Module - 1 22/9/23

Control Engineering Systems

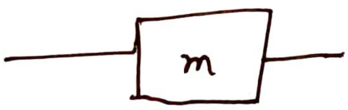
Transfer function models of mechanical systems.

* Linear time varying.

$$\text{Transfer function (TF)} = \left.\frac{LT \text{ of } o/p}{LT \text{ of } i/p}\right|_{\text{with zero initial conditions}}$$

Models : Translational

i) mass



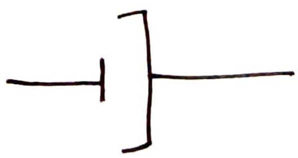
Assuming ⊙ of body

ii) Spring

K

Elastic deformation

iii) Dashpot



B

friction in rotating mechanical system

Symbols used

x = Displacement, m

$v = \frac{dx}{dt}$ = velocity, m/s

$a = \frac{dv}{dt} = \frac{d^2x}{dt^2}$ = acceleration, m/s^2

f = force applied, N

f_m = opposing force by mass, N

f_b = opposing force by dashpot, N

f_K = opposing force by spring, N

M = Mass, Kg

K = stiffness of spring, N/m

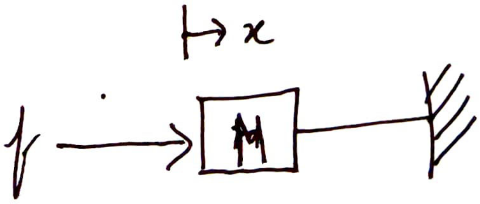
B = Viscous friction co-efficient, N-sec/m

All lowercase indicates it is time function.

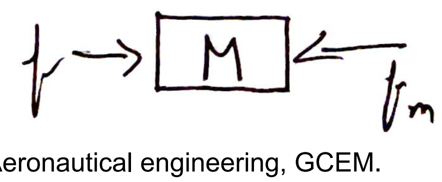
① Case 1

'N' Second law

$f \propto \frac{d^2x}{dt^2}$



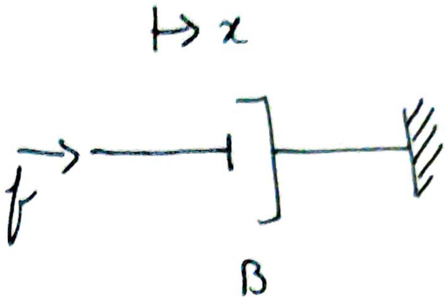
$f = f_m = M\frac{d^2x}{dt^2}$



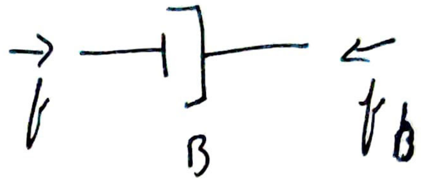
Case ii)

$f_b \propto$ velocity

$f_b \propto \frac{dx}{dt}$

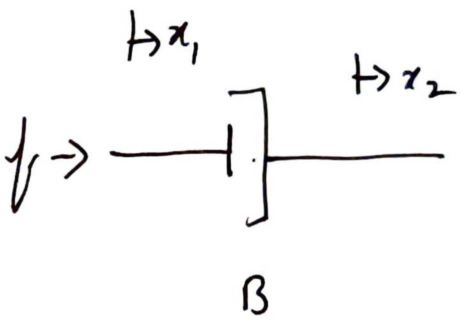


$f = f_b = B\frac{dx}{dt}$



Case iii)

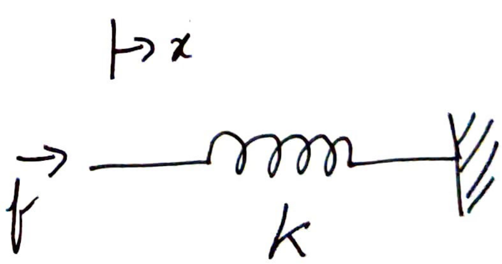
Change in dis $(x_1 - x_2)$



$\therefore \; f_b \propto \frac{d}{dt}(x_1 - x_2)$

$f = f_b = B\frac{d}{dt}(x_1 - x_2)$

Case iv)



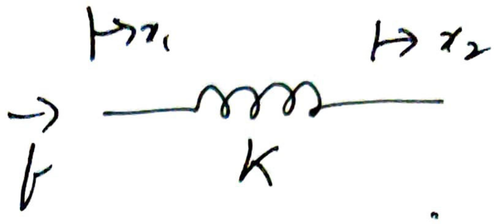
$f_k \propto$ distance changed.

$f_k \propto x$

$\therefore \; f_k = Kx$

Case v)

$\frac{dis}{\cdot} \quad (x_1 - x_2)$



$f_k \propto (x_1 - x_2)$

$f = f_k = K(x_1 - x_2)$

* Laplace transform

$$x(t) = \mathcal{L}\{x(t)\} = X_{(s)}$$

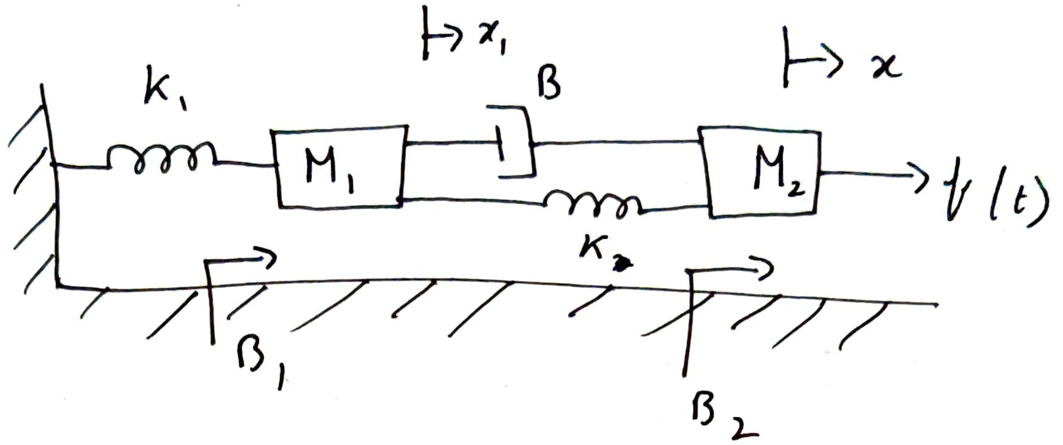
$$\frac{dx}{dt}(t) = \mathcal{L}\left\{\frac{d}{dx}x(t)\right\} = s\dot{X}_{(s)}$$

$$\frac{d^2x}{dt^2}(t) = \mathcal{L}\left\{\frac{d2}{dt^2}x(t)\right\} = s^2 X_{(s)}$$

(zero initial conditn.)

