

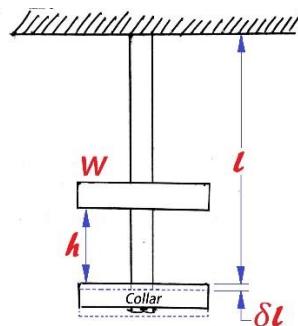
## Module 2

July/August 2021

### 3.a) What is impact stress? (05 Marks)

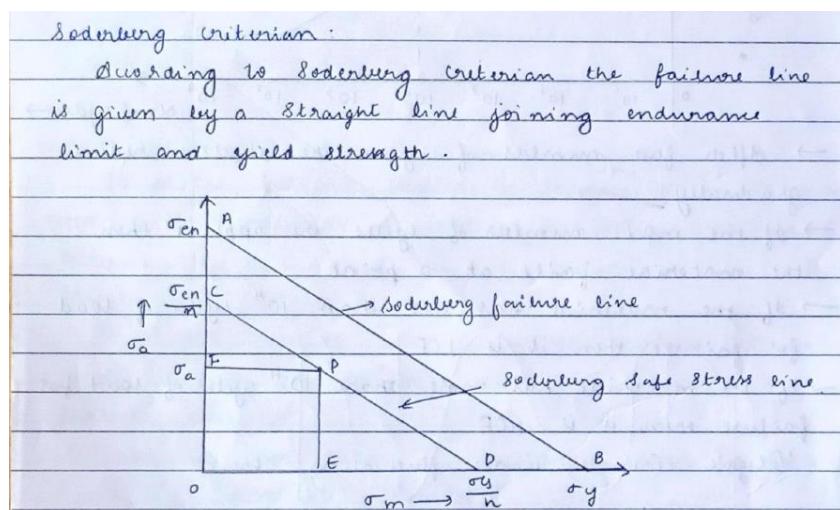
Impact stress, also known as dynamic stress, is the stress that occurs when a body is subjected to a sudden and rapid loading or unloading. It is a type of stress that is caused by the application of a high force or shock to a material or structure in a short period of time.

When an impact force is applied to a material or structure, it creates a stress wave that travels through the material. The stress wave can cause the material to deform, fracture, or even fail completely, depending on the material properties and the magnitude of the impact force. Impact stress can occur in many different situations, such as when a car collides with a barrier, when a hammer strikes a nail, or when a baseball bat hits a ball. Understanding the impact stress that a material or structure can withstand is important for designing and building structures and products that are durable and safe. Figure shows the effect of impact load on an axial bar.



### 4.a) Explain Soderberg and Goodman lines with neat graph. (05 Marks)

Soderberg and Goodman diagrams are graphical representations of the fatigue limit or endurance limit of a material. These diagrams are commonly used in engineering to predict the fatigue life of a material under cyclic loading conditions.



Soderberg Equation under steady load conditions is given by,

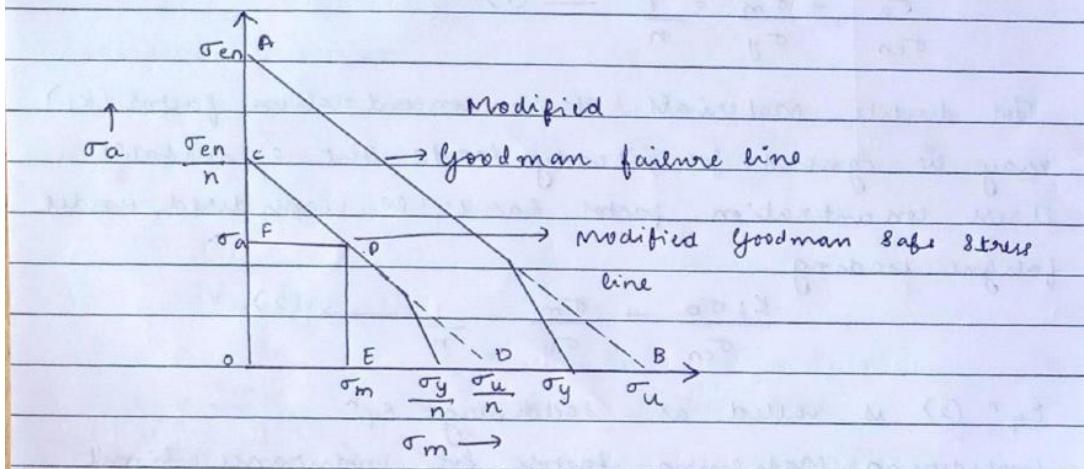
$$\frac{\sigma_a}{\sigma_{en}} + \frac{\sigma_m}{\sigma_y} = \frac{1}{n} \quad (1)$$

