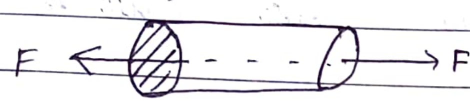
Date 7/6/23 Page

MODULE-1:

BASICS OF LINEAR ELASTICITY, CONCEPT OF STRESS AND STRAIN.

★ STRESS : is defined as resisting force per unit cross-sectional area. The unit of stress in metric system is N/mm^2.

1. NORMAL STRESS :



When the external force is perpendicular to the cross-section of the body, the induced stress is known as Normal stress.

Two Types. — Tensile stress / Compressive stress.

i) TENSILE STRESS :-

P is force.

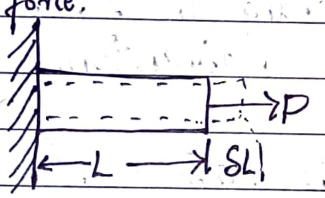


Fig. 1.1.

It is the stress that causes extension of the body as shown in fig 1.1.

Mathematically tensile stress

$$\sigma_t = +\frac{P}{A} \; N/mm^2$$

ii) COMPRESSIVE STRESS :-

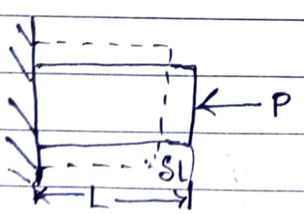


Fig. 2.2.

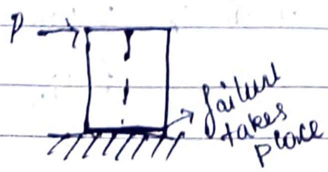
It is the stress that causes contraction of the body as shown in fig 2.2.

Mathematically Compressive stress

$$\sigma_c = -\frac{P}{A} \; N/mm^2$$

2) SHEAR STRESS :-

It is the stress that causes distortion of the body as shown in fig 1.3.



Mathematically shear stress

$$\tau = \frac{P}{A}$$

SURYA Gold
Date ____ Page ____

STRAIN:- It is measure of deformation in the body as a result of applied stress.
Mathematically strain is defined as ratio of change in dimension to original dimension.

i) LINEAR STRAIN:- Also called as Longitudinal strain,

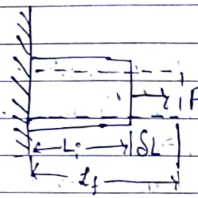
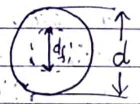


fig. 1.4.


$\delta d = d_i - d_f$
fig 1.6.

★ It is the ratio of change in linear dimension to original linear dimension.

ii) LATERAL STRAIN:- It is the ratio of change in lateral dimension to original lateral dimension

★ With reference to figure 1.4 Linear strain Epsalon.

$$\epsilon_{lin} = \frac{\delta l}{L} = \frac{L_f - L_i}{L_i}$$

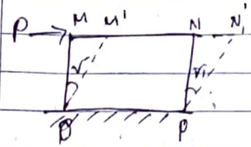
• Lateral strain $\epsilon_{lat} = \frac{\delta d}{d} = \frac{d_i - d_f}{d_i}$

POISON'S RATIO :- It is the property of the material which is defined as the ratio of lateral strain to linear strain.
Mathematically poison's ratio $\gamma = \frac{\epsilon_{lat}}{\epsilon_{lin}}$

No unit.

SURYA Gold
Date ____ Page ____

iii) SHEAR STRAIN:- It is the measure of angle through which a body undergoes distortion. when it is subjected to shear stress.


Distortion. fig 1.6

Let m.n.o.p be a plane element subjected to shear force as shown in fig 1.6

M' → M prime

Let MN'OP be the distored shape.
From the figure $\tan\gamma = \frac{NN'}{NP}$

When γ is very small $\tan\gamma$ is approximately equal to γ. Therefore $\tan\gamma \simeq \gamma$.
Shear strain $\gamma = \frac{NN'}{NP}$.

1/6/23.