Example 1

Write differential equations governing the mechanical system & determine the transfer function.



So our i/p is "$f(t)$" & o/p is "x"

Laplace transform of i/p, $f(t) = \mathcal{L}\{f(t)\} = F(s)$

o/p, $x = \mathcal{L}\{x\} = X(s)$

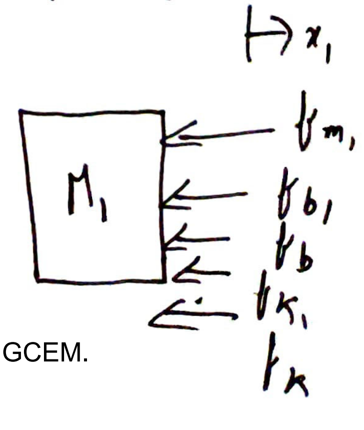
$x_1 = \mathcal{L}\{x_1\} = X_1(s)$

$$\therefore \quad TF = \frac{X(s)}{F(s)}$$

Two nodes M_1 & M_2

Consider node M_1 & displacement is x_1

opposing forces $f_{m_1}, f_{b_1}, f_b, f_{k_1}, f_k$



$$f_{m_1} = M_1 \frac{d^2 x_1}{dt^2} \; ; \quad f_{b_1} = B_1 \frac{dx_1}{dt} \; ; \quad f_{k_1} = k_1 x_1$$

$$f_b = B \frac{d}{dt}(x_1 - x) \; ; \quad f_k = k(x_1 - x)$$

$$f_{m_1} + f_{b_1} + f_b + f_{k_1} + f_k = 0$$

$$\therefore \; M_1 \frac{d^2 x_1}{dt^2} + B_1 \frac{dx_1}{dt} + B \frac{d}{dt}(x_1 - x) + k_1 x_1 + k(x_1 - x) = 0$$

Laplace transform.

$$M_1 s^2 X_1(s) + B_1 s X_1(s) + B s [X_1(s) - X(s)]$$
$$+ K_1 X_1(s) + K [X_1(s) - X(s)] = 0$$

$$X_1(S) [M_1 s^2 + (B_1 + B) s + (K_1 + k)] - X(s) [Bs + k] = 0$$

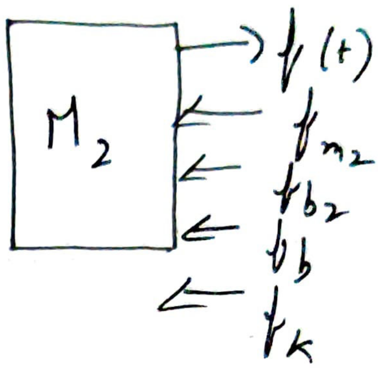
$$X_1(s) [M_1 s^2 + (B_1 + B) s + (K_1 + k)] = X(s) [Bs + k]$$

$$X_1(s) = X(s) \frac{[Bs + k]}{[M_1 s^2 + (B_1 + B) s + (K_1 + k)]} \quad \text{--- (1)}$$

Similarly taking node 'M_2' displacement 'x'.

opposing forces f_{m_2}, f_{b_2}, f_b & f_k

$$f_{m_2} = M_2 \frac{dx^2}{dt^2} \, ; \quad f_{b_2} = B_2 \frac{dx}{dt}$$



$$f_b = B \frac{d}{dt} (x - x_1) \, ;$$

$$f_k = k (x - x_1)$$

$$f_{m_2} + f_{b_2} + f_b + f_k = f(t)$$

$$M_2 \frac{dx^2}{dt^2} + B_2 \frac{dx}{dt} + B \frac{d}{dt}(x - x_1) + k(x - x_1) = f(t)$$

Laplace Transform

$$M_2 s^2 X(s) + B_2 s X(s) + B s \left[X(s) - X_1(s)\right]$$

$$+ k\left[X(s) - X_1(s)\right] = F(s)$$

$$X(s)\left[M_2 s^2 + (B_2 + B) s + k\right] - X_1(s)\left[Bs + k\right] = F(s) \quad \text{—(2)}$$

Substitute '$X_1(s)$' from eq^n (1) to (2)

$$X(s)\left[M_2 s^2 + (B_2 + B)s + k\right] - \frac{X(s)\,(Bs+k)^2}{M_1 s^2 + (B_1 + B)s + (k_1 + k)}$$

$$= F(s)$$

$$X(s)\left[\frac{\left[M_2 s^2 + (B_2 + B)s + k\right]\left[M_1 s^2 + (B_1 + B)s + (k_1 + k)\right] - (Bs+k)^2}{M_1 s^2 + (B_1 + B)s + (k_1 + k)}\right]$$

$$= F(s)$$

$$\frac{X(s)}{F(s)} = \frac{M_1 s^2 + (B_1 + B)s + (k_1 + k)}{\left[M_2 s^2 + (B_2 + B)s + k\right]\left[M_1 s^2 + (B_1 + B)s + (k_1 + k)\right] - (Bs+k)^2}$$

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Example 2

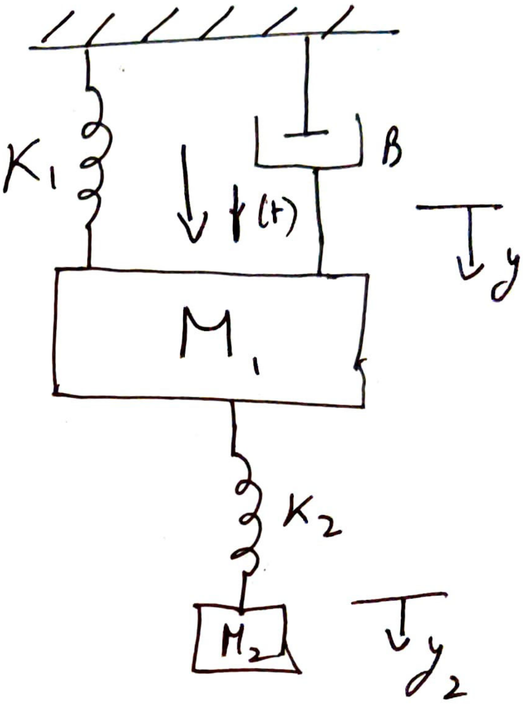
Determine transfer function

i/p $f(t)$

LT $\to F(s)$

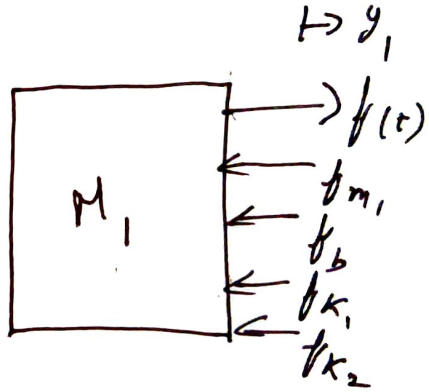
O/P y_2 LT $\to Y_2(s)$

To find, TF $= \frac{Y_2(s)}{F(s)}$



Two nodes M_1 & M_2

Take node M_1.