For ductile materials stress concentration factor ( $K_t$ ) may be ignored for steady loads but a suitable stress concentration factor has to be considered under fatigue loading.

$$\frac{K_t \sigma_a}{\sigma_{en}} + \frac{\sigma_m}{\sigma_y} = \frac{1}{n} \quad (2)$$

Goodman Criteria :

According to Goodman Criteria a failure line is given by a straight line joining endurance limit and ultimate stress.



Goodman Equation under steady load conditions is given by,

$$\frac{\sigma_a}{\sigma_{en}/n} = 1 - \frac{\sigma_m}{\sigma_u/n}$$

For ductile materials stress concentration factor ( $K_t$ ) may be ignored for steady loads but a suitable stress concentration factor has to be considered under fatigue load.

$$\text{For } \frac{K_t \sigma_a}{\sigma_{en}} + \frac{\sigma_m}{\sigma_u} = \frac{1}{n} \quad (2)$$

For brittle material stress concentration factor applied for both mean stress and stress amplitude

$$\frac{K_t \sigma_a}{\sigma_{en}} + \frac{K_t^2 \sigma_m}{\sigma_u} = \frac{1}{n} \quad (3)$$

3.a) Derive the equation for instantaneous stress due to axial impact on bars. (12 Marks)

Impact stress due to axial load :

consider a bar carrying a load 'W' at a height 'h' and falling on the collar provided at the lower end.

Let  $A$  be the cross sectional area of the bar  $E$  be the young's modulus of material of the bar

$l$  = length of the bar

$\delta e$  = Deformation of the bar

$P$  = Force at which the deflection  $\delta e$  is produced

$\sigma_i$  = Stress induced in the bar due to application of impact load

$h$  = height from which load is acting

WKT, Energy gained by the system in the form

$$\text{strain energy} = \frac{1}{2} P \delta e \quad \text{--- (1)}$$

Potential energy lost by the weight =  $mgh = W(h + \delta e)$  --- (2)

since the energy gained by the system is equal to the potential energy lost by the weight,

$$\text{negt, } \frac{1}{2} P \delta e = W(h + \delta e) \quad \text{--- (3)}$$

$$\text{WKT, } \sigma_i = \frac{P}{A} \Rightarrow P = \sigma_i A$$

$$\delta e = \frac{P e}{A E} \Rightarrow \frac{\sigma_i A e}{A E} \Rightarrow \delta e = \frac{\sigma_i L}{E}$$

$$\frac{1}{2} \frac{\sigma_i A}{E} \frac{\sigma_{il}}{E} = w \left( h + \frac{\sigma_{il}}{E} \right)$$

$$\frac{Al}{2E} (\sigma_i)^2 = wh + \frac{we}{E} \sigma_i$$

$$\frac{Al}{2E} (\sigma_i)^2 - \frac{we \sigma_i}{E} - wh = 0 \quad \text{--- (4)}$$

