

summer 2018 16M

shaft T_{max} T_{min}

M_{bmax} M_{bmin}

$D = ?$

$\sigma_u, \sigma_y, n, B, C$

$\sigma_{en} = \sigma_u/2$

$\frac{M}{I} = \frac{\sigma}{c} \Rightarrow \sigma_{ba} = \frac{M_{ba}}{Z}$

$M_{ba} = \frac{M_{bmax} - M_{bmin}}{2}$

$\sigma_{bm} = \frac{M_{bm}}{Z}$

The equivalent max. shear stress,

$$\tau_{eq(max)} = \sqrt{\left(\frac{1}{2}\sigma_{eq\text{-}n}\right)^2 + (\tau_{eq})^2} = \tau_{yp}/n \quad (\text{eq. } 2.23(a) \text{ Pg. } 26)$$

$$\sigma_{eq\text{-}n} = \sigma_m + \left(\frac{\sigma_{yp}}{\sigma_{en}}\right)\frac{K_{tf}\,\sigma_a}{ABC} \quad (\text{eqn. } 2.22(a) \text{ Pg. } 26)$$

$$\frac{M_{bm}}{I} = \frac{\sigma_{bm}}{C}.$$

$$\frac{M_{bmax} + M_{bmin}}{2} = M_{bm}.$$

$M_{bm} = 110000$ N-mm.

$$\frac{(440-220)\times10^3}{2} = 110000$$

$$\sigma_{bm} = \frac{M_{bm}}{Z} = \frac{110000}{\frac{\pi D^3}{32}} \quad (\text{Pg. } 13)$$

$$\frac{M_{ba}}{I} = \frac{\sigma_{ba}}{C}.$$

$$\frac{M_{bmax} - M_{bmin}}{2} = M_{ba}.$$

$$\sigma_{ba} = \frac{M_{ba}}{C} = \frac{330000}{\frac{\pi D^3}{32}}$$

$M_{ba} = 330000$ N-mm.

(Assume $K_{tf} = 1$.) (A → bending. Pg. 25)

$$\sigma_{eqn} = \frac{110000}{\frac{\pi D^3}{32}} + \left(\frac{410}{0.5\times550}\right)\frac{1\times\frac{330000}{\frac{\pi D^3}{32}}}{1\times0.85\times0.62}.$$

$$\sigma_{eq\text{-}n} = \frac{10.6298\times10^6}{D^3} \text{ N/mm}^2.$$

$$\tau_{eq} = \tau_m + \left(\frac{\tau_{yp}}{\tau_{en}}\right)\frac{K_{sf}\,\tau_a}{ABC}$$

($K_{sf} = 1$ assume) ($A \to 0.5$ Pg. 25. torsional).

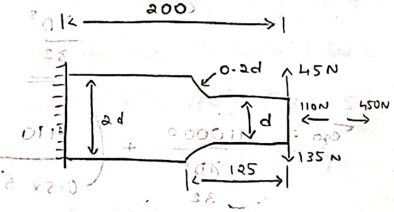
$$\tau = \frac{T}{Z_p}K_s \quad (\text{Pg. } 10.\ 2.1(c))$$

$$T_a = \frac{T_{max} - T_{min}}{2}, \quad T_m = \frac{T_{max} + T_{min}}{2}$$

$$\tau = \frac{T}{Z_p}K_s, \quad Z_p \to \frac{\pi D^3}{32}$$

3) A steel cantilever member shown in fig. is subjected to a transverse load at its end that varies from 45 N up to 135 N down. As an axial load varies from 110 N compression to 450 N tension. Determine the required diameter at the change of section for infinite life using a FOS of 2. The strength properties of the material are $\sigma_u = 550$ MPa, $\sigma_y = 470$ MPa, $\sigma_{en} = 275$ MPa. Notch sensitivity index $= 1$.

→)



$F_{max} = 450N$.

$F_{min} = -110N$.

$M_{b_{min}} = -45 \times 125 = \underline{\underline{-5625 \text{ N-mm}}}$.

$M_{b_{max}} = 135 \times 125 = \underline{\underline{16875 \text{ N-mm}}}$.