$$f_{m_1} + f_b + f_{K_1} + f_{K_2} = f(t)$$

~~L.T~~

$$M_1 \frac{d^2 y_1}{dt^2} + B \frac{dy_1}{dt} + K_1 y_1 + K_2 (y_1 - y_2) = f(t)$$

L.T

$$M_1 s^2 Y_1(s) + B s Y_1(s) + K_1 Y_1(s) + K_2 [Y_1(s) - Y_2(s)] = F(s)$$

$$Y_1(s) [M_1 s^2 + Bs + (K_1 + K_2)] - Y_2(s) K_2 = F(s) \quad \text{--- } (1)$$

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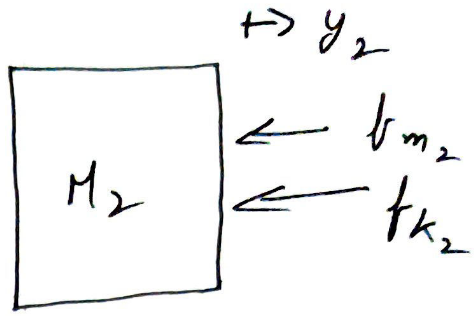
Take node M_2



$$f_{m_2} + f_{k_2} = 0$$

$$M_2 \frac{d^2 y_2}{dt^2} + k_2 (y_2 - y_1) = 0$$

L.T

$$M_2 s^2 Y_2(s) + K_2 [Y_2(s) - Y_1(s)] = 0$$

$$Y_2(s) [M_2 s^2 + K_2] - Y_1(s) K_2 = 0$$

$$Y_1(s) = Y_2(s) \frac{M_2 s^2 + K_2}{K_2} \quad \text{— (2)}$$

Put Y_1 from eqn (2) in (1)

$$Y_2(s) \left[\frac{M_2 s^2 + K_2}{K_2}\right] [M_1 s^2 + Bs + (K_1 + K_2)] - Y_2(s) K_2 = F(s)$$

$$\therefore \frac{Y_2(s)}{F(s)} = \frac{K_2}{[M_1 s^2 + Bs + (K_1 + K_2)][M_2 s^2 + K_2] - K_2^2}$$

Mechanical Rotational System

Moment of inertia (J) ($\rightarrow$ mass $\rightarrow$ CG)
Dashpot (B)
Torsional Spring (K)

Symbols to know :

θ = Angular displacement, rad

$\frac{d\theta}{dt}$ = Angular velocity, rad/sec

$\frac{d^2\theta}{dt^2}$ = Angular acceleration, rad/sec^2

T = Applied torque, N-m

J = Moment of inertia, kg-m^2/rad.

B = Rotational frictional coeffecient, N-m (rad/sec)

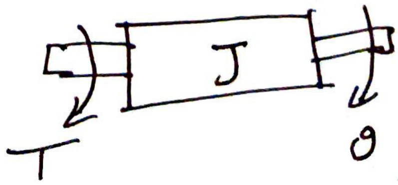
K = Stiffness of spring, N-m/rad.

Torque Balance eq^{n}s

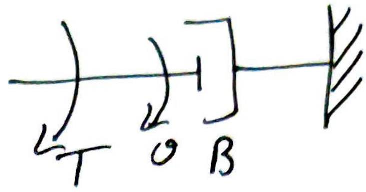
Element 1 :

$$T_j \propto \frac{d^2\theta}{dt^2}$$

$$T_j = T = J\frac{d^2\theta}{dt^2}$$



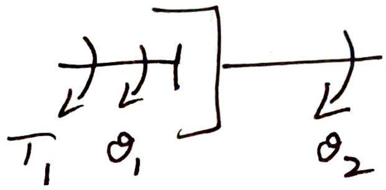
Element 2 :



$$T_b \propto \frac{d\theta}{dt}$$

$$T_b = B\frac{d\theta}{dt} = T$$

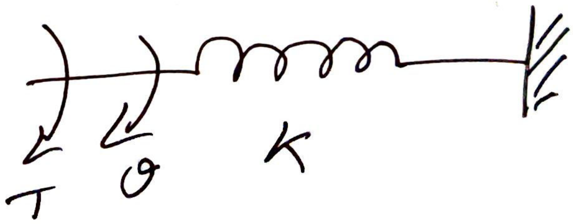
Element 3 :



$$T_b \propto \frac{d}{dt}(\theta_1 - \theta_2)$$

$$T_b = B\frac{d}{dt}(\theta_1 - \theta_2) = T$$

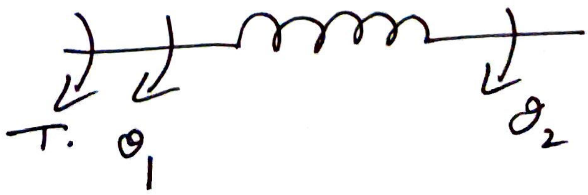
Element 4 :



$$T_k \propto \theta$$

$$T_k = K\theta = T$$

Element 5 :



$$T_k \propto (\theta_1 - \theta_2)$$

$$T_k = K(\theta_1 - \theta_2)$$

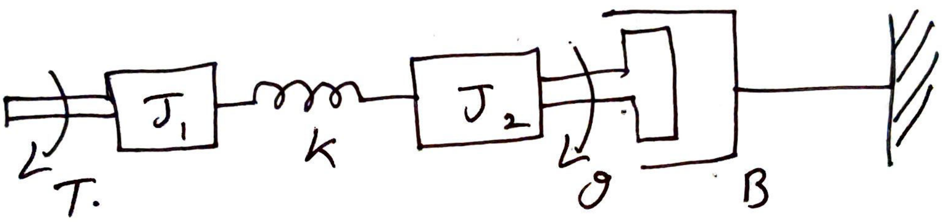
L T of $\theta = \mathcal{L}(\theta) = \theta(s)$

L T of $\frac{d\theta}{dt} = \mathcal{L}\left\{\frac{d\theta}{dt}\right\} = s\,\theta(s)$

L T of $\frac{d^2\theta}{dt^2} = \mathcal{L}\left\{\frac{d^2\theta}{dt^2}\right\} = s^2\theta(s)$

