Gopalan College of Engineering and Management

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MODULE-WISE SOLUTIONS

Year / Semester	II / IV
Course Code	21AE44
Course Name	Mechanics of Materials

Module 3: Deflection of Beams, Torsion of Circular Shafts

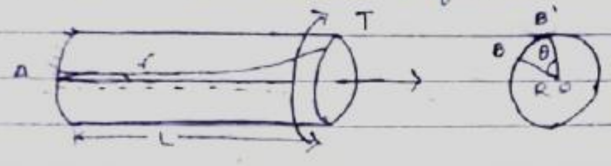
1. **List the assumption of pure torsion and derive torsion equation. (10 Marks) Aug./Sept.2020, 18AE34**

★ ASSUMPTIONS IN THE THEORY OF PURE TORSION:-

(some properties)

1. The material is Homogeneous and isotropic.
2. Cross-sections which are plane before applying twisting moment remains plane even after applying twisting moment.
3. The stresses are within elastic limit i.e shear stress is proportional to shear strain.
4. Radial lines remain radial even after applying twisting moment.
5. Torsion along the shaft is uniform.

DERIVATION OF TORSION EQUATION.



Consider a shaft of length 'L' and radius 'R' Subjected to twisting moment 'T' as shown in the fig.
Let 'O' be the center of circular Section and 'B' be any point on the surface.
Let AB is a line parallel to axis of the shaft
From geometry of the fig.

$$BB' = R\theta = L\gamma \rightarrow (1)$$

From the defination of shear modulus. Shear strain

$$\gamma = \frac{\tau}{G} \rightarrow (2).$$

where τ is the shear stress at radius 'R' & G is the shear modulus.

Substitute Eqn (2) in (1).

$$R\theta = L\frac{\tau}{G}$$

$$\frac{\tau}{R} = \frac{G\theta}{L} \rightarrow (3)$$

If point 'B' is taken a any radius 'r' it can be shown that.

$$\frac{\tau_r}{r} = \frac{G\theta}{L} \rightarrow (4)$$

from Eqn (3) & (4)

$$\frac{\tau_r}{r} = \frac{\tau}{R}$$

$$\tau_r = \tau\frac{r}{R} \rightarrow (5).$$

Consider an elemental area dA at a distance r from the center as shown in fig 2.

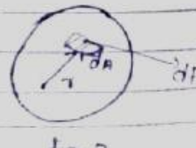


fig. 2.

Torque applied. $\rightarrow dT = dF \times r$.

Tangential Force $\rightarrow dF = \tau_r dA$.

$$dT = \tau_r r\,dA \rightarrow (6)$$

Substitute Eq (5) in (6).

$$dT = \frac{\tau}{R} r^2 dA.$$

$$T = \frac{\tau}{R}\int r^2 dA.$$

From defination polar moment of inertia

$$J = \int r^2 dA.$$

$$T = \frac{\tau J}{R}$$

$$\frac{\tau}{R} = \frac{T}{J} \rightarrow (7).$$

From Eqn (4) & (7).

$$\frac{T}{J} = \frac{\tau}{R} = \frac{G\theta}{L} \rightarrow (8)$$

Eqn (8) is the torsional equation for Circular Shaft.

2. **A solid circular shaft has to transmit a power of 1000 KW at 120 rpm. Find the diameter of the shaft, if the shear stress of the material must not exceed 80 N/mm^2. The maximum torque 1.25 times of its mean, what percentage of saving in material would be obtained if the shaft is replaced by hollow one whose internal diameter is 0.6 times its external diameter, the length, material and maximum shear stress being same. (10 Marks) Aug./Sept.2020, 18AE34**

Given :

