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(ISO 9001:2015)
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MODULE-WISE SOLUTIONS

Year / Semester	II / IV
Course Code	21AE44
Course Name	Mechanics of Materials

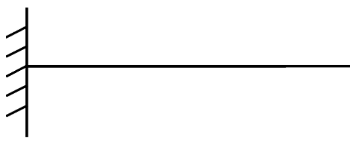
Module 2: Bending Moment and Shear Force in Beams, Euler-Bernoulli Beam Theory

1. **Explain types of beams, loads and supports with neat illustrations. (06 Marks) Aug./Sept.2020, 18AE34**

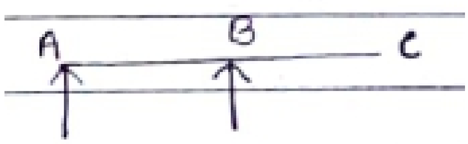
Types of Beams:

(i) **Simply Supported Beam:** Supported at both ends, allowing rotation and vertical displacement.

(ii) **Cantilever Beam:** Fixed at one end, resists loads primarily through bending moments.

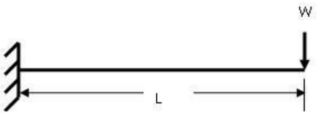


(iii) **Overhanging Beam:** A beam that extends beyond its supports, creating one or more sections with unsupported ends

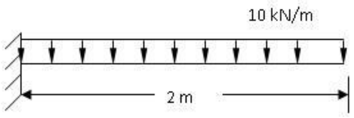


Types of Loads:

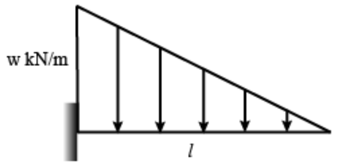
(i) Point Load: Concentrated force applied at a specific location.



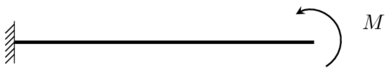
(ii) **Uniformly Distributed Load (UDL):** Even load distribution along the beam's length.



(iii) **Uniformly Varying Load (UVL):** Linearly varying load, forming a triangular distribution.

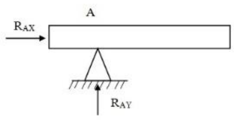


(iv) **Moment:** Bending moment applied to the beam, causing it to deflect.

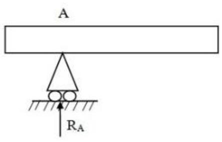


Types of Supports:

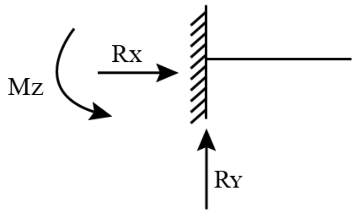
(i) **Hinged Support:** Allows rotation but not translation.



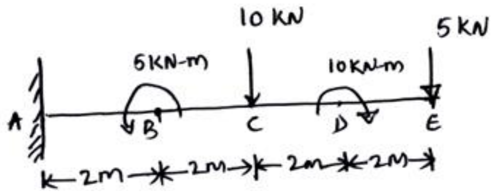
(ii) **Roller Support:** Allows rotation and translation in one direction.

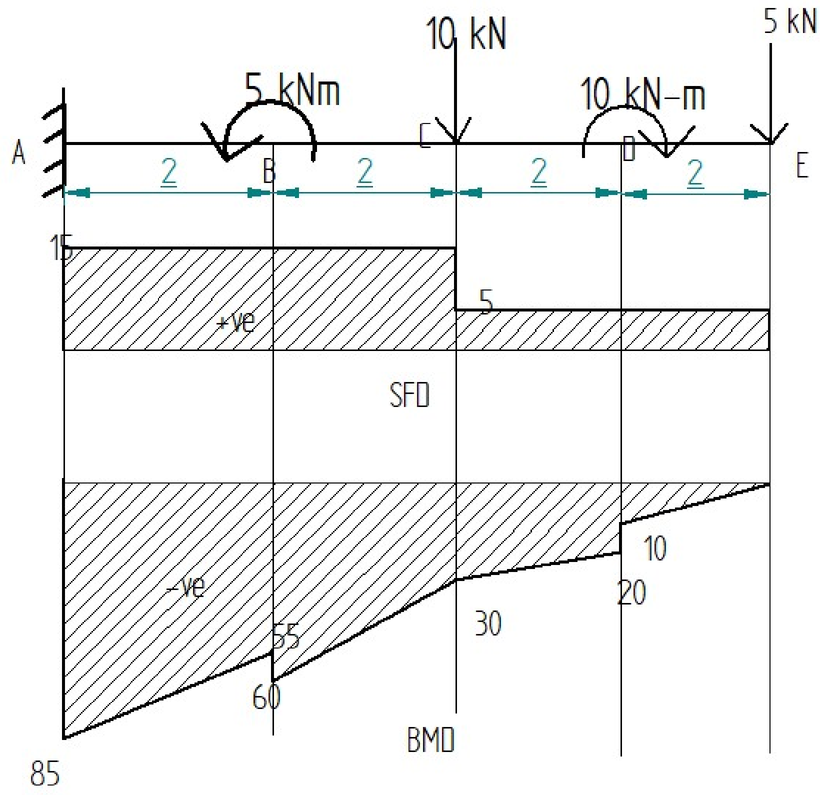


(iii) **Fixed Support:** Prevents both rotation and translation.



2. **Find the reactions at the fixed end and draw SFD and BMD for the Cantilever beam shown in Fig.Q3(b). (07 Marks) Aug./Sept.2020, 18AE34**





Reaction Force:

Total Vertical load = 10 +5 = 15 kN

Vertical Reaction at A = 15 kN (upwards)

Shear Forces:

Shear Force at A and B = 15 kN

Shear Force at C, D & E = 5 kN

Bending Moments:

$Bending\ Moment\ at\ E = 0$

$Bending\ Moment\ at\ RHS\ of\ D = -5 \times 2 = -10\ kN - m$

$Bending\ Moment\ at\ LHS\ of\ D = -10 - 10 = -20\ kN - m$

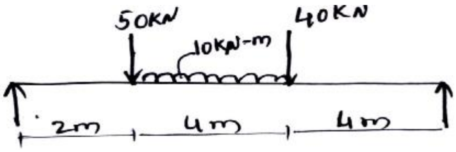
$Bending\ Moment\ at\ C = (-5 \times 4) - 10 = -30\ kN - m$