Example 3

T.F & differential governing eqn's



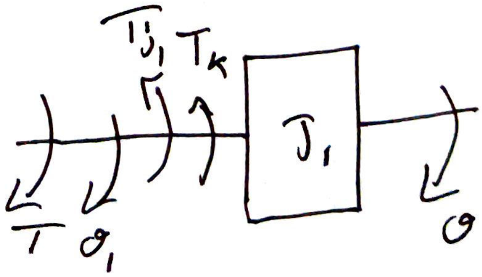
i/p $T \xrightarrow{LT} T(s)$

o/p θ $\mathcal{L}(\theta) = \theta(s)$

To find : $\frac{\theta(s)}{T(s)}$

Two nodes J_1 & J_2

i) J_1



$T = T_{J_1} + T_K$

$$T = J_1 \frac{d^2\theta_1}{dt^2} + K(\theta_1 - \theta)$$

$$T = J_1 \frac{d^2\theta_1}{dt^2} + K\theta_1 - K\theta$$

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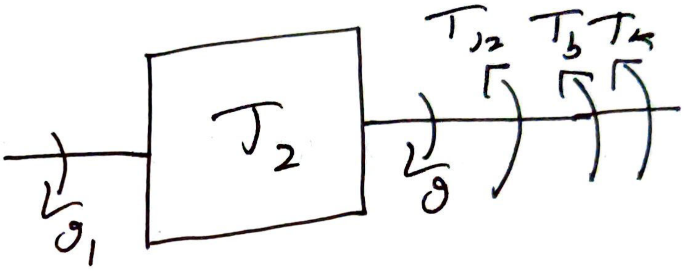
L T

$$J_1 s^2 \theta_1(s) + k\theta_1(s) - k\theta(s) = T(s)$$

$$(J_1 s^2 + k)\theta_1(s) - k\theta(s) = T(s) \quad \text{--- (1)}$$

ii) J_2 node



$$T_{J_2} + T_b + T_k = 0$$

$$J_2 \frac{d^2\theta}{dt^2} + B\frac{d\theta}{dt} + k(\theta - \theta_1) = 0$$

Taking L T

$$J_2 s^2\theta(s) + Bs\theta(s) + k\theta(s) - k\theta_1(s) = 0$$

$$(J_2 s^2 + Bs + k)\theta(s) - k\theta_1(s) = 0$$

$$\theta_1(s) = \frac{J_2 s^2 + Bs + k}{k}\theta(s) \quad \text{--- (2)}$$

Substit $\theta_1(s)$ from (2) in (1)

$$(J_1 s^2 + k)\frac{(J_2 s^2 + Bs + k)}{k}\theta(s) - k\theta(s) = T(s)$$

$$\therefore \frac{\theta(s)}{T(s)} = \frac{k}{(J_1 s^2 + k)(J_2 s^2 + Bs + k) - k^2}$$

Ex. 4

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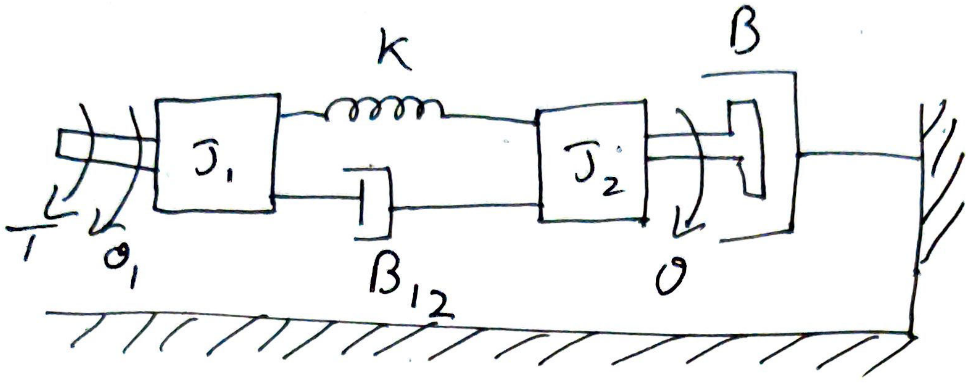
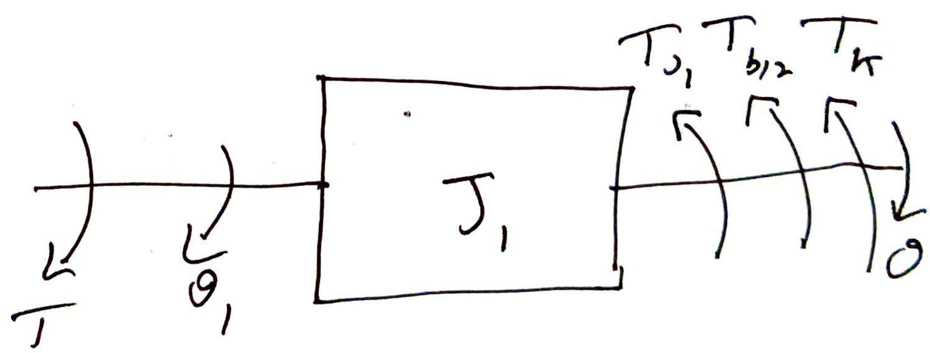


i/p T $\rightarrow$ LT $\rightarrow$ $T(s)$

o/p 'θ' $\rightarrow$ LT $\rightarrow$ $\theta(s)$

To find $TF = \dfrac{\theta(s)}{T(s)}$

Take J_1 & draw free body diagram.



$$\therefore T = T_{J_1} + T_{b_{12}} + T_K$$

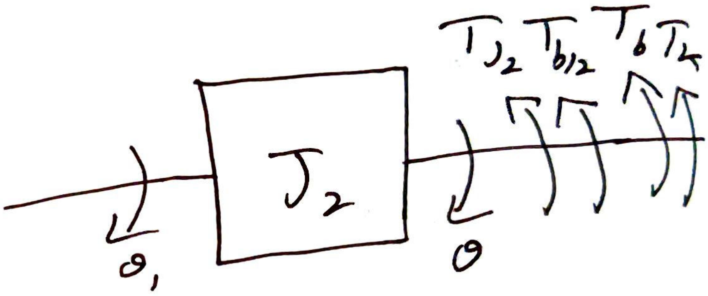
$$T = J_1 \frac{d^2\theta}{dt^2} + B_{12}\frac{d}{dt}(\theta_1 - \theta) + K(\theta_1 - \theta)$$

Take Laplace Transform

$$T_{(s)} = J_1 s^2 \theta_1(s) + s B_{12}\left[\theta_1(s) - \theta(s)\right] + k\theta_1(s) - k\theta(s)$$

$$T_{(s)} = \theta_1(s)\left[J_1 s^2 + sB_{12} + k\right] - \theta(s)\left[sB_{12} + k\right] \quad \text{--- (1)}$$

