Gopalan College of Engineering and Management

(ISO 9001:2015)

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Affiliated to Visvesvaraya Technological University (VTU), Belagavi, Karnataka

Recognised by Govt. of Karnataka

Address: 181/1, 182/1, Sonnenahalli, Hoodi, K.R.Puram, Whitefield, Bangalore, Karnataka - 560 048

Phone No: (080) - 42229748 **Email:** gcem@gopalancolleges.com **Website:** www.gopalancolleges.com/gcem

MODULE-WISE SOLUTIONS

Year / Semester	II / IV
Course Code	21AE44
Course Name	Mechanics of Materials

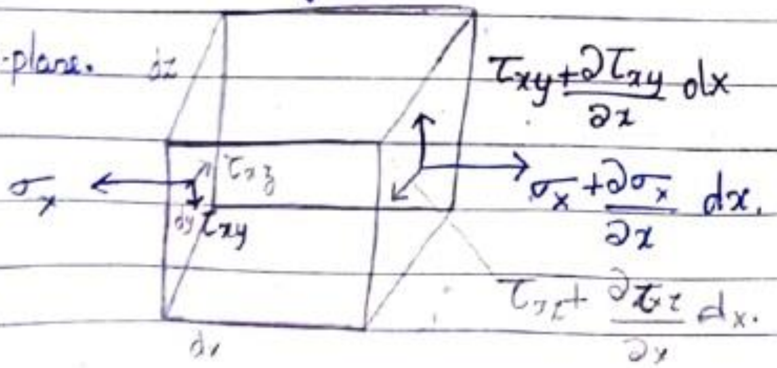
Module 1: Basics of Linear Elasticity, Simple & Compound Stresses

1. Analyze the stress at a point in 3D Elastic body and derive 3D stress equilibrium equations at that point. Deduce the equations to plane stress conditions. (10 Marks) Aug./Sept.2020, 18AE33

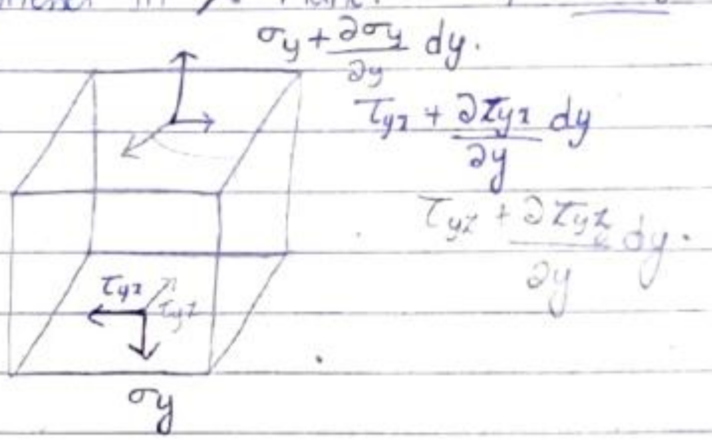
Consider an infinitesimally small 3-dimentional element of dimensions dx, dy & dz.

Stresses in x-plane. dz

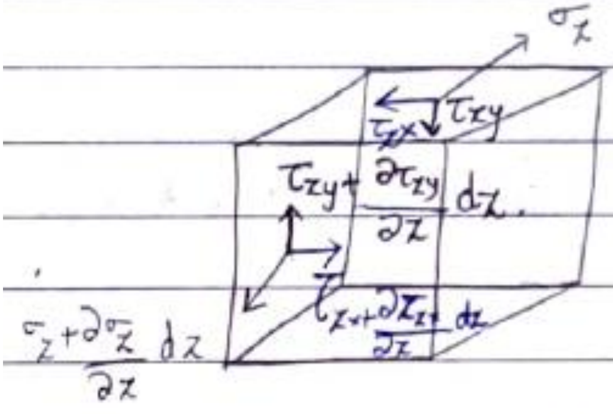
$dydz$

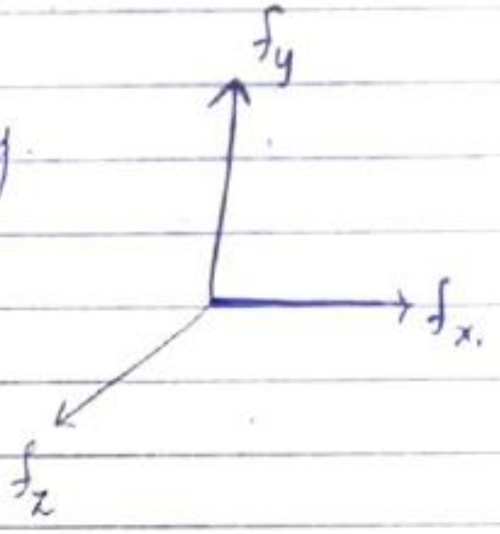


Stresses in X-Plane. $\rightarrow y \rightarrow dxdz$



Stresses in Z-plane. $\rightarrow dxdy$





Let the element be Subjected to $\sigma_x, \sigma_y, \sigma_z$. Shear stresses $\tau_{xy}, \tau_{yz}, \tau_{zx}$ & body forces. F_x, F_y & F_z.

For the equillibrium of the element the conditions are:

$\Sigma F_x = 0$, $\Sigma F_y = 0$, $\Sigma F_z = 0$.

Considering $\Sigma F_x = 0$.

$$\left[\cancel{\sigma_x} + \frac{\partial \sigma_x}{\partial x} dx\right] dydz - \cancel{\sigma_x} dydz + \left[\cancel{\tau_{yx}} + \frac{\partial \tau_{yx}}{\partial y} dy\right] dxdz$$

$$- \cancel{\tau_{yx}} dxdz + \left[\cancel{\tau_{zx}} + \frac{\partial \tau_{zx}}{\partial z} dz\right] dxdy - \cancel{\tau_{zx}} dxdy + f_x\, dxdydz = 0$$

$$\left(\frac{\partial \sigma_x}{\partial x} + \frac{\partial \tau_{yx}}{\partial y} + \frac{\partial \tau_{zx}}{\partial z} + f_x\right) dxdydz = 0.$$

$$\because dxdydz \neq 0.$$

$$\frac{\partial \sigma_x}{\partial x} + \frac{\partial \tau_{yx}}{\partial y} + \frac{\partial \tau_{zx}}{\partial z} + f_x = 0 \longrightarrow (1).$$

$$\Sigma F_y = 0.$$

$$\left[\tau_{xy} + \frac{\partial \tau_{xy}}{\partial x} dx\right] dydz - \tau_{xy}\, dydz$$

$$+ \left[\sigma_y + \frac{\partial \sigma_y}{\partial y} dy\right] dxdz - \sigma_y\, dxdz.$$

$$+ \left[\tau_{zy} + \frac{\partial \tau_{zy}}{\partial z} dz\right] dxdy - \tau_{zy}\, dxdy + f_y\, dxdydz = 0.$$

$$\left[\frac{\partial \tau_{xy}}{\partial x} + \frac{\partial \sigma_y}{\partial y} + \frac{\partial \tau_{zy}}{\partial z} + f_y\right] dxdydz = 0.$$

$$\because dxdydz \neq 0.$$

$$\frac{\partial \tau_{xy}}{\partial x} + \frac{\partial \sigma_y}{\partial y} + \frac{\partial \tau_{xy}}{\partial z} + f_y = 0 \rightarrow (2).$$

Considering $\Sigma F_z = 0.$

$$\left[\tau_{xz} + \frac{\partial \tau_{xz}}{\partial x} dx\right] dydz - \tau_{xz}\, dydz$$

$$+ \left[\tau_{yz} + \frac{\partial \tau_{yz}}{\partial y} dy\right] dxdz - \tau_{yz}\, dxdz.$$

$$+ \left[\sigma_z + \frac{\partial \sigma_z}{\partial z} dz\right] dxdy - \sigma_z\, dxdy + f_z\, dxdydz = 0.$$

$$= \left[\frac{\partial \tau_{xz}}{\partial x} + \frac{\partial \tau_{yz}}{\partial y} + \frac{\partial \sigma_z}{\partial z} + f_z\right] dxdydz = 0.$$

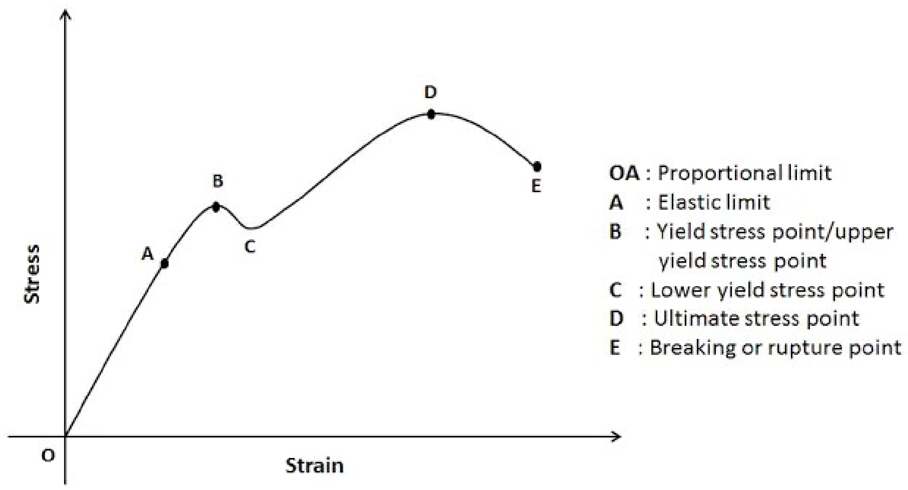
$$\because dxdydz \neq 0.$$

$$\frac{\partial \tau_{xz}}{\partial x} + \frac{\partial \tau_{yz}}{\partial y} + \frac{\partial \sigma_z}{\partial z} + f_z = 0 \rightarrow (3).$$