$\tau = 80\ N/mm^2$

$N = 120 rpm$

$T_{max} = 1.25\ T_{mean}$

$P = 1000\ kW = 1000 \times 10^3\ W$

$d_i = 0.6\ d_o$

$$P = \frac{2\pi N T}{60}$$

$$1000 \times 10^3 = \frac{2\pi \times 120 \times T}{60}$$

$$T = \frac{60 \times 1000 \times 10^3}{2 \times 120 \times \pi}$$

$$T = \frac{250 \times 10^3}{\pi}$$

$$T = 79.5 \times 10^3\ Nm$$

$$T_{mean} = 79.5 \times 10^6\ Nmm$$

$$T_{max} = 1.25\ T_{mean}$$

$$T_{max} = 1.25\,(79.5 \times 10^6)$$

$$\frac{\tau}{R} = \frac{T}{J} \quad \text{[From torsional equation]}$$

$$\frac{80}{d/2} = \frac{99.875 \times 10^6}{\pi/32 \times d^4}$$

$$\frac{80 \times 2}{d} = \frac{99.375 \times 10^6 \times 32}{\pi \times d^4}$$

$$d^3 = \frac{99.875 \times 10^6 \times 32}{\pi \times 160}$$

$$d = 1.8495\ m$$

$$d = 184.94\ mm$$

For hollow shaft

$$d_i = 0.6 d_o$$

$$\frac{80}{d_o/2} = \frac{99.87 \times 10^6}{\frac{\pi}{32}\left[d_o^4 - (0.6 d_o)^4\right]}$$

$$\frac{d_o^4}{d_o} = \frac{32 \times 99.87 \times 10^6}{\pi [0.8704][160]}$$

$$d_o^3 = 7268026.974$$

$$d_o = 193.70\ mm$$

$$d_i = 0.6\ d_o$$

$$d_i = 0.6\ [193.7]$$

$$d_i = 116.22\ mm$$

Area of solid shaft $= \frac{\pi}{4}(184.94)^2$

$= 26862.81\ mm^2$

Area of hollow shaft $= \frac{\pi}{4}\left[d_o^2 - d_i^2\right] = \frac{\pi}{4}\left[193.71^2 - 116.22^2\right]$

$= 18859.45\ mm^2$

% of saving $= \frac{A_s - A_h}{A_s} = \frac{26862.81 - 18859.45}{26862.81}$

$= \underline{0.29\%}$

3. A hollow shaft is to transmit 300 kW at 80 rpm. If the shear stress is not to exceed 50MN/m2 and diameter ration is 3/7. Find the external and internal diameter if the twist is 1.2º and length is 2m. Assuming maximum torque is 20% greater than mean. Take G = 80GN/m2. (12 Marks) Jan./Feb. 2021, 18AE34

Data:

$P = 300 \times 10^3\ W$

N = 80 rpm

$\tau = 50\ \ N/mm^2$

$\frac{d_i}{d_o} = \frac{3}{7}$

$d_i = 0.42 d_o$

$\theta = 1.2^{\circ} = 1.2 \times \frac{\pi}{180} = 0.02\ rad$

L = 2000 mm

$T = 1.2 T_m$

$G = 80 \times 10^3\ \ N/mm^2$

Solution:

$$Power\ Transmitted, P = \frac{2\pi N T_m}{60}\ W$$

$$300 \times 10^3 = \frac{2\pi \times 80 \times T_m}{60}$$

$$T_m = 34180.2\ N - m = 34.18 \times 10^6\ N - mm$$

$$T = 1.2 \times T_m$$

$$T = 1.2 \times 34.18 \times 10^6 = 41.01 \times 10^6\ N - mm$$

Diameter based on strength:

$$R = \frac{d_o}{2}$$

$$J = \frac{\pi\left({d_o}^4 - {d_i}^4\right)}{32}$$

$d_i = 0.42 d_o$

$$J = \frac{\pi\left({d_o}^4 - (0.42 d_o)^4\right)}{32}$$

$$J = 0.0951\ {d_o}^4$$

$$\frac{T}{J} = \frac{\tau}{R}$$

$$\frac{41.01 \times 10^6}{0.0951\, {d_o}^4} = \frac{50}{0.5 d_o}$$

$${d_o}^3 = 4311402$$

$$d_0 = 162.75\, mm$$

Diameter based on rigidity:

$$\frac{T}{J} = \frac{G\theta}{L}$$

$$\frac{41.01 \times 10^6}{0.0951\, {d_o}^4} = \frac{80 \times 10^3 \times 0.02}{2000}$$

$${d_o}^4 = 539.03 \times 10^6$$

$$d_o = 152.37\, mm$$

Since diameter based on strength is greater than diameter based on rigidity, $d_0 = 162.75$ mm is recommended.

$$\boldsymbol{d_0 = 162.75\, mm}$$

$$\boldsymbol{d_i = 0.42 \times 162.75 = 68.36\, mm}$$

4. **What are the assumptions made in theory of pure torsion? (04 Marks) Jan./Feb. 2021, 18AE34**

★ ASSUMPTIONS IN THE THEORY OF PURE TORSION:-

(some properties)

1. The material is Homogeneous and isotropic.
2. Cross-sections which are plane before applying twisting moment remains plane even after applying twisting moment.
3. The stresses are within elastic limit i.e shear stress is proportional to shear strain.
4. Radial lines remain radial even after applying twisting moment.
5. Torsion along the shaft is uniform.