Draw free body diagram for J_2



$$\therefore T_{J_2} + T_{b12} + T_b + T_k = 0$$

$$J_2 \frac{d^2\theta}{dt^2} + B_{12}\frac{d}{dt}(\theta - \theta_1) + B\frac{d\theta}{dt} + k(\theta - \theta_1) = 0$$

Taking L.T

$$J_2 s^2 \theta(s) - B_{12} s\theta_1(s) + s\theta(s)\left[B_{12} + B\right] + k\theta(s) - k\theta_1(s) = 0$$

$$\theta(s)\left[s^2 J_2 + s(B_{12} + B) + k\right] - \theta_1(s)\left[sB_{12} + k\right] = 0$$

$$\theta_1(s) = \frac{\left[s^2 J_2 + s(B_{12} + B) + k\right]}{\left[sB_{12} + k\right]}\,\theta(s) \quad \text{(2)}$$

Substitute $\theta_1(s)$ in eqn ①

$$T(s) = \left[J_1 s^2 + sB_{12} + K\right] \frac{\left[J_2 s^2 + s(B_{12}+B) + K\right]\theta(s)}{(sB_{12}+K)} - (sB_{12}+K)\,\theta(s)$$

$$T(s) = \theta(s)\left[\frac{(J_1 s^2 + sB_{12} + K)(J_2 s^2 + s(B_{12}+B) + K) - (sB_{12}+K)^2}{(sB_{12}+K)}\right]$$

$$\therefore \frac{\theta(s)}{T(s)} = \frac{sB_{12}+K}{(J_1 s^2 + sB_{12} + K)\left[J_2 s^2 + s(B_{12}+B) + K\right] - (sB_{12}+K)^2}$$

Electrical Systems

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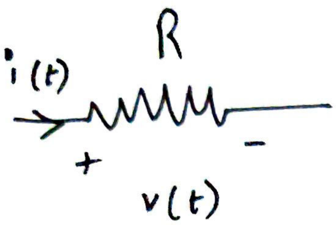
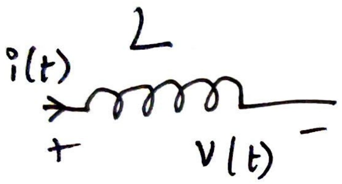
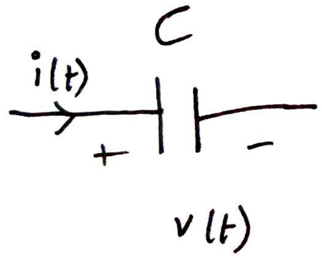
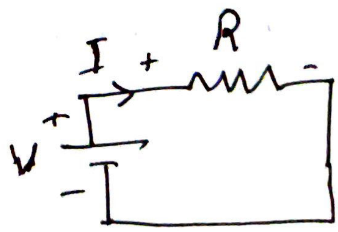
Element	Voltage A/c Element	Current through Element
	$v(t) = R\,i(t)$	$i(t) = \frac{v(t)}{R}$
	$V(t) = L\frac{d\,i(t)}{dt}$	$i(t) = \frac{1}{L}\int v(t)\,dt$
	$V(t) = \frac{1}{C}\int i(t)\,dt$	$i(t) = C\frac{dv(t)}{dt}$

Resistor → Resistance → Ohm's (Ω)

Inductor → Inductance → Henry (H)

Capacitor → Capacitance → farad (F)

Ohm's Law :



$$V = RI$$

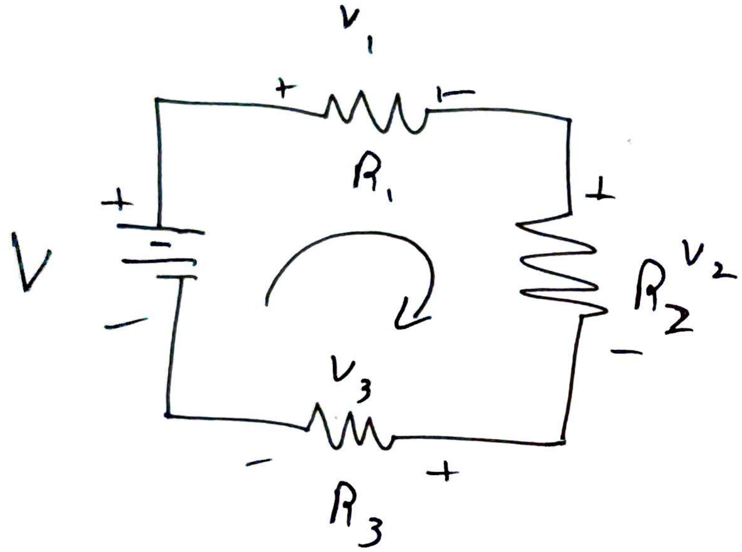
$$I = \frac{V}{R}$$

$$R = \frac{V}{I}$$

Kirchoff's Voltage law (KVL):

"The algebraic sum of all voltages in a loop must equal zero"

* current gets divided in ~~series~~ Parallel circuit
* Voltage gets divided in ~~parallel~~ series circuit.

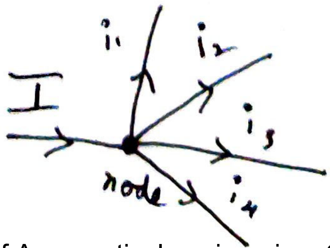


$$\therefore V = V_1 + V_2 + V_3$$

$$V - V_1 + V_2 + V_3 = 0$$

Kirchoff's current law (KCL)

"The algebraic sum of current entering a node is equal to zero"



$$\therefore I - i_1 - i_2 - i_3 - i_4 = 0$$

$$I = i_1 + i_2 + i_3 + i_4$$

Ex 5

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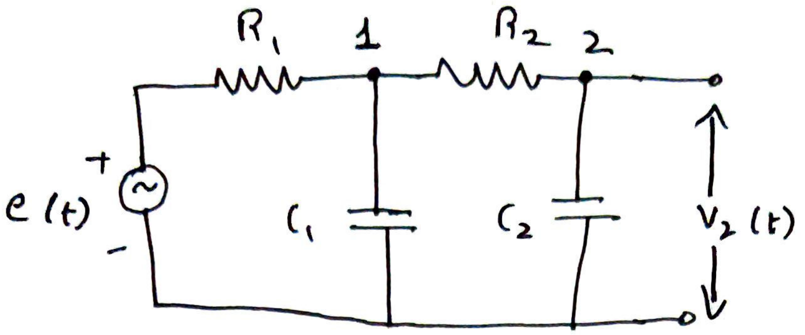