$$ax^2 + bx + c = 0$$

$$a = \frac{Al}{2E}, \quad b = -\frac{we}{E}, \quad c = -wh.$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\sigma_i = -\left(\frac{we}{E}\right) \pm \sqrt{\left(\frac{we}{E}\right)^2 + \frac{2}{Al} \left(\frac{Al}{2E}\right) (-wh)}$$

$$\frac{\cancel{Al}}{\cancel{2E}}$$

$$\sigma_i = \frac{we}{E} + \sqrt{\left(\frac{we}{E}\right)^2 + \frac{2Alwh}{E}}$$

$$\frac{Al}{E}$$

$$\sigma_i = \frac{we}{E} \times \frac{E}{Al} + \sqrt{\left(\frac{we}{E}\right)^2 \left(\frac{E}{Al}\right)^2 + \frac{2Alwh}{E} \left(\frac{E}{Al}\right)^2}$$

$$= \frac{w}{A} + \sqrt{\left(\frac{w}{A}\right)^2 + \frac{2whe}{Al}} \quad \text{multiply and divide by } \left(\frac{w}{A}\right)^2 + \left(\frac{h}{w}\right)^2$$

$$= \frac{w}{A} + \frac{w}{A} \sqrt{1 + \frac{2hEA}{we}}$$

$$\sigma_i = \frac{w}{A} \left[ 1 + \sqrt{1 + \frac{2hEA}{we}} \right]$$

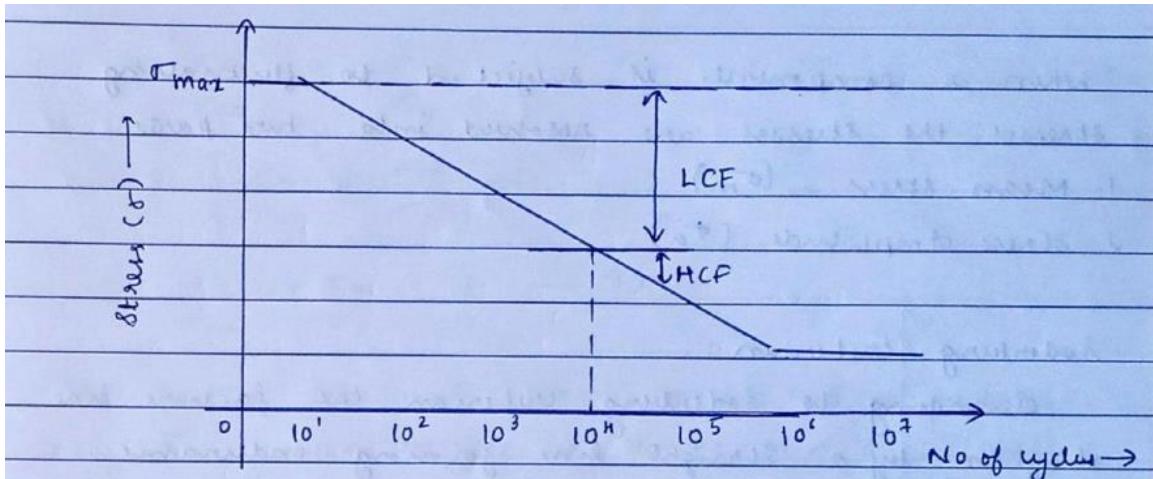
$$\sigma_i = \sigma \left[ 1 + \sqrt{1 + \frac{2h}{\delta E}} \right]$$

$$\sigma_i = \sigma \cdot \text{IF}$$

#### 4.a) Explain with a neat sketch SN diagram and endurance limit. (10 Marks)

The **SN diagram**, is a graphical representation of the relationship between the cyclic stress amplitude ( $\sigma_a$ ) and the number of cycles to failure (N) of a material. The SN diagram is an essential tool in evaluating the fatigue behavior of a material, which is important for designing and manufacturing components and structures that can withstand cyclic loading conditions. The SN diagram is typically represented as a plot of stress amplitude ( $\sigma_a$ ) on the y-axis and the number of cycles to failure (N) on the x-axis, as shown in the following sketch:

→ as the load is applied the stress remains constant at initially upto the limit of proportionality



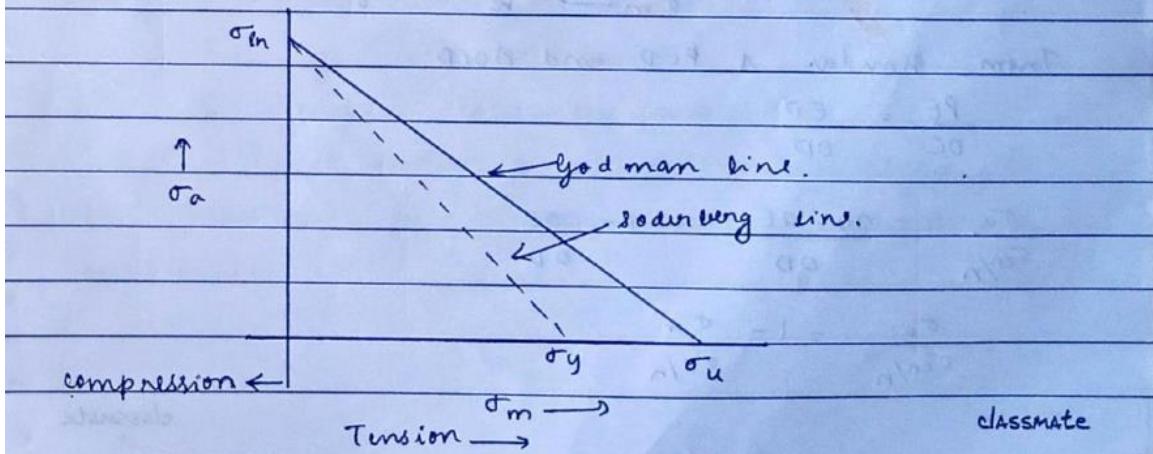
→ after few number of cycles the stress reduces gradually

→ If the max. number of cycles are applied then the material fails at a point

→ If the material has less than  $10^4$  cycles of load for failure then it is LCF

→ If the material has more than  $10^4$  cycle of load for failure then it is HCF

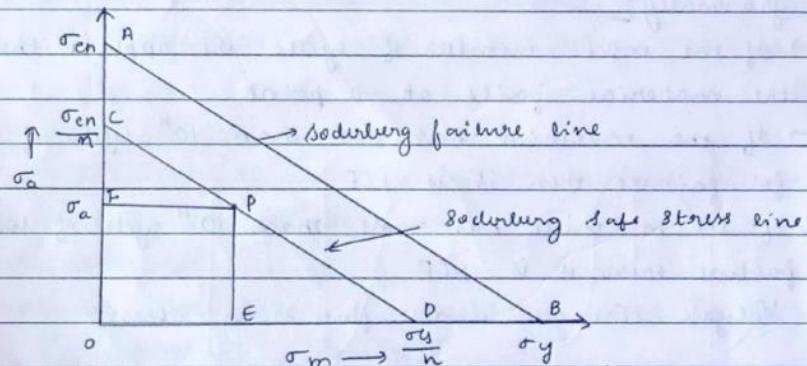
Fatigue strength under fluctuating stress



**The endurance limit** is defined as the maximum stress amplitude that a material can withstand without failing due to fatigue, even after an infinite number of cycles. The fatigue limit is an important parameter that engineers use to design components and structures that will be subjected to cyclic loading conditions. For some materials, such as steel, the endurance limit can be determined experimentally by subjecting a sample to cyclic loading and measuring the stress amplitude at which the material fails. For other materials, such as aluminum, the endurance limit is not well-defined, and the SN diagram shows a continuously decreasing curve, with no distinct fatigue limit. In general, materials with a higher endurance limit are more desirable for applications that involve cyclic loading, as they can withstand more cycles before failure.

