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MODULE-1:

BASICS OF LINEAR ELASTICITY, CONCEPT OF STRESS AND STRAIN.

★ STRESS: is defined as resisting force per unit cross-sectional area. The unit of stress in metric system is N/mm^2.

1. NORMAL STRESS :

When the external force is perpendicular to the cross-section of the body, the induced stress is known as Normal stress.

Two Types. — Tensile stress
Compressive stress.

i) TENSILE STRESS :-

P is force.

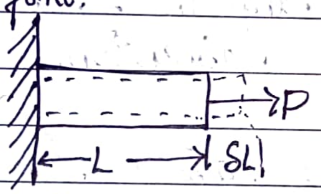


Fig. 1.1.

It is the stress that causes extension of the body as shown in fig 1.1.

Mathematically tensile stress

$$\sigma_t = +\frac{P}{A} \ N/mm^2.$$

ii) COMPRESSIVE STRESS :-

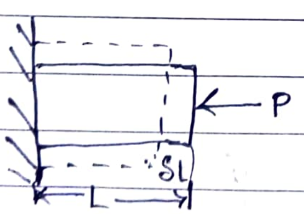


Fig. 2.2.

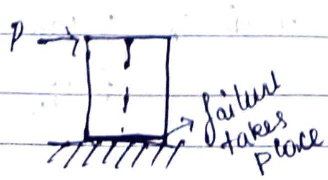
It is the stress that causes contraction of the body as shown in fig 2.2.

Mathematically Compressive stress

$$\sigma_c = -\frac{P}{A} \ N/mm^2.$$

2) SHEAR STRESS :-

It is the stress that causes distortion of the body, as shown in fig 1.3.



Mathematically shear stress

$$\tau = \frac{P}{A}$$

STRAIN:- It is measure of deformation in the body as a result of applied stress.
Mathematically strain is defined as ratio of change in dimension to original dimension.

i) **LINEAR STRAIN:-** Also called as Longitudinal strain,

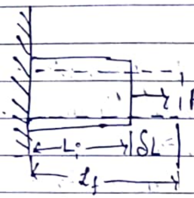
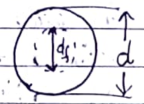


fig. 1.4.



$\delta d = d_i - d_f$

fig 1.6.

★ It is the ratio of change in Linear dimension to original linear dimension.

★ ii) **LATERAL STRAIN:-** It is the ratio of change in Lateral dimension to original lateral dimension

★ With reference to figure 1.4 Linear strain Epsalon.

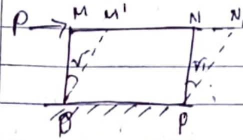
$$\epsilon_{lin} = \frac{\delta L}{L} = \frac{L_f - L_i}{L_i}$$

• Lateral Strain $\epsilon_{lat} = \frac{\delta d}{d} = \frac{d_i - d_f}{d_i}$

POISON'S RATIO :-> It is the property of the material which is defined as the ratio of lateral strain to linear strain.
Mathematically poison's ratio $\gamma = \frac{\epsilon_{lat}}{\epsilon_{lin}}$

No unit.

iii) **SHEAR STRAIN:-** It is the measure of angle through which a body undergoes distortion when it is subjected to shear stress.



Distortion. fig 1.6 Let m.n.o.p be a plane element subjected to shear force as shown in fig 1.6

M' -> M prime

Let MN'OP be the distored Shape.
From the figure $\tan\gamma = \frac{NN'}{NP}$

When γ is very small $\tan\gamma$ is approximately equal to γ. Therefore $\tan\gamma \simeq \gamma$.
Shear strain $\gamma = \frac{NN'}{NP}$.

1/6/23. **UNITS OF STRESS:-**

★ In Metric System the unit of stress is N/mm^2.
★ In SI System, the unit of stress N/m^2 or Pascal.

$$1MPa = 10^6 N/m^2$$

$$= \frac{10^6 N}{(10^3 mm)^2} = \frac{10^6 N}{10^6 mm^2}$$

★ $1MPa = 1 N/mm^2$.

★ **STATE OF STRESS AT A POINT:-**

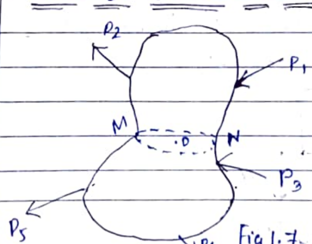
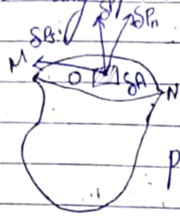


Fig 1.7.

★ Consider an arbitarily shaped 3-dimensional body which is in equilibrium under the action of forces P_1, P_2, P_3 etc....

Let MN be any plane passing through point O.



Let δA be the infinitesimally in area over point O.

Fig - 1.8

Let ∂P be the resultant force is distributed over area δA.

The resultant force can be resolved into 2 Components

a) Normal Component (δP_n)

b) Tangential Component (δP_s)

NORMAL STRESS AT POINT O.

$$\sigma = \lim_{\delta A \to 0} \frac{\delta P A}{\delta A}$$

Shear Stress at Point O.

$$\tau = \lim_{\delta A \to 0} \frac{\delta P_s}{\delta A}$$

RESULTANT STRESS AT POINT O IS EQUAL TO :-

$$R = \sqrt{\sigma^2 + \tau^2}$$

STATE OF STRAIN AT A POINT :- (STRAIN DISPLACEMENT RELATIONS)

Consider a 2D element of dimensions ∂x & ∂y as shown in the figure

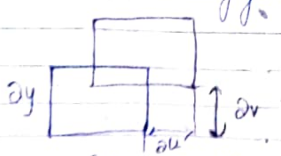


Fig 1.9.

* Let ∂u & ∂v be the displacements of element is x & y directions respectively.

* NORMAL STRAIN in X direction

$$\epsilon_x = \frac{\partial u}{\partial x} \rightarrow (1)$$

* NORMAL STRAIN IN Y DIRECTION.

$$\epsilon_y = \frac{\partial v}{\partial y} \rightarrow (2)$$

* Similarly Normal Strain in Z direction

$$\epsilon_z = \frac{\partial w}{\partial z} \rightarrow (3)$$

*

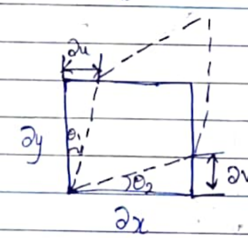


Fig : 1.10.