UNITS OF STRESS:-

★ In Metric System the unit of stress is N/mm^2.
★ In SI System, the unit of stress N/m^2 or Pascal.

$1MPa = 10^6 N/m^2$

$$= \frac{10^6 N}{(10^3 mm)^2} = \frac{10^6 N}{10^6 mm^2}$$

★ $1 MPa = 1 N/mm^2$.

★ STATE OF STRESS AT A POINT:-

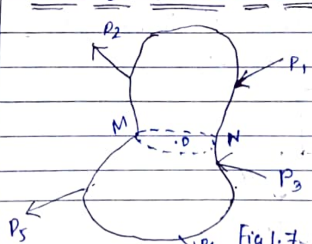


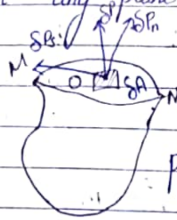
Fig 1.7.

★ Consider an arbitarily shaped 3-dimensional body which is in equilibrium under the action of forces P_1, P_2, P_3 etc....

SURYA Gold

Date____ Page____

Let MN be any plane passing through point O.



Let δA be the infinitesimally small area over point O.

Fig - 1.8

Let δP be the resultant force is distributed over area δA.

The resultant force can be resolved into 2 components

a) Normal Component (δP_n)

b) Tangential Component (δP_s)

NORMAL STRESS AT POINT O.

$$\sigma = \lim_{\delta A \to 0} \frac{\delta P_A}{\delta A}$$

Shear Stress at Point O.

$$\tau = \lim_{\delta A \to 0} \frac{\delta P_s}{\delta A}$$

RESULTANT STRESS AT POINT O IS EQUAL TO:-

$$R = \sqrt{\sigma^2 + \tau^2}$$

STATE OF STRAIN AT A POINT :- (STRAIN DISPLACEMENT RELATIONS)

Consider a 2D element of dimensions ∂x & ∂y as shown in the figure.

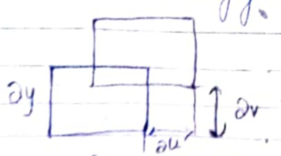


Fig 1.9.

SURYA Gold

Date____ Page____

* Let ∂u & ∂v be the displacements of element in x & y directions respectively.

* NORMAL STRAIN in X direction

$$\epsilon_x = \frac{\partial u}{\partial x} \rightarrow (1)$$

* NORMAL STRAIN IN Y DIRECTION.

$$\epsilon_y = \frac{\partial v}{\partial y} \rightarrow (2)$$

* Similarly Normal Strain in Z direction

$$\epsilon_z = \frac{\partial w}{\partial z} \rightarrow (3)$$

*

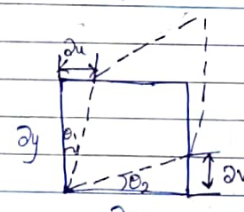


Fig : 1.10.

From fig 1.10

$\tan\theta_1 \simeq \theta_1 = \frac{\partial u}{\partial y}$

$\tan\theta_2 \simeq \theta_2 = \frac{\partial v}{\partial x}$

Shear Strain γ_{xy} is $= \theta_1 + \theta_2$.

$$\gamma_{xy} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \quad — (4).$$

Similarly $\gamma_{yz} = \frac{\partial v}{\partial z} + \frac{\partial w}{\partial y} \rightarrow (5)$

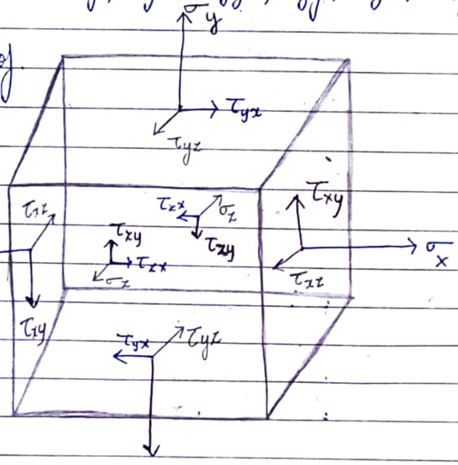
$$\gamma_{xz} = \frac{\partial u}{\partial z} + \frac{\partial w}{\partial x} \rightarrow (6).$$

20/6/23.