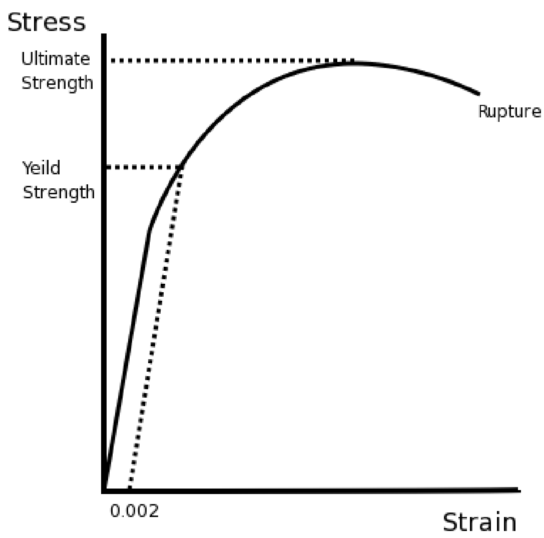
Eqn ① ② & ③ represents stress Equilibrium Eq^n in 3-D, In 2-D the Equilibrium equations reduces to

$$\left.\begin{aligned} \frac{\partial \sigma_x}{\partial x} + \frac{\partial \tau_{xy}}{\partial y} + f_x = 0. \\ \frac{\partial \tau_{xy}}{\partial x} + \frac{\partial \sigma_y}{\partial y} + f_y = 0. \end{aligned}\right\} \text{2-D}$$

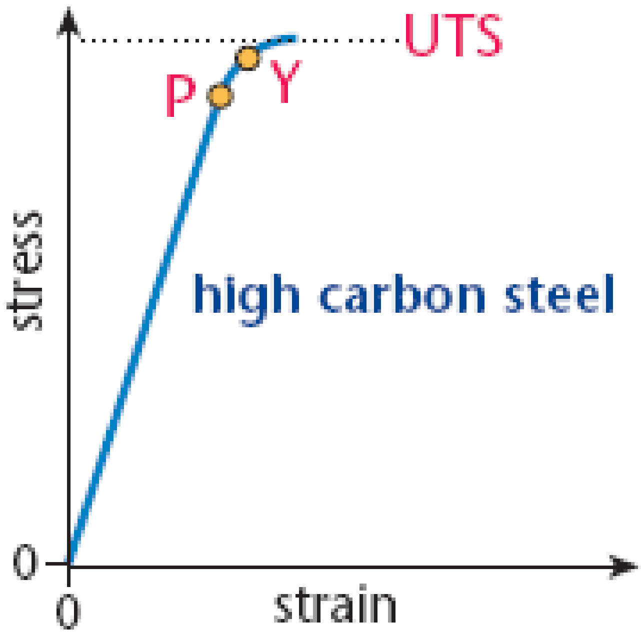
2. **Draw the stress-strain curves for the below mentioned materials mentioning/illustrating salient features of stress-strain curves: (i) Mild steel (ii) Aluminium (iii) High carbon steel (iv) Cast iron (v) Glass (05 Marks) Aug./Sept.2020, 18AE33**



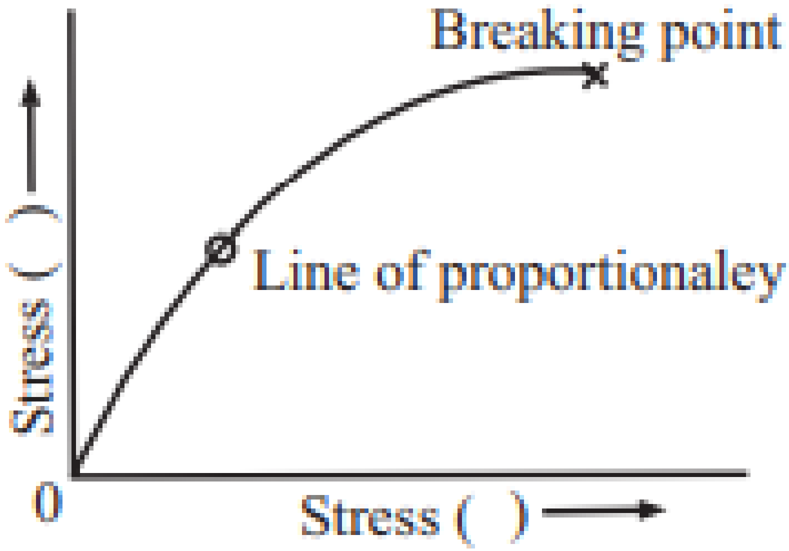
(i) Mild steel



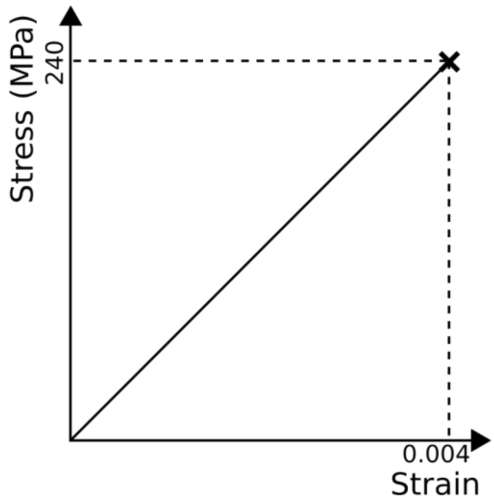
(ii) Aluminium



(iii) High carbon steel



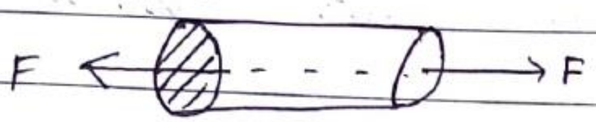
(iii) Cast iron



(v) Glass

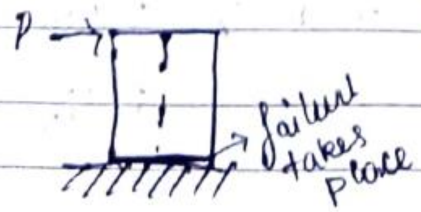
3. Define the following with example: (i) Normal stress (ii) Shear stress (iii) Bending stress (iv) Torsional stress (v) Bearing stress (05 Marks) Aug./Sept.2020, 18AE33

1. NORMAL STRESS :



When the external force is perpendicular to the cross-section of the body, the induced stress is known as Normal Stress.

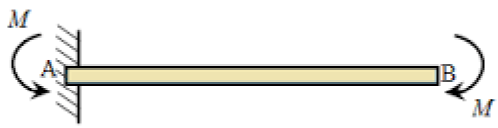
SHEAR STRESS :- It is the stress that causes distortion of the body as shown in fig 1.3. Mathematically shear stress



$$\tau = \frac{P}{A}$$

(iii) Bending Stress: Bending stress is the internal resistance generated within a component when an external bending moment or force is applied.

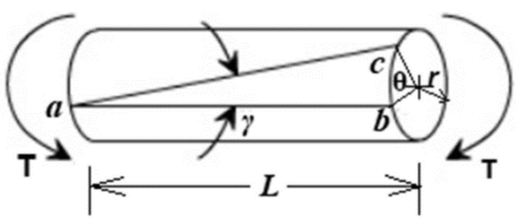
Bending stress, $\sigma = \frac{My}{I}$



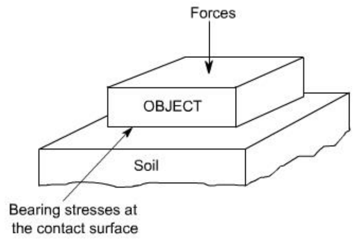
(a) Cantilever subjected to moment at its end

(iv) Torsional stress Torsional shear stress is the shear stress offered by the body against torsional load or twisting load.

Torsional stress, $\tau = \frac{TR}{J}$



(v) Bearing stress: The stresses developed when two elastic bodies are forced together are termed bearing stresses.