$\sigma_u = 550$ MPa, $\sigma_y = 470$ MPa, $\sigma_{en} = 275$ MPa, FOS $= 2$, $q = 1$.

equivalent normal stress,

$$\sigma_{eq\cdot n} = \sigma_m + \left(\frac{\sigma_{yp}}{\sigma_{en}}\right)\frac{k_{tf}\,\sigma_a}{ABC}.$$

Fig 2.23 → axial

Fig. 2.25 → bending

Axial.

$\sigma_m = \dfrac{F_m}{A}$. $\quad F_m = \dfrac{F_{max} + F_{min}}{2} = \underline{\underline{170N}}$.

$\sigma_a = \dfrac{F_{max} - F_{min}}{2}$.

$= \dfrac{170}{\frac{\pi d^2}{4}} = \underline{\underline{\dfrac{216.45}{d^2}}}$.

$\dfrac{M_{ba}}{I} = \dfrac{\sigma_{ba}}{c} \Rightarrow \sigma_{ba} = \dfrac{M_{ba}}{Z}$, $Z = \dfrac{\pi d^3}{32}$

$M_{ba} = \dfrac{M_{b\,max} - M_{b\,min}}{2}$ (135×125, -45×125)

$F_a = \dfrac{F_{max} - F_{min}}{2} = \underline{\underline{280N}}$.

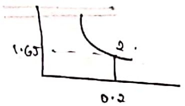
$\sigma_a = \dfrac{F_a}{A} = \dfrac{356.50}{d^2}$ //.

$A = 0.7$ for $\overset{\text{axial}}{\text{bending}}$. (Pg. 25)

$B = 0.85$ (d not given so take middle value in table)

$c = 0.88$. (Pg. 32).

(Pg. 42. 2.23)



$$\frac{r}{d} = \frac{0.2d}{d} = \underline{\underline{0.2}}.$$

$$\frac{D}{d} = \frac{2d}{d} = \underline{\underline{2}}.$$

$\underline{\underline{K_t = 1.65}}$

$K_{tf} = 1 + q\,(K_t - 1)$.

$= 1 + 1\,(1.65 - 1) = \underline{\underline{1.65}}$.

$$\tau_{eqn} = \frac{216.45}{d^2} + \left(\frac{470}{275}\right) \frac{1.65 \times \frac{356.50}{d^2}}{0.7 \times 0.85 \times 0.88}.$$

$\tau_{eqn} = 2136.48 / \underline{\underline{d^2}}\ N/mm^2$.

<u>Bending</u>

$$\frac{M_{bm}}{I} = \frac{\tau_{bm}}{c} \Rightarrow \tau_{bm} = \frac{M_{bm}}{Z}.$$

$$\frac{M_{bmax} + M_{bmin}}{2} = M_{bm}.$$

$$\tau_{bm} = \frac{5625}{Z} = \underline{\underline{\frac{5625}{\frac{\pi d^3}{32}}}}$$

$M_{bm} = \underline{\underline{5625\ N\text{-}mm}}$.

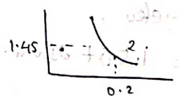
$$\frac{M_{ba}}{I} = \frac{\tau_{ba}}{c} \Rightarrow \tau_{ba} = \frac{M_{ba}}{Z}$$

$$\frac{M_{bmax} - M_{bmin}}{2} = M_{ba}.$$

$M_{ba} = \underline{\underline{11250\ N\text{-}mm}}$.

$$\tau_{ba} = \underline{\underline{\frac{11250}{\frac{\pi d^3}{32}}}}.$$

$$\therefore \tau_{eqn} = \frac{5625}{\frac{\pi d^3}{32}} + \left(\frac{470}{275}\right) \times \frac{1.45 \times \frac{11250}{\frac{\pi d^3}{32}}}{0.7 \times 0.85 \times 0.88}.$$



(Pg. 2.25)
Pg. 43.

$\underline{\underline{K_t = 1.45}}$

$K_{tf} = 1 + q(K_t - 1)$.

$K_{tf} = \underline{\underline{1.45}}$

$$\tau_{eqn} = \underline{\underline{436946.4722}}/d^3\ N/mm^2$$

$\tau_{eqn} = \tau_{eqn\,(axial)} + \tau_{eqn\,(bending)}$.

$235 = 2136.48/d^2 + \dfrac{436946.4722}{d^3}$.

$\therefore d = \underline{\underline{12.5430\ mm}}$.

$\boxed{\tau_{eqn} = \dfrac{\tau_{yp}}{n}} = \underline{\underline{235\ N/mm^2}}.$