* **Body Force**: It is a force that acts on entire volume of the Body.
  Eg: Gravitational force, Magnetic force etc..
  Mathematically Body force is given by $f = \frac{F}{V}$ $\frac{N}{mm^3}$
  where, F is Net force & V is Volume of the body.

* **Stresses In 3-D.**
  Figure shows a 3-dimensional body subjected to normal stress $\sigma_x, \sigma_y, \sigma_z$.
  Shear stress $\tau_{xy}, \tau_{yx}, \tau_{yz}, \tau_{zy}, \tau_{zx}, \tau_{xz}$

From Equality of cross shear
$\tau_{ij} = \tau_{ji}$

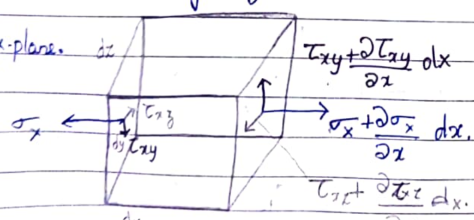


$\rightarrow \tau_{xy} = \tau_{yx}$
$\rightarrow \tau_{yz} = \tau_{zy}$
$\rightarrow \tau_{xz} = \tau_{zx}$

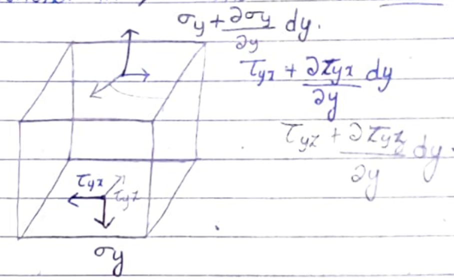
* **Equillibrium Equation's In 3-D.**

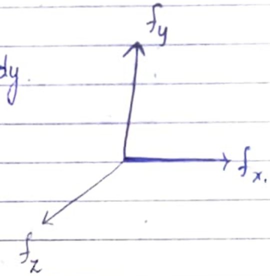
Consider an infinitesimally small 3-dimentional element of dimension's dx, dy & dz.

stresses in x-plane. $dydz$

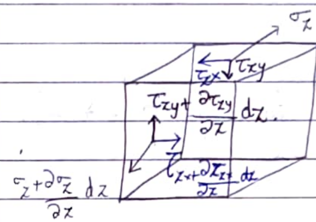


Stresses in Y-Plane. $\rightarrow Y \rightarrow dxdz$





Stresses in Z-plane. $\rightarrow dxdy$



Let the element be subjected to $\sigma_x, \sigma_y, \sigma_z$. Shear stresses $\tau_{xy}, \tau_{yz}, \tau_{zx}$ & body forces f_x, f_y & f_z.
For the equillibrium of the element the conditions are:
$\Sigma F_x = 0$, $\Sigma F_y = 0$, $\Sigma F_z = 0$.
Considering $\Sigma F_x = 0$.

$$\rightarrow \left[\cancel{\sigma_x} + \frac{\partial \sigma_x}{\partial x} dx\right] dydz - \cancel{\sigma_x} dydz + \left[\cancel{\tau_{yx}} + \frac{\partial \tau_{yx}}{\partial y} dy\right] dxdz - \cancel{\tau_{yx}} dxdz + \left[\cancel{\tau_{zx}} + \frac{\partial \tau_{zx}}{\partial z} dz\right] dxdy - \cancel{\tau_{zx}} dxdy + f_x\, dxdydz = 0$$

$$\rightarrow \left(\frac{\partial \sigma_x}{\partial x} + \frac{\partial \tau_{yx}}{\partial y} + \frac{\partial \tau_{zx}}{\partial z} + f_x\right) dxdydz = 0.$$

$$\because dxdydz \neq 0.$$

$$\frac{\partial \sigma_x}{\partial x} + \frac{\partial \tau_{yx}}{\partial y} + \frac{\partial \tau_{zx}}{\partial z} + f_x = 0 \longrightarrow (1).$$