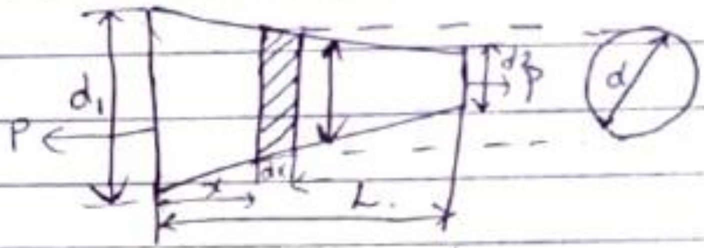


4. **Show that total Elongation in an uniformly tapering circular section bar is $\delta = \frac{4PL}{\pi E d_1 d_2}$. (06 Marks)**

Consider a uniformly tapering circular bar with diameter varying from d_1 to d_2 over a length L.

Rate of change of diameter

$K = \frac{d_1 - d_2}{L} \rightarrow (1).$



Consider an element of length dx at a distance x from one of the ends.

Diameter at this section $d = d_1 - Kx$.

Cross-Sectional area of the element $A = \frac{\pi}{4}(d_1 - Kx)^2$.

Extention of the element $\rightarrow d\delta = \frac{P\,dx}{\frac{\pi}{4}(d_1 - Kx)^2 E}$

Extension of the bar, $\delta = \frac{4P}{\pi E}\int_0^L \frac{dx}{(d_1 - kx)^2}$

$$\int \frac{1}{(a+bx)^2}\,dx = \frac{-1}{b(a+bx)}$$

$$\delta = \frac{4P}{\pi E K}\left[\frac{1}{(d_1 - Kx)}\right]_0^L$$

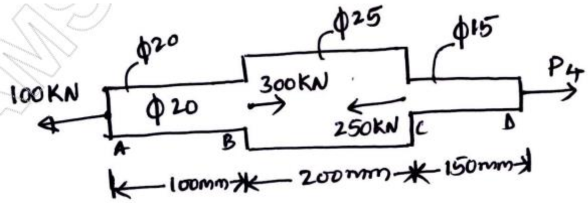
$$\delta = \frac{4P}{\pi E K}\left[\frac{1}{d_2} - \frac{1}{d_1}\right] \rightarrow (2)$$

Substitute eq (1) in (2).

$$\delta = \frac{4PL}{\pi E (d_1 - d_2)}\left[\frac{d_1 - d_2}{d_1 d_2}\right].$$

$$\delta = \frac{4PL}{\pi d_1 d_2 E}, \quad \delta = \frac{PL}{\frac{\pi d_1 d_2 E}{4}} \rightarrow (3).$$

5. **Determine the stresses in various segments of the circular bar shown in Fig.Q2(b). Compute the total Elongation taking Young's modulus to be 195 GPa. (08 Marks) Aug./Sept.2020, 18AE33**

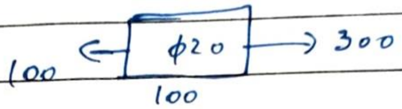


$\Sigma F = 0$

$-100 + 300 - 250 + P_4 = 0$

$P_4 = 50 kN$

Section ①



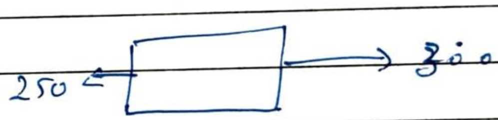
$P_1 = 200 kN$

$A_1 = \frac{\pi d^2}{4} = \frac{\pi (20)^2}{4} = 314.15 mm^2$

$L_1 = 100m$, $E = 195 \times 10^3 N/mm^2$

$\delta_1 = \frac{P_1 L_1}{AE_1} = \frac{200 \times 10^3 \times 100}{314.15 \times 195 \times 10^3} = 0.326 mm$

δ_2 $P_2 = 50 kN$



$L_2 = 200 m$, $A_2 = \frac{\pi \times 25^2}{4} = 490.87 mm^2$

$E = 195 \times 10^3 N/mm^2$

$\delta_2 = \frac{P_2 L_2}{A_2 E} = \frac{50 \times 10^3 \times 200}{490.82 \times 195 \times 10^3} = 0.104 mm.$

$P_3 = 200 kN$

$A_3 = \frac{\pi d^2}{4} = \frac{\pi (15)^2}{4} = 176.71 mm^2$, $L_3 = 150 mm$, $E = 195 \times 10^3$

$\delta_3 = \frac{PL}{AE} = \frac{200 \times 150 \times 10^3}{176.71 \times 195 \times 10^3} = 0.870 mm$

The total elongation

$\delta = \delta_1 + \delta_2 + \delta_3$

$= 0.326 + 0.104 + 0.870$

$\delta = 1.3 \text{ mm}$

6. Define Hooke's law. Draw and explain the stress-strain curves for ductile and brittle materials. (10 Marks) Jan./Feb. 2021, 18AE33

Hooke's Law States that stress is propotional to strain within elastic limit.
The ratio of stress to strain within elastic limit is characteristic of the material.
From Hooke's Law within elastic limit $\sigma \propto \epsilon$

$\sigma = E\epsilon$ $E \rightarrow$ propotional Constant known as Young's Modulus.

26/6/25 Stress - Strain Curve For Ductile Materials :-

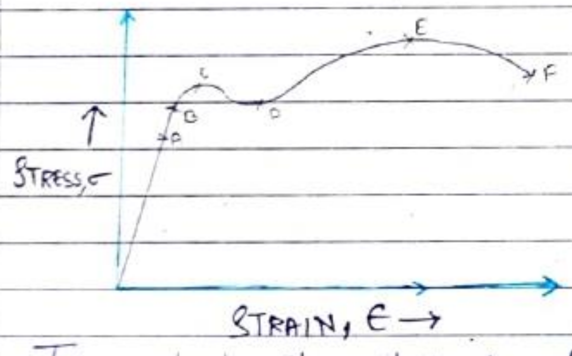


Figure show's stress-strain curve for a typical mild steel specimen. The following salient points are observed in the stress-strain curve.

(i) Propotionality Limit (A) :- It is the limiting value of stress upto which stress is propotional to strain.

(ii) Elastic Limit (B) :- It is the limiting value of stress upto which upon the release of stress the material retains it's original dimensions.

(iii) Upper Yield point (C) :- It is the stress at which the force starts reducing with increase in extension. This phenomenon is called as. yielding.

→ For mild steel this stress is about 250 N/mm^2.

(iv) Lower Yield point (D) :- It is the constant value of stress at which the strain increases for sometime.

(v) Ultimate Stress (E) :- It is the maximum value of stress the material can withstand.

Necking takes place at this point, for mild steel this stress is about (370 – 400) N/mm^2.

(vi) Breaking Stress (F) :- It is the stress at which finally the specimen fails.

STRESS - STRAIN CURVE FOR BRITTLE MATERIAL :

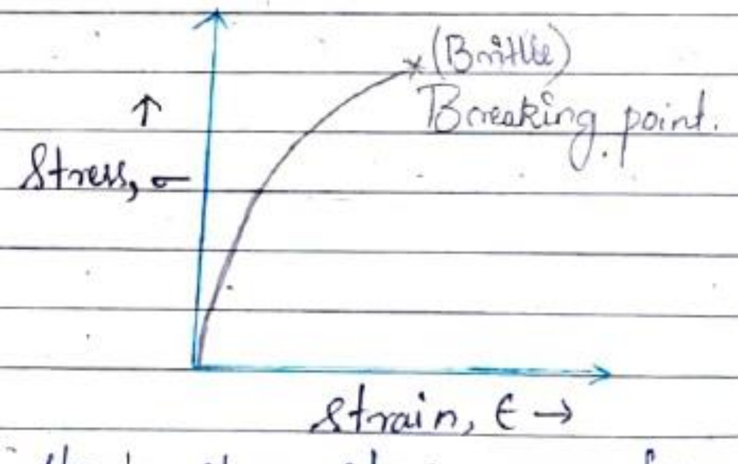


fig show's stress-strain curve for a material (Brittle) like cast iron.

The following Salient features are observed:-

i) There is no appreciable change in rate of strain

ii) There is no yield point & no necking takes place.

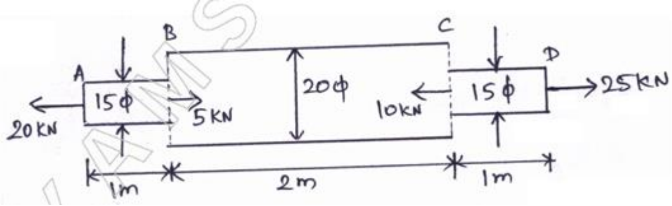
iii) The ultimate stress coincides with breaking Stress

iv) The strain at failure is very small.