5. **Define torsional rigidity and write the equation for power transmitted by the shaft. (04 Marks) Jan./Feb. 2021, 18AE34**

TORSIONAL RIGIDITY :-

From Torsional equation.

$$\frac{T}{J} = \frac{G\theta}{L} \Rightarrow GJ = \frac{T}{\theta/L}$$

Where GJ is called as torsional rigidity which is defined as torque required to produce unit angle of twist in unit length.

POWER TRANSMITTED BY THE SHAFT :-

Consider a shaft rotating at a speed of N rpm subjected to torque 'T' angular velocity of the shaft.

$$\omega = \frac{2\pi N}{60} \text{ rad/s} \rightarrow (1)$$

power transmitted $P = T\omega \rightarrow \omega \rightarrow (2)$.

From Eq (1) & (2).

$$P = \frac{2\pi NT}{60}$$

6. **A hollow shaft is to transmit 200 KW at 80 rpm. If the shear stress is not to exceed 60 MPa and internal diameter is 0.6 of the external diameter, find the diameters of the shaft. (06 Marks) Feb./Mar. 2022, 18AE34**

Given,

$P = 200kW = 200 \times 10^3$

$N = 80$ rpm

$\tau = \cancel{60MPa}$ 60 N/mm^2

$= \cancel{60 \times 10^6}$ $d_1 = 0.6 d_0$

$$P = \frac{2\pi NT}{60}$$

$$T = \frac{200 \times 10^2 \times 60}{2 \times \pi \times 80}$$

$$T = 23.8 \times 10^{+3} \text{ N-M}$$

$$= 23.8 \times 10^6 \text{ N-MM}$$

$$\frac{T}{J} = \frac{\tau}{R}$$

$$\frac{23.8 \times 10^6}{\frac{\pi}{2}\left(d_o^4 - (0.6 d_o)^4\right)} = \frac{60}{\frac{d_o}{2}}$$

$$\frac{23.8 \times 10^6 \times 32}{\pi \times 2 \times 60 \times 0.8704} = d_o^3$$

$$d_o^3 = 2821009.5$$

$$d_o = 132.40 \text{ mm}$$

$$d_i = 79.44 \text{ MM}$$

7. **A solid shaft of 200 mm diameter has the same cross-sectional area as a hollow shaft of the same material with inside diameter of 150 mm. Find the ration of: (i) Power transmitted by both the shafts at the same angular velocity. (ii) Angles of twist in equal lengths of these shafts, when stressed to the same velocity. (08 Marks) Feb./Mar. 2022, 18AE34**

$d = 200\text{mm}$ $\qquad \omega_s = \omega_h = \omega$

$d_i = 150\text{mm}$ $\qquad T_H = T_S = T$

$$A_s = A_h$$

$$\frac{\pi}{4}\left[d_o^2 - d_i^2\right] = \frac{\pi}{4} d^2$$

$$d_o^2 = d^2 + d_i^2$$

$$d_o^2 = 200^2 + (150)^2$$

$$d_o = 250\text{mm}$$

i) $\frac{P_H}{P_s} = \frac{T_H}{T_s} = \frac{\frac{\pi}{16}\left[\frac{d_o^4 - d_i^4}{d_o}\right]}{\frac{\pi}{16} d^3} = \frac{Z_{PH}}{Z_{PS}}$

$$Z_{PH} = \frac{\pi}{16}\left[\frac{250^4 - 150^4}{250}\right]$$

$$Z_{PH} = 2670353.75 = 26.7 \times 10^5 \text{ mm}^3$$

$$\frac{P_H}{P_s} = \frac{26.7 \times 10^5}{15.7 \times 10^5}$$

$$\frac{P_H}{P_s} = 1.7$$

Hollow shaft supplies 7% more power than solid shaft.

