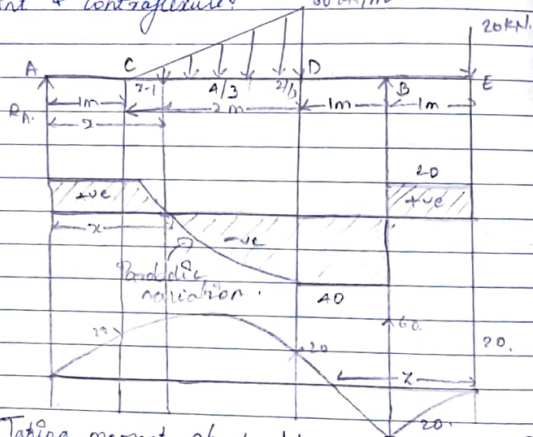


4/9/23.

7. Draw SFD & BMD for over hanging beam shown in fig. Indicate all significant values, including the point & contraflexure. 60 kN/m .



Taking moment about $M_A = 0$, $R_B = 60 \text{ kN}$, $R_A = 20 \text{ kN}$
 $+20 - \frac{1}{2} (x-1) \left(\frac{60(x-1)}{2} \right) = 0$

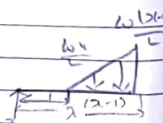
$$20 - 15(x-1)^2 = 0$$

$$15(x-1)^2 = 20 \Rightarrow (x-1)^2 = 1.33$$

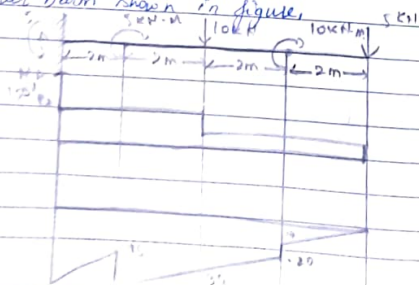
$$x-1 = 1.153 \Rightarrow x = 2.153$$

$$M = 20x - \frac{1}{2} (x-1) \left(\frac{60(x-1)}{2} \right)$$

$$M = 27.69$$



Find the relation of fixed end and SFD & BMD for the continuous beam shown in figure.



MODULE - 4.

ENERGY METHODS.

CONSERVATIVE FORCES:- Forces that are done in moving a particle from one point to another point are called conservative forces.

Eg: Gravitational force, Magnetic force etc...

STRAIN ENERGY:- Energy absorbed by the body when it is strained within its elastic limit.

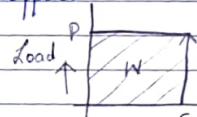
It is denoted by U . Mathematically strain energy stored in the body is equal to work done by the externally applied load. $\Rightarrow U = W$.

Law of Conservation of Energy.

Work done by the Externally applied load:-

a) **SUDDENLY Applied load.**

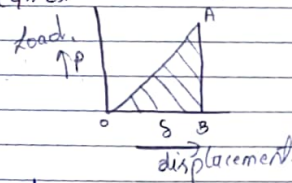
$$W = PS$$



b) **GRADUALLY Applied Load (GAL)**

WD by GAL = Area of Load.

$$W = \frac{1}{2} PS$$



c) **STRAIN ENERGY DUE TO AXIAL LOAD.**

(Strain Energy Stored in the bar).



Fig. 2.



Fig. 1.

Consider a bar of length L , cross sectional area A & Young's Modulus E . Subjected to an axial pull P as shown in fig. 1.

Let S be the deformation due to applied Load and σ be the stress induced.

Fig. 2 shows load deformation curve for the given bar.

Work done by the applied load is equal (work done) to area of triangle OAB.
 $W = \frac{1}{2} P \delta \rightarrow (1)$

Since strain energy stored in the bar is equal to work done by the external load, $\rightarrow U = W$.
 $\therefore U = \frac{1}{2} P \delta \rightarrow (2)$

$$\left. \begin{aligned} P &= \sigma A \\ \delta &= \frac{\sigma L}{E} \end{aligned} \right\} \rightarrow (3)$$

Substituting (3) in (2).

$$U = \frac{1}{2} (\sigma A) \left(\frac{\sigma L}{E} \right)$$

$$U = \frac{\sigma^2 A L}{2E}$$

$$U = \frac{\sigma^2 V}{2E} \rightarrow (4)$$

(4) is the Exp for strain energy (or) due to Axial Loading

STRAIN ENERGY DUE TO TORSION (Strain E. Stored in the Shaft)



Fig. 1

Consider a shaft of length 'L' dia - 'd' & rigidity modulus 'G'. Subjected to externally applied torque - 'T' as shown in Fig 1.