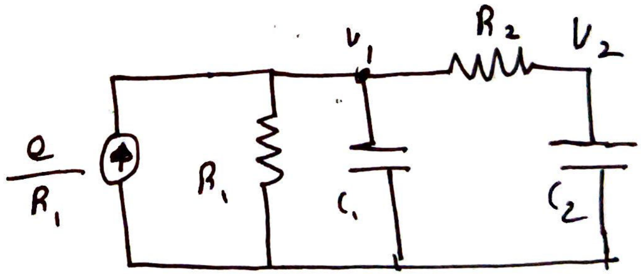
i/p $\quad e(t) = \mathcal{L}\, e(t) = E(s)$

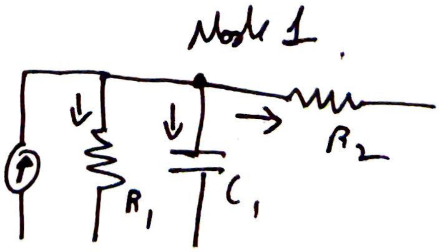
o/p $\quad v_2(t) = \mathcal{L}\, v_2(t) = V_2(s)$

$\therefore$ To find $TF = \dfrac{V_2(s)}{E(s)}$

Step (1) Take node 1 & write circuit in terms of current. Apply KCL

$$\frac{e}{R_1} = \frac{v_1}{R_1} + C_1 \frac{dv_1}{dt} + \frac{v_1 - v_2}{R_2}$$





Taking LT

$$\frac{V_1(s)}{R_1} + C_1 s V_1(s) + \frac{V_1(s)}{R_2} - \frac{V_2(s)}{R_2} = \frac{E(s)}{R_1}$$

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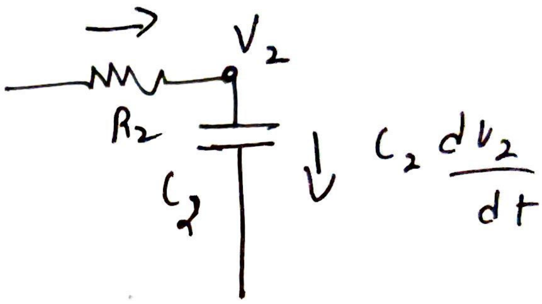
$$V_1(s)\left[\frac{1}{R_1} + sC_1 + \frac{1}{R_2}\right] - \frac{V_2(s)}{R_2} = \frac{E(s)}{R_1} \quad \text{—} \; (1)$$

Step (2) Taking Node (2)

Apply KCL



$$\frac{V_2 - V_1}{R_2} + C_2 \frac{dV_2}{dt} = 0$$

Taking LT

$$\frac{V_2(s)}{R_2} - \frac{V_1(s)}{R_2} + C_2 s V_2(s) = 0$$

$$\therefore V_1(s) = [1 + sC_2R_2]V_2(s) \quad \text{—} \; (2)$$

$\therefore$ Substitut $V_1(s)$ in eqn (1)

$$[1 + sC_2R_2]V_2(s)\left[\frac{1}{R_1} + sC_1 + \frac{1}{R_2}\right] - \frac{V_2(s)}{R_2} = \frac{E(s)}{R_1}$$

$$\therefore \frac{V_2(s)}{E(s)} = \frac{R_2}{(1 + sR_2C_2)[R_1 + R_2 + sC_1R_1R_2) - R_1]}$$

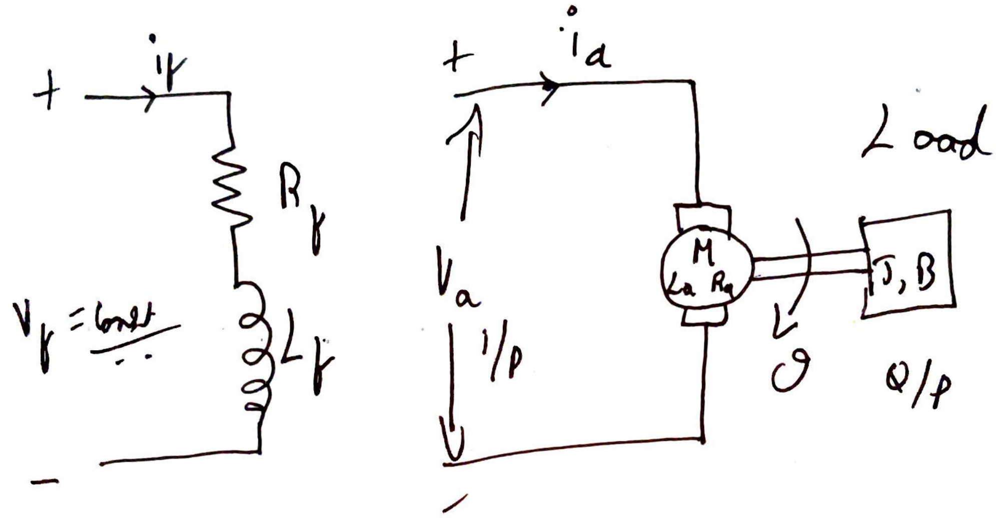
Module - 1

3/10/23

Armature Controlled DC Motor

Speed of DC Motor $\propto$ Armature Voltage

" — " $\frac{1}{\propto}$ flux in field winding.



Let, $R_a \to$ Armature resistance, Ω

$L_a \to$ " — " Inductance, H

$i_a \to$ " — " current, A

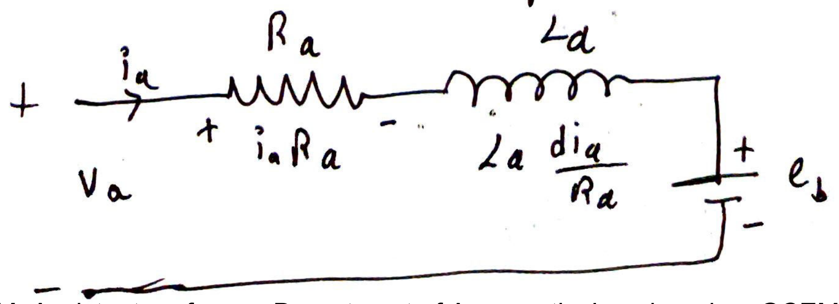
$V_a \to$ " — " Voltage, V

$e_b \to$ Back emf, V

$K_t \to$ Torque constant, N-m/A

$K_b \to$ Back emf constant, V/(rad/sec)

Step ① Equivalent ~~ckt~~ circuit of armature



By KVL

$$i_a R_a + L_a \frac{di_a}{dt} + e_b = V_a \quad \text{—} (1)$$

Step (2)