#### 4.b) Derive the equation for Soderberg relationship. (10 Marks)

According to Soderberg criterion the failure line is given by a straight line joining endurance limit and yield strength.



From similar triangles PED and DOCD

$$\frac{PE}{OC} = \frac{ED}{OD}$$

$$\frac{\sigma_a}{\sigma_{en}/n} = \frac{OD - OE}{OD} = 1 - \frac{OE}{OD}$$

$$\frac{\sigma_a}{\sigma_{en}/n} = 1 - \frac{\sigma_m}{\sigma_y/n}$$

$$\frac{n \sigma_a}{\sigma_{en}} + \frac{n \sigma_m}{\sigma_y} = 1$$

$$n \left[ \frac{\sigma_a}{\sigma_{en}} + \frac{\sigma_m}{\sigma_y} \right] = 1$$

$$\frac{\sigma_a}{\sigma_{en}} + \frac{\sigma_m}{\sigma_y} = \frac{1}{n} \quad (1)$$

For ductile materials stress concentration factor ( $K_t$ ) may be ignored for steady loads but a suitable stress concentration factor has to be considered under fatigue loading.

$$\frac{K_t \sigma_a}{\sigma_{en}} + \frac{\sigma_m}{\sigma_y} = \frac{1}{n} \quad (2)$$

Eq. (2) is called as Soderberg Eq.

Considering modifying factors for endurance limit we get

$$\frac{K_t \sigma_a}{ABC \sigma_{en}} + \frac{\sigma_m}{\sigma_y} = \frac{1}{n} \quad (3)$$

where A = load collection factor

B = size correction factor

C = surface correction factor

∴ Eq. (3) is known as Modified Soderberg Eq.

Note: Soderberg Eq. is only suitable for ductile materials

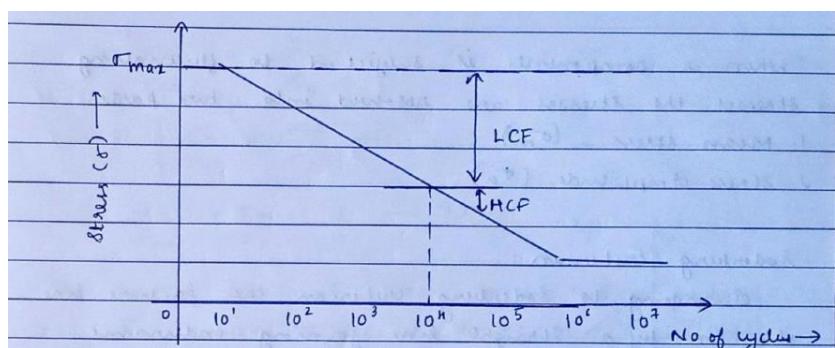
## Feb./Mar. 2022

### 3.a) Define fatigue. With neat sketch explain S-N diagram. (10 Marks)

**Fatigue:** Fatigue is a phenomenon where a material or a structure experiences a gradual and progressive failure under repeated or cyclic loading, even though the stress levels are well below its yield or ultimate strength. Fatigue failure can occur due to a combination of factors, including the presence of defects, changes in the material's microstructure, environmental factors, and applied cyclic loading. Fatigue failure often occurs in components or structures that are subjected to repeated loading cycles, such as aircraft wings, bridges, and rotating machinery.