Let θ be the angle of twist due to applied torque & τ be the shear stress induced.

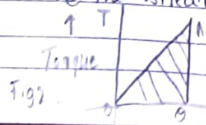


Fig 2 shows Torque V.S. Angle of twist for the given shaft

Work done by the applied torque is equal to Area of triangle OAB.
 $W = \frac{1}{2} T \theta \rightarrow (1)$

Since strain energy stored in the shaft is equal to work done by the applied torque $U = W$.

$$U = \frac{1}{2} T \theta \rightarrow (2)$$

$$\text{From torsion Eqn } T = \frac{CJ}{R}, \theta = \frac{TL}{GR} \rightarrow (3)$$

Where J is polar moment of inertia.

R - maximum radius.

Substituting eq. (3) in (2).

$$U = \frac{1}{2} \frac{CJ}{R} \cdot \frac{TL}{GR}$$

$$U = \frac{C^2 J L}{2 G R^2} \rightarrow (4)$$

$$\text{WKT. } R = \frac{d}{2}, J = \frac{\pi d^4}{32} \rightarrow (5)$$

$$U = \frac{C^2 (\pi d^4 / 32) L}{2 G (d/2)^2}$$

$$U = \frac{C^2 L}{2 G} \times \frac{\pi d^4}{32} \times \frac{4}{d^2}$$

$$U = \frac{C^2 \pi d^2 L}{4 G}$$

$$U = \frac{\tau^2 V}{4 G} \rightarrow (6)$$

Eqn (6) is the Exp for strain energy due to torsion.

STRAIN ENERGY DUE TO BENDING (STRAIN ENERGY STORED IN THE BEAM)

* Note: -

Bending Eqn is given by $\frac{E}{R} = \frac{M}{I} = \frac{\sigma}{y}$.

Where E is Young's Modulus, R is radius of curvature. M is applied moment, I is Moment of Inertia of beam. y is distance from neutral axis. (Cross section).

Derivation →

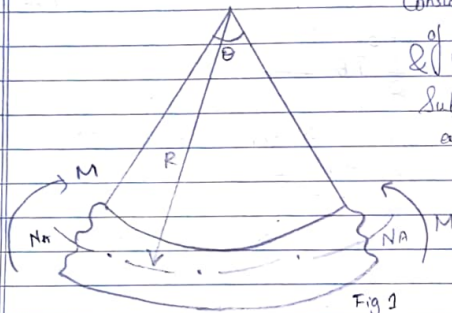
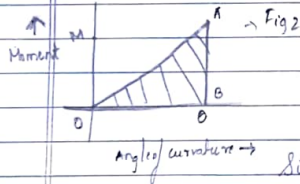


Fig 1

Consider a beam segment of length 'L', MOI → I. & Young's Modulus = E. Subjected to. Externally applied Moment 'M' as shown in fig 1. Let θ be the angle subtended by the curvature.

Fig 2 Shows Moment Vs θ curve for the given beam segment.



Work done by the applied moment is equal to area of $\Delta AOB = \frac{1}{2} \times \theta \times M = W$. Since strain energy stored in the beam is equal to work done by the applied moment. $U = W \Rightarrow U = \frac{1}{2} M \theta \rightarrow (2)$



From Geometry of fig 1 angle of curvature $\theta = \frac{L}{R} \rightarrow (3)$. Substituting (3) in (2).

$$U = \frac{1}{2} \times M \times \frac{L}{R} \Rightarrow U = \frac{1}{2} \frac{M L}{R} \rightarrow (4)$$

From Bending eqn. $\Rightarrow \frac{E}{R} = \frac{M}{I} \Rightarrow R = \frac{E I}{M} \rightarrow (5)$

Subst. (5) in (4)

$$U = \frac{1}{2} \frac{M L}{\frac{E I}{M}} \rightarrow (6)$$

Eq (6) is the expression for Strain energy due to bending.