From fig 1.10

$\tan\theta_1 \simeq \theta_1 = \frac{\partial u}{\partial y}$

$\tan\theta_2 \simeq \theta_2 = \frac{\partial v}{\partial x}$

Shear Strain γ_{xy} is $= \theta_1 + \theta_2$.

$$\gamma_{xy} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \text{ --- } (4).$$

Similarly $\gamma_{yz} = \frac{\partial v}{\partial z} + \frac{\partial w}{\partial y} \rightarrow (5)$

$$\gamma_{xz} = \frac{\partial u}{\partial z} + \frac{\partial w}{\partial x} \rightarrow (6).$$

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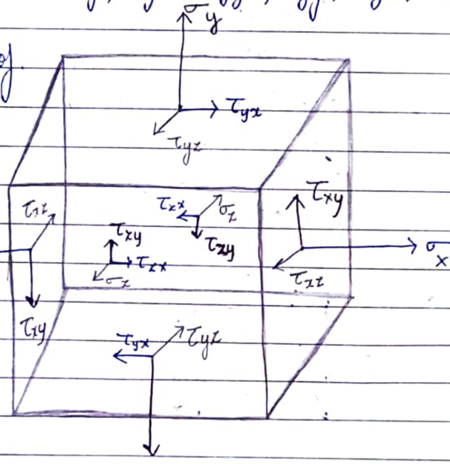
* Body Force: It is a force that acts on entire volume of the Body.
Eg: Gravitational force, Magnetic force etc..
Mathematically Body force is given by $f = \frac{F}{V}$ $\frac{N}{mm^3}$
where, F is Net force & V is Volume of the body.

* Stresses In 3-D.

Figure shows a 3-dimensional body subjected to normal stress $\sigma_x, \sigma_y, \sigma_z$.
Shear stress $\tau_{xy}, \tau_{yx}, \tau_{yz}, \tau_{zy}, \tau_{zx}, \tau_{xz}$

From Equality of cross shear
$\tau_{ij} = \tau_{ji}$

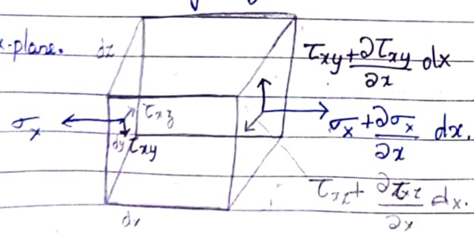


→ $\tau_{xy} = \tau_{yx}$
→ $\tau_{yz} = \tau_{zy}$
→ $\tau_{xz} = \tau_{zx}$

** Equilibrium Equation's In 3-D.

Consider an infinitesimally small 3-dimensional element of dimension's dx, dy & dz.

stresses in x-plane. dydz



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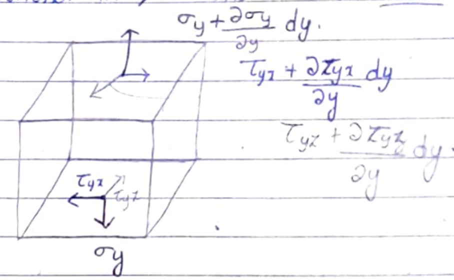
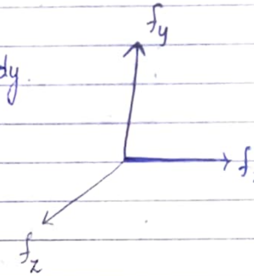
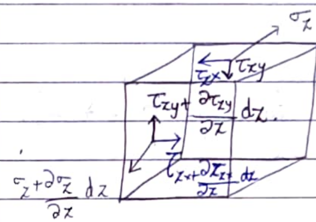
Stresses in Y-Plane. → Y → dxdz



Stresses in Z-plane. → dxdy





Let the element be subjected to $\sigma_x, \sigma_y, \sigma_z$. Shear stresses $\tau_{xy}, \tau_{yz}, \tau_{zx}$ & body forces F_x, F_y & F_z.
For the equilibrium of the element the conditions are:
$\Sigma F_x = 0$, $\Sigma F_y = 0$, $\Sigma F_z = 0$.
Considering $\Sigma F_x = 0$.

→ $\left[\cancel{\sigma_x} + \frac{\partial \sigma_x}{\partial x} dx\right] dydz - \cancel{\sigma_x} dydz + \left[\cancel{\tau_{yx}} + \frac{\partial \tau_{yx}}{\partial y} dy\right] dxdz$
$- \cancel{\tau_{yx}} dxdz + \left[\cancel{\tau_{zx}} + \frac{\partial \tau_{zx}}{\partial z} dz\right] dxdy - \cancel{\tau_{zx}} dxdy + f_x\, dxdydz = 0$

→ $\left(\frac{\partial \sigma_x}{\partial x} + \frac{\partial \tau_{yx}}{\partial y} + \frac{\partial \tau_{zx}}{\partial z} + f_x\right) dxdydz = 0.$

$\because dxdydz \neq 0.$

$$\frac{\partial \sigma_x}{\partial x} + \frac{\partial \tau_{yx}}{\partial y} + \frac{\partial \tau_{zx}}{\partial z} + f_x = 0 \longrightarrow (1).$$

$\Sigma F_y = 0.$

$$\left[\tau_{xy} + \frac{\partial \tau_{xy}}{\partial x} dx\right] dy\,dz - \tau_{xy}\, dy\,dz$$
$$+ \left[\sigma_y + \frac{\partial \sigma_y}{\partial y} dy\right] dx\,dz - \sigma_y\, dx\,dz.$$
$$+ \left[\tau_{zy} + \frac{\partial \tau_{zy}}{\partial z} dz\right] dx\,dy - \tau_{zy}\, dx\,dy + f_y\, dx\,dy\,dz = 0.$$

$$= \left[\frac{\partial \tau_{xy}}{\partial x} + \frac{\partial \sigma_y}{\partial y} + \frac{\partial \tau_{zy}}{\partial z} + f_y\right] dx\,dy\,dz = 0.$$