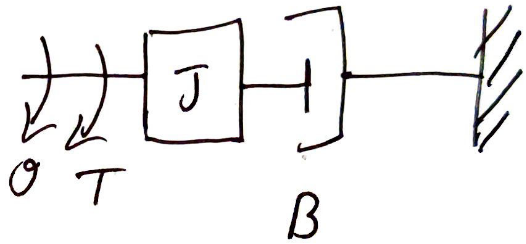
Torque $\quad T \propto$ Product of flux & current

$T \propto i_a$ $\quad$ ($\because$ flux is constant)

$$T = k_t i_a \quad \text{—} (2)$$

Step (3)

$$T = J\frac{d^2\theta}{dt^2} + B\frac{d\theta}{dt} \quad \text{—} (3)$$



Step (4)

Back emf $\propto \dot{\theta}$

$$\therefore e_b \propto \frac{d\theta}{dt} \qquad e_b = K_b \frac{d\theta}{dt} \quad \text{—} (4)$$

Taking LT on eqn (1) (2) (3) & (4)

$$I_a(s) R_a + L_a s I_a + E_b(s) = V_a(s) \quad \text{—} (5)$$

$$T(s) = K_t I_a(s) \quad \text{—} (6)$$

$$J s^2 \theta(s) + B s(\theta(s)) = T(s) \quad \text{—} (7)$$

$$E_b(s) = k_b s \theta(s) \quad \text{—} (8)$$

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Eqⁿ (6) & (7)

$$K_t I_a(s) = (Js^2 + Bs)\,\theta(s)$$

$$I_a(s) = \frac{(Js^2 + Bs)}{K_t}\,\theta(s) \quad \text{--- (9)}$$

Eqⁿ (5) can be written has

$$(R_a + sL_a) I_a(s) + E_b(s) = V_a(s) \quad \text{--- (10)}$$

Substitut $E_b(s)$ & $I_a(s)$ from (8) & (9) in (10)

$$(R_a + sL_a)\frac{(Js^2 + Bs)}{K_t}\,\theta(s) + K_b s\,\theta(s) = V_a(s)$$

$$\left[\frac{(R_a + sL_a)(Js^2 + Bs) + K_b K_t s}{K_t}\right]\theta(s) = V_a(s)$$

$$TF \therefore \frac{\theta(s)}{V_a(s)} = \frac{K_t}{(R_a + sL_a)(Js^2 + Bs) + K_b K_t s}$$

(or)

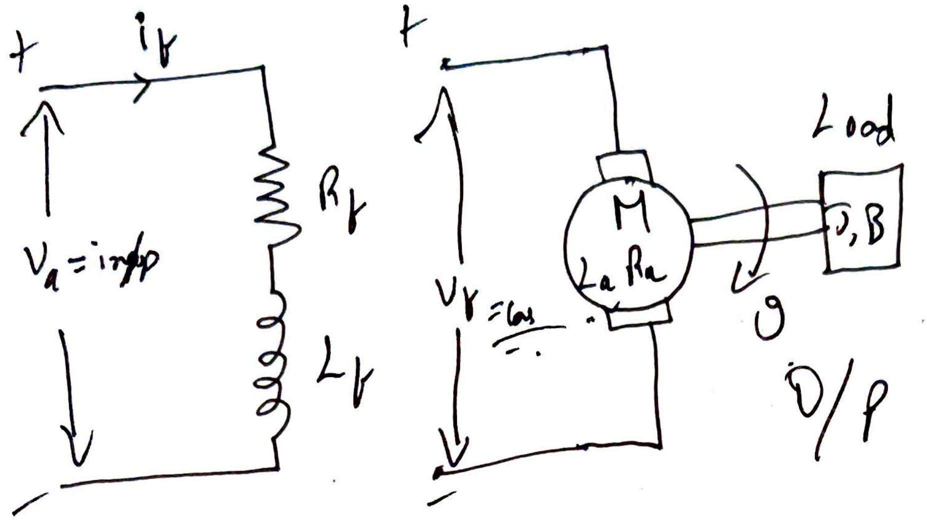
$$= \frac{K_t}{R_a\left(\frac{sL_a}{R_a} + 1\right) Bs\left(1 + \frac{Js^2}{Bs}\right) + K_b K_t s}$$

where $\frac{La}{Ra} = Ta$ = Electric time constant

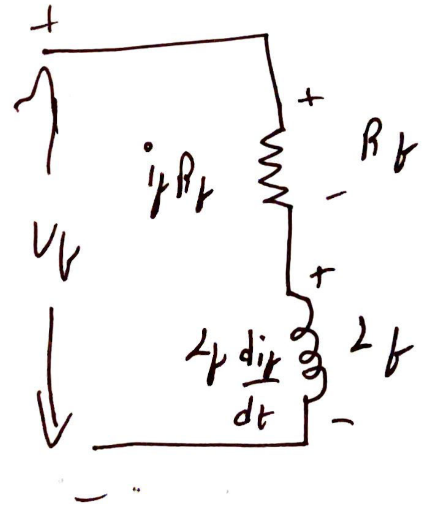
$\frac{J}{B} = Tm$ = Mechanical time constant.

T F of field controlled DC Motor.

4/10/23



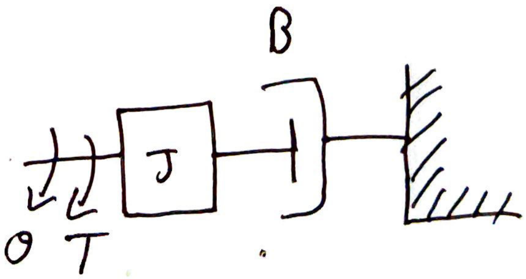
i) Step Equivalent circuit



∴ KVL

$$R_f i_f + L_f \frac{di_f}{dt} = V_f \quad \text{— (1)}$$

ii)



$T \propto i_f \times i_a$

i_a is constant

$\therefore T \propto i_f$

$$T = K_{tf} \cdot i_f \quad \text{— (2)}$$

$K_{tf} \rightarrow$ Torque constant N-m/A

iii) Differential governing eqn for Mechanical System.

$$J \frac{d^2\theta}{dt^2} + B\frac{d\theta}{dt} = T \quad -(3)$$

taking LT for above eqn

$$\cancel{R_f i_f + L_f \frac{di_f}{dt} = V_f}$$

$$R_f I_f(s) + L_f s I_f(s) = V_f(s) \quad -(4)$$

$$T(s) = K_{tf} I_f(s) \quad -(5)$$

$$J s^2 \theta(s) + B s \theta(s) = T(s) \quad -(6)$$

Equate (5) & (6)

$$K_{tf} I_f(s) = J s^2 \theta(s) + B s \theta(s)$$

$$I_f(s) = s \frac{(Js + B)}{K_{tf}} \theta(s) \quad -(7)$$

Equate I_f in eqn (4)

$$I_f(s)(R_f + sL_f) = V_f(s)$$

$$(R_f + sL_f) s \left(\frac{Js + B}{K_{tf}}\right) \theta(s) = V_f(s)$$

$$\therefore \frac{\theta(s)}{V_f(s)} = \frac{K_{tf}}{s(R_f + sL_f)(B + sJ)}$$

Also

$$\frac{\theta(s)}{V_f(s)} = \frac{K_{tf}}{s R_f \left(1 + \frac{sL_f}{R_f}\right) B\left(1 + \frac{sJ}{B}\right)}$$

where,

$$K_m = \frac{K_{tf}}{R_f B} = \text{Motor gain constant}$$

$$T_f = \frac{L_f}{R_f} = \text{Field time constant}$$

$$T_m = \frac{J}{B} = \text{Mechanical time constant}$$

4/10/23

Electrical Analogous of Mechanical Systems

* System is in analogous if TF are in identical form.