If BM is not uniform throughout the beam segment. Consider a small strip of length dx , strain energy stored in the strip $\Rightarrow dU = \frac{M^2 dx}{2 I E}$

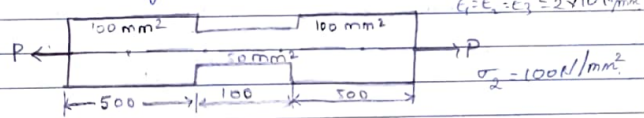
$$U = \int_0^L \frac{M^2 dx}{2 E I} \rightarrow (7)$$

Numerical Problem on Strain Energy Due to Axial Load. List of formulae:-

$$1. U = \frac{\sigma^2 n L}{2 E}$$

$$2. \sigma = \frac{P}{A}$$

1. The Max stress produced by a pull in a bar of 1100mm Length. is 100 N/mm^2 . The area of cross section as shown in fig. Calculate the strain energy stored in the bar. If E is equal to $2 \times 10^5 \text{ N/mm}^2$.



$$A_1 = A_3 = 100 \text{ mm}^2, A_2 = 50 \text{ mm}^2, L_1 = L_3 = 500 \text{ mm}, L_2 = 100 \text{ mm}$$

$$\sigma_2 = \frac{P}{A_2} \Rightarrow P = 100 \times 50 \Rightarrow P = 5000 \text{ N.}$$

$$\sigma_1 = \sigma_3 = \frac{5000}{100} = 50 \text{ N/mm}^2.$$

$$U_3 = U_1 = \frac{\sigma^2 A_1 L_1}{2 E_1} = \frac{50^2 (100) (500)}{2 \times 2 \times 10^5} = 312.5 \text{ N-mm.}$$

$$U_2 = 125 \text{ N-mm.}$$

$$= U_1 + U_2 + U_3 = 750 \text{ N-mm.}$$

$$U = 0.75 \text{ J}$$

2) STRAIN ENERGY Due To TORSION:-

$$\text{Formula:} \rightarrow U = \frac{\tau^2 A L}{4 G}, \tau = \frac{T R}{J}, A = \frac{\pi d^2}{4}, R = \frac{d}{2}$$

$$J = \frac{\pi d^4}{32}, A = \frac{\pi (d_o^2 - d_i^2)}{4}, R = \frac{d_o}{2}$$

- 2) A Solid & hollow shaft of same material at same Max stress are subjected to same Max Torque. The ratio of inner to outer diameter for the hollow shaft is 0.75. Find the ratio of strain Energy in the two shafts.

Soln: $G_s = G_h$, $\tau_s = \tau_h$, $T_s = T_h$.
 $\frac{d_i}{d_o} = 0.75$ $\rightarrow \frac{U_s}{U_h} = ?$

Let $d \rightarrow$ diameter of solid shaft, d_i & d_o be the inner & outer dia of hollow shaft.

Shear stress induced in the solid shaft. $\tau_s = \left(\frac{TR}{J} \right)_s$
 $\tau_s = \frac{T d/2}{\pi d^4/32} \rightarrow \tau_s = \frac{16T}{\pi d^3} \rightarrow (1)$

Shear stress induced in the hollow shaft.

$$\tau_h = \left(\frac{TR}{J} \right)_h$$

$$\tau_h = T d_o/2 \rightarrow \tau_h = \frac{16T}{\pi (d_o^4 - d_i^4)} \rightarrow \tau_h = \frac{16T}{\pi (d_o^4 - (0.75 d_o)^4)}$$

$$\tau_h = \frac{16T}{0.68 \pi d_o^3} \rightarrow (2)$$

from data (1) = (2).

$$\tau_s = \tau_h$$

$$\frac{16T}{\pi d^3} = \frac{16T}{0.68 \pi d_o^3} \rightarrow d_o^3 = 1.47 d^3$$

$$[d_o = 1.137 d] \rightarrow (3)$$

Strain Energy in Solid Shaft.

$$U_s = \left(\frac{\tau^2 AL}{4G} \right)_s \rightarrow U_s = \frac{\tau^2 \pi d^2 L}{16G} \rightarrow (4)$$