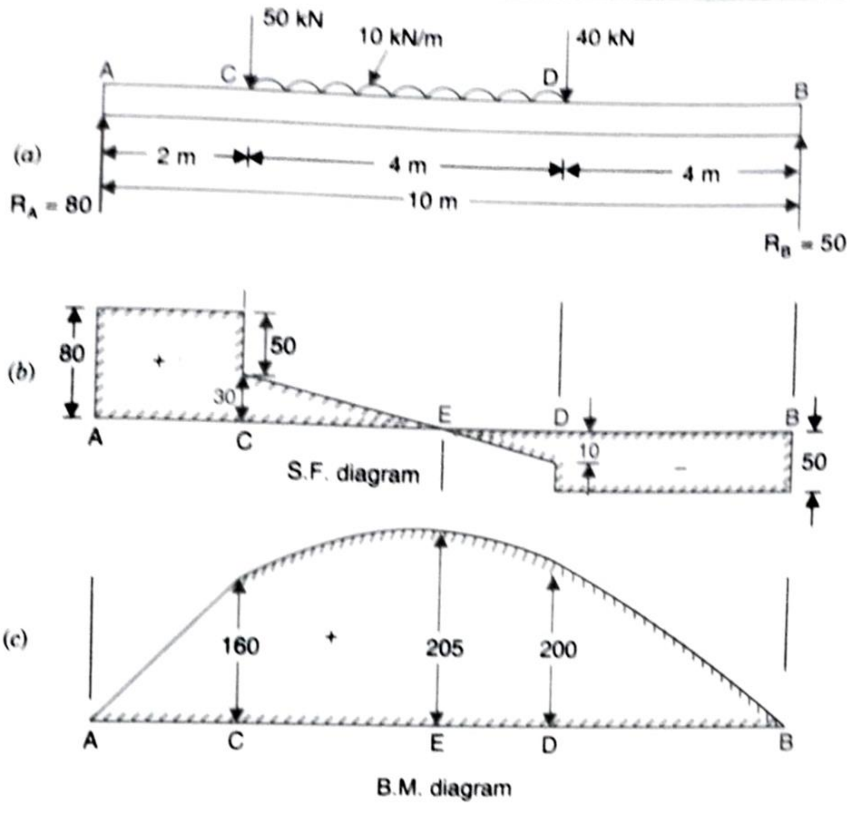
$Bending\ Moment\ at\ RHS\ of\ D = (-5 \times 6) - 10 - (10 \times 2) = -60\ kN - m$

$Bending\ Moment\ at\ LHS\ of\ D = -60 + 5 = -55\ kN - m$

$Bending\ Moment\ at\ A = (-5 \times 8) - 10 - (10 \times 4) + 5 = -85\ kN - m$

3. **Draw the SFD and BMD of the simply supported beam loaded as shown in Fig. and locate maximum bending moment. (07 Marks) Aug./Sept.2020, 18AE34**





Reaction Forces:

$M_A = 0$

$R_B(10) - (40 \times 6) - (40 \times 4) - (50 \times 2) = 0$

$R_B = 50\ kN$

$R_A + R_B = 50 + 40 + 40 = 130\ kN$

$R_A = 80\ kN$

Shear Forces:

$Shear\ Force\ at\ A = 80\ kN$

$Shear\ Force\ at\ B = -50\ kN$

$Shear\ Force\ at\ C = 30\ kN$

$Shear\ Force\ at\ D = -50\ kN$

Shear Force at E = 0

Let x be the distance from A to E,

$Shear\ Force\ at\ E = 80 - 50 - 10(x - 2) = 0$

$x = 5\ m$

Bending Moments:

$Bending\ Moment\ at\ A = 0$

$Bending\ Moment\ at\ B = 0$

$Bending\ Moment\ at\ C = 80 \times 2 = 160\ kN - m$

$Bending\ Moment\ at\ D = 50 \times 4 = 200\ kN - m$

$Bending\ Moment\ at\ E = (80 \times 5) - (50 \times 3) - (10 \times 3 \times 1.5) = 205\ kN - m$

4. Mention the assumptions of Euler-Bernoulli's beam theory and derive the bending stress equation. (10 Marks) Aug./Sept.2020, 18AE34

8.4 ASSUMPTIONS IN PURE BENDING (SIMPLE BENDING)

The following assumptions are made in the theory of pure bending (simple bending) :

1. The material of the beam is perfectly homogeneous (i.e., of the same kind throughout) and isotropic (i.e., same elastic properties in all directions).
2. The value of Young's modulus (E) is same in tension and compression.
3. The beam material is stressed within its elastic limit and thus, obeys Hooke's law.
4. The stress is purely longitudinal.
5. The beam is initially straight and every layer of it is free to expand or contract.
6. The transverse sections, which were plane before bending, remains plane ever after bending.
7. The radius of curvature of beam is very large compared to its depth.
8. The resultant pull or thrust on a transverse section of the beam is zero (i.e., the beam is in equilibrium).

8.5 DERIVATION OF BENDING EQUATION

8.5.1 Relationship between bending stress and radius of curvature

Consider an elemental length 'δx' of a beam subjected to a simple bending or pure bending [Fig. 8.5]. Due to the action of this bending, let this elemental length of beam bend into an arc of a circle with O as centre of curvature as shown in Fig. 8.6.

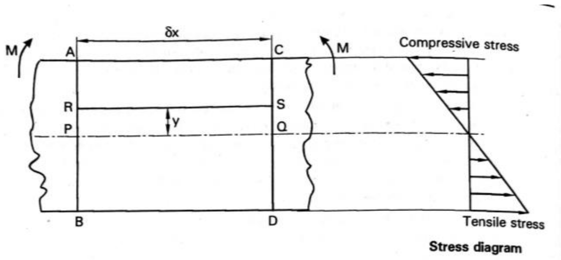


Fig. 8.5

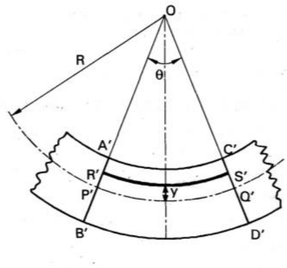


Fig. 8.6

Let

R = Radius of neutral layer P'Q'

θ = Angle subtended by the arc at the centre

M = Moment acting on the beam.

Consider *a* layer *RS* at *a* distance *y* from the neutral axis *PQ* of the beam. Let this layer be compressed to *R'S'* after bending. Decrease in length of this layer *RS*, $\delta_l = RS - R'S'$

$$\text{Strain in the layer } RS,\ \epsilon = \frac{\text{Change in length of RS}}{\text{Original length of RS}}$$

$$= \frac{RS - R'S'}{RS} = 1 - \frac{R'S'}{RS}$$

$$= 1 - \frac{(R-y)\theta}{R\theta} \quad \text{where } RS = PQ = P'Q' = R\theta$$

$$= 1 - \frac{R-y}{R} = \frac{R-R+y}{R} = y/R$$

$$\therefore \text{ Strain in the layer } RS,\ \epsilon = \frac{y}{R}$$

As R is constant, strain is directly proportional to its distance from the neutral layer.