$\because dx\,dy\,dz \neq 0.$

$$\frac{\partial \tau_{xy}}{\partial x} + \frac{\partial \sigma_y}{\partial y} + \frac{\partial \tau_{xy}}{\partial z} + f_y = 0 \rightarrow (2).$$

Considering $\Sigma F_z = 0.$

$$\left[\tau_{xz} + \frac{\partial \tau_{xz}}{\partial x} dx\right] dy\,dz - \tau_{xz}\, dy\,dz$$
$$+ \left[\tau_{yz} + \frac{\partial \tau_{yz}}{\partial y} dy\right] dx\,dz - \tau_{yz}\, dx\,dz.$$
$$+ \left[\sigma_z + \frac{\partial \sigma_z}{\partial z} dz\right] dx\,dy - \sigma_z\, dx\,dy + f_z\, dx\,dy\,dz = 0.$$

$$= \left[\frac{\partial \tau_{xz}}{\partial x} + \frac{\partial \tau_{yz}}{\partial y} + \frac{\partial \sigma_z}{\partial z} + f_z\right] dx\,dy\,dz = 0.$$

$\because dx\,dy\,dz \neq 0.$

$$\frac{\partial \tau_{xz}}{\partial x} + \frac{\partial \tau_{yz}}{\partial y} + \frac{\partial \sigma_z}{\partial z} + f_z = 0 \rightarrow (3).$$

Eqn (1) (2) & (3) represents stress Equilibrium Eqn in 3-D, In 2-D the Equilibrium equations reduces to

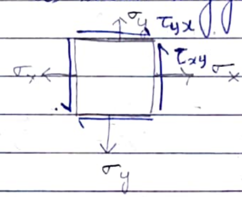
$$\left.\begin{aligned} \frac{\partial \sigma_x}{\partial x} + \frac{\partial \tau_{xy}}{\partial y} + f_x &= 0. \\ \frac{\partial \tau_{xy}}{\partial x} + \frac{\partial \sigma_y}{\partial y} + f_y &= 0. \end{aligned}\right\} \text{2-D}$$

* PLANE STRESS:-

It is state of stress in which normal & shear stress components perpendicular to z plane or assumed to be zero.

This implies $\Rightarrow \sigma_z = \tau_{xz} = \tau_{yz} = 0.$ (plane direction)

Ex: Thin planar element subjected to inplane loading as shown in the figure.



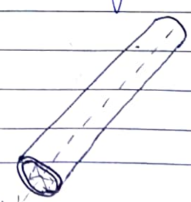
* PLANE STRAIN:-

It is a state of strain in which normal & shear strain components perpendicular to z plane are assumed to be zero.

$\gamma \Rightarrow$ gamma

$\Rightarrow \epsilon_z = \gamma_{xz} = \gamma_{yz} = 0.$

Ex: Long Hallow Cylinder Subjected to inplane loading.



$$\epsilon_z = \frac{\Delta z}{z} \simeq 0.$$

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St. Venant's Strain Compatibility Relations:-

Strain Compatability Eqn's establishes relationship b/w normal & shear strain components.

* Normal strain in x-direction

$$\epsilon_x = \frac{\partial u}{\partial x}$$, where u is displacement in x-direction.

Differentiating twice w.r.t 'y'.

$$\frac{\partial \epsilon_x}{\partial y} = \frac{\partial^2 u}{\partial x \partial y} \Rightarrow \frac{\partial^2 \epsilon_x}{\partial y^2} = \frac{\partial^3 u}{\partial x \partial y^2} \rightarrow (1)$$

* Normal strain in Y-direction.

$$\epsilon_y = \frac{\partial v}{\partial y}$$, where v is displacement in y-direction.

Diff. twice w.r.t 'x'.

$$\frac{\partial \epsilon_y}{\partial x} = \frac{\partial^2 v}{\partial x \partial y} \Rightarrow \frac{\partial^2 \epsilon_y}{\partial x^2} = \frac{\partial^3 v}{\partial x^2 \partial y} \rightarrow (2)$$

Shear strain in x-y plane.

$$\gamma_{xy} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}$$

* Diff w.r.t 'x'.

$$\frac{\partial \gamma_{xy}}{\partial x} = \frac{\partial^2 u}{\partial x \partial y} + \frac{\partial^2 v}{\partial x^2}$$

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* Diff w.r.t 'y'

$$\frac{\partial^2 \gamma_{xy}}{\partial x \partial y} = \frac{\partial^3 u}{\partial y^2 \partial x} + \frac{\partial^3 v}{\partial x^2 \partial y} \rightarrow (3)$$

From Eqn (1), (2) & (3)

$$\frac{\partial^2 \gamma_{xy}}{\partial x \partial y} = \frac{\partial^2 \epsilon_x}{\partial y^2} + \frac{\partial^2 \epsilon_y}{\partial x^2} \rightarrow (A).$$

Simillarly, $$\frac{\partial^2 \gamma_{yz}}{\partial y \partial z} = \frac{\partial^2 \epsilon_y}{\partial z^2} + \frac{\partial^2 \epsilon_z}{\partial y^2} \rightarrow (B)$$

$$\frac{\partial^2 \gamma_{xz}}{\partial x \partial z} = \frac{\partial^2 \epsilon_x}{\partial z^2} + \frac{\partial^2 \epsilon_z}{\partial x^2} \rightarrow (C)$$

Eqn's A, B & C are first set of strain Compatability Eqn's.

Shear strain in x-y-plane $\gamma_{xy} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}$

* Diff. w.r.t 'z'.

$$\frac{\partial \gamma_{xy}}{\partial z} = \frac{\partial^2 u}{\partial y \partial z} + \frac{\partial^2 v}{\partial x \partial z} \rightarrow (4)$$

Shear Strain in y-z plane

$$\gamma_{yz} = \frac{\partial v}{\partial z} + \frac{\partial w}{\partial y}.$$

Diff w.r.t 'x'.

$$\frac{\partial \gamma_{yz}}{\partial x} = \frac{\partial^2 v}{\partial x \partial z} + \frac{\partial^2 w}{\partial x \partial y} \rightarrow (5)$$

Di Shear strain in X-Z plane.

$$\gamma_{xz} = \frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}$$

diff. w.r.t 'y'

$$\frac{\partial \gamma_{xz}}{\partial y} = \frac{\partial^2 u}{\partial y \partial z} + \frac{\partial^2 w}{\partial x \partial y} \rightarrow (6)$$

Adding Eq^n's (5) & (6) & Sub (4).