ii) Angle of twist in equal length

$$\frac{\tau}{R} = \frac{G\theta_1}{L}$$

For solid shaft

$$\theta_s = \frac{\tau L}{R_s G} \quad \text{——— (1)}$$

For Hollow shaft

$$\theta_h = \frac{\tau L}{R_H G} \quad \text{——— (2)}$$

from (1) & (2)

$$\frac{\theta_H}{\theta_s} = \frac{1/R_H}{1/R_s} = \frac{R_s}{R_H}$$

$$\frac{\theta_H}{\theta_s} = \frac{200/2}{250/2} = \underline{0.8}$$

8. **A solid shaft of 80 mm diameter is to be replaced by a hollow shaft of external diameter 100 mm. Determine the internal diameter of the hollow shaft if the same power is to be transmitted by both the shafts at the same angular velocity and shear stress. (06 Marks) Feb./Mar. 2022, 18AE34**

$d = 80\,mm$ - (solid)

$d_o = 100\,mm$ - (hollow)

$d_i = ?$

$P_H = P_S \qquad C_H = C_S$

$T_H = Z_{PH}\tau \qquad T_S = Z_{PS}\tau$

$Z_{PH} = Z_{PS}$

$$\Rightarrow \frac{\pi}{16}\left[\frac{d_o^4 - d_i^4}{d_o}\right] = \frac{\pi}{16} d^3$$

$$\frac{d_o^4 - d_i^4}{d_o} = d^3$$

$$\frac{(100)^4 - d_i^4}{100} = (80)^3$$

$$(100)^4 - d_i^4 = (80)^3(100)$$

$$d_i^4 = 1\times10^9 - 512\times10^5$$

$$d_i^4 = 488\times10^5$$

$$\boxed{d_i = 83.58\,mm}$$

Internal diameter of hollow shaft is 83.58 mm

9. **Determine the diameter of solid shaft which will transmit 440 kW at 280 rpm. If maximum torsional shear stress is 40 N/mm^2. Take G = 84 kN/mm^2. (04 Marks) July/August 2021, 18AE34**

$\theta/L = 1°/m = \pi/180/10^3$ rad/mm. $\quad P = \frac{2\pi NT}{60}$

$\tau = 40\,N/mm^2$.

$G = 84\times10^3\,N/mm^2$, $\quad \frac{T}{J} = \frac{\tau}{\rho}$

$P = \frac{2\pi NT}{60}$, $\quad 440\times10^3 = \frac{2\pi(220)T}{60}$

$440\times10^3 = 29.32\,T$, $\Rightarrow T = 15\times10^3$ N-m

15×10^6 N-mm.

$\frac{T}{J} = \frac{\tau}{R}$

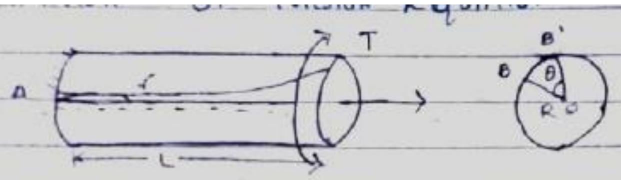
$\frac{15\times10^6}{\frac{\pi d^4}{32}} = \frac{40}{d/2} = \frac{32\times15\times10^6}{\pi d^4} = \frac{2\times40}{d}$

$= \frac{32\times15\times10^6}{80\pi} = d^3 \Rightarrow 1.9\times10^6 = d^3$

$\sqrt[3]{1.9\times10^6} = d$.

$\therefore d = 123.85\,mm$

10. Determine the rate of twist and shear stress distribution in a circular section bar of radius 'R' which is subjected to equal and opposite torque 'T' at each of its free end. (08 Marks) Dec.2019/Jan.2020, 18AE34



Consider a shaft of length 'L' and radius 'R' subjected to twisting moment 'T' as shown in the fig.
Let 'O' be the center of circular section and 'B' be any point on the surface.
Let AB is a line parallel to axis of the shaft.
From geometry of the fig.

$BB' = R\theta = L\gamma \;-(1)$.

From the defination of shear modulus. Shear strain

$\gamma = \frac{\tau}{G} \rightarrow (2)$.

where τ is the shear stress at radius 'R' & G is the shear modulus.