Strain Energy hollow shaft

$$U_h = \left(\frac{\tau^2 AL}{4G} \right)_h \Rightarrow \frac{\tau^2 \pi (d_o^2 - d_i^2) L}{16G} \Rightarrow \frac{\tau^2 \pi (d_o^2 - (0.75 d_o)^2) L}{16G}$$

$$U_h = \frac{0.4375 \tau^2 \pi d_o^2 L}{16G} \rightarrow (5)$$

$$\frac{U_s}{U_h} = \frac{\tau^2 \pi d^2 L / 16G}{0.4375 \tau^2 \pi d_o^2 L / 16G} \rightarrow \frac{d^2}{0.4375 (1.137 d)^2}$$

$$\frac{U_s}{U_h} = 1.76$$

h //

- 3) STRAIN ENERGY DUE TO BENDING.

$$U = \int_0^L \frac{M^2 dx}{2EI}, \quad W = \frac{1}{2} W \delta, \quad \delta \rightarrow \text{deflection}$$

3. A cantilever beam of uniform cross section carries a point load at free end. determine i) strain energy stored by the cantilever beam & deflection at free end.

ii) If load W is equal to 200 kN, $E = 2 \times 10^8 \text{ kN/m}^2$.

$L = 3 \text{ m}$, $I = 10^3 \text{ m}^4$. determine the above values.

$$\Rightarrow U = ? \quad \delta = ?$$

$$W = 200 \text{ kN}$$

$$L = 3 \text{ m}$$

$$E = 2 \times 10^8 \text{ kN/m}^2$$

$$I = 10^3 \text{ m}^4$$

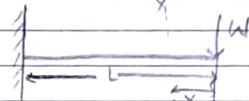


Fig shows a cantilever beam of length L subjected to point load at the free end.

Consider a section $x-x$ at a distance x from the free end. Moment at section at $x-x$.

$$M = Wx$$

Strain Energy stored in the beam $U = \int_0^L \frac{M^2 dx}{2EI}$

$$U = \int_0^L \frac{(Wx)^2 dx}{2EI} = \int_0^L \frac{W^2 x^2 dx}{2EI} = \frac{W^2}{2EI} \int_0^L x^2 dx$$

$$U = \frac{W^2}{2EI} \left[\frac{x^3}{3} \right]_0^L \Rightarrow U = \frac{W^2 L^3}{6EI} \rightarrow (1)$$

$$W = \frac{1}{2} W \delta, \quad \delta = \frac{1}{2} W \delta = \frac{W^2 L^3}{6EI}$$

6/9/23

$$\delta = \frac{WL^3}{3EI} \rightarrow (2)$$

ii) Substitute (1) & (2)

$$U = \frac{(200 \times 10^3)^2 (3)^3}{2 \times 3 (2 \times 10^8) (10^{-3})} \quad W = 200 \times 10^3 \text{ N}$$

$$L = 3 \text{ m}$$

$$U = \frac{6 \times 10^8}{12 \times 10^{11}} \Rightarrow U = 900 \text{ J} \quad E = 2 \times 10^8 \times 10^3 \text{ N/m}^2$$

$$I = 10^{-3}$$

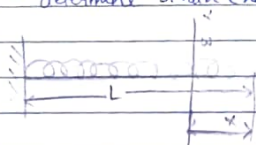
$$\delta = 9 \times 10^{-3} \text{ m}$$

$$= 0.009 \text{ m}$$

A Cantilever beam of length L carries UDL of intensity w over its entire length as shown in fig.

Determine i) U stored by Cantilever beam.

ii) If $w = 10 \text{ kN/m}$, $L = 2 \text{ m}$, $EI = 2 \times 10^5 \text{ kN-m}^2$ determine strain energy stored.



Consider a section $x-x$ at a distance x from the free end.

$U = ?$ Moment at x section $x-x$

$$M = \frac{wx^2}{2}$$

Strain energy stored in the beam

$$U = \int_0^L \frac{M^2 dx}{2EI} \Rightarrow \int_0^L \frac{w^2 x^4 dx}{8EI}$$

$$U = \frac{w^2 L^5}{40EI}$$

$$= 0.4 \text{ J}$$

A simply supported beam of span L carries a point load F at mid span. Determine strain energy stored by beam and deflection at its mid span.

fig shows a simply supported beam of span L subjected to point load F at mid span.