(5) + (6) − (4).

$$\frac{\partial \gamma_{yz}}{\partial x} + \frac{\partial \gamma_{xz}}{\partial y} - \frac{\partial \gamma_{xy}}{\partial z} = \frac{2\partial^2 w}{\partial x \partial y}$$

Diff. w.r.t 'z'.

$$\frac{\partial}{\partial z}\left[\frac{\partial \gamma_{yz}}{\partial x} + \frac{\partial \gamma_{xz}}{\partial y} - \frac{\partial \gamma_{xy}}{\partial z}\right] = \frac{\partial}{\partial z}\left[\frac{2\partial^2 w}{\partial x \partial y}\right].$$

$$\frac{\partial}{\partial z}\left[\frac{\partial \gamma_{yz}}{\partial x} + \frac{\partial \gamma_{xz}}{\partial y} - \frac{\partial \gamma_{xy}}{\partial z}\right] = \frac{2\partial^3 w}{\partial x \partial y \partial z} \rightarrow (7)$$

Normal strain in z-direction.

$$\epsilon_z = \frac{\partial w}{\partial z}$$

diff w.r.t 'x'.

$$\frac{\partial \epsilon_z}{\partial x} = \frac{\partial^2 w}{\partial x \partial z}$$

diff. w.r.t 'y'

$$\frac{\partial^2 \epsilon_z}{\partial x \partial y} = \frac{\partial^3 w}{\partial x \partial y \partial z} \rightarrow (8)$$

Substituting Eq^n (8) in Eq^n (7)

$$\frac{\partial}{\partial z}\left[\frac{\partial \gamma_{yz}}{\partial x} + \frac{\partial \gamma_{xz}}{\partial y} - \frac{\partial \gamma_{xy}}{\partial z}\right] = \frac{2\partial^2 \epsilon_z}{\partial x \partial y} \rightarrow (D).$$

Similarly $\frac{\partial}{\partial x}\left[\frac{\partial \gamma_{xy}}{\partial z} + \frac{\partial \gamma_{xz}}{\partial y} - \frac{\partial \gamma_{yz}}{\partial x}\right] = \frac{2\partial^2 \epsilon_x}{\partial y \partial z} \rightarrow (E).$

$$\frac{\partial}{\partial y}\left[\frac{\partial \gamma_{xy}}{\partial z} + \frac{\partial \gamma_{yz}}{\partial x} - \frac{\partial \gamma_{xz}}{\partial y}\right] = \frac{2\partial^2 \epsilon_y}{\partial x \partial z} \rightarrow (F)$$

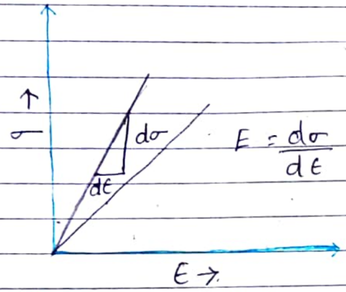
Eq^n's D, E & F are Second set of strain Compatability relation's.

Hooke's Law

* Hooke's Law States that stress is propotional to strain within elastic limit.
* The ratio of stress to strain within elastic limit is characteristic of the material.
* From Hooke's Law within elastic limit $\sigma \propto \epsilon$

  $\sigma = E\epsilon$ $E \rightarrow$ propotional Constant

  known as Young's Modulus.
* Modulus of elasticity is the slope of linear region of stress-strain diagram as shown in diagram.



$$E = \frac{d\sigma}{d\epsilon}$$

However present-day experiment's show that for mild steel Hooke's Law hold's good upto proportionality Limit which is very close to Elastic limit.

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STRESS - STRAIN CURVE FOR DUCTILE MATERIALS :-

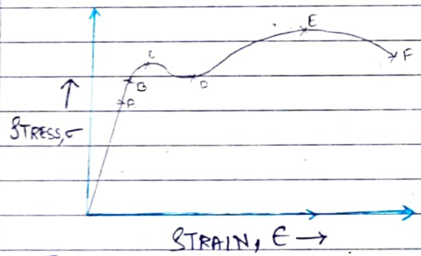

Figure show's stress-strain curve for a typical mild steel specimen. The following salient points are observed in the stress-strain curve.

(i) Proportionality Limit (A) :- It is the limiting value of stress upto which stress is propotional to strain.

(ii) ELASTIC Limit (B) :- It is the limitting value of stress upto which upon the release of stress the material retains it's original dimensions.

(iii) Upper Yield point (C) :- It is the stress at which the force starts reducing with increase in extension. This phenomenon is called as yielding.

→ For mild steel this stress is about 250 N/mm^2.

(iv) Lower Yield point (D) :- It is the constant value of stress at which the strain increases for sometime.

(v) Ultimate Stress (E) :- It is the maximum value of stress the material can withstand.

Necking takes place at this point, for mild steel this stress is about (370 – 400) N/mm^2.

(vi) Breaking Stress (F) :- It is the stress at which finally the specimen fails.

STRESS - STRAIN CURVE FOR BRITTLE MATERIAL :

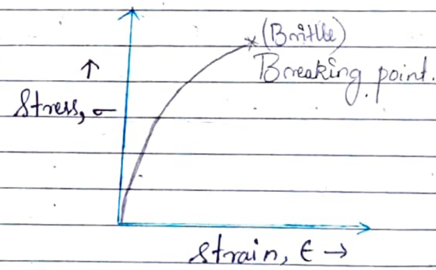

fig show's stress-strain curve for a material (Brittle) like cast iron.

The following Salient features are observed:-

i) There is no appreciable change in rate of strain

ii) There is no yield point & no necking takes place.

iii) The ultimate stress coincides with breaking Stress

iv) The strain at failure is very small.