**SN Diagram:** The SN diagram, is a graphical representation of the relationship between the cyclic stress amplitude ( $\sigma_a$ ) and the number of cycles to failure (N) of a material. The SN diagram is an essential tool in evaluating the fatigue behavior of a material, which is important for designing and manufacturing components and structures that can withstand cyclic loading conditions. The SN diagram is typically represented as a plot of stress amplitude ( $\sigma_a$ ) on the y-axis and the number of cycles to failure (N) on the x-axis, as shown in the following sketch:

→ as the load is applied the stress remains constant at initially upto the limit of proportionality



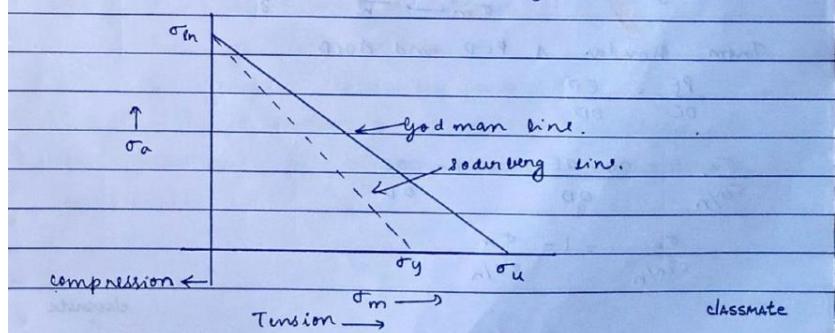
→ after few numbers of cycles the stress reduces gradually

→ if the max. number of cycles are applied then the material fails at a point

→ if the material has less than  $10^4$  cycles of load for failure then it is LCF

→ if the material has more than  $10^4$  cycles of load for failure then it is HCF

Fatigue strength under fluctuating stress



**3.b) What is endurance limit? Explain the important modifying factors effects the endurance limit. (10 Marks)**

#### **Endurance Limit:**

The endurance limit is defined as the maximum stress amplitude that a material can withstand without failing due to fatigue, even after an infinite number of cycles. The fatigue limit is an important parameter that engineers use to design components and structures that will be subjected to cyclic loading conditions.

For some materials, such as steel, the endurance limit can be determined experimentally by subjecting a sample to cyclic loading and measuring the stress amplitude at which the material fails. For other materials, such as aluminum, the endurance limit is not well-defined, and the SN diagram shows a continuously decreasing curve, with no distinct fatigue limit.

In general, materials with a higher endurance limit are more desirable for applications that involve cyclic loading, as they can withstand more cycles before failure.

#### **Modifying Factors Effecting Endurance Limit:**

Several factors can modify the endurance limit, making it essential to consider these factors when designing and testing materials. Some of the most important modifying factors that affect the endurance limit are:

- a) ***Loading frequency:*** The endurance limit decreases as the frequency of cyclic loading increases. At high frequencies, the material experiences more cycles of loading and unloading, leading to faster crack growth and reduced endurance limit.
- b) ***Size effect:*** The size of the component or structure can also affect the endurance limit. Smaller components tend to have a lower endurance limit than larger components.
- c) ***Surface finish:*** The surface finish of a material affects its resistance to fatigue failure. Rough surfaces can initiate and accelerate crack growth, reducing the material's endurance limit. Smooth surfaces reduce the likelihood of crack initiation, thereby increasing the endurance limit.
- d) ***Notches and surface treatments:*** Notches and surface treatments, such as shot peening, can increase the endurance limit by reducing stress concentrations and inducing compressive residual stresses in the material.
- e) ***Mean stress:*** The endurance limit is affected by the level of mean stress present in the cyclic loading. Materials subjected to cyclic loading with a high mean stress level have a reduced endurance limit.

- f) **Material defects:** Material defects, such as voids, inclusions, or cracks, can act as stress concentration points, leading to premature failure. The presence of such defects reduces the material's endurance limit.
- g) **Environmental conditions:** Environmental factors, such as temperature, humidity, and corrosive agents, can accelerate the crack growth rate, reducing the endurance limit.