Considering beam segment AB . Let $x-x$ be the section at a distance x from A .

$$\text{Moment at section } x-x, \quad M = \frac{Fx}{2} \Rightarrow U_{AB} = \int_0^{L/2} \frac{M^2 dx}{2EI}$$

$$U_{AB} = \int_0^{L/2} \frac{(Fx/2)^2 dx}{2EI}$$

$$U_{AB} = \int_0^{L/2} \frac{F^2 x^2 dx}{8EI} \Rightarrow U_{AB} = \frac{F^2}{8EI} \int_0^{L/2} x^2 dx$$

$$U_{AB} = \frac{F^2}{8EI} \left[\frac{x^3}{3} \right]_0^{L/2}$$

$$U_{AB} = \frac{F^2}{24EI} \left[\frac{L}{2} \right]^3 \Rightarrow U_{AB} = \frac{F^2 L^3}{192EI}$$

from symmetry of the beam.

$$U_{AB} = U_{BC} = \frac{F^2 L^3}{192EI}$$

Total strain energy in the beam $U = U_{AB} + U_{BC}$

$$\boxed{U = \frac{F^2 L^3}{96EI}}$$

Work done by the applied load, $W = \frac{1}{2} \times F \delta$

$\therefore U = W$ Since Strain Energy = Work done.

$$\frac{1}{2} F \delta = \frac{F^2 L^3}{96EI} \Rightarrow \boxed{\delta = \frac{F L^3}{48EI}}$$

Energy Theorem -

SURYA

Date

Page

CASTIGLIANO'S Ist THEOREM

Statement $\rightarrow$ In an elastic member, partial derivative of strain energy w.r.t to deflection is equal to load acting at that point, deflection being measured in the direction of load.

To prove $\rightarrow \frac{\partial U}{\partial \delta_1} = P_1$



Consider a simply supported beam subjected to point loads P_1 & P_2 as shown in the figure. Let δ_1 & δ_2 be the corresponding deflections. Strain Energy shown in the beam,

$$U = \frac{1}{2} P_1 \delta_1 + \frac{1}{2} P_2 \delta_2 \rightarrow (1)$$

Let ∂P_1 & ∂P_2 be the additional loads applied in such a proportion that additional deflection is produced at point 1 alone.

Additional strain energy due to loads ∂P_1 & ∂P_2 .

$$\partial U = \frac{1}{2} \partial P_1 \partial \delta_1 + \frac{1}{2} \partial P_2 \partial \delta_2$$

Neglecting ∂^2 higher order terms. $\partial U = P_1 \partial \delta_1$

$$\frac{\partial U}{\partial \delta_1} = P_1 \quad (\text{or}) \quad \text{ingen. mod.} \quad \frac{\partial U}{\partial \delta_1} = P_1$$

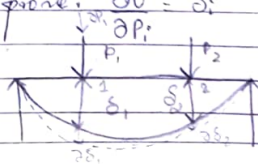
Hence Castigliano's first theorem is proved.

CASTIGLIANO'S SECOND (II) THEOREM

Statement - In an elastic member, partial derivative of strain energy with respect to load at a point is equals deflection at that point deflection being measured in direction of load.

$\Rightarrow$ To prove: $\frac{\partial U}{\partial P_1} = \delta_1$

Proof:-



Consider a simply supported beam subjected to point load P , δ_1 as shown in the fig.

Let δ_1 & δ_2 be the corresponding deflections. Strain Energy in the beam

$$U = \frac{1}{2} P_1 \delta_1 + \frac{1}{2} P_2 \delta_2 \rightarrow (1)$$

Let $\partial P_1 \rightarrow$ a small load at 1. $\rightarrow$ increasing deflection $\partial \delta_1$ at point 1. Additional strain energy due to ∂P_1 .

$$\partial U = \frac{1}{2} \partial P_1 \partial \delta_1 + \frac{1}{2} P_2 \partial \delta_2 \rightarrow (2)$$

Neglecting the higher order terms

$$\partial U = P_1 \partial \delta_1 + \frac{1}{2} P_2 \partial \delta_2 \rightarrow (3)$$

Total strain energy

$$U' = U + \partial U$$

$$U' = \frac{1}{2} P_1 \delta_1 + \frac{1}{2} P_2 \delta_2 + \frac{1}{2} \partial P_1 \delta_1 + \frac{1}{2} P_2 \partial \delta_2 \rightarrow (4)$$