→ NUMERICAL PROBLEMS:- (ON STRAIN DISPLACEMENT RELATIONS)

Nomenclature:-

→ u, v, w → displacements in X, Y & Z directions (respectively)
→ U → resultant displacement.
→ $\epsilon_x, \epsilon_y, \epsilon_z$ → Normal strain in X, Y & Z directions
→ $\gamma_{xy}, \gamma_{yz}, \gamma_{zx}$ → shear strain in xy, yz & zx plane respectively

★ LIST OF FORMULAE:-

i) Resultant displacement $U = ui + vj + wk$ (vector's displacement)

2) Magnitude of Resultant displacement $U = \sqrt{u^2+v^2+w^2}$

3) Normal strains, $\epsilon_x = \frac{\partial u}{\partial x}$, $\epsilon_y = \frac{\partial v}{\partial y}$, $\epsilon_z = \frac{\partial w}{\partial z}$

4) Shear strains, $\gamma_{xy} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}$

$\gamma_{yz} = \frac{\partial v}{\partial z} + \frac{\partial w}{\partial y}$ $\quad \gamma_{xz} = \frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}$

5) Strain Components in matrix form:- $\epsilon_{ij} = \begin{bmatrix} \epsilon_x & \gamma_{xy} & \gamma_{xz} \\ \gamma_{xy} & \epsilon_y & \gamma_{yz} \\ \gamma_{zx} & \gamma_{yz} & \epsilon_z \end{bmatrix}$

PROBLEMS:-

1. Consider the displacement field $U = [y^2 i + 3yz j + (4+6x^2)k]10^{-2}$ → ①
Determine the strain Components at the points P (1,0,2)

Sol^n Given displacement field $U = [y^2 i + 3yz j + (4+6x^2)k]10^{-2}$
displacement field is generally expressed as
$U = ui + vj + wk$ → ②.

Comparing Eq's ① & ②
$u = y^2 \times 10^{-2}$, $v = 3yz \times 10^{-2}$, $w = (4+6x^2)10^{-2}$

Normal Strain → $\epsilon_x = \frac{\partial u}{\partial x} \Rightarrow \frac{\partial}{\partial x}(y^2 \times 10^{-2}) \Rightarrow \boxed{\epsilon_x = 0}$

$\epsilon_y = \frac{\partial v}{\partial y} \Rightarrow \frac{\partial}{\partial y}(3yz \times 10^{-2}) \Rightarrow \boxed{\epsilon_y = 3z \times 10^{-2}}$

$\epsilon_z = \frac{\partial w}{\partial z} \Rightarrow \frac{\partial}{\partial z}(4+6x^2)10^{-2} \Rightarrow \boxed{\epsilon_z = 0}$

Shear strain → $\gamma_{xy} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}$

→ $\gamma_{xy} = \frac{\partial}{\partial y}(y^2 \times 10^{-2}) + \frac{\partial}{\partial x}(3yz \times 10^{-2})$

$\gamma_{xy} = 2y \times 10^{-2} + 0 \Rightarrow \boxed{\gamma_{xy} = 2y \times 10^{-2}}$

→ $\gamma_{yz} = \frac{\partial v}{\partial z} + \frac{\partial w}{\partial y}$

$= \frac{\partial}{\partial z}(3yz \times 10^{-2}) + \frac{\partial}{\partial y}[(4+6x^2)10^{-2}]$

$\boxed{\gamma_{yz} = 3y \times 10^{-2}}$

→ $\gamma_{xz} = \frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}$

$= \frac{\partial}{\partial z}(y^2 \times 10^{-2}) + \frac{\partial}{\partial x}[(4+6x^2)10^{-2}]$

$= 0 + (12x)10^{-2}$

$\boxed{\gamma_{xz} = (12x)10^{-2}}$

The strain components are given by $\epsilon_x = 0$, $\epsilon_y = 3z \times 10^{-2}$, $\epsilon_z = 0$, $\gamma_{xy} = 2y \times 10^{-2}$, $\gamma_{yz} = 3y \times 10^{-2}$, & $\gamma_{xz} = (12x) \times 10^{-2}$.

The strain Components at Point (1, 0, 2) are given by.

$\epsilon_x = 0$, Unit $\qquad \gamma_{xy} = 0$

$\epsilon_y = 6\times10^{-2}$ Units $\qquad \gamma_{yz} = 0$

$\epsilon_z = 0$ unit. $\qquad \gamma_{xz} = 12\times10^{-2}$ Units.

2) The Displacement Components in a strained body are as follows., $u = 0.01xy + 0.02y^2$

$v = 0.02x^2 + 0.01z^3y$; $w = 0.01xy^2 + 0.05z^2$.

Determine the strain Components at point (3, 2, −5)

Soln: Normal strain :→

$\epsilon_x = \frac{\partial u}{\partial x} \Rightarrow \frac{\partial}{\partial x}(0.01xy + 0.02y^2)$

$\epsilon_x \Rightarrow 0.01y.$

$\epsilon_y = \frac{\partial v}{\partial y} \Rightarrow \frac{\partial}{\partial y}(0.02x^2 + 0.01z^3y)$

$\epsilon_y \Rightarrow 0.01z^3$

$\epsilon_z = \frac{\partial w}{\partial x} \Rightarrow \frac{\partial}{\partial x}(0.01xy^2 + 0.05z^2)$

$\epsilon_z = 0.05(2z) \Rightarrow \underline{0.1z}$

$\gamma_{xy} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \Rightarrow \frac{\partial}{\partial y}(0.01xy + 0.02y^2) + \frac{\partial}{\partial x}(0.02x^2 + 0.01z^3y)$

$= 0.01x + 0.02(2y) + 0.02(2x)$

$= 0.01x + 0.04y + 0.04x$

$= 0.05x + 0.04y.$

$\gamma_{yz} = \frac{\partial v}{\partial z} + \frac{\partial w}{\partial y} \Rightarrow \frac{\partial}{\partial z}(0.02x^2 + 0.01z^3y) + \frac{\partial}{\partial y}(0.01xy^2 + 0.05z^2)$

$= 0.01(3z^2)y + 0.01x(2y)$

$= 0.03yz^2 + 0.02xy.$

$\gamma_{xz} = \frac{\partial u}{\partial z} + \frac{\partial w}{\partial x} \Rightarrow \frac{\partial}{\partial z}(0.01xy + 0.02y^2) + \frac{\partial}{\partial x}(0.01xy^2 + 0.05z^2)$

$\Rightarrow \cancel{0.05(2z)} \Rightarrow \cancel{0.1z} \Rightarrow 0.01\times(y^2) \Rightarrow 0.01y^2$

The strain Components are given by $\epsilon_x = 0.01y$, $\epsilon_y = 0.01z^3$, $\epsilon_z = 0.1z$, $\gamma_{xy} = 0.05x + 0.04y$, $\gamma_{yz} = 0.03yz^2 + 0.02xy$, $\gamma_{xz} = 0.01y^2$.