**4.a) Define fluctuating stress. Explain the Goodman and Soderberg relationship with relevant sketches and equations. (10 Marks)**

**Fluctuating Stress:**

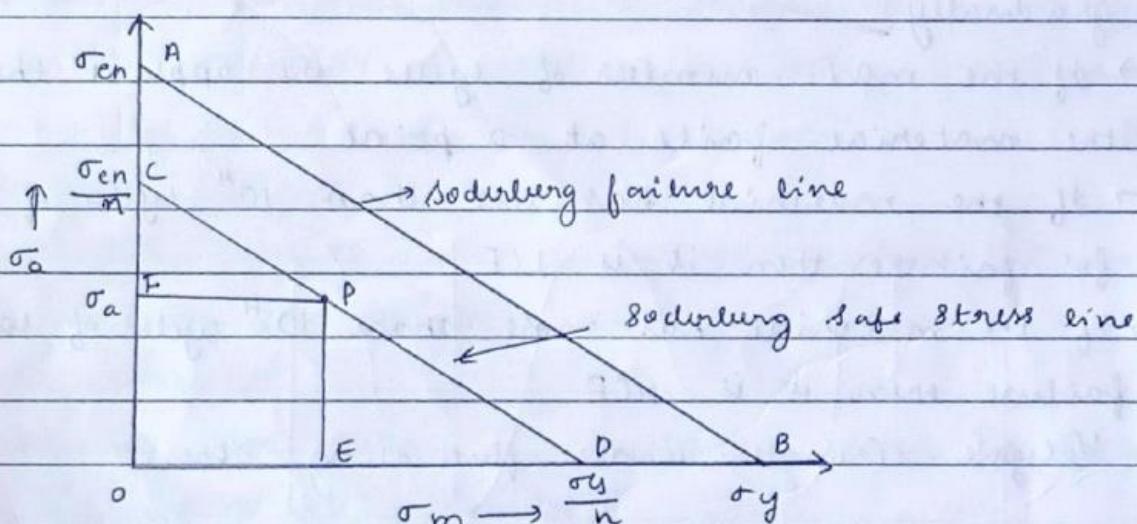
Fluctuating stress, also known as alternating stress, is a type of stress that varies in magnitude over time, as opposed to static stress, which remains constant. Fluctuating stress occurs when a material or structure is subjected to cyclic loading, such as in the case of a rotating shaft or a bridge deck that is exposed to vehicular traffic. The magnitude of the fluctuating stress varies between a minimum value and a maximum value, with the difference between the two values known as the stress amplitude.

**Goodman and Soderberg Relationships:**

Soderberg and Goodman diagrams are graphical representations of the fatigue limit or endurance limit of a material. These diagrams are commonly used in engineering to predict the fatigue life of a material under cyclic loading conditions.

*Soderberg criterian :*

*According to Soderberg criterian the failure line is given by a straight line joining endurance limit and yield strength .*



Soderberg Equation under steady load conditions is given by,

$$\frac{\sigma_a}{\sigma_{en}} + \frac{\sigma_m}{\sigma_y} = \frac{1}{n} \quad (1)$$

For ductile materials stress concentration factor ( $K_t$ ) may be ignored for steady loads but a suitable stress concentration factor has to be considered under fatigue loading.

$$\frac{K_t \sigma_a}{\sigma_{en}} + \frac{\sigma_m}{\sigma_y} = \frac{1}{n} \quad (2)$$

Eq<sup>n</sup> (2) is called as Soderberg Eq<sup>n</sup> considering modifying factors for endurance limit we get

$$\frac{A B C \sigma_a}{\sigma_{en}} + \frac{\sigma_m}{\sigma_y} = \frac{1}{n} \quad (3)$$