Let the loads P_1 , P_2 & ∂P_1 simultaneously applied on the beam. Total strain energy U'

$$U' = \frac{1}{2} (P_1 + \partial P_1) (\delta_1 + \partial \delta_1) + \frac{1}{2} P_2 (\delta_2 + \partial \delta_2)$$

$$U' = \frac{1}{2} P_1 \delta_1 + \frac{1}{2} P_1 \partial \delta_1 + \frac{1}{2} \partial P_1 \delta_1 + \frac{1}{2} \partial P_1 \partial \delta_1$$

$$+ \frac{1}{2} P_2 \delta_2 + \frac{1}{2} P_2 \partial \delta_2 \rightarrow (5)$$

Equations (2) & (5)

$$P_1 \partial \delta_1 + \frac{1}{2} P_2 \partial \delta_2 = \frac{1}{2} P_1 \partial \delta_1 + \frac{1}{2} P_2 \partial \delta_2 + \frac{1}{2} \partial P_1 \delta_1$$

$$+ \frac{1}{2} P_1 \partial \delta_1 + \frac{1}{2} P_2 \partial \delta_2 = \frac{1}{2} \partial P_1 \delta_1$$

Substitute eq (3) in (6)

$$\partial U = \partial P_1 \delta_1$$

$$\frac{\partial U}{\partial P_1} = \delta_1$$

Hence Castigliano's IInd theorem is proved.

PROBLEMS ON Castigliano's Theorem.

$$1. U = \int_0^L \frac{M^2 dx}{2EI}$$

$$2. \delta = \frac{\partial U}{\partial P}$$

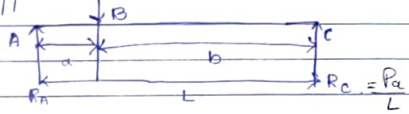
1. Determine the deflection at the free end of cantilever beam subject to point load at its free end.

Using Castigliano's theorem.

$$1. U = \frac{P^2 L^3}{6EI} \quad \delta = \frac{\partial U}{\partial P}$$

$$\delta = \frac{\partial}{\partial P} \left(\frac{P^2 L^3}{6EI} \right) = \frac{2PL^3}{6EI} \Rightarrow \boxed{\delta = \frac{PL^3}{3EI}}$$

2. A simply supported beam is loaded as shown in the figure.



Moment at A.

$$M_A = 0.$$

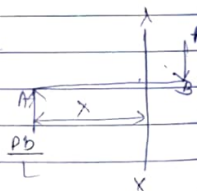
$$R_C L - P a = 0.$$

$$R_C = \frac{P a}{L} \Rightarrow R_A + R_C = P \Rightarrow R_A = P - \frac{P a}{L} = P \left(1 - \frac{a}{L} \right) = P \left(\frac{L-a}{L} \right) = \frac{P b}{L}$$

Considering section A-B of the beam
Let x-x be the section at a distance x from A.

Moment at section x-x

$$M = \frac{P b}{L} x$$



Strain energy in section AB.

$$U_{AB} = \int_0^a \frac{M^2 dx}{2EI}$$

$$= \int_0^a \frac{P^2 b^2 x^2}{2EI L^2} dx = \frac{P^2 b^2 a^3}{6EI L^2} = \frac{P^2 b^2 a^3}{2EI L^2}$$

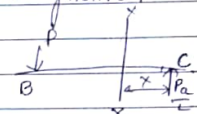
Considering segment BC of the beam.

Let x-x be the section at a distance from C.

Moment at section x-x.

$$M = \frac{P a x}{L} \Rightarrow U_{BC} = \int_0^L \frac{M^2 dx}{2EI}$$

$$U_{BC} = \frac{P^2 a^2 b^3}{6EI L^2}$$



Total strain energy.

$$U = U_{AB} + U_{BC}$$

$$= \frac{P^2 b^2 a^3}{6EI L^2} + \frac{P^2 a^2 b^3}{6EI L^2}$$

$$= \frac{P^2 a^2 b^2}{6EI L^2} (a+b) = \frac{P^2 a^2 b^2}{6EI L^2} (L)$$

$$U = \frac{P^2 a^2 b^2}{6EI L}$$

Deflection for the given beam.

$$\delta = \frac{\partial U}{\partial P}$$

$$\delta = \frac{\partial}{\partial P} \left(\frac{P^2 a^2 b^2}{6EI L} \right) = \frac{2P a^2 b^2}{6EI L}$$

MAXWELL RECIPROCAL THEOREM.