Let σ = Bending stress in the layer

E = Young's modulus of the beam

$$\therefore \text{ Young's modulus } E = \frac{\text{Stress}}{\text{Strain}}$$

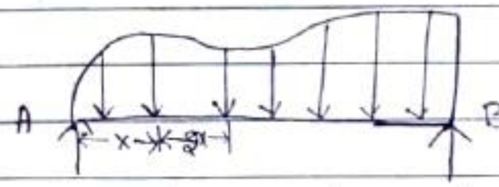
$$\text{i.e., } E = \frac{\sigma}{y/R}$$

$$\therefore\ \sigma = \frac{E}{R} \cdot y \qquad \text{--- (8.1)}$$

As E and R are constants, stress in the layer *RS* is directly proportional to the distance of the layer from the neutral layer. In Fig. 8.6, all layers below the neutral layer are subjected to tensile stresses where as the layers above the neutral layer are subjected to compressive stresses. The equation 8.1 can also be written as

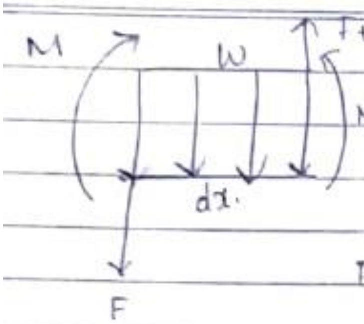
$$\frac{\sigma}{y} = \frac{E}{R} \qquad \text{--- (8.2)}$$

5. Derive the relationship between shear force and bending moment. (04 Marks) Jan./Feb. 2021, 18AE34



This figure shows a Simply Support beam subjected to general loading.

Consider an elemental length dx at a distance x from one of the support.



where F is the shear force, M is the bending Moment, w is the load intensity.

For the equillibrium of force, Sum of forces = 0.

The above figure shows free body diagram of elemental length.

$$F + dF - F - w\,dx = 0.$$

$$dF = w\,dx$$

$$\frac{dF}{dx} = w \rightarrow (1)$$

For the equillibrium of moments,

Taking moment about left face of the element

$$M + dM - M + (F + dF)\,dx - w\,dx \times \frac{dx}{2} = 0 \rightarrow .$$

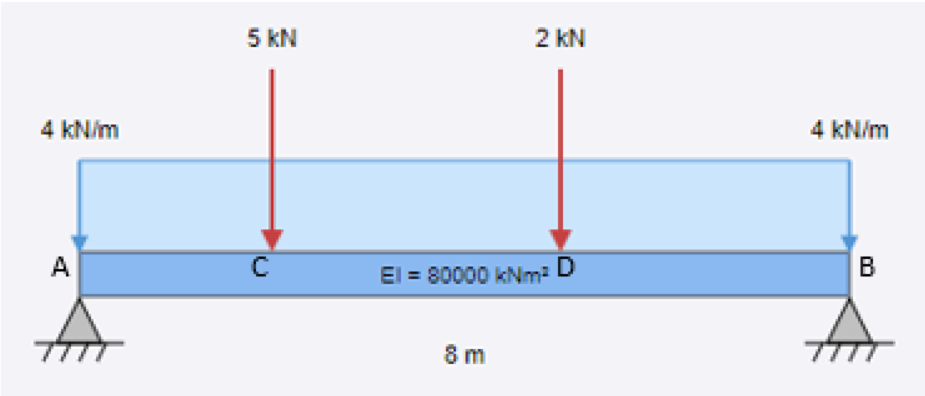
$$dM + F\,dx + dF\,dx - \frac{w\,dx^2}{2} = 0$$ neglecting the higher order term

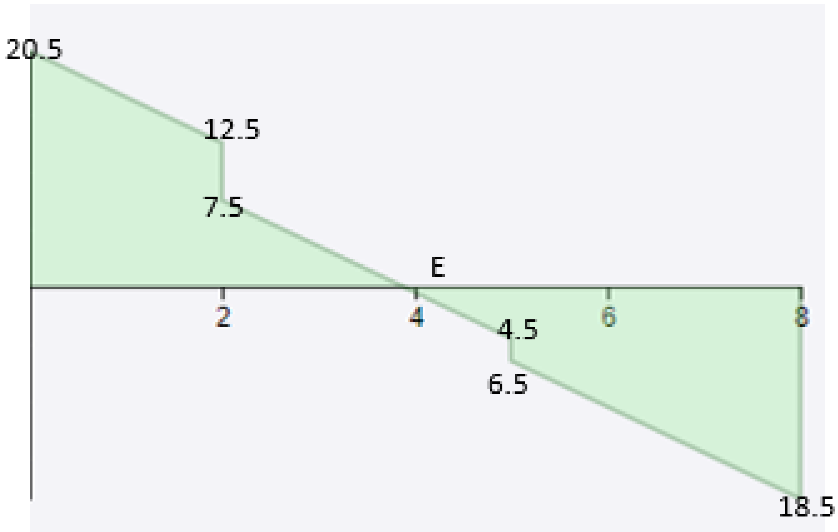
$$dM = -F\,dx$$

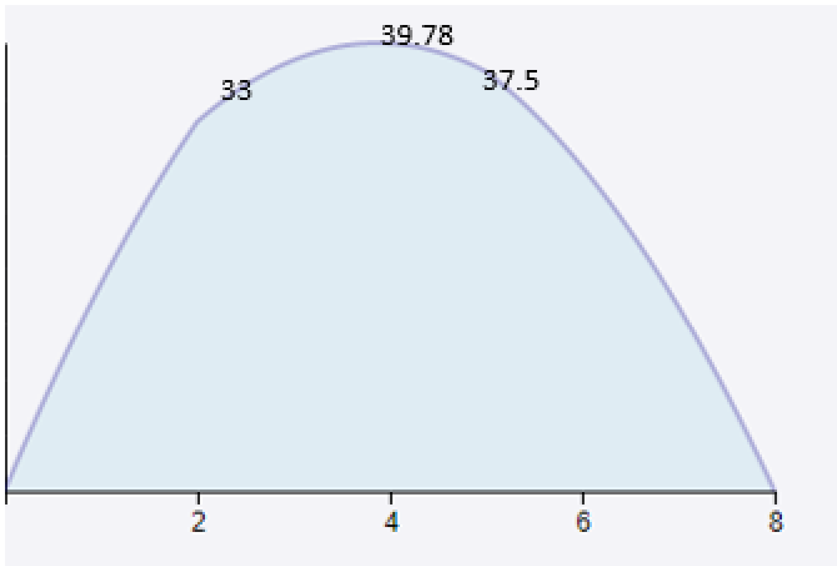
$$\frac{dM}{dx} = -F \rightarrow (2).$$

Eqn (1) & (2) gives relation b/w load intensity, shear force

6. **A beam of 8 m span simply supported at its end carries loads of 2kN and 5kN at a distance of 3m and 6m respectively from right support. In addition, the beam carries a UDL of 4kN/m for its entire length. Draw the SF and BM diagram. (12 Marks) Jan./Feb. 2021, 18AE34**







Reaction Forces:

$M_A = 0$

$R_B(8) - (2 \times 5) - (5 \times 2) - (8 \times 4 \times 4) = 0$

$R_B = 18.5\ kN$

$R_A + R_B = 2 + 5 + 32 = 39\ kN$

$R_A = 20.5\ kN$

Shear Forces:

$Shear\ Force\ at\ A = 20.5\ kN$

$Shear\ Force\ at\ D = 7.5\ kN$

$Shear\ Force\ at\ C = -6.5\ kN$

$Shear\ Force\ at\ B = -18.5\ kN$

Shear Force at E = 0

Let x be the distance from A to E,

$Shear\ Force\ at\ E = 20.5 - 8 - 5 - 4(x - 2) = 0$

$x = 3.875\ m$

Bending Moments:

$Bending\ Moment\ at\ A = 0$

$Bending\ Moment\ at\ B = 0$

$Bending\ Moment\ at\ C = (20.5 \times 2) - (4 \times 2 \times 1) = 33\ kN - m$

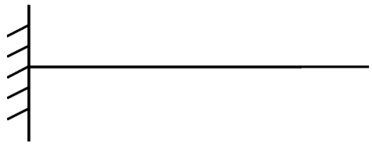
$Bending\ Moment\ at\ D = (18.5 \times 3) - (4 \times 3 \times 1.5) = 37.5\ kN - m$

$$Bending\ Moment\ at\ E = (20.5 \times 3.875) - (8 \times 2.875) - (5 \times 1.875) - (4 \times 1.875 \times 0.9375) = 39.78\ kN - m$$

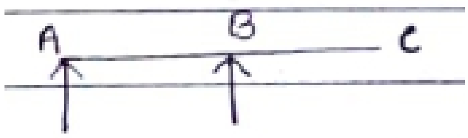
7. **What are the different types of beams? (04 Marks) Jan./Feb. 2021, 18AE34**

(i) **Simply Supported Beam:** Supported at both ends, allowing rotation and vertical displacement.

(ii) **Cantilever Beam:** Fixed at one end, resists loads primarily through bending moments.



(iii) **Overhanging Beam:** A beam that extends beyond its supports, creating one or more sections with unsupported ends



8. Derive the Euler-Bernoulli beam theory equations. (10 Marks) Jan./Feb. 2021, 18AE34

8.5 DERIVATION OF BENDING EQUATION

8.5.1 Relationship between bending stress and radius of curvature

Consider an elemental length 'δx' of a beam subjected to a simple bending or pure bending [Fig. 8.5]. Due to the action of this bending, let this elemental length of beam bend into an arc of a circle with O as centre of curvature as shown in Fig. 8.6.

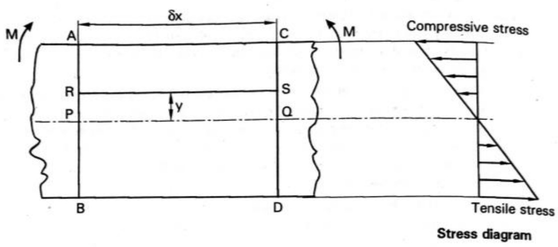


Fig. 8.5

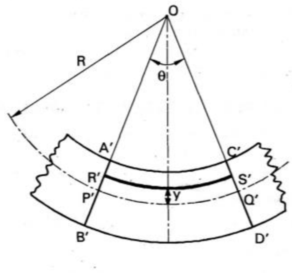


Fig. 8.6

Let

R = Radius of neutral layer P'Q'

θ = Angle subtended by the arc at the centre

M = Moment acting on the beam.

Consider a layer RS at a distance y from the neutral axis PQ of the beam. Let this layer be compressed to $R'S'$ after bending. Decrease in length of this layer RS, $\delta_l = RS - R'S'$

$$\text{Strain in the layer } RS, \epsilon = \frac{\text{Change in length of RS}}{\text{Original length of RS}}$$

$$= \frac{RS - R'S'}{RS} = 1 - \frac{R'S'}{RS}$$

$$= 1 - \frac{(R-y)\theta}{R\theta} \text{ where } RS = PQ = P'Q' = R\theta$$

$$= 1 - \frac{R-y}{R} = \frac{R-R+y}{R} = y/R$$

$\therefore$ Strain in the layer RS, $\epsilon = \frac{y}{R}$

As R is constant, strain is directly proportional to its distance from the neutral layer.

Let σ = Bending stress in the layer

E = Young's modulus of the beam

$\therefore$ Young's modulus $E = \frac{\text{Stress}}{\text{Strain}}$

i.e., $E = \frac{\sigma}{y/R}$

$$\therefore \sigma = \frac{E}{R} \cdot y \qquad \text{--- (8.1)}$$

As E and R are constants, stress in the layer RS is directly proportional to the distance of the layer from the neutral layer. In Fig. 8.6, all layers below the neutral layer are subjected to tensile stresses where as the layers above the neutral layer are subjected to compressive stresses. The equation 8.1 can also be written as

$$\frac{\sigma}{y} = \frac{E}{R} \qquad \text{--- (8.2)}$$

8.5.3 Relation between bending moment and radius of curvature

Fig. 8.8 shows an element area dA at a distance y from the neutral axis i.e., z-axis. Let σ_x (i.e., σ) be the stress at this element. The element of force $\sigma_x dA$ (i.e., σdA) acting on the element of area dA is in the positive direction of the x-axis when σ_x (i.e., σ) is positive and in the negative direction when σ_x (i.e., σ) is negative. As the element dA is located above the neutral axis, a positive stress σ_x (i.e., σ) acting on that element produces an element of moment equal to $\sigma_x y\, dA$ (i.e., $\sigma y dA$).

This element of moment acts opposite in direction to the positive bending moment M shown in Fig. 8.8.

$\therefore$ Elemental moment, $dM = -\sigma y\, dA$

$$= -\left(-\frac{E}{R}y\right)y\,dA \quad \left[\because \sigma = -\frac{E}{R}\times \text{ and } \rho = R\right]$$

$$= \frac{E}{R}y^2 dA \text{ or } +\frac{E}{\rho}y^2 dA$$

The integral or algebraic sum of all such elemental moments over the entire cross-sectional area A must be equal to the bending moment.

$$\therefore M = \sum \frac{E}{R} y^2\, dA \text{ or } \int_A \frac{E}{R} y^2\, dA$$

$$= \frac{E}{R}\sum y^2\, dA \text{ or } \frac{E}{R}\int_A y^2\, dA \qquad \text{---- (8.5)}$$

where, $\sum y^2\, dA$ or $\int_A y^2\, dA$ is the second moment of area about centroid.

i.e., Moment of inertia about centroidal axis (i.e., z-axis or neutral axis),

$$I = \sum y^2\, dA \text{ or } \int_A y^2\, dA$$

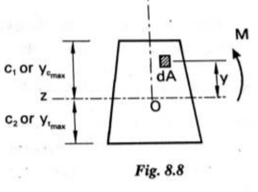


Fig. 8.8

$$\therefore M = \frac{E}{R} I$$

$$\text{i.e., } \frac{M}{I} = \frac{E}{R} \text{ or } \frac{M}{I} = \frac{E}{\rho} \qquad \text{---- (8.6)}$$