$\Sigma F_y = 0.$

$$\left[\tau_{xy} + \frac{\partial \tau_{xy}}{\partial x} dx\right] dy\,dz - \tau_{xy}\,dy\,dz$$
$$+ \left[\sigma_y + \frac{\partial \sigma_y}{\partial y} dy\right] dx\,dz - \sigma_y\,dx\,dz.$$
$$+ \left[\tau_{zy} + \frac{\partial \tau_{zy}}{\partial z} dz\right] dx\,dy - \tau_{zy}\,dx\,dy + f_y\,dx\,dy\,dz = 0.$$

$$= \left[\frac{\partial \tau_{xy}}{\partial x} + \frac{\partial \sigma_y}{\partial y} + \frac{\partial \tau_{zy}}{\partial z} + f_y\right] dx\,dy\,dz = 0.$$

$\because dx\,dy\,dz \neq 0.$

$$\frac{\partial \tau_{xy}}{\partial x} + \frac{\partial \sigma_y}{\partial y} + \frac{\partial \tau_{xy}}{\partial z} + f_y = 0 \rightarrow (2).$$

Considering $\Sigma F_z = 0.$

$$\left[\tau_{xz} + \frac{\partial \tau_{xz}}{\partial x} dx\right] dy\,dz - \tau_{xz}\,dy\,dz$$
$$+ \left[\tau_{yz} + \frac{\partial \tau_{yz}}{\partial y} dy\right] dx\,dz - \tau_{yz}\,dx\,dz.$$
$$+ \left[\sigma_z + \frac{\partial \sigma_z}{\partial z} dz\right] dx\,dy - \sigma_z\,dx\,dy + f_z\,dx\,dy\,dz = 0.$$

$$= \left[\frac{\partial \tau_{xz}}{\partial x} + \frac{\partial \tau_{yz}}{\partial y} + \frac{\partial \sigma_z}{\partial z} + f_z\right] dx\,dy\,dz = 0.$$

$\because dx\,dy\,dz \neq 0.$

$$\frac{\partial \tau_{xz}}{\partial x} + \frac{\partial \tau_{yz}}{\partial y} + \frac{\partial \sigma_z}{\partial z} + f_z = 0 \rightarrow (3).$$

Eqn (1) (2) & (3) represents stress Equilibrium Eq^n in 3-D, In 2-D the Equilibrium equations reduces to

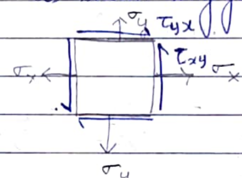
$$\left.\begin{aligned} \frac{\partial \sigma_x}{\partial x} + \frac{\partial \tau_{xy}}{\partial y} + f_x = 0. \\ \frac{\partial \tau_{xy}}{\partial x} + \frac{\partial \sigma_y}{\partial y} + f_y = 0. \end{aligned}\right\} \text{2-D}$$

* PLANE STRESS:-

It is state of stress in which normal & shear stress components perpendicular to z plane or assumed to be zero. plane direction

This implies $\Rightarrow \sigma_z = \tau_{xz} = \tau_{yz} = 0.$

Ex: Thin planar element subjected to inplane loading as shown in the figure.

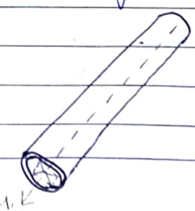


* PLANE STRAIN:-

It is a state of strain in which normal & shear strain components perpendicular to z plane are assumed to be zero.

$\gamma \Rightarrow$ gamma $\Rightarrow \epsilon_z = \gamma_{xz} = \gamma_{yz} = 0.$

Ex: Long Hallow Cylinder Subjected to inplane loading.



$\epsilon_z = \frac{\Delta z}{z} \approx 0.$

* St. Venant's Strain Compatibility Relations:-

Strain Compatability Eqn's establishes relationship b/w normal & shear strain components.

* Normal strain in x-direction

$$\epsilon_x = \frac{\partial u}{\partial x}$$, where u is displacement in x-direction.