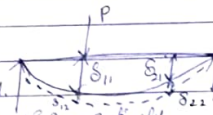
STATEMENT :- In an elastic member, deflection at point i due to load applied at point j is equal to deflection at point j due to load applied at i.

To prove: $\delta_{ij} = \delta_{ji}$

Proof: Case 1:-

Consider a SPSB subjected to load P at point i. Let δ_{i1} & δ_{j1} be the deflections at points 1 & 2 respectively.

Strain energy stored in the beam $U = \frac{1}{2} P \delta_{i1} \rightarrow (1)$



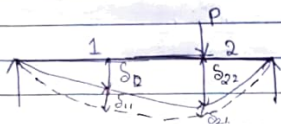
Let load P be applied at point 2. δ_{12} & δ_{22} are the corresponding deflections at point 1 & 2 respectively. Additional strain energy due to this load.

$$U_2 = \frac{1}{2} P \delta_{22} + P \delta_{12} \rightarrow (2)$$

Total strain energy $\Rightarrow U = U_1 + U_2$.

$$U = \frac{1}{2} P \delta_{11} + \frac{1}{2} P \delta_{22} + P \delta_{12} \rightarrow (3)$$

Case II:-



Consider the same beam subjected to load P at Point 2. Let δ_{12} & δ_{22} be the deflections at point 1 & 2 respectively. Strain energy stored in a beam.

$$U_2 = \frac{1}{2} P \delta_{22} \rightarrow (4)$$

Let Additional strain energy due to this load.

$$U_1 = \frac{1}{2} P \delta_{11} + P \delta_{21} \rightarrow (5)$$

Total strain energy $\Rightarrow U = U_1 + U_2$.

$$U = \frac{1}{2} P \delta_{22} + \frac{1}{2} P \delta_{11} + P \delta_{21} \rightarrow (6)$$

Since strain energy is same in both the cases, equating (3) & (6).

$$\frac{1}{2} P \delta_{11} + \frac{1}{2} P \delta_{22} + P \delta_{12} = \frac{1}{2} P \delta_{11} + \frac{1}{2} P \delta_{22} + P \delta_{21}$$

$$P \delta_{12} = P \delta_{21}$$

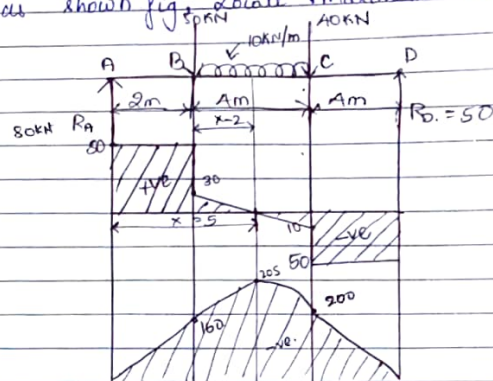
$$\delta_{12} = \delta_{21}$$

or in general $\Rightarrow \delta_{ij} = \delta_{ji}$

hence Maxwell's Reciprocal theorem is proved.

Q.P.

Draw the SFD & BMD of the simply supported beam as shown fig. Locate maximum bending Moment.



$$M_A = 0$$

$$R_D(10) - 40(6) - 40(4) - 50(2) + R_A(0) = 0$$

$$R_D = 50 \text{ kN}$$

$$R_A + R_D = 50 + 40 + 40 \Rightarrow R_A + 50 = 130 \Rightarrow R_A = 80 \text{ kN}$$

$$\sum F_x = +80 - 50 - 10(x-2) = 0 \Rightarrow 30 - 10x + 20 = 0 \Rightarrow 50 - 10x = 0 \Rightarrow 50 = 10x \Rightarrow x = 5$$

$$M_A = 0 = M_0 \Rightarrow M_0 = 160 \text{ kN-m}$$

$$M_x = 80(5) - 50(3) - 10(x-2)(x-2) \quad x = 5 = 205$$

