GOPALAN COLLEGE
OF ENGINEERING AND MANAGEMENT
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Control Engineering
18AE732
Credits: 3
Department of Aeronautical Engineering

MODULE-3

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Routh-Hurwitz Stability Criterion

It is having one necessary condition and one sufficient condition for stability. If any control system doesn't satisfy the necessary condition, then we can say that the control system is unstable. But, if the control system satisfies the necessary condition, then it may or may not be stable. So, the sufficient condition is helpful for knowing whether the control system is stable or not.

Necessary Condition for Routh-Hurwitz Stability :
The necessary condition is that the coefficients of the characteristic polynomial should be positive. This implies that all the roots of the characteristic equation should have negative real parts. Consider the characteristic equation of the order 'n' is

$$a_0 s^n + a_1 s^{n-1} + a_2 s^{n-2} + \ldots + a_{n-1} s^1 + a_n s^0 = 0$$

Note that, there should not be any term missing in the $\mathbf{n}_{th}$ order characteristic equation. This means that the $\mathbf{n}_{\mathbf{th}}$ order characteristic equation should not have any coefficient that is of zero value.

Routh Array Method: If all the roots of the characteristic equation exist to the left half of the 's' plane, then the control system is stable. If at least one root of the characteristic equation exists to the right half of the 's' plane, then the control system is unstable. So, we have to find the roots of the characteristic equation to know whether the control system is stable or unstable. But, it is difficult to find the roots of the characteristic equation as order increases.

Routh-Hurwitz Stability Criterion

Routh array method: In this method, there is no need to calculate the roots of the characteristic equation. First formulate the Routh table and find the number of the sign changes in the first column of the Routh table. The number of sign changes in the first column of the Routh table gives the number of roots of characteristic equation that exist in the right half of the 's' plane and the control system is unstable.

Follow this procedure for forming the Routh table.

Fill the first two rows of the Routh array with the coefficients of the characteristic polynomial as mentioned in the table below. Start with the coefficient of s^n and continue up to the coefficient of s^0 .

Fill the remaining rows of the Routh array with the elements as mentioned in the table below. Continue this process till you get the first column element of row s^0 is a_n. Here a_n, is the coefficient of s^0 in the characteristic polynomial.

Note − If any row elements of the Routh table have some common factor, then you can divide the row elements with that factor for the simplification will be easy.

Routh-Hurwitz Stability Criterion

The following table shows the Routh array of the n^{th} order characteristic polynomial.

$$a_0 s^n + a_1 s^{n-1} + a_2 s^{n-2} + \ldots + a_{n-1} s^1 + a_n s^0$$

s^n	a_0	a_2	a_4	a_6	...	...
s^{n-1}	a_1	a_3	a_5	a_7	...	...
s^{n-2}	$b_1 = \frac{a_1 a_2 - a_3 a_0}{a_1}$	$b_2 = \frac{a_1 a_4 - a_5 a_0}{a_1}$	$b_3 = \frac{a_1 a_6 - a_7 a_0}{a_1}$	...	...	...
s^{n-3}	$c_1 = \frac{b_1 a_3 - b_2 a_1}{b_1}$	$c_2 = \frac{b_1 a_5 5 - b_3 a_1}{b_1}$	$\vdots$			
$\vdots$	$\vdots$	$\vdots$	$\vdots$			
s^1	$\vdots$	$\vdots$				
s^0	a_n					

Root-Locus Plots

The six rules of the root locus:

1.Find the number of poles, zeroes, number of branches, etc., from the given transfer functions.
2.Draw the plot that shows the poles and zeroes marked on it.
3.Calculate the angle of asymptotes and draw a separate sketch.
4.Find the centroid and draw a separate sketch.
5.Find the breakaway points. These points can also be in the form of complex numbers. We can use the angle condition to verify such points in the complex form.
6.Calculate the intersection points of the root locus with the imaginary axis (or y-axis).
7.Calculate the angle of arrival and departure if applicable.
8.Draw the final sketch of the root locus by combining all the above sketches.
9.We can also predict the stability and performance of the given system using the root locus technique.

*Most of the steps can be confusing. Let's discuss an example that will help us to understand the method to draw the root locus plot with the explanation.

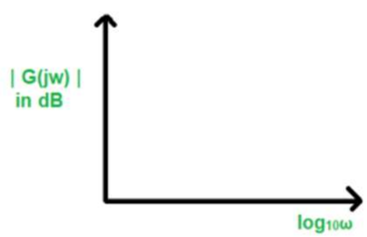
Frequency Response Analysis Using Bode Plots

Bode plots describe linear time-invariant systems' frequency response (change in magnitude and phase as a function of frequency). It helps in analyzing the stability of the control system. It applies to the minimum phase transfer function i.e. (poles and zeros should be in the left half of the s-plane).

Types of Bode Plot

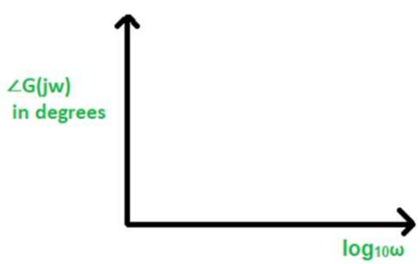
1. Gain plot:

It represents the magnitude response of the system as a function of frequency. It is plotted on a logarithmic scale



2. Phase plot:

It depicts the frequency-dependent phase shift of the system's output signal compared to its input signal. It's also drawn on a logarithmic scale.



Bode plot representation for the open loop system is:

$20 \log|G(j\omega)|$

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Frequency Response Analysis Using Bode Plots

Advantages

1.It helps in identifying the stability of the system.
2.It helps in identifying phases and gaining margins with minimum calculation.
3.It can be used to calculate the system's transfer function.
4.It can show the amplification and attenuation in the gain plot which is helpful in designing the filters.

Disadvantages

1.It is only applicable to LTI (linear time-invariant) system.
2.It is not suitable for the system having extremely high or low frequencies.
3.It focuses on the frequency response without considering the transient time effect.