7. Derive the equilibrium equations for a 3-D stress system. (10 Marks) Jan./Feb. 2021, 18AE33

(Refer to solution of Q1)

8. A steel bar ABCD 4m long is subjected to forces as shown in Fig.Q.2(a). Find the elongation of the bar. Take E for the steel as 200 GPa. (10 Marks) Jan./Feb. 2021, 18AE33



$$E = 200 \times 10^3 \ N/mm^2$$

$$\delta_1 = \frac{PL}{AE}$$

$$= \frac{20 \times 10^3 \times 1000}{\pi/4 (15)^2 \times 200 \times 10^3}$$

$$\boxed{\delta_1 = 0.565 mm}$$

$$\delta_2 = \frac{PL}{AE}$$

$$= \frac{30 \times 10^3 \times 2000}{\frac{\pi}{4}(20)^2 \times 200 \times 10^3}$$

$$\boxed{\delta_2 = 0.9549 mm}$$

$$\delta_3 = \frac{PL}{AE}$$

$$= \frac{25 \times 10^3 \times 1000}{\frac{\pi}{4}(15)^2 \times 200 \times 10^3}$$

$$\boxed{\delta_3 = 0.707 \text{ mm}}$$

$$\delta = \delta_1 + \delta_2 + \delta_3$$

$$\delta = 0.565 + 0.9549 + 0.707$$

$$\boxed{\delta = 2.2269 \text{ mm}}$$

9. Derive the equilibrium equations for the state of stress in 3-dimensions. (10 Marks) Feb./Mar. 2022, 18AE33

(Refer to solution of Q1)

10. Define the following: (i) True stress (ii) Engineering stress (iii) Hooke's law (iv) Poisson's ratio (v) Volumetric strain (10 Marks) Feb./Mar. 2022, 18AE33

(i) True stress: True stress is determined by dividing the load by the instantaneous area.

$$True\ Stress = \frac{Load}{Instantaneous\ Area}$$

(ii) Engineering stress: Engineering stress is determined by dividing the load by the original area.

$$Engineering\ Stress = \frac{Load}{Original\ Area}$$

(iii) Hooke's law:

Hooke's Law States that stress is propotional to strain within elastic limit.

From Hooke's Law within elastic limit $\sigma \propto \epsilon$

$\sigma = E\epsilon$ $E \rightarrow$ propotional Constant known as Young's Modulus.

(iv) Poisson's ratio:

Poison's Ratio :→ It is the property of the material which is defined as the ratio of lateral strain to linear strain.

Mathematically poison's ratio $\gamma = \frac{\epsilon_{lat}}{\epsilon_{lin}}$

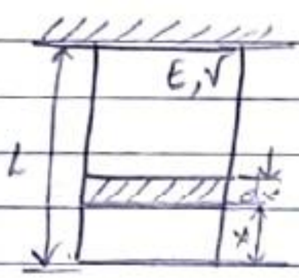
(v) Volumetric strain: Volumetric strain is defined as the change in volume divided by the original volume.

Volumetric Strain, $\varepsilon_v = \frac{\delta V}{V}$

11. Draw and explain stress-strain curves for Brittle and Ductile materials. (10 Marks) July/August 2021, 18AE33

(Refer to solution of Q6)

12. Show that extension produced due to self weight of a bar of uniform C/S fixed at one end and suspended vertically is equal to half the extension produced by a load equal to self weight applied at freed end. (10 Marks) July/August 2021, 18AE33



Consider an elemental length dx at a distance x from the free end.

Extension of this element:

$$d\delta = \frac{P dx}{AE} = \frac{\gamma A x dx}{AE}$$

$$\delta = \int_0^L \frac{\gamma x dx}{E} = \frac{\gamma}{E}\left[\frac{x^2}{2}\right]_0^L \Rightarrow \delta = \frac{\gamma L^2}{2E}.$$

Extension produced by a load equal to self -weight applied at freed end,

$$\delta = \frac{PL}{AE}$$

$$\delta = \frac{(\gamma AL)L}{AE}$$

$$\delta = \frac{\gamma L^2}{E} \qquad (2)$$

From (1) and (2), Extension produced due to self-weight of a bar of uniform C/S fixed at one end and suspended vertically is equal to half the extension produced by a load equal to self-weight applied at freed end.

13. Derive the equilibrium equations for a 3-D stress system. (08 Marks) Dec.2019/Jan.2020, 18AE33

(Refer to solution of Q1)

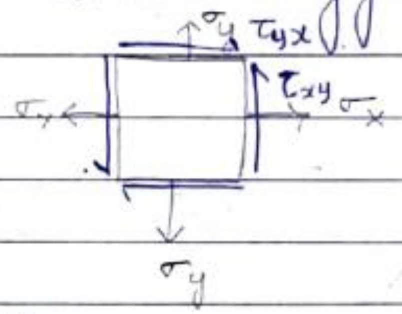
14. Define plane stress and plane strain with equations. (04 Marks) Dec.2019/Jan.2020, 18AE33

* PLANE STRESS :-

It is state of stress in which normal & shear stress components perpendicular to z plane or assumed to be zero. plane direction

This implies ⇒ $\sigma_z = \tau_{xz} = \tau_{yz} = 0$.

Ex: Thin planar element subjected to inplane loading as shown in the figure.

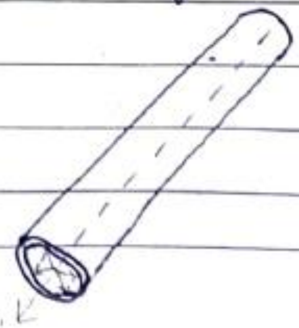


* PLANE STRAIN :-

It is a state of strain in which normal & shear strain components perpendicular to z plane are assumed to be zero.

γ ⇒ gamma ⇒ $\epsilon_z = \gamma_{xz} = \gamma_{yz} = 0$.

Ex: Long Hallow Cylinder Subjected to inplane loading.



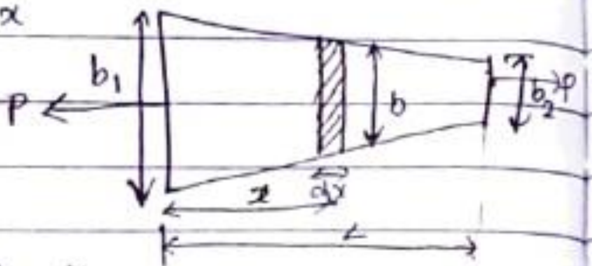
$\epsilon_z = \frac{\Delta z}{z} \simeq 0$.

15. Derive the total extension in a uniformly tapering rectangular bat with neat sketch. (08 Marks) Dec.2019/Jan.2020, 18AE33

Consider a tapering rectangular bar with width varying from b_1 to b_2 over a length L subjected to axial load P.

Rate of change of width $K = \dfrac{b_1 - b_2}{L} \rightarrow (1)$.

Consider an element of length dx at a distance x from one of the ends.



Width at this section $b = b_1 - Kx$

Cross-Sectional area of the element $A = (b_1 - Kx)t$.

Extension of the element $\rightarrow d\delta = \dfrac{P\,dx}{(b_1 - Kx)tE}$

Total extension of the bar

$$\delta = \frac{P}{tE} \int_0^L \frac{dx}{(b_1 - Kx)}$$

$$\left[\int \frac{1}{(a+bx)}\,dx = \frac{1}{b} \log_e (a+bx). \right] \rightarrow \text{formula}$$

$$\delta = \frac{P}{tE}\left(\frac{-1}{K}\right)\Big[\log_e (b_1 - Kx)\Big]_0^L$$

$$\delta = \frac{-P}{tEK}\Big[\log_e b_2 - \log_e b_1\Big]$$

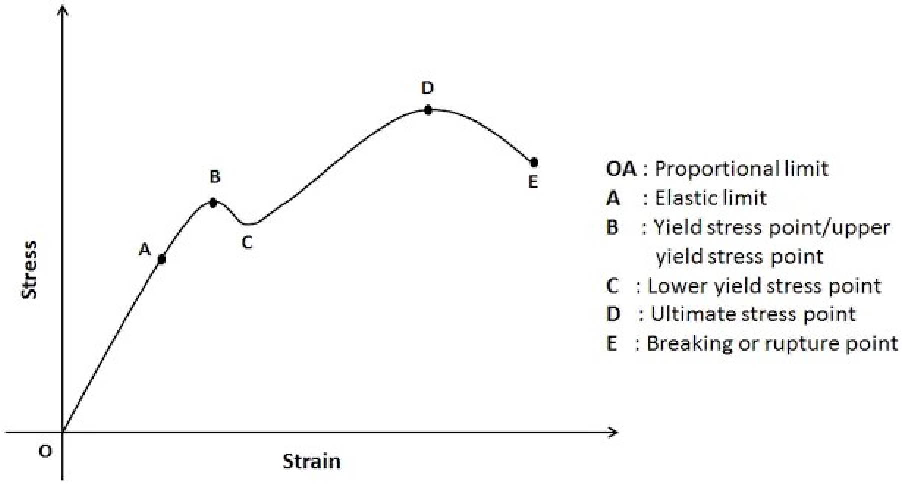
$$\delta = \frac{P}{tEK}\Big[\log_e b_1 - \log_e b_2\Big]$$

$$\delta = \frac{P}{tEK}\log_e\left(\frac{b_1}{b_2}\right) \rightarrow (2).$$

$$\delta = \frac{PL}{(b_1 - b_2)tE}\log_e\left(\frac{b_1}{b_2}\right) \rightarrow (3)$$

This is the expression for extension of tapered bar for rectangular cross-section.