~~Mass~~ Spring → Capacitor / Capacitance

Dash-pot → Resistance / Resistor

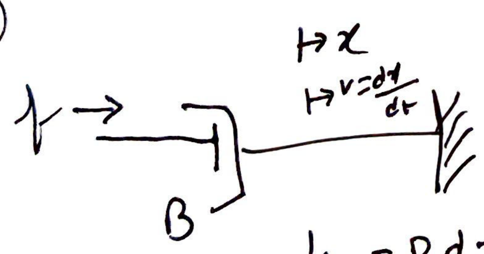
~~Spring~~ Mass → Inductor. / Inductance

(1) Force - Voltage Analogy

Elements

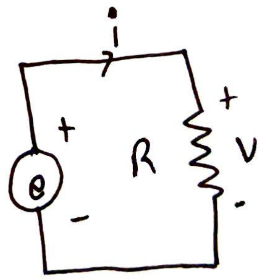
M S	E S
i/p : Force	i/p : Voltage
o/p : Velocity	o/p : Current through element

(1)



$$f = B\frac{dx}{dt}$$

$$f = Bv$$

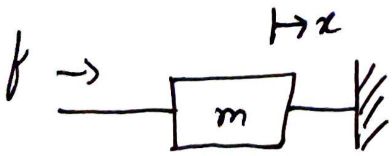


$$e = v$$

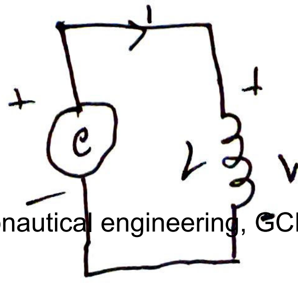
$$v = Ri$$

$$e = Ri$$

(2)



$$f = M\frac{d^2x}{dt^2} = M\frac{dv}{dt}$$

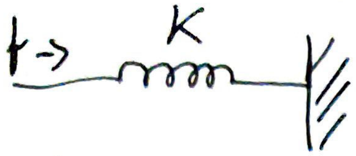


$$e = v$$

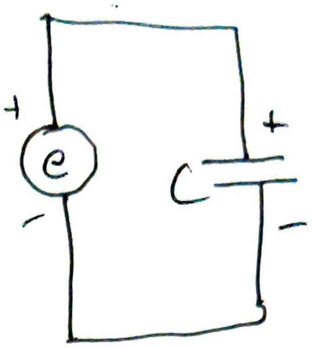
$$e = L\frac{di}{dt}$$

(3)

$\mapsto x = \int v\,dt$



$$f = Kx = k\int v\,dt$$



$$e = v = \frac{1}{C}\int i\,dt$$

$$\therefore\ e = \frac{1}{c}\int i\,dt$$

F - V Analogous Quantity

Item	Mechanical System	Electrical System
(1) Independent (i/p)	Force, f	Voltage, v, e
(2) Dependent (O/P)	Velocity (or) Displacement	Current (or) Charge
(3) Dissipative element	B	R
(4) Storage Element	M, K	L, 1/C
(5) Physical law	Newton's 2nd law $\Sigma f = 0$	KVL $\Sigma v = 0$
(6) Changing the level of independent variable	Lever $\frac{f_1}{f_2} = \frac{l_1}{l_2}$	Transformer $\frac{e_1}{e_2} = \frac{N_1}{N_2}$

Force - Current Analogy : (KCL)

Elements

M. S

i/p : F.

o/p : Velocity

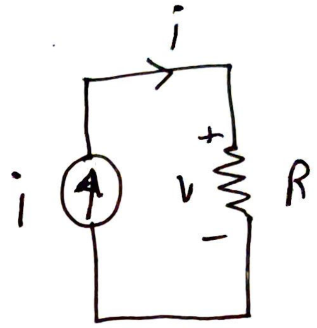
E. S

i/p : i

o/p : Voltage across circuit.

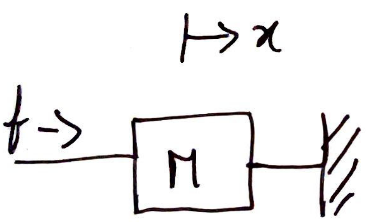
(1)

$$f = B\frac{dx}{dt} = Bv$$



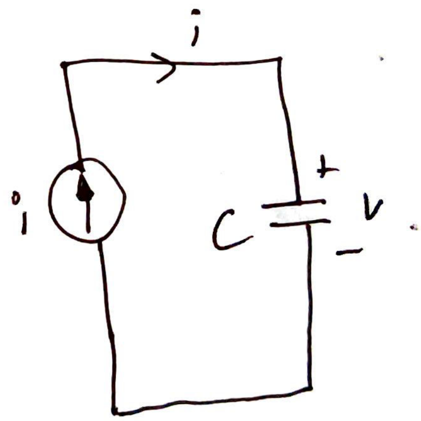
$$i = \frac{1}{R}v$$

(2)



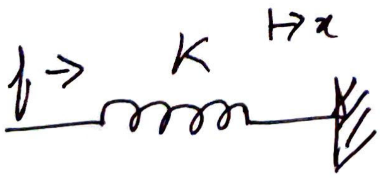
$$f = M\frac{d^2x}{dt^2}$$

$$f = M\frac{dv}{dt}$$

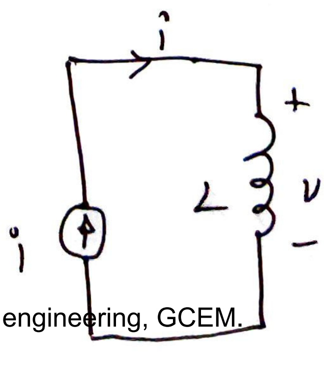


$$i = C\frac{dv}{dt}$$

(3)



$$f = Kx = K\int v\,dt$$



$$i = \frac{1}{L}\int v\,dt$$