The strain Components at Point (3, 2, −5) are given by.

$\epsilon_x = 0.01(2) \Rightarrow 0.02$, $\gamma_{xy} = 0.05(3) + 0.04(2) = 0.23$

$\epsilon_y = 0.01(-5)^3 = -1.25$, $\gamma_{yz} = 0.03(2)(-5)^2 + 0.02(3)(2) = 1.62$

$\epsilon_z = 0.1(-5) = -0.5$, $\gamma_{xz} = 0.01(2)^2 = 0.04$

3) If the displacement field in the body is specified as. $u = (x^2+3)10^{-3}$, $v = (3y^2z)10^{-3}$, $w = (x+3z)\times10^{-3}$

Determine the strain Components at a point (1, 2, 3)

Soln: Normal strain:-

$\epsilon_x = \frac{\partial u}{\partial x} \Rightarrow \frac{\partial}{\partial x}(x^2+3)10^{-3} \Rightarrow (2x + \cancel{3})10^{-3} \Rightarrow (2x)10^{-3}$

$\epsilon_y = \frac{\partial v}{\partial y} = \frac{\partial}{\partial y}(3y^2z)10^{-3} = (3(2y)z)10^{-3} \Rightarrow (6yz)10^{-3}$

$\epsilon_z = \frac{\partial w}{\partial z} = \frac{\partial}{\partial z}(x+3z)10^{-3} = (3)10^{-3}$

→ $\gamma_{xy} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \Rightarrow \frac{\partial}{\partial y}(x^2+3)10^{-3} + \frac{\partial}{\partial x}(3y^2z)10^{-3}$

$\gamma_{xy} = 0.$

→ $\gamma_{yz} = \frac{\partial v}{\partial z} + \frac{\partial w}{\partial y} \Rightarrow \frac{\partial}{\partial z}(3y^2z)10^{-3} + \frac{\partial}{\partial y}(x+3z)10^{-3}$

$\gamma_{yz} = (3y^2)10^{-3}.$

→ $\gamma_{xz} = \frac{\partial u}{\partial z} + \frac{\partial w}{\partial x} = \frac{\partial}{\partial z}(x^2+3)10^{-3} + \frac{\partial}{\partial x}(x+3z)\times10^{-3}$

$\gamma_{xz} = 1\times10^{-3}$

$\epsilon_x = 2\times10^{-3}$ units.
$\epsilon_y = 36\times10^{-3}$ units
$\epsilon_z = 3\times10^{-3}$
$\gamma_{xy}=0,\ \gamma_{yz}=(3y^2)10^{-3} = 12\times10^{-3},\ \gamma_{xz}=10^{-3}$ units.

PROBLEMS ON EQUILIBRIUM EQUATIONS:-

Nomenclature: $\sigma_x, \sigma_y, \sigma_z$ Normal stress in x, y & z.
$\tau_{xy}, \tau_{yx}, \tau_{xz}$ shear stress in xy, yx & xz plane.
f_x, f_y & f_z body forces in x, y & z direction respectively.

LIST OF FORMULAE:→

Equilibrium Equations in 3D.

* $\dfrac{\partial\sigma_x}{\partial x} + \dfrac{\partial\tau_{xy}}{\partial y} + \dfrac{\partial\tau_{xz}}{\partial z} + f_x = 0.$

* $\dfrac{\partial\tau_{xy}}{\partial x} + \dfrac{\partial\sigma_y}{\partial y} + \dfrac{\partial\tau_{yz}}{\partial z} + f_y = 0.$

* $\dfrac{\partial\tau_{xz}}{\partial x} + \dfrac{\partial\tau_{yz}}{\partial y} + \dfrac{\partial\sigma_z}{\partial z} + f_z = 0.$

1. A State of stress at a point is given by $\sigma_x = x^3yz + xy^2$, $\sigma_y = 3y^2z + yz$, $\sigma_z = x^2y^2z^2 + xz$. $\tau_{xy} = x^2yz$, $\tau_{yz} = xy^2z$, $\tau_{zx} = xyz^2$. In the absence of body forces, determine whether the equilibrium conditions are satisfied or not at point $(3, -4, 2)$. If not then determine the suitable body forces required at this point so that these stress components are under equilibrium.

Solⁿ: In Abs of fs The equilibrium eqⁿs are given by
So, f_x, f_y & $f_z = 0$.

$\dfrac{\partial\sigma_x}{\partial x} + \dfrac{\partial\tau_{xy}}{\partial y} + \dfrac{\partial\tau_{xz}}{\partial z} = 0 \longrightarrow (1)$

$\dfrac{\partial\tau_{xy}}{\partial x} + \dfrac{\partial\sigma_y}{\partial y} + \dfrac{\partial\tau_{yz}}{\partial z} = 0 \longrightarrow (2)$

$\dfrac{\partial\tau_{xz}}{\partial x} + \dfrac{\partial\tau_{yz}}{\partial y} + \dfrac{\partial\sigma_z}{\partial z} = 0 \longrightarrow (3)$

Diff.→

$\dfrac{\partial\sigma_x}{\partial x} = 3x^2yz + 2xy^2$, $\dfrac{\partial\tau_{xy}}{\partial x} = 2xyz$.

$\dfrac{\partial\tau_{xy}}{\partial y} = x^2z$, $\dfrac{\partial\sigma_y}{\partial y} = 6yz + z$.

$\dfrac{\partial\tau_{xz}}{\partial z} = 2xyz$, $\dfrac{\partial\tau_{yz}}{\partial z} = xy^2$

$\dfrac{\partial\tau_{xz}}{\partial x} = yz^2$, $\dfrac{\partial\tau_{yz}}{\partial y} = 2xyz$.