16. **Draw a stress-strain diagram for ductile material and mention the salient points. (04 Marks) Dec.2019/Jan.2020, 18AE33**



17. **The tensile test was conducted on a mild steel bar. The following data was obtained from the test:**
Diameter of steel bar = 16 mm
Gauge length = 80 mm
Load at proportionality limit = 72 kN
Extension at a load of 60 kN = 0.115 mm
Load at failure = 80 kN
Final gauge length = 104 mm
Diameter of rod at failure = 12 mm
Determine: (i) Young's modulus (ii) Proportionality limit (iii) True breaking stress (iv) Percentage elongation (08 Marks) Dec.2019/Jan.2020, 18AE33

Data:

d = 16 mm

L = 80 mm

δ at 60 kN = 0.115 mm

Solution:

(i) Young's Modulus

Area of cross-section,

$$A = \frac{\pi d^2}{4} = \frac{\pi (16)^2}{4}$$

$$A = 201.062\ mm^2$$

$$Stress, \sigma = \frac{P}{A} = \frac{60 \times 10^3}{201.062}$$

$$\sigma = 298.4\ \ N/mm^2$$

$$Strain, \varepsilon = \frac{\delta}{L} = \frac{0.115}{80}$$

$$\varepsilon = 0.0014375$$

$$Young's\ Modulus, E = \frac{\sigma}{\varepsilon} = \frac{298.4}{0.0014375}$$

$$\boldsymbol{E = 207.58 \times 10^3\ N/mm^2}$$

(ii) Proportionality limit

$$Proportionality\ Limit = \frac{Load\ at\ Proportionality\ Limit}{Original\ Area}$$

$$Proportionality\ Limit = \frac{60 \times 10^3}{201.062}$$

$$Proportionality\ Limit = 298.4\ N/mm^2$$

(iii) True breaking stress

$$True\ Breaking\ Stress = \frac{Load\ at\ Failure}{Instantaneous\ Area}$$

$$Instantaneous\ Area = \frac{\pi(12)^2}{4} = 113.09\ mm^2$$

$$True\ Breaking\ Stress = \frac{80 \times 10^3}{113.09}$$

$$\boldsymbol{True\ Breaking\ Stress = 707.4\ N/mm^2}$$

(iv) Percentage elongation

$$\%\ Elongation = \frac{L_f - L_i}{L_i} \times 100$$

$$\%\ Elongation = \frac{104 - 80}{80} \times 100$$

$$\%\ \boldsymbol{Elongation = 30}$$

18. Derive the equilibrium equation for a 3-dimensional stress system. (10 Marks) Aug./Sept.2020, 17AE34

(Refer to solution of Q1)

19. Draw stress strain curve for mild steel and mention the salient point. (05 Marks) Aug./Sept.2020, 17AE34

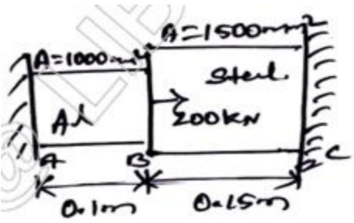
(Refer to solution of Q16)

20. Write a note on constitutive laws for anisotropic materials. (05 Marks) Aug./Sept.2020, 17AE34

Constitutive laws for anisotropic materials describe how materials respond to external forces and deformations, considering their directional dependencies:

Constitutive laws for anisotropic materials are essential in materials science and engineering. Anisotropic materials exhibit varying properties along different axes, necessitating tensorial representations for their behavior. These laws relate stress to strain through stiffness or compliance tensors. Anisotropy levels include orthotropic, transversely isotropic, and fully anisotropic materials, each requiring specific constitutive models. These laws are crucial in applications such as composites, geophysics, and aerospace engineering, enabling accurate predictions of material responses in diverse scenarios.

21. A composite bar is shown in Fig.Q2(c). Determine stress developed in each member. Eal = 0.7×10^5 N/mm^2, Esteel = 2 ×10^5 N/mm^2. (10 Marks) Aug./Sept.2020, 17AE34



for Al.

$A = 1000 \text{ mm}^2$.

$L = 0.1 \text{ m} = 100 \text{ mm}$

$E_{Al} = 0.7 \times 10^5 \text{ N/mm}^2$

$$\delta = \frac{PL}{AE}$$

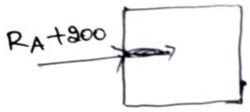
$$\delta = \frac{-R_A \times 100}{1000 \times 0.7 \times 10^5}$$

$\delta = -R_A \times 1.4285 \times 10^{-6}$ mm.

for Steel.

$A = 1500 \text{ mm}^2$

$L = 150 \text{ mm}$ $\quad E_{Steel} = 2 \times 10^5 \text{ N/mm}^2$



$$\delta = \frac{PL}{AE}$$

$$= \frac{(R_A + 200) \times 150}{1500 \times 2 \times 10^5}$$

$\delta = (R_A + 200)\, 5 \times 10^{-7}$ mm.

$\delta = \delta_1 + \delta_2$

$0 = (-R_A \times 1.4285 \times 10^{-6}) + (R_A + 200) 5 \times 10^{-7}$.

$$R_A(5\times10^{-7}) + 1\times10^{-4} - R_A \times 1.4285\times10^{-6} = 0.$$

$$R_A = 108.04 \text{ KN}.$$

$$R_A = 108.04\times10^3 \text{ N}.$$

$$R_B = 200 + 108.04.$$

~~$R_A = 200 + 308.$~~

$$R_B = 308.04\times10^3 \text{ N}.$$

~~$\sigma_{Al} = -RA$~~

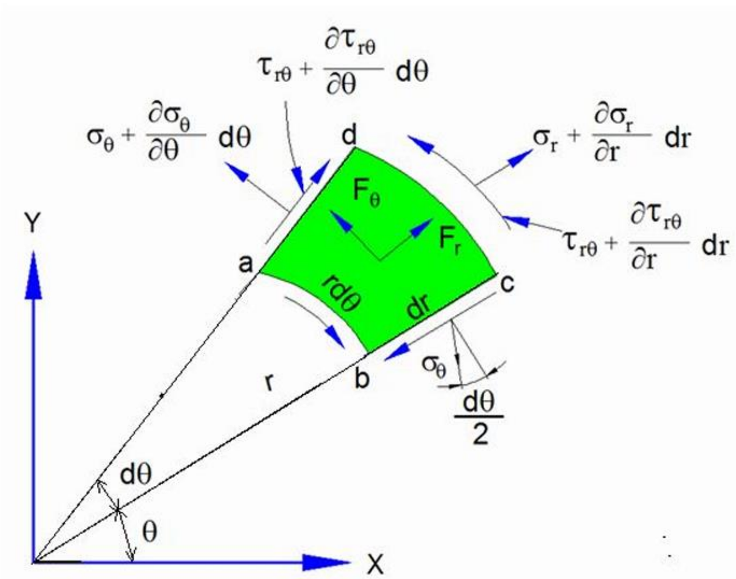
$$\sigma_{Al} = \frac{P}{A} = \frac{-108.04\times10^3}{1000.}$$

$$\sigma_{Al} = -108.04 \text{ N/mm}^2.$$

$$\sigma_{Steel} = \frac{P}{A} = \frac{308.04\times10^5}{1500}$$

$$\sigma = 205.36 \text{ N/mm}^2.$$

22. **Derive the equilibrium equations in polar co-ordinate for a two dimensional state of stress. (10 Marks) Dec.2018/Jan.2019, 17AE34**



The polar coordinate system (r, θ) and the cartesian system (x, y) are related by the following expressions:

$$x = r\cos\theta, \qquad r^2 = x^2 + y^2$$

$$y = r\sin\theta, \qquad \theta = \tan^{-1}\left(\frac{y}{x}\right) \tag{2.43}$$

Consider the state of stress on an infinitesimal element abcd of unit thickness described by the polar coordinates as shown in the Figure 2.16. The body forces denoted by F_r and F_θ are directed along r and θ directions respectively.