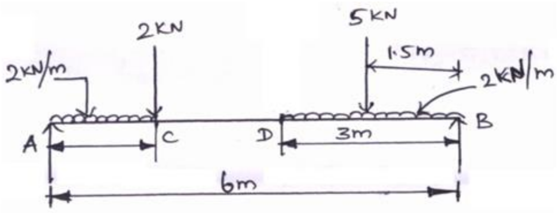
Equation 8.6 can be rearranged to express the curvature in terms of the bending moment in the beam

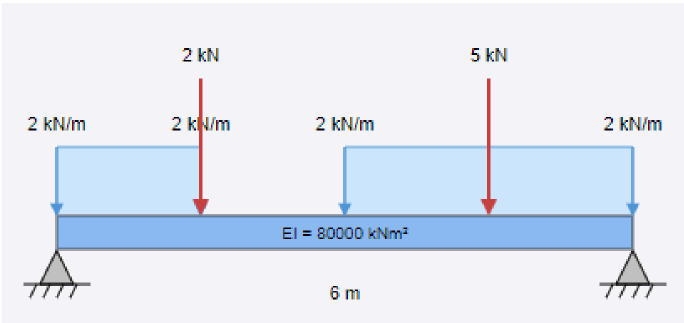
$$\frac{1}{R} = \frac{M}{EI} \qquad \text{---- (8.7)}$$

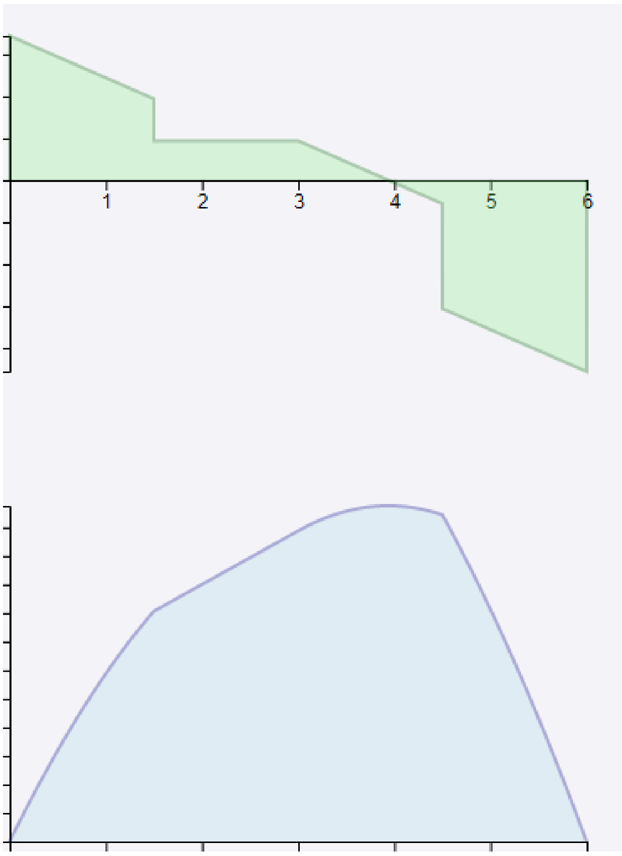
From (2) and (7),

$$\frac{E}{R} = \frac{M}{I} = \frac{\sigma}{y} \qquad (8)$$

9. A simply supported beam AB, 6 m long is loaded as shown in Fig.Q3(a). Draw the shear force and bending moment diagrams for the beam. (10 Marks) Feb./Mar. 2022, 18AE34







Reaction Forces:

$M_A = 0$

$R_B(6) - (2 \times 3 \times 4.5) - (5 \times 4.5) - (2 \times 1.5) - (2 \times 1.5 \times 0.75) = 0$

$R_B = 9.13\ kN$

$R_A + R_B = 3 + 2 + 6 + 5 = 16\ kN$

$R_A = 6.88\ kN$

Shear Forces:

$Shear\ Force\ at\ A = 6.88\ kN$

$Shear\ Force\ at\ C = 1.88\ kN$

$Shear\ Force\ at\ D = 1.88\ kN$

$Shear\ Force\ at\ E = -6.13\ kN$

$Shear\ Force\ at\ B = -9.13\ kN$

Bending Moments:

$Bending\ Moment\ at\ A = 0$

$Bending\ Moment\ at\ B = 0$

$Bending\ Moment\ at\ C = (6.88 \times 1.5) - (2 \times 1.5 \times 0.75) = 8.07\ kN - m$

$Bending\ Moment\ at\ D = (6.88 \times 3) - (2 \times 1.5 \times 2.25) - (2 \times 1.5)$

$= 10.89\ kN - m$

$Bending\ Moment\ at\ E = (9.13 \times 1.5) - (2 \times 1.5 \times 0.75) = 11.45\ kN - m$

10. A beam ABCD, 4 m long is overhanging by 1 m and carries load as shown in Fig.Q3(b). Draw the shear force and bending moment diagrams for the beam and locate the point of contraflexure. (10 Marks) Feb./Mar. 2022, 18AE34

Reaction Forces:

$M_D = 0$

$R_B(3) - (2 \times 1 \times 3.5) - (4 \times 2) = 0$

$R_B = 5\ kN$

$R_D + R_B = 2 + 4 = 6\ kN$

$R_D = 1\ kN$

Shear Forces:

$Shear\ Force\ at\ A = 0$

$Shear\ Force\ at\ B = 3\ kN$

$Shear\ Force\ at\ C = -1\ kN$

$Shear\ Force\ at\ D = -1\ kN$

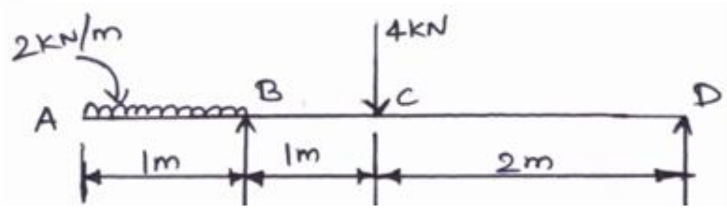
Bending Moments:

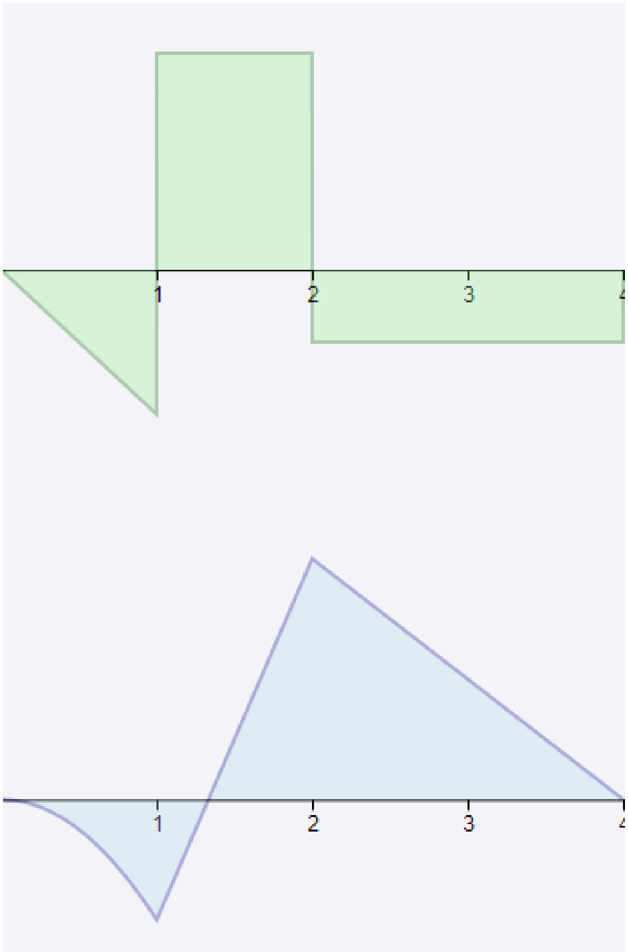
$Bending\ Moment\ at\ A = 0$

$Bending\ Moment\ at\ B = (-2 \times 1) = -2\ kN - m$

$Bending\ Moment\ at\ C = (1 \times 2) = 2\ kN - m$

$Bending\ Moment\ at\ D = 0$

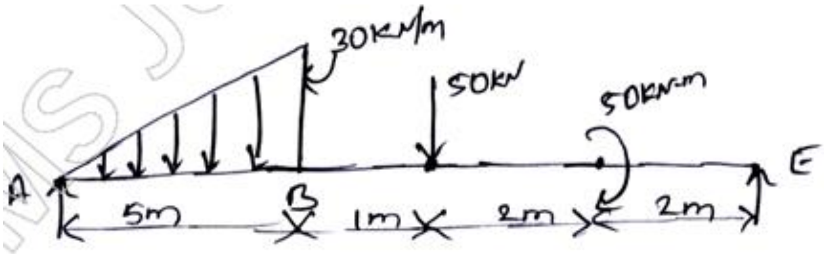


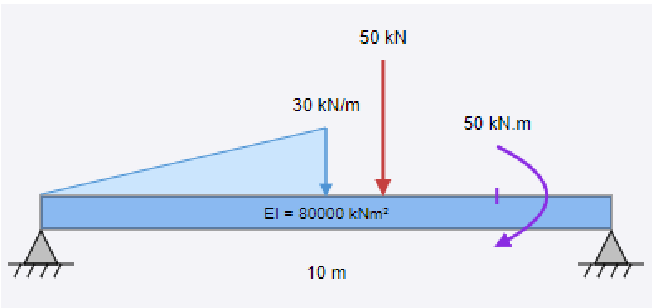


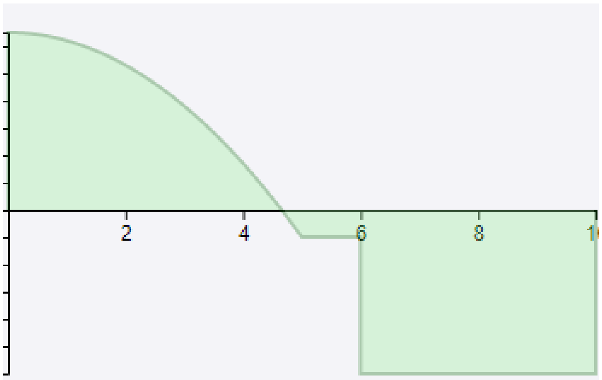
11. **What are the assumptions made in theory of simple bending? Derive an equation for bending stress. (10 Marks) Feb./Mar. 2022, 18AE34**

(Refer to solution of Q4)

12. **Draw SFD and BMD for a simply supported beam shown in Fig.Q.3. (20 Marks) July/August 2021, 18AE34**







Reaction Forces:

$M_A = 0$

$R_E(10) - 50 - (50 \times 6) - \left(0.5 \times 5 \times 30 \times \frac{2 \times 5}{3}\right) = 0$

$R_E = 60\ kN$

$R_A + R_E = 75 + 50 = 125\ kN$

$R_A = 65\ kN$

Shear Forces:

$Shear\ Force\ at\ A = 65\ kN$

$Shear\ Force\ at\ B = -10\ kN$

$Shear\ Force\ at\ C = -60\ kN$

$Shear\ Force\ at\ D = -60\ kN$

$Shear\ Force\ at\ E = -60\ kN$

Bending Moments:

$Bending\ Moment\ at\ A = 0$

$Bending\ Moment\ at\ E = 0$

$Bending\ Moment\ at\ B = (65 \times 5) - (75 \times \frac{5}{3}) = 200\ kN - m$

$Bending\ Moment\ at\ C = (65 \times 6) - (75 \times \frac{8}{3}) = 190\ kN - m$

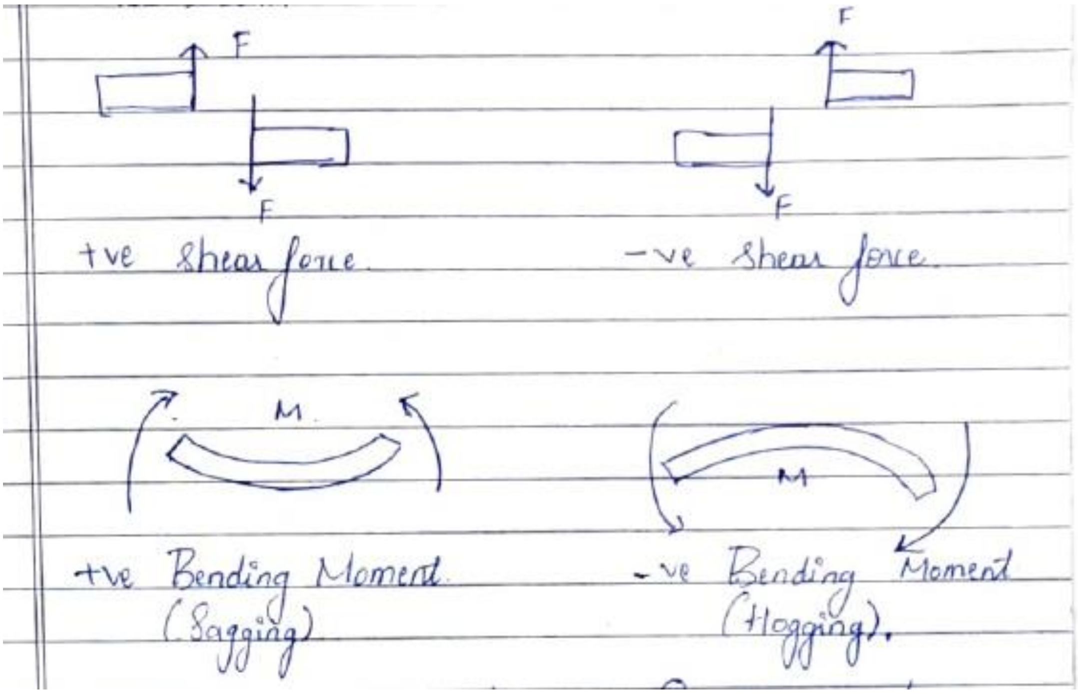
$Bending\ Moment\ at\ LHS\ of\ D = (65 \times 8) - \left(75 \times \frac{14}{3}\right) - (50 \times 2) = 70\ kN - m$