$\dfrac{\partial\sigma_z}{\partial z} = 2x^2y^2z + x.$

Substituting the above Expn's.
At point $(3, -4, 2)$

$\dfrac{\partial\sigma_x}{\partial x} = 3(3)^2(-4)(2) + 2(3)(-4)^2 = -120.$

$\dfrac{\partial\tau_{xy}}{\partial x} = 2(3)(-4)(2) = -48.$, $\dfrac{\partial\tau_{xy}}{\partial y} = (3)^2(2) = 18.$ $\Big\}\ -150 \rightarrow (4)$

$\dfrac{\partial\sigma_y}{\partial y} = 6(-4)(2) + (2) = -46$, $\dfrac{\partial\tau_{xz}}{\partial z} = 2(3)(-4)(2) = -48.$

$\dfrac{\partial\tau_{yz}}{\partial z} = (3)(-4)^2 = 48.$ $\quad -46 \rightarrow (5)$

$\dfrac{\partial\tau_{xz}}{\partial x} = (-4)(2)^2 = -16$, $\dfrac{\partial\tau_{yz}}{\partial y} = 2(3)(-4)(2) = -48.$

$\dfrac{\partial\sigma_z}{\partial z} = 2(3)^2(-4)^2(2) + 3 = 579.$ $\quad 515 \rightarrow (6)$

Substituting Eqn (4) in (1).
$-120 + 18 - 48 = \underline{\underline{-150}} \neq 0.$

Sub Eq (5) in (2).

$-48 + (-46) + 48 = -46 \neq 0.$

Sub eq (6) in (3).

$-16 + (-48) + 579 = 515 \neq 0.$

∴ The equilibrium equations are not Satisfied. for the given stress Components at point $(3,-4,2)$.

The body forces that are to be added to equilibrium equations so as to satisfied. them are

$f_x = 150$ units, $f_y = 46$ units.

$f_z = -515$ units.

2/6/23.

2. The stress components at a point in a body are given by $\sigma_x = 3xy^2z + 2x$, $\sigma_y = 5xyz + 3y$, $\sigma_z = x^2y + y^2z$, $\tau_{xy} = 0$. $\tau_{yz} = \tau_{zx} = 3xy^2z + 2xy$. In the absence of body forces, determine whether equilibrium Conditions are satisfied are not at $(1,-1,2)$. If not, then determine the suitable body forces required at this point so that these stress Components are under equillibrium.

Soln: In the Absence of body forces, The equillibrium eqns are given by Eqn (1), (2), (3).

$$\frac{\partial \sigma_x}{\partial x} + \frac{\partial \tau_{xy}}{\partial y} + \frac{\partial \tau_{zx}}{\partial z} = 0 \rightarrow (1).$$

$$\frac{\partial \tau_{xy}}{\partial x} + \frac{\partial \sigma_y}{\partial y} + \frac{\partial \tau_{yz}}{\partial z} = 0 \rightarrow (2).$$

$$\frac{\partial \tau_{zx}}{\partial x} + \frac{\partial \tau_{yz}}{\partial y} + \frac{\partial \sigma_z}{\partial z} = 0 \rightarrow (3).$$

$$\frac{\partial(3xy^2z + 2x)}{\partial x} = 3y^2z + 2.$$

$$\frac{\partial \tau_{xy}}{\partial y} = \frac{\partial(0)}{\partial y} = 0$$

$$\frac{\partial \tau_{zx}}{\partial z} = \frac{\partial}{\partial z}(3xy^2z + 2xy) = 3xy^2$$

At $(1,-1,2)$. ⇒

$$\left.\begin{aligned} &\frac{\partial \sigma_x}{\partial x} = 3(1)(-1)^2(2) + 2 = 3(1)(2)+2 = 8 \\ &\frac{\partial \tau_{xy}}{\partial y} = 0, \quad \frac{\partial \tau_{zx}}{\partial z} = 3(1)(-1)^2 = 3 \end{aligned}\right\} (4).$$

$8 + 0 + 3 = 0 \Rightarrow 11 \neq 0$. ∴ The Eqln not Satisfied.

$$\frac{\partial \tau_{xy}}{\partial x} = 0, \quad \frac{\partial \sigma_y}{\partial y} = \frac{\partial}{\partial y}(5xyz + 3y) = 5xz + 3.$$

$$\frac{\partial \tau_{yz}}{\partial z} = \frac{\partial}{\partial z}(3xy^2z + 2xy) = 3xy^2.$$

$$\left.\begin{aligned} &\text{At } (1,-1,2) \Rightarrow \frac{\partial \tau_{xy}}{\partial x} = 0, \quad \frac{\partial \sigma_y}{\partial y} = 5(1)(2)+3 = 13. \\ &\frac{\partial \tau_{yz}}{\partial z} = 3(1)(-1)^2 = 3. \end{aligned}\right\} (5)$$

$13 + 0 + 3 = 0 \quad 16 \neq 0$ ∴ The Eqln not Satisfied

$$\frac{\partial \tau_{zx}}{\partial x} = \frac{\partial}{\partial x}(3xy^2z + 2xy) = 3y^2z + 2y.$$

$$\frac{\partial \tau_{yz}}{\partial y} = \frac{\partial}{\partial y}(3xy^2z + 2xy) = 3x(2y)z + 2x = 6xyz + 2x.$$

$$\frac{\partial \sigma_z}{\partial z} = \frac{\partial}{\partial z}(x^2y + y^2z) = y^2$$

$$\left.\begin{aligned} &\text{At } (1,-1,2) \Rightarrow \frac{\partial \tau_{zx}}{\partial x} = 3(-1)^2(2) + 2(-1) = 6 - 2 = 4. \\ &\frac{\partial \tau_{yz}}{\partial y} = 6(1)(-1)(2) + 2(1) = -12 + 2 = -10. \\ &\frac{\partial \sigma_z}{\partial z} = (-1)^2 = 1 \Rightarrow 4 + (-10) + 1 = -5 \quad -5 \neq 0. \end{aligned}\right\} \rightarrow (6)$$

∴ All the 3 equillibrium Eqn not satisfied for the given stress components at point $(1,-1,2)$.

The suitable body forces required at this point so that these stress components are under equillibrium are given by $f_x = -11$ units, $f_y = -16$ units, $f_z = 5$ units.

* Previous QP Problems.