Substitute Eqn (2) in (1).

$R\theta = L\frac{\tau}{G}$

$\frac{\tau}{R} = \frac{G\theta}{L} \longrightarrow (3)$

of point 'B' is taken a any radius 'r' it can be shown that.

$$\frac{\tau_r}{r} = \frac{G\theta}{L} \rightarrow (4)$$

from Eqn (3) & (4)

$$\frac{\tau_r}{r} = \frac{\tau}{R}$$

$$\tau_r = \tau \frac{r}{R} \rightarrow (5)$$

Consider an elemental area dA at a distance r from the center as shown in fig 2.

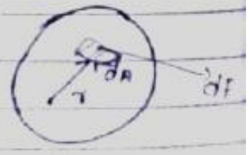


Fig. 2.

Torque applied. $\rightarrow dT = dF \times r$.

Tangential Force $\rightarrow dF = \tau_r dA$.

$$dT = \tau_r r dA \rightarrow (6)$$

Substitute Eq (5) in (6).

$$dT = \frac{\tau}{R} r^2 dA.$$

$$T = \frac{\tau}{R} \int r^2 dA.$$

From defination polar moment of inertia

$$J = \int r^2 dA.$$

$$T = \frac{\tau J}{R}$$

$$\frac{\tau}{R} = \frac{T}{J} \rightarrow (7).$$

From Eqn (4) & (7).

$$\frac{T}{J} = \frac{\tau}{R} = \frac{G\theta}{L} \rightarrow (8)$$

Eqn (8) is the torsional equation for circular Sha

11. A 2m long hollow cylinder shaft has 80 mm outer diameter and 10 mm wall thickness. When the torsional load on the shaft is 6 kN-m, determine: (i) Maximum shear stress induced (ii) Angle of twist. Also draw the distribution of shear stress in the wall of the shaft. Take G = 80 GPa. (12 Marks) Dec.2019/Jan.2020, 18AE34

$L = 200\,mm \quad T = 6\times10^6\ N\text{-}mm \quad G = 80\times10^3 \quad \tau = ?$

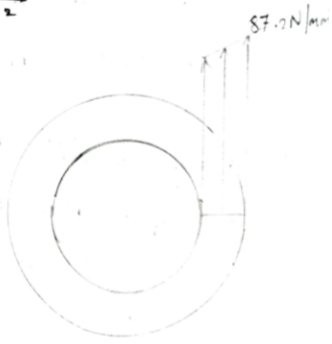
$d_o = 80mm \quad t = 10mm \quad J = \frac{\pi}{32}\left[80^4 - d_i^4\right] \quad \theta = ?$

$\frac{T}{J} = \frac{\tau}{R} \qquad t = \frac{d_o - d_i}{2} = \frac{80 - d_i}{2}$

$10 = \frac{80 - d_i}{2}$

$20 = 80 - d_i$

$d_i = 60mm$



$J = \frac{\pi}{32}\left[80^4 - 60^4\right]$

$J = \frac{\pi}{32}\left[15040\times10^3\right]$

$J = 278.8\times10^4\ mm^4$

i) $\frac{\tau}{R} = \frac{T}{J}$ [From torsional equation]

$\tau = \cancel{80}\ \frac{6\times10^6\times40}{274.8\times10^4}$

$\tau = 87.33\ N/mm^2$ [maximum shear stress]

ii) $\frac{T}{J} = \frac{G\theta}{L}$

$\theta = \frac{6\times10^6\times2000}{274.8\times10^4\times80\times10^3}$

$\theta = 3.12^\circ$

iii) Torsional stiffness

$K = \frac{GJ}{L}$

$= \frac{80\times10^3\times278.4\times10^4}{2000}$

$= 1113.6\times10^5\ N\text{-}mm$

$K = 1113.6\times10^2\ Nm$

$k = 111.36\times10^3\ Nm$ (or) $k = 111.36\ kNm$

12. Obtain the relationship between torque and shear stress in solid circular shaft. (10 Marks) Aug./Sept.2020, 17AE34

(Refer to the solution of Q10)

13. Compare the weight, strength and stiffness of hollow shaft of same external diameter as that of solid shaft. The inner diameter of hollow shaft is half the external diameter. Both the shafts have the same material and length. (10 Marks) Aug./Sept.2020, 17AE34