$Bending\ Moment\ at\ RHS\ of\ D = (60 \times 2) = 120\ kN - m$

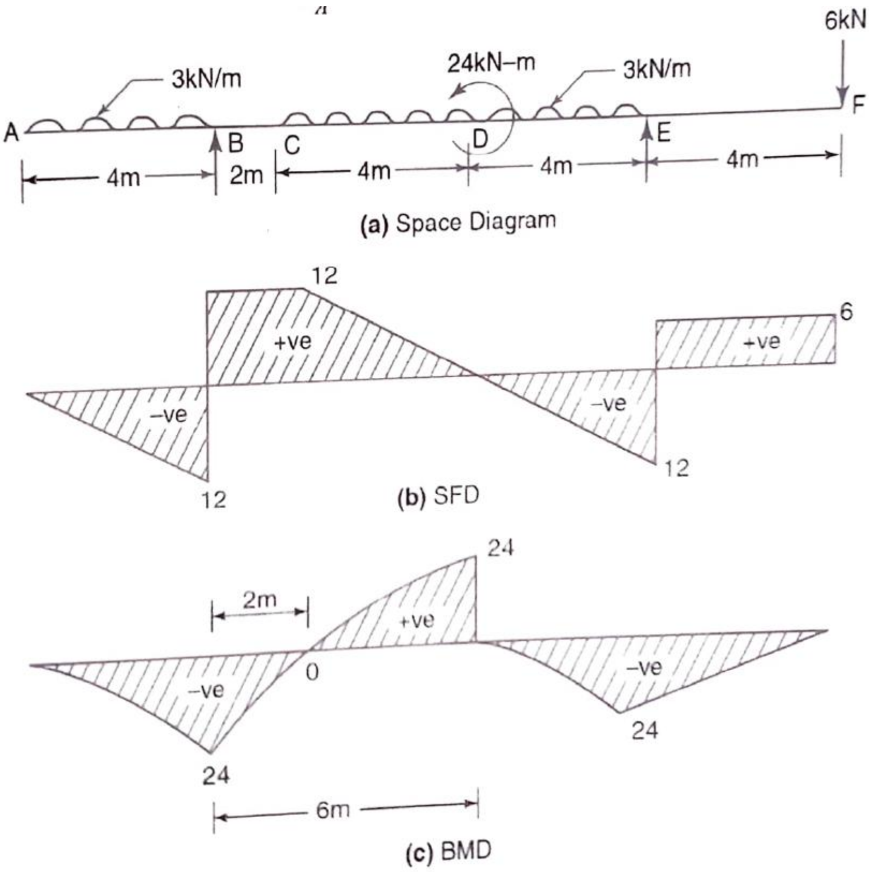
13. Mention the sign conventions in SFD and BMD. (04 Marks) Dec.2019/Jan.2020, 18AE34

Shear force & bending movement are vector quantities & the following sign conventions are used: a) Shear force is positive if it tends to move the left portion upward relative to the right portion.

b) Bending movement if it tends to sag the beam.



14. **Draw the bending moment and shear force diagrams for the beams shown in Fig.Q3(b). Indicate the salient values on the diagram. Fig.Q3(b) (12 Marks) Dec.2019/Jan.2020, 18AE34**



(a) Space Diagram
(b) SFD
(c) BMD

Reaction Forces:

$Taking\ moments\ about\ E,$

$R_B(10) - (3 \times 4 \times 12) - (3 \times 8 \times 4) - 24 = (-6 \times 4)$

$R_B = 24\ kN$

$R_E + R_B = 12 + 24 + 6 = 42\ kN$

$R_E = 18\ kN$

Shear Forces:

$Shear\ Force\ at\ A = 0$

$Shear\ Force\ at\ B = 12\ kN$

$Shear\ Force\ at\ C = 12\ kN$

$Shear\ Force\ at\ D = 0$

$Shear\ Force\ at\ E = 6\ kN$

$Shear\ Force\ at\ F = 6\ kN$

Bending Moments:

$Bending\ Moment\ at\ A\ \&\ F = 0$

$Bending\ Moment\ at\ B = -(3 \times 4 \times 2) = -24\ kN - m$

$Bending\ Moment\ at\ C = -(3 \times 4 \times 4) + (24 \times 2) = 0$

$Bending\ Moment\ at\ D = -(3 \times 4 \times 8) + (24 \times 6) - (3 \times 4 \times 2) = 24\ kN - m$

$Bending\ Moment\ at\ E = -(6 \times 4) = -24\ kN - m$

15. Derive the relationship between load, shear force and bending moment. (04 Marks) Dec.2019/Jan.2020, 18AE34

(Refer to solution of Q5)

16. What are the Euler-Bernoulli assumptions? (04 Marks) Dec.2019/Jan.2020, 18AE34

8.4 ASSUMPTIONS IN PURE BENDING (SIMPLE BENDING)

The following assumptions are made in the theory of pure bending (simple bending) :

1. The material of the beam is perfectly homogeneous (i.e., of the same kind throughout) and isotropic (i.e., same elastic properties in all directions).
2. The value of Young's modulus (E) is same in tension and compression.
3. The beam material is stressed within its elastic limit and thus, obeys Hooke's law.
4. The stress is purely longitudinal.
5. The beam is initially straight and every layer of it is free to expand or contract.
6. The transverse sections, which were plane before bending, remains plane ever after bending.
7. The radius of curvature of beam is very large compared to its depth.
8. The resultant pull or thrust on a transverse section of the beam is zero (i.e., the beam is in equilibrium).

17. **What are the Euler-Bernoulli assumptions? (05) Aug./Sept.2020, 17AE34**

(Refer to solution of Q17)

18. **Explain in detail the implications of Euler Bernoulli assumptions and derive the expression for the same. (8) Dec.2018/Jan.2019, 17AE34**

Below are the key assumptions of the Euler-Bernoulli beam theory and their implications:

(i) Beam Shape: This theory assumes beams are straight and uniform in cross-section.
Implication: Beams can have slight curvature or varying cross-sections, particularly under bending or torsion.

(ii) Small Deformations: It assumes beam deformations are small relative to their length.
Implication: Not suitable for analyzing large deflections or rotations in highly flexible or slender beams.

(iii) Linear Elastic Material: Assumes materials follow Hooke's law throughout loading.
Implication: Real materials, especially under extreme loads, may exhibit non-linear behaviors like plasticity.

(iv) Neglects Shear Deformation: Ignores shear deformation effects within beams.
Implication: May lead to inaccuracies in predicting shear-related stresses and deformations for slender beams.