Dec 18-19 [1] Displacement field at a point on a body is given as follows, $U=(x^2+y)$, $V=(3+z)$; $w=(x^2+2y)$. Determine strain components at $(3,1,-2)$ and express them in matrix form.

Sol^n: $\epsilon_x = \frac{\partial u}{\partial x} = \frac{\partial}{\partial x}(x^2+y) = 2x$, Point $2(3) = 6 = \epsilon_x = 6$ units

$\epsilon_y = \frac{\partial v}{\partial y} = \frac{\partial}{\partial y}(3+z) = 0$, $\epsilon_y = 0$ units

$\epsilon_z = \frac{\partial w}{\partial z} = \frac{\partial}{\partial z}(x^2+2y) = 0$, $\epsilon_z = 0$ units.

$\gamma_{xy} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} = \frac{\partial}{\partial y}(x^2+y) + \frac{\partial}{\partial x}(3+z) = \cancel{x}\,1 \Rightarrow (\phi)^2 = \phi$ units

$\gamma_{yz} = \frac{\partial v}{\partial z} + \frac{\partial w}{\partial y} = \frac{\partial}{\partial z}(3+z) + \frac{\partial}{\partial y}(x^2+2y) = 1+2 = 3$ units

$\gamma_{xz} = \frac{\partial u}{\partial z} + \frac{\partial w}{\partial x} = \frac{\partial}{\partial z}(x^2+y) + \frac{\partial}{\partial x}(x^2+2y) = 2x = 2(3) = 6$ units

$$\epsilon_{ij} = \begin{bmatrix} \epsilon_x & \gamma_{xy} & \gamma_{xz} \\ \gamma_{xy} & \epsilon_y & \gamma_{yz} \\ \gamma_{zx} & \gamma_{yx} & \epsilon_z \end{bmatrix} = \begin{bmatrix} 6 & (1)\,\phi & 6 \\ \phi\,(1) & 0 & 3 \\ 6 & 3 & 0 \end{bmatrix}$$

2. A structure is loaded and displacements associated with the dyown state are mapped. Find all the strains associated with the following observed displacement field at point $(0.6, -0.75, 1)$ m.

$u = (-x^4+3x-3y^2+8yz+5)\times10^{-3}$ m.
$v = (-4y^4+y+8xz+1)\times10^{-3}$ m
$w = (2z^2+3z-8xy+8)\times10^{-3}$ m.

Sol^n Normal strain :- $\epsilon_x = \frac{\partial}{\partial x}(-x^4+3x-3y^2+8yz+5)\times10^{-3}$ m

$\epsilon_x = (-4x^3+3)\times10^{-3}$ m.

$\epsilon_y = \frac{\partial}{\partial y}(-4y^4+y+8xz+1)\times10^{-3}$ m $= [-4(4y^3)+1]\times10^{-3}$ m

$\epsilon_y = (-16y^3+1)10^{-3}$ m

$\epsilon_z = \frac{\partial}{\partial z}(2z^2+3z-8xy+8)\times10^{-3}$ m $= (2(2z)+3)10^{-3}$ m

$\epsilon_z = (4z+3)10^{-3}$ m.

$\gamma_{xy} = \frac{\partial}{\partial y}(-x^4+3x-3y^2+8yz+5)\times10^{-3}$ m $+ \frac{\partial}{\partial x}(-4y^4+y+8xz+1)\times10^{-3}$ m

$= (-3(2y)+8z)10^{-3}$ m $+ (8z)\times10^{-3}$ m $\Rightarrow (-6y+8z)10^{-3} + 8z\times10^{-3}$ m

$= (-6y+16z)\times10^{-3}$ m.

$\gamma_{yz} = \frac{\partial}{\partial z}(-4y^4+y+8xz+1)\times10^{-3}$ m $+ \frac{\partial}{\partial y}(2z^2+3z-8xy+8)\times10^{-3}$ m.

$\gamma_{yz} = (8x)\times10^{-3}$ m $+ (-8x)\times10^{-3}$ m $= 0$

$\gamma_{xz} = \frac{\partial}{\partial z}(-x^4+3x-3y^2+8yz+5)\times10^{-3}$ m $+ \frac{\partial}{\partial x}(2z^2+3z-8xy+8)\times10^{-3}$ m

$\gamma_{xz} = (8y)\times10^{-3}$ m $+ (-8y)\times10^{-3}$ m $= 0$.

At point → $\epsilon_x = (-4(0.6)^3+3)\times10^{-3}$ m $= 2.136\times10^{-3}$ units

$\epsilon_y = (-16y^3+1)10^{-3}$ m $\Rightarrow (-16(-0.75)^3+1)10^{-3}$ m $= 7.75\times10^{-3}$ units

$\epsilon_z = (4z+3)10^{-3}$ m $\Rightarrow (4(1)+3)10^{-3}$ m $\Rightarrow 7\times10^{-3}$ units.

$\gamma_{xy} = [(-6)(-0.75)+16(1)]\times10^{-3}$ m $= 20.5\times10^{-3}$ units.

$\gamma_{yz} = \gamma_{xz} = 0$ units.

9am

*3 Displacement field at a point on a body is given as follows $u=(x^2yz+z^2)$, $v=(xy^2z+y^2)$, $w=(xyz^2+x^2)$ determine strain components at $(2,1,2)$ and express them in matrix form.

$\epsilon_x = \frac{\partial u}{\partial x} = \frac{\partial}{\partial x}(x^2yz+z^2) = 2xyz$, $\gamma_{xy} = x^2z+y^2z$.

$\epsilon_y = \frac{\partial v}{\partial y} = 2xyz+2y$, $\gamma_{yz} = xy^2+xz^2$

$\epsilon_z = \frac{\partial w}{\partial z} = 2xyz$, $\gamma_{xz} = x^2y+2z+yz^2+2x$.

at point $(2,1,2)$.

$\epsilon_x = 8$ $\quad \gamma_{xy} = 10$ $\quad$ The matrix form.

$\epsilon_y = 10$ $\quad \gamma_{yz} = 10$

$\epsilon_z = 8$. $\quad \gamma_{xz} = 16$.

$$= \begin{bmatrix} 8 & 10 & 16 \\ 10 & 10 & 10 \\ 16 & 10 & 8 \end{bmatrix}$$