Resolving the forces in the r-direction, we have for equilibrium, $\Sigma F_r = 0$,

$$-\sigma_r \times r d\theta + \left(\sigma_r + \frac{\partial\sigma_r}{\partial r}dr\right)(r + dr)d\theta - \sigma_\theta dr \sin\frac{d\theta}{2} + F_r - \left(\sigma_\theta + \frac{\partial\sigma_\theta}{\partial\theta}d\theta\right)dr$$

$$\sin\frac{d\theta}{2} - \tau_{r\theta} dr\cos\frac{d\theta}{2} + \left(\tau_{r\theta} + \frac{\partial\tau_{r\theta}}{\partial\theta}d\theta\right)dr\cos\frac{d\theta}{2} = 0$$

Since $d\theta$ is very small,

$$\sin\frac{d\theta}{2} = \frac{d\theta}{2} \text{ and } \cos\frac{d\theta}{2} = 1$$

Neglecting higher order terms and simplifying, we get

$$r\frac{\partial\sigma_r}{\partial r} dr\, d\theta + \sigma_r dr\, d\theta - \sigma_\theta dr\, d\theta + \frac{\partial\tau_{r\theta}}{\partial\theta} dr\, d\theta = 0$$

on dividing throughout by $r d\theta\ dr$, we have

$$\frac{\partial\sigma_r}{\partial r} + \frac{1}{r}\frac{\partial\tau_{r\theta}}{\partial\theta} + \frac{\sigma_r - \sigma_\theta}{r} + F_r = 0 \tag{2.44}$$

Similarly resolving all the forces in the θ - direction at right angles to r - direction, we have

$$-\sigma_\theta dr\cos\frac{d\theta}{2} + \left(\sigma_\theta + \frac{\partial\sigma_\theta}{\partial\theta}d\theta\right)dr\cos\frac{d\theta}{2} + \tau_{r\theta} dr\sin\frac{d\theta}{2} + \left(\tau_{r\theta} + \frac{\partial\tau_{r\theta}}{\partial\theta}d\theta\right)dr$$

$$\sin\frac{d\theta}{2} - \tau_{r\theta} r d\theta + (r + dr)d\theta\left(\tau_{r\theta} + \frac{\partial\tau_{r\theta}}{\partial r}dr\right) + F_\theta = 0$$

On simplification, we get

$$\left(\frac{\partial\sigma_\theta}{\partial\theta} + \tau_{r\theta} + \tau_{r\theta} + r\frac{\partial\tau_{r\theta}}{\partial r}\right)d\theta\, dr = 0$$

Dividing throughout by $r d\theta\, dr$, we get

$$\frac{1}{r}\cdot\frac{\partial\sigma_\theta}{\partial\theta} + \frac{\partial\tau_{r\theta}}{dr} + \frac{2\tau_{r\theta}}{r} + F_\theta = 0 \tag{2.45}$$

In the absence of body forces, the equilibrium equations can be represented as:

$$\frac{\partial\sigma_r}{\partial r} + \frac{1}{r}\frac{\partial\tau_{r\theta}}{\partial\theta} + \frac{\sigma_r - \sigma_\theta}{r} = 0 \tag{2.46}$$

$$\frac{1}{r}\frac{\partial\sigma_\theta}{\partial\theta} + \frac{\partial\tau_{r\theta}}{\partial r} + \frac{2\tau_{r\theta}}{r} = 0$$

23. Displacement field at a point on a body is given as follows :

$$u = (x^2 + y),\ v = (3 + z),\ w = (x^2 + 2y)$$

Determine strain components at (3, 1, -2) and express them in Matrix form. (06 Marks) Dec.2018/Jan.2019, 17AE34

Solⁿ: $\epsilon_x = \frac{\partial u}{\partial x} = \frac{\partial}{\partial x}(x^2+y) = 2x$, $2(3) = 6 = \epsilon_x = 6$ units

$\epsilon_y = \frac{\partial v}{\partial y} = \frac{\partial}{\partial y}(3+z) = 0$, $\epsilon_y = 0$ units

$\epsilon_z = \frac{\partial w}{\partial z} = \frac{\partial}{\partial z}(x^2+2y) = 0$, $\epsilon_z = 0$ units

$\gamma_{xy} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} = \frac{\partial}{\partial y}(x^2+y) + \frac{\partial}{\partial x}(3+z) = 1 \Rightarrow (1)^2 = 1$ units

$\gamma_{yz} = \frac{\partial v}{\partial z} + \frac{\partial w}{\partial y} = \frac{\partial}{\partial z}(3+z) + \frac{\partial}{\partial y}(x^2+2y) = 1 + 2 = 3$ units

$\gamma_{xz} = \frac{\partial u}{\partial z} + \frac{\partial w}{\partial x} = \frac{\partial}{\partial z}(x^2+y) + \frac{\partial}{\partial x}(x^2+2y) = 2x = 2(3) = 6$ units

$$\epsilon_{ij} = \begin{bmatrix} \epsilon_x & \gamma_{xy} & \gamma_{xz} \\ \gamma_{xy} & \epsilon_y & \gamma_{yz} \\ \gamma_{zx} & \gamma_{yz} & \epsilon_z \end{bmatrix} = \begin{bmatrix} 6 & 1 & 6 \\ 1 & 0 & 3 \\ 6 & 3 & 0 \end{bmatrix}$$

24. Define plane stress and plane strain. (04 Marks) Dec.2018/Jan.2019, 17AE34

PLANE STRESS :-

It is state of stress in which normal & shear stress components perpendicular to z plane or assumed to be zero.

PLANE STRAIN :-

It is a state of strain in which normal & shear strain components perpendicular to z plane are assumed to be zero.

$\Rightarrow \epsilon_z = \gamma_{xz} = \gamma_{yz} = 0.$

25. Draw stress – strain curve for ductile material and mention the salient points. (06 Marks) Dec.2018/Jan.2019, 17AE34

(Refer to solution of Q16)

26. Define the term load factor and allowable stress. (04 Marks) Dec.2018/Jan.2019, 17AE34

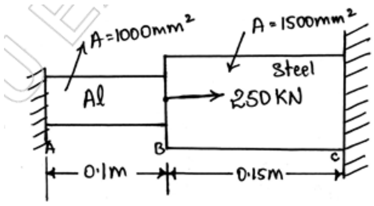
Load Factor is defined as ratio of allowable stress to the maximum stress.

For ductile materials, $Load\ Factor = \frac{Allowable\ Stress}{Yield\ Stress}$

For brittle materials, $Load\ Factor = \frac{Allowable\ Stress}{Ultimate\ Stress}$

Allowable stress, or allowable strength, is the maximum stress that can be safely applied to a structure.

27. A composite bar is shown in Fig Q2(c). Determine the stress developed in each member. Take E_{al} = 70 GPa; E_{steel} = 200 GPa (10 Marks) Dec.2018/Jan.2019, 17AE34



(Refer to solution of Q21)

28. Analyze the stress at point in 3D elastic material and reduce the stress equations to two dimensional state. (10 Marks) June/July 2019, 17AE34

(Refer to solution of Q1)

29. A structure is loaded and displacements associated with the deformed state are mapped. Find all the strains associated with the following observed displacement field at point (0.6, −0.75, 1) m. m 10)

$$u = (-x^4 + 3x - 3y^2 + 8yz + 5) \times 10^{-3}\,m$$
$$v = (-4y^4 + y + 8xz + 1) \times 10^{-3}\,m$$
$$w = (2z^2 + 3z - 8xy + 8) \times 10^{-3}\,m$$

(10 Marks) June/July 2019, 17AE34

Normal strain :- $\epsilon_x = \frac{\partial}{\partial x}(-x^4+3x-3y^2+8yz+5)\times 10^{-3} m$

$\epsilon_x = (-4x^3+3)\times 10^{-3} m.$

$\epsilon_y = \frac{\partial}{\partial y}(-4y^4+y+8xz+1)\times 10^{-3} m = [-4(4y^3)+1]\times 10^{-3} m$

$\epsilon_y = (-16y^3+1)10^{-3} m$

$\epsilon_z = \frac{\partial}{\partial z}(2z^2+3z-8xy+8)\times 10^{-3} m = (2(2z)+3)\,10^{-3} m$

$\epsilon_z = (4z+3)\,10^{-3} m.$

$\gamma_{xy} = \frac{\partial}{\partial y}(-x^4+3x-3y^2+8yz+5)\times10^{-3}m + \frac{\partial}{\partial x}(-4y^4+y+8xz+1)\times10^{-3}m$

$= (-3(2y)+8z)10^{-3}m + (8z)\times10^{-3}m \Rightarrow (-6y+8z)10^{-3}+8z\times10^{-3}m$

$= (-6y+16z)\times10^{-3}m.$

$\gamma_{yz} = \frac{\partial}{\partial z}(-4y^4+y+8xz+1)\times10^{-3}m + \frac{\partial}{\partial y}(2z^2+3z-8xy+8)\times10^{-3}m.$

$\gamma_{yz} = (8x)\times10^{-3}m + (-8x)\times10^{-3}m = \underline{\underline{0}}$

$\gamma_{xz} = \frac{\partial}{\partial z}(-x^4+3x-3y^2+8yz+5)\times10^{-3}m+\frac{\partial}{\partial x}(2z^2+3z-8xy+8)\times10^{-3}m$

$\gamma_{xz} = (8y)\times10^{-3}m + (-8y)\times10^{-3}m = 0.$

At point → $\epsilon_x = (-4(0.6)^3+3)\times10^{-3}m = 2.136\times10^{-3}$ units

$\epsilon_y = (-16y^3+1)10^{-3}m \Rightarrow (-16(-0.75)^3+1)10^{-3}m = 7.75\times10^{-3}$ units

$\epsilon_z = (4z+3)10^{-3}m \Rightarrow (4(1)+3)10^{-3}m \Rightarrow 7\times10^{-3}$ units.

$\gamma_{xy} = [(-6)(-0.75)+16(1)]\times10^{-3}m = 20.5\times10^{-3}$ units.

$\gamma_{yz} = \gamma_{xz} = 0$ units.