Data:

$$d = d_o;\quad d_i = \frac{1}{2}d_o = 0.5\,d_o;\quad l_H = l_S;\quad \tau_H = \tau_S;\quad G_H = G_S;\quad w_H = w_S\ (\because \text{Same material})$$

Solution :

(i) Comparison of weight

Weight = Volume × Specific weight of the material = Area × Length × Specific weight of the material

$$\frac{W_H}{W_S} = \frac{\frac{\pi}{4}\left(d_o^2 - d_i^2\right)\times l_H \times w_H}{\frac{\pi}{4}d^2 \times l_S \times w_S} = \frac{\left(d_o^2 - d_i^2\right)}{d^2}\quad (\because l_H = l_S; w_H = w_S)$$

$$= \frac{d_o^2\left\{1-\left(d_i/d_o\right)^2\right\}}{d_o^2} = \left(1-0.5^2\right) = 0.75\quad (\because d_o = d)$$

$$\therefore\ \frac{W_H}{W_S} = 0.75$$

(ii) Comparison of strength

Torsional strength $\quad \frac{T}{\tau} = Z_P$

$$\frac{(Z_P)_H}{(Z_S)_S} = \frac{\frac{\pi}{16}\frac{\left(d_o^4 - d_i^4\right)}{d_o}}{\frac{\pi}{16}d^3} = \frac{d_o^4\left\{1-\left(\frac{d_i}{d_o}\right)^4\right\}}{d_o \cdot d^3} = \frac{d^3\left(1-0.5^4\right)}{d^3} = 0.9375\quad (\because d_o = d)$$

$$\therefore\ \frac{\text{Torsional strength of hollow shaft}}{\text{Torsional strength of solid shaft}} = \mathbf{0.9375}$$

(iii) Comparison of stiffness

$$\frac{\text{Torsional stiffness of hollow shaft}}{\text{Torsional stiffness of solid shaft}} = \frac{G_H J_H}{G_S J_S} = \frac{J_H}{J_S}\quad (\because G_H = G_S)$$

$$= \frac{\frac{\pi}{32}\left(d_0^4 - d_i^4\right)}{\frac{\pi}{32}d^4} = \frac{d_0^4\left\{l-\left(\frac{d_i}{d_0}\right)^4\right\}}{d^4}$$

$$= \frac{d^4\left(l-0.5^4\right)}{d^4}\quad (\because d_o = d)$$

$$= \mathbf{0.9375}$$

14. A 2 meters long hollow cylinder shaft has 80mm outer diameter and 10mm wall thickness. When the torsional load on the shaft is 6kN-m, determine: i) Maximum shear stress induced ii) Angle of twist. Also draw the distribution of shear stress in the wall of the shaft. Take G as 80 Gpa. Also find torsional stiffness. (12 Marks) Dec.2018/Jan.2019, 17AE34

(Refer to the solution of Q11)

15. Show that maximum shear stress in the shaft under torsion is given by $\tau_{max} = \frac{TR}{J}$. (10 Marks) June/July 2019, 17AE34

(Refer to the solution of Q10)

16. A solid circular shaft has to transmit a power of 1000 kW at 120 rpm. Find the diameter of the shaft if the shear stress of the material must not exceed 80 N/mm^2. The maximum torque 1.25 times of its mean. What percentage of saving in material would be obtained. If the shaft is replaced by the hallow shaft is replaced by the hallow shaft having internal diameter of 0.6 times its external diameter, the length, material and the maximum shear stress being same. (10 Marks) Jan./Feb.2021, 17AE34

(Refer to the solution of Q2)

17. A hollow shaft is to transmit 200kW at 80rpm. If the shear stress is not to exceed 60 MPa and internal diameter is 0.6 of the external diameter, find the diameters of the shaft. (06 Marks) Feb./Mar. 2022, 17AE34

Data:

$P = 200 \times 10^3\ W$

N = 80 rpm

$\tau = 60\ \ N/mm^2$

$d_i = 0.6d_o$

Solution:

$$Power\ Transmitted, P = \frac{2\pi NT}{60}\ W$$

$$200 \times 10^3 = \frac{2\pi \times 80 \times T}{60}$$

$$T = 23873.24\ N - m = 23.873 \times 10^6\ N - mm$$

$$R = \frac{d_o}{2}$$

$$J = \frac{\pi\left({d_o}^4 - {d_i}^4\right)}{32}$$

$d_i = 0.6d_o$

$$J = \frac{\pi\left({d_o}^4 - (0.6d_o)^4\right)}{32}$$

$$J = 0.0855\ {d_o}^4$$

$$\frac{T}{J} = \frac{\tau}{R}$$

$$\frac{23.873 \times 10^6}{0.0855\ {d_o}^4} = \frac{60}{0.5d_o}$$

$${d_o}^3 = 2.327 \times 10^6$$

$$\boldsymbol{d_0 = 132.51\ mm}$$

$$\boldsymbol{d_i = 0.6 \times 132.51 = 79.5\ mm}$$

18. A solid shaft of 200mm diameter has the same cross-sectional area as a hollow shaft of the same material with inside diameter of 150 mm. Find the ratio of i) Power transmitted by both the shafts at the same angular velocity. ii) Angles of twist in equal lengths of there, shafts when stressed to the same velocity. (08 Marks) Feb./Mar. 2022, 17AE34

(Refer to the solution of Q7)

19. A solid shaft of 80mm diameter is to be replaced by a hollow shaft of external diameter 100mm. Determine the internal diameter of the hollow shaft if the same power is to be transmitted by both the shafts at the same angular velocity and shear stress. (06 Marks) Feb./Mar. 2022, 17AE34

(Refer to the solution of Q8)

20. A 2 meters long hollow cylinder shaft has 80mm outer diameter and 10mm wall thickness. When the torsional load on the shaft is 6kN-m, determine i) maximum shear stress induced and ii) angle of twist. Also draw the distribution of shear stress in the wall of the shaft. Take G as 80GPa. (10 Marks) Dec.2016/Jan.2017, 15AE34

(Refer to the solution of Q11)

21. A hollow shaft is subjected to a torque 8 kNm. The angle of twist in the shaft is to be limited to 1.7° in a length equal to twenty times the outer diameter. Taking the inner diameter to outer diameter ratio as 0.7, determine: (i) Inner diameter and outer diameter, and (ii) Maximum shear stress induced. Take G as 80 GPa. (10 Marks) Dec.2017/Jan.2018, 15AE34

DATA :

$T = 8\ KN\text{-}m = 8 \times 10^3\ N\text{-}m = 8 \times 10^6\ N/mm^2$

$\theta = 1.7^\circ = 1.7 \times \frac{\pi}{180}$ rad

$L = 20\, d_o$

$\frac{d_i}{d_o} = 0.7 \Rightarrow d_i = 0.7\, d_o$

(i) $d_i = ?$ and $d_o = ?$

(ii) $\tau = ?$

$G = 80\ GPa = 80 \times 10^3\ N/mm^2$

Soln:

Design based on strength

$$\frac{T}{J} = \frac{\tau}{R}$$

$$J = \frac{\pi}{32}\left[d_o^4 - d_i^4\right] = \frac{\pi}{32}\left[d_o^4 - (0.7\, d_o)^4\right] = \frac{\pi}{32}\left(0.7599\, d_o^4\right)$$

$$J = 0.0746\, d_o^4$$

$$\frac{T}{0.0746\, d_o^4} = \frac{\tau}{d_o/2}$$

$$\Rightarrow \frac{8 \times 10^6}{0.0746\, d_o^4} = \frac{2\tau}{d_o/2}$$

$$\Rightarrow \frac{T}{J} = \frac{G\theta}{L}$$

$$\Rightarrow \frac{8\times10^6}{0.0746\, d_o^{43}} = \frac{80\times10^3 \times 1.7 \times \pi}{180 \times 20 \times \cancel{d_o}}$$

$$\Rightarrow \frac{8\times10^6}{0.0746\, d_o^3} = \frac{80\times10^3 \times 1.7 \times \pi}{180\times20}$$