where A = load collection factor

B = size correction factor

C = surface correction factor

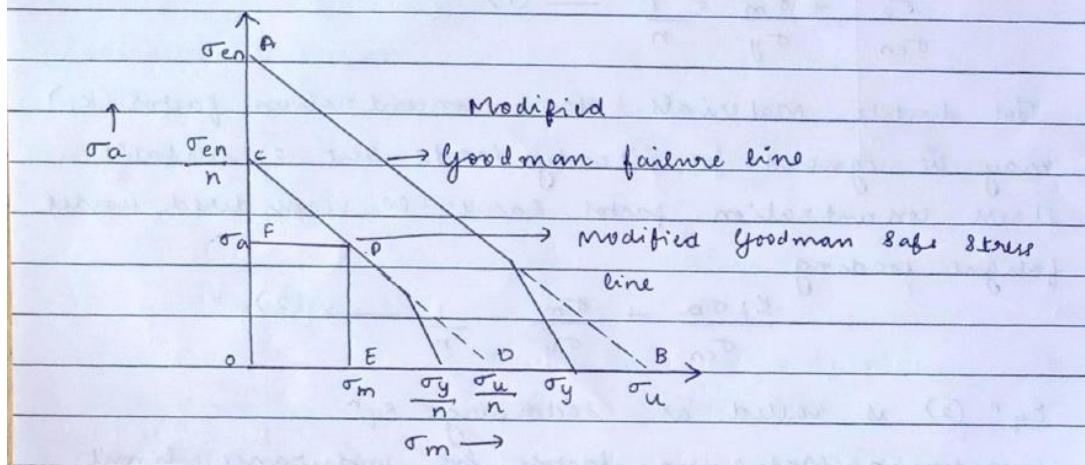
∴ Eq<sup>n</sup> (3) is known as modified Soderberg Eq<sup>n</sup>

Note: Soderberg Eq<sup>n</sup> is only suitable for ductile materials

Any combination of mean stress and alternating stress that falls below the Soderberg line represents a safe operating region for the material. Any combination of stresses that falls above the Soderberg line may cause the material to yield or fail.

Goodman Criteria :

According to Goodman Criteria a failure line is given by a straight line joining endurance limit and ultimate stress.



Goodman Equation under steady load conditions is given by,

$$\frac{\sigma_a}{\sigma_{en/n}} = 1 - \frac{\sigma_m}{\sigma_u/n}$$

For ductile materials stress concentration factor ( $K_t$ ) may be ignored for steady loads but a suitable stress concentration factor has to be considered under fatigue load

$$\text{For } \frac{K_t \sigma_a}{\sigma_{en}} + \frac{\sigma_m}{\sigma_u} = \frac{1}{n} \quad (2)$$

For Brittle material stress concentration factor applied for both mean stress and stress amplitude

$$\frac{K_t \sigma_a}{\sigma_{en}} + \frac{K_t' \sigma_m}{\sigma_u} = \frac{1}{n} \quad (3)$$

Considering Modifying factor for load size and surface for the endurance limit we get,

$$\text{Ductile } \frac{K_t \sigma_a}{ABC \sigma_{en}} + \frac{\sigma_m}{\sigma_u} = \frac{1}{n} \quad (4)$$

$$\text{Brittle } \frac{K_t \sigma_a}{ABC \sigma_{en}} + \frac{K_t' \sigma_m}{\sigma_u} = \frac{1}{n} \quad (5)$$

Any combination of mean stress and alternating stress that falls below the Goodman line represents a safe operating region for the material. However, any combination of stresses that falls above the Goodman line may cause the material to fail due to fatigue.

In general, the Soderberg and Goodman diagrams are used to evaluate the fatigue life of a material under cyclic loading conditions.

#### 4.c) Define stress concentration and explain the same. (04 Marks)

Stress concentration:

It is a point where the stress is significantly greater than its surrounding area

Stress concentration factor:

It is the ratio of highest stress in the body to the reference stress (Normal stress) It is the dimension factor that is used to quantify how concentrated the stress is in a mechanical part

$$K_t = \frac{\sigma_{max}}{\sigma_{ref}}$$

### Characteristic of stress concentration:

- Function of the geometry or shape of the part but not its size or material composition
- The type of load applied to the part
- The specific geometric stress raised (flute, radius, notch, hole) in the part
- always defined with respect to the typically particular nominal stress
- Typically assumes a linear elastic, homogeneous a isotropic material.