30. Discuss and illustrate the stress-strain curves for the following materials (i) Mild Steel (ii) Aluminum (iii) Cast Iron, with salient features. (10 Marks) June/July 2019, 17AE34

(i) Mild Steel

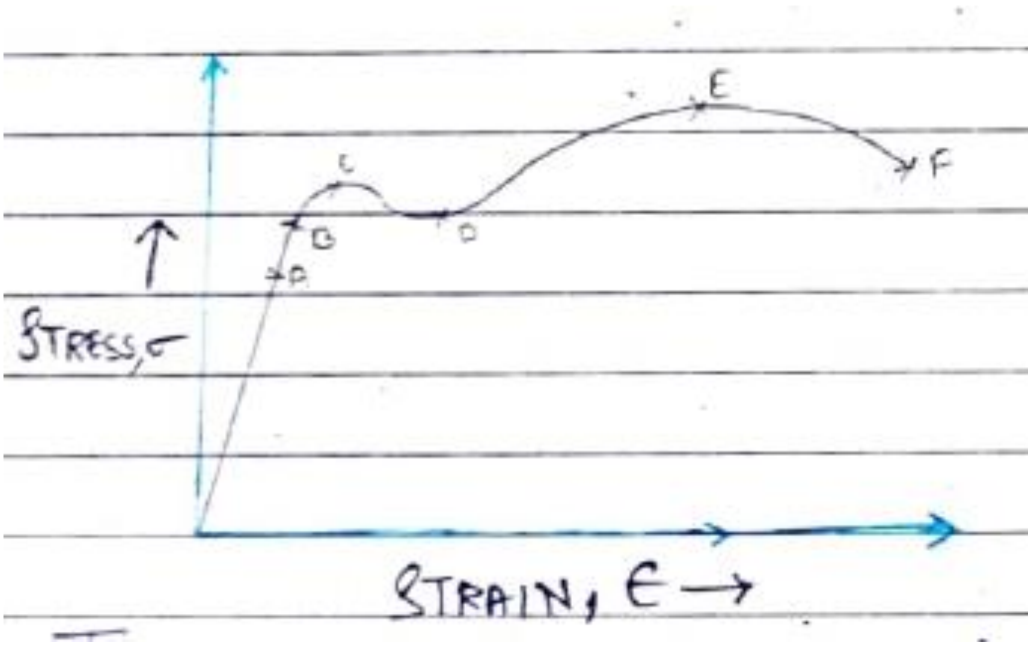


Figure shows stress-strain curve for a typical mild steel specimen. The following salient points are observed in the stress-strain curve.

(i) Proportionality Limit (A):- It is the limiting value of stress upto which stress is proportional to strain.

(ii) Elastic Limit (B):- It is the limiting value of stress upto which upon the release of stress the material retains it's original dimensions.

(iii) Upper Yield point (C):- It is the stress at which the force starts reducing with increase in extension. This phenomenon is called as yielding.

→ For mild steel this stress is about 250 N/mm^2.

(iv) Lower Yield point (D):- It is the constant value of stress at which the strain increases for sometime.

(v) Ultimate Stress (E):- It is the maximum value of stress the material can withstand.

Necking takes place at this point, for mild steel this stress is about (370 – 400) N/mm^2.

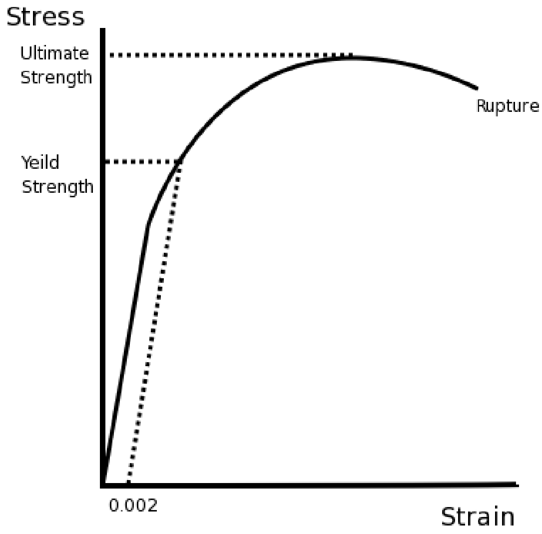
(vi) Breaking Stress (F):- It is the stress at which finally the specimen fails.

(ii) Aluminum:

(a) There is no clear yield point.

(b) Necking takes place at ultimate stress.

(c) Failure stress is below ultimate stress.



(iii) Cast Iron

STRESS - STRAIN CURVE FOR BRITTLE MATERIAL:

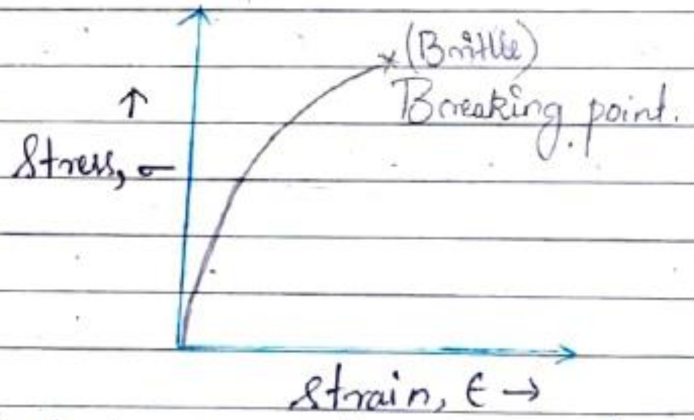


fig show's stress-strain curve for a material (Brittle) like cast iron.

The following Salient features are observed:-

i) There is no appreciable change in rate of strain

ii) There is no yield point & no necking takes place.

iii) The ultimate stress coinsides with breaking Stress

iv) The strain at failure is very small.

31. Derive equilibrium equations for a 3D stress system. (12 Marks) Jan./Feb.2021, 17AE34

(Refer to solution of Q1)

32. With a neat sketch explain all the salient points of a stress-strain diagram for mild steel. (08 Marks) Jan./Feb.2021, 17AE34

(Refer to solution of Q16)

33. Discuss material selection for structural performance. (12 Marks) Jan./Feb.2021, 17AE34

1. Consider load Requirements
Determine the expected loads and stresses the Structure will endure. including static, dynamic and environmental loads.

2. Strength and Stiffness: Choose materials with high strength and stiffness to support the applied loads and prevent deformation.

3. Durability and Corrosion Resistance
Assess the environment and exposure conditions to select materials that resist corrosion, degradation, and other potential hazards.

4. Temperature & Thermal properties: Factor in the operating temperature range & thermal expansion characteristics of materials to ensure structural stability.

5. Weight & density: Optimize material weight & density to achieve an efficient design, balancing Strength & weight savings.

6. Cost & Avalability :- Consider the material cost, avalability, and fabrication process to stay within budget and timeline constraints.

7. Fatigue Resistance :- Evaluate the materials ability to withstand repetitive loading cycles without failure.

8. Creep & deformation :- Examine the long-term behavior of materials under constant stress to avoid excessive deformation.

9. Environmental impact :- Assess the ecological footprint of materials to choose sustainable & environmentally friendly options.

10. Joining & Assembly :- Consider compatibility with joining methods and assembly techniques for care of construction.

11. Material Testing & Certification :- Rely on certified materials with well documented mechanical properties & performance data.

12. ~~Pre~~ Maintainance and Service life :- Evaluate the care of maintanance and the expected service life of the selected materials.

34. Derive the equilibrium equations for the state of stress in 3-Dimensions. (10 Marks) Feb./Mar. 2022, 17AE34

(Refer to solution of Q1)

35. Define the following: i) True stress ii) Engineering stress iii) Hooke's law iv) Poisson's ratio v) Volumetric strain. (10 Marks) Feb./Mar. 2022, 17AE34

(Refer to solution of Q10)

36. Write the equilibrium equations for a 3-dimensional stress system. (03 Marks) Dec.2016/Jan.2017, 15AE34

(Refer to solution of Q1)

37. Define Plane stress and Plane strain. (04 Marks) Dec.2016/Jan.2017, 15AE34

(Refer to solution of Q24)

38. Displacement field at a point on a body is given as follows : u = (x^2yz + z^2), v = (xy^2z + y^2), w = (xyz^2 + x^2). Determine strain components at (2, 1, 2) and express them in a matrix form. (09 Marks) Dec.2016/Jan.2017, 15AE34

$\epsilon_x = \frac{\partial v}{\partial x} = \frac{\partial}{\partial x}(x^2yz + z^2) = 2xyz$, $\gamma_{xy} = x^2z + y^2z$.

$\epsilon_y = \frac{\partial v}{\partial y} = 2xyz + 2y$, $\gamma_{yz} = xy^2 + xz^2$

$\epsilon_z = \frac{\partial w}{\partial z} = 2xyz$, $\gamma_{xz} = x^2y + 2z + yz^2 + 2x$.

at point (2, 1, 2).

$\epsilon_x = 8$ $\quad \gamma_{xy} = 10$ $\quad$ The matrix form.