$$\frac{8\times10^6}{8.8537} = d_o^3$$

$$d_o^3 = 903.57$$

$$\therefore \boxed{d_o = 96.67 \text{ mm}}$$

$$d_i = 0.7\,(96.67)$$

$$\therefore \boxed{d_i = 67.66 \text{ mm}}$$

$\rightarrow$ We have

$$\frac{T}{J} = \frac{\tau}{d_o/2} \qquad \because R = d_o/2$$

$$\frac{T}{J} = \frac{2\tau}{d_o}$$

$$\Rightarrow \frac{8\times10^6}{0.0746\, d_o^{34}} = \frac{2\tau}{96.67}$$

$$\frac{8\times10^6}{6.514\times10^6 \times 0.0207} = \tau$$

$$\therefore \boxed{\tau = 59.37}$$

22. **Find the diameter of the shaft required to transmit 60 KW at 150 rpm if the maximum torque is 25% more than the mean torque for a maximum permissible shear – stress of 60MN/m2. Find also the angle of twist for a length of 4m. Take G = 80GPa. (10 Marks) Dec.2018/Jan.2019, 15AE34**

Given,

Power (P) = 60 kW = 60×10^3 watt

$N = 150$ rpm

$T_{max} = 1.25\ T_{mean}$

$\tau = 60\ N/MM^2$

$L = 4m = 4 \times 10^3\ MM$

$\theta = ?$

$$P = \frac{2\pi N T}{60}$$

$$60 \times 10^3 = \frac{2 \times \pi \times 150 \times T_{mean}}{60}$$

$T_{mean} = 38.19 \times 10^2\ N\text{-}M$

$= 38.19 \times 10^5\ N\text{-}MM$

$T_{max} = 47.72 \times 10^5\ N\text{-}MM$

$$\frac{T}{J} = \frac{\tau}{R}$$

$$\frac{47.73 \times 10^3}{\frac{\pi}{32} d^4} = \frac{60}{\frac{d}{2}}$$

$$\therefore \frac{47.73 \times 10^5 \times 32}{\pi \times 60 \times 2} = d^3$$

$d^3 = 405144.8$

$d = 73.99\ MM$

$$\frac{G\theta}{L} = \frac{T}{J}$$

$$\frac{80 \times 10^3 \times \theta}{4000} = \frac{47.73 \times 10^5}{\frac{\pi}{32}(73.99)^4}$$

$$\theta = \frac{47.73 \times 10^5 \times 4000 \times 32}{\pi (73.99)^4 \times 80 \times 10^3}$$

$\theta = 0.081\ rad = 4.6°$

23. A solid shaft rotating at 500 rpm transmits 30 kWatts power. Maximum torque is 20% more than the mean torque. Material of the shaft has allowable shear stress of 65 MPa. Modulus of rigidity is 81 GPa. Angle of twist in shaft should not exceed 1 deg. in 1m length. Determine the diameter of the shaft. (10 Marks) Aug./Sept.2020, 15AE34

Data:

$P = 30 \times 10^3\ W$

N = 500 rpm

$\tau = 65\ N/mm^2$

$\theta = 1 = \frac{\pi}{180}\ rad$

L = 1000 mm
$T = 1.2T_m$
$G = 81 \times 10^3 \ N/mm^2$

Solution:

$$Power\ Transmitted, P = \frac{2\pi N T_m}{60}\ W$$

$$30 \times 10^3 = \frac{2\pi \times 500 \times T_m}{60}$$

$$T_m = 572.96\ N - m = 572.96 \times 10^3\ N - mm$$

$$T = 1.2 \times T_m$$

$$T = 1.2 \times 572.96 \times 10^3 = 687.54 \times 10^3\ N - mm$$

Diameter based on strength:

$$R = \frac{d}{2}$$

$$J = \frac{\pi d^4}{32}$$

$$\frac{T}{J} = \frac{\tau}{R}$$

$$\frac{32 \times 687.54 \times 10^3}{\pi d^4} = \frac{65}{0.5d}$$

$$d^3 = 53871.7$$

$$\boldsymbol{d = 37.77\ mm}$$

Diameter based on rigidity:

$$\frac{T}{J} = \frac{G\theta}{L}$$

$$\frac{32 \times 687.54 \times 10^3}{\pi d^4} = \frac{81 \times 10^3 \times \pi}{1000 \times 180}$$

$$d^4 = 4.953 \times 10^6$$

$$\boldsymbol{d = 47.18\ mm}$$

Since diameter based on rigidity is greater than diameter based on strength, $\boldsymbol{d = 47.18\ mm}$ is recommended.