(v) Bernoulli's Hypothesis: Assumes cross-sections remain planar during deformation.
Implication: warping may occur, especially in non-slender or torsionally loaded beams.

(vi) Stress-Strain Linearity: Assumes constant material properties throughout the beam.
Implication: Real materials can have property variations affecting beam behavior.

(vii) Loading and Boundary Conditions: Assumes well-defined, simple loading.
Implication: Real-world beams often face complex loading scenarios, including concentrated loads and temperature effects.

8.5 DERIVATION OF BENDING EQUATION

8.5.1 Relationship between bending stress and radius of curvature

Consider an elemental length 'δx' of a beam subjected to a simple bending or pure bending [Fig. 8.5]. Due to the action of this bending, let this elemental length of beam bend into an arc of a circle with O as centre of curvature as shown in Fig. 8.6.

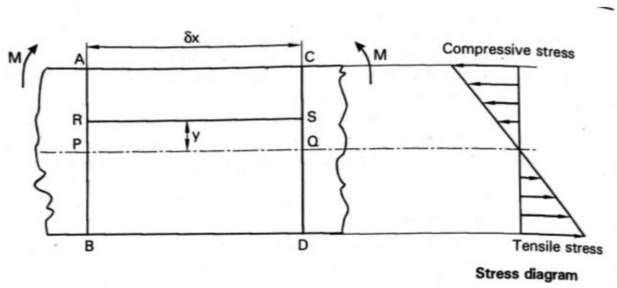


Fig. 8.5

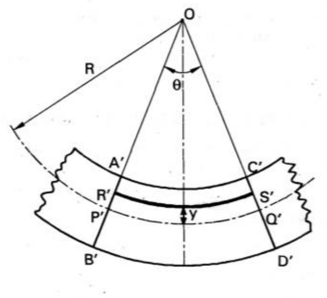


Fig. 8.6

Let

R = Radius of neutral layer P'Q'

θ = Angle subtended by the arc at the centre

M = Moment acting on the beam.

Consider *a* layer *RS* at *a* distance *y* from the neutral axis *PQ* of the beam. Let this layer be compressed to *R'S'* after bending. Decrease in length of this layer *RS*, $\delta_l = RS - R'S'$

$$\text{Strain in the layer } RS, \in \; = \frac{\text{Change in length of RS}}{\text{Original length of RS}}$$

$$= \frac{RS - R'S'}{RS} = 1 - \frac{R'S'}{RS}$$

$$= 1 - \frac{(R-y)\theta}{R\theta} \quad \text{where } RS = PQ = P'Q' = R\theta$$

$$= 1 - \frac{R-y}{R} = \frac{R-R+y}{R} = y/R$$

$$\therefore \text{ Strain in the layer } RS, \; \in \; = \frac{y}{R}$$

As *R* is constant, strain is directly proportional to its distance from the neutral layer.

Let σ = Bending stress in the layer

E = Young's modulus of the beam

$$\therefore \text{ Young's modulus } E = \frac{\text{Stress}}{\text{Strain}}$$

$$\text{i.e., } E = \frac{\sigma}{y/R}$$

$$\therefore \; \sigma = \frac{E}{R} \cdot y \qquad \text{--- (8.1)}$$

As *E* and *R* are constants, stress in the layer *RS* is directly proportional to the distance of the layer from the neutral layer. In Fig. 8.6, all layers below the neutral layer are subjected to tensile stresses where as the layers above the neutral layer are subjected to compressive stresses. The equation 8.1 can also be written as

$$\frac{\sigma}{y} = \frac{E}{R} \qquad \text{--- (8.2)}$$

19. List and explain the implications of Euler-Bernoulli's assumptions. (06 Marks) June/July 2019, 17AE34

Below are the key assumptions of the Euler-Bernoulli beam theory and their implications:

(viii) Beam Shape: This theory assumes beams are straight and uniform in cross-section.
Implication: Beams can have slight curvature or varying cross-sections, particularly under bending or torsion.

(ix) Small Deformations: It assumes beam deformations are small relative to their length.
Implication: Not suitable for analyzing large deflections or rotations in highly flexible or slender beams.

(x) Linear Elastic Material: Assumes materials follow Hooke's law throughout loading.
Implication: Real materials, especially under extreme loads, may exhibit non-linear behaviors like plasticity.

(xi) Neglects Shear Deformation: Ignores shear deformation effects within beams.
Implication: May lead to inaccuracies in predicting shear-related stresses and deformations for slender beams.

(xii) Bernoulli's Hypothesis: Assumes cross-sections remain planar during deformation.
Implication: warping may occur, especially in non-slender or torsionally loaded beams.

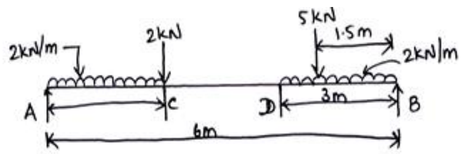
(xiii) Stress-Strain Linearity: Assumes constant material properties throughout the beam.
Implication: Real materials can have property variations affecting beam behavior.

(xiv) Loading and Boundary Conditions: Assumes well-defined, simple loading.
Implication: Real-world beams often face complex loading scenarios, including concentrated loads and temperature effects.

20. List the Euler-Bernoulli assumptions and explain its implications. (10 Marks) Jan./Feb.2021, 17AE34

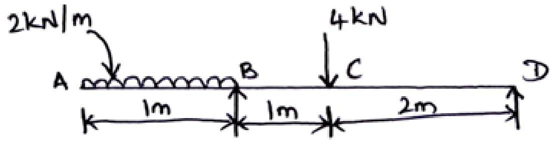
(Refer solution of Q19)

21. A simply supported beam AB, 6m long is loaded as shown in Fig.Q.3(a). Draw the shear force and bending moment diagrams for the beam. (10 Marks) Feb./Mar. 2022, 17AE34



(Refer solution of Q9)

22. **A beam ABCD, 4m long is overhanging by 1m and carries load as shown in Fig.Q.3(b). Draw the shear force and bending moment diagrams for the beam and locate the point of contra flexure. (10 Marks) Feb./Mar. 2022, 17AE34**



(Refer solution of Q10)

23. **What are the assumptions made in theory of simple bending? Derive an equation for bending stress. (10 Marks) Feb./Mar. 2022, 17AE34**

(Refer solution of Q4)

24. **What are the Euler – Bernoulli assumptions and its implications? (06 Marks) Dec.2016/Jan.2017, 15AE34**

(Refer solution of Q19)

25. **List out the Euler-Bernoulli assumptions and its implications. (06 Marks) Dec.2017/Jan.2018, 15AE34**

(Refer solution of Q19)

26. **What are the Eyler – Bernoulli assumptions and its implications? (06 Marks) Dec.2018/Jan.2019, 15AE34**

(Refer solution of Q19)

27. **Explain in detail the implications of Euler-Bernoulli assumptions and derive the expression for the same. (08 Marks) Aug./Sept.2020, 15AE34**

(Refer solution of Q18)