$\epsilon_y = 10$ $\quad \gamma_{yx} = 10$

$\epsilon_z = 8$ $\quad \gamma_{xz} = 16$

$$= \begin{bmatrix} 8 & 10 & 16 \\ 10 & 10 & 10 \\ 16 & 10 & 8 \end{bmatrix}$$

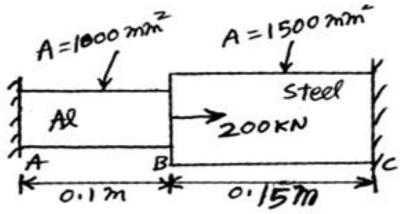
39. Write a note on constitutive laws for anisotropic materials. (04 Marks) Dec.2016/Jan.2017, 15AE34

(Refer to solution of Q20)

40. Draw a stress – strain diagram for ductile material and mention the salient points. (04 Marks) Dec.2016/Jan.2017, 15AE34

(Refer to solution of Q16)

41. A composite bar is a shown in fig. Q2(c). Determine the stress developed in each member. Take $E_{a\ell} = 0.7 \times 10^5 N/mm^2$; $E_{steel} = 2 \times 10^5$ N/mm^2. (08 Marks) Dec.2016/Jan.2017, 15AE34



(Refer to solution of Q21)

42. Derive equilibrium equations for a 3D stress system. (10 Marks) Dec.2017/Jan.2018, 15AE34

(Refer to solution of Q1)

43. State of stress at a point is given follows:

$$\sigma_x = x^3yz + x^2y^2, \quad \sigma_y = 3y^2z + yz, \quad \sigma_z = x^2y^2z^2 + xz,$$
$$\tau_{xy} = x^2yz, \quad \tau_{yz} = xy^2z, \quad \tau_{xz} = xyz^2.$$

In the absence of body forces determine the equilibrium conditions are satisfied or not at points (3, -4, 2). (06 Marks) Dec.2017/Jan.2018, 15AE34

In the absence of body forces, equilibrium equations are given by,

$$\frac{\partial \sigma_x}{\partial x} + \frac{\partial \tau_{xy}}{\partial y} + \frac{\partial \tau_{xz}}{\partial z} = 0 \longrightarrow (1)$$

$$\frac{\partial \tau_{xy}}{\partial x} + \frac{\partial \sigma_y}{\partial y} + \frac{\partial \tau_{yz}}{\partial z} = 0 \longrightarrow (2)$$

$$\frac{\partial \tau_{xz}}{\partial x} + \frac{\partial \tau_{yz}}{\partial y} + \frac{\partial \sigma_z}{\partial z} = 0 \longrightarrow (3)$$

Diff $\rightarrow$

$$\frac{\partial \sigma_x}{\partial x} = 3x^2yz + 2xy^2, \quad \frac{\partial \tau_{xy}}{\partial x} = 2xyz.$$

$$\frac{\partial \tau_{xy}}{\partial y} = x^2z, \quad \frac{\partial \sigma_y}{\partial y} = 6yz + z.$$

$$\frac{\partial \tau_{xz}}{\partial z} = 2xyz, \quad \frac{\partial \tau_{yz}}{\partial z} = xy^2$$

$$\frac{\partial \tau_{xz}}{\partial x} = yz^2 \quad \frac{\partial \tau_{yz}}{\partial y} = 2xyz.$$

$$\frac{\partial \sigma_z}{\partial z} = 2x^2y^2z + x.$$

Substituting the above Expn's.
At point (3, -4, 2)

$$\frac{\partial \sigma_x}{\partial x} = 3(3)^2(-4)(2) + 2(3)(-4)^2 = -120.$$

$$\frac{\partial \tau_{xy}}{\partial x} = 2(3)(-4)(2) = -48., \quad \frac{\partial \tau_{xy}}{\partial y} = (3)^2(2) = 18.$$

$-150 \rightarrow (4)$

$$\frac{\partial \sigma_y}{\partial y} = 6(-4)(2)+(2) = -46, \quad \frac{\partial \tau_{xz}}{\partial z} = 2(3)(-4)(2) = -48.$$

$$\frac{\partial \tau_{yx}}{\partial x} = (3)(-4)^2 = 48.$$

$-46 \rightarrow (5).$

$$\frac{\partial \tau_{zx}}{\partial x} = (-4)(2)^2 = -16, \quad \frac{\partial \tau_{yz}}{\partial y} = 2(3)(-4)(2) = -48.$$

$$\frac{\partial \sigma_z}{\partial z} = 2(3)^2(-4)^2(2)+3 = 579.$$

$515 \rightarrow (6).$

Substituting. Eqn (4) in (1).

$$-128 + 18 - 48 = \underline{-150} \neq 0.$$

Sub Eq (5) in (2).

$$-48 + (-46) + 48 = -46 \neq 0.$$

Sub eq (6) in (3).

$$-16 + (-48) + 579 = 515 \neq 0.$$

∴ The equillibrium equations are not Satisfied for the given stress Components at point (3, -4, 2).

The body forces that are to be added to equillibrium equations so as to satisfied them are

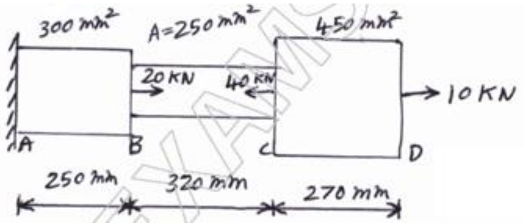
$$f_x = 150 \text{ units}, f_y = 46 \text{ units}.$$

$$f_z = -515 \text{ units}.$$

44. Define the terms load factor and allowable stress. (04 Marks) Dec.2017/Jan.2018, 15AE34

(Refer to solution of Q26)

45. A stepped bar has a fixed support at one end and loads acting are as shown in fig. Q2(a). Determine the stresses and deformations induced in each portion. Also find the net deformation induced. Take E = 200 GPa. (10 Marks) Dec.2018/Jan.2019, 15AE34



Sol'. Given's

$E = 200\ GPa$

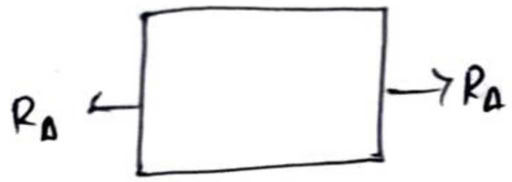
$A_1 = 300\ mm^2$ $\qquad L_1 = 250\ mm$

$A_2 = 250\ mm^2$ $\qquad L_2 = 320\ mm$

$A_3 = 450\ mm^2$ $\qquad L_3 = 270\ mm$

$-R_A + 20 + 10 - 40 = 0 \Rightarrow R_A = 10\ KN.$

Section ①



$$\delta_1 = \frac{P_1 L_1}{A_1 E} = \frac{10 \times 10^3 \times 250}{300 \times 2 \times 10^5} = 0.041\ mm$$

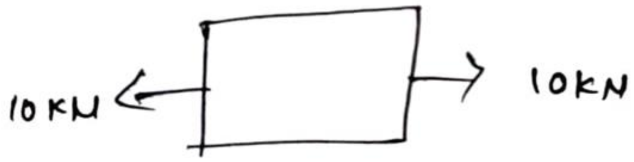
$$\sigma_1 = \frac{P_1}{A_1} = \frac{10 \times 10^3}{300} = 433.33\ N/mm^2$$

Section ②

$$\delta_2 = \frac{P_2 L_2}{A_2 E} = \frac{30 \times 10^3 \times 320}{250 \times 2 \times 10^5} = 0.192\,mm$$

$$\sigma_2 = \frac{P_2}{A_2} = \frac{30 \times 10^3}{250} = 120\ N/mm^2$$

Section ③



$$\delta_3 = \frac{P_3 L_3}{A_3 E} = \frac{10 \times 10^3 \times 270}{450 \times 2 \times 10^5} = 0.030\,mm$$

$$\sigma_3 = \frac{P_3}{A_3} = \frac{10 \times 10^3}{450} = 22.222\ N/mm^2$$

$$\delta = \delta_1 + \delta_2 + \delta_3$$

$$\delta = 0.041 + 0.192 + 0.030$$

$$\boxed{\delta = 0.263\,mm}$$

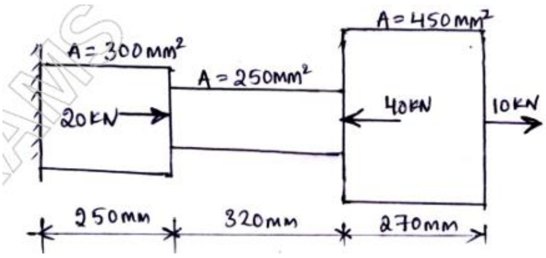
46. Derive equilibrium equation for three-dimensional state of stress in rectangular coordinate system. (08 Marks) Aug./Sept.2020, 15AE34

(Refer to solution of Q1)

47. **Draw stress-strain curve for a ductile material and mention the salient points. (08 Marks) Aug./Sept.2020, 15AE34**

(Refer to solution of Q16)

48. **A stepped bar shown in Fig.Q2(b). Determine the stresses induced and deformation induced in each portion. Also find the net deformation is stepped bar. Take E = 200 GPa. Fig.Q2(b) (08 Marks) Aug./Sept.2020, 15AE34**



(Refer to solution of Q45)