Differentiating twice w.r.t 'y'.

$$\frac{\partial \epsilon_x}{\partial y} = \frac{\partial^2 u}{\partial x \partial y} \Rightarrow \frac{\partial^2 \epsilon_x}{\partial y^2} = \frac{\partial^3 u}{\partial x \partial y^2} \rightarrow (1)$$

* Normal strain in Y-direction.

$$\epsilon_y = \frac{\partial v}{\partial y}$$, where v is displacement in y-direction.

Diff. twice w.r.t 'x'.

$$\frac{\partial \epsilon_y}{\partial x} = \frac{\partial^2 v}{\partial x \partial y} \Rightarrow \frac{\partial^2 \epsilon_y}{\partial x^2} = \frac{\partial^3 v}{\partial x^2 \partial y} \rightarrow (2)$$

Shear strain in x-y plane.

$$\gamma_{xy} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}$$

* Diff w.r.t 'x'.

$$\frac{\partial \gamma_{xy}}{\partial x} = \frac{\partial^2 u}{\partial x \partial y} + \frac{\partial^2 v}{\partial x^2}$$

* Diff w.r.t 'y'

$$\frac{\partial^2 \gamma_{xy}}{\partial x \partial y} = \frac{\partial^3 u}{\partial y^2 \partial x} + \frac{\partial^3 v}{\partial x^2 \partial y} \rightarrow (3)$$

From Eqn (1), (2) & (3)

$$\frac{\partial^2 \gamma_{xy}}{\partial x \partial y} = \frac{\partial^2 \epsilon_x}{\partial y^2} + \frac{\partial^2 \epsilon_y}{\partial x^2} \rightarrow (A).$$

Similarly $\Rightarrow$ $$\frac{\partial^2 \gamma_{yz}}{\partial y \partial z} = \frac{\partial^2 \epsilon_y}{\partial z^2} + \frac{\partial^2 \epsilon_z}{\partial y^2} \rightarrow (B)$$

$$\frac{\partial^2 \gamma_{xz}}{\partial x \partial z} = \frac{\partial^2 \epsilon_x}{\partial z^2} + \frac{\partial^2 \epsilon_z}{\partial x^2} \rightarrow (C)$$

Eqn's A, B & C are first set of strain Compatability Eqn's.

Shear strain in x-y-plane $\gamma_{xy} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}$

* Diff. w.r.t 'z'.

$$\frac{\partial \gamma_{xy}}{\partial z} = \frac{\partial^2 u}{\partial y \partial z} + \frac{\partial^2 v}{\partial x \partial z} \rightarrow (4)$$

Shear Strain in y-z plane

$$\gamma_{yz} = \frac{\partial v}{\partial z} + \frac{\partial w}{\partial y}.$$

Diff w.r.t 'x'.

$$\frac{\partial \gamma_{yz}}{\partial x} = \frac{\partial^2 v}{\partial x \partial z} + \frac{\partial^2 w}{\partial x \partial y} \rightarrow (5)$$

Di Shear strain in X-Z plane.

$$\gamma_{xz} = \frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}$$

diff. w.r.t 'y'

$$\frac{\partial \gamma_{xz}}{\partial y} = \frac{\partial^2 u}{\partial y \partial z} + \frac{\partial^2 w}{\partial x \partial y} \rightarrow (6)$$

Adding Eq^n's (5) & (6) & Sub (4).

(5) + (6) − (4).

$$\frac{\partial \gamma_{yz}}{\partial x} + \frac{\partial \gamma_{xz}}{\partial y} - \frac{\partial \gamma_{xy}}{\partial z} = \frac{2\partial^2 w}{\partial x \partial y}$$

Diff. w.r.t 'z'.

$$\frac{\partial}{\partial z}\left[\frac{\partial \gamma_{yz}}{\partial x} + \frac{\partial \gamma_{xz}}{\partial y} - \frac{\partial \gamma_{xy}}{\partial z}\right] = \frac{\partial}{\partial z}\left[\frac{2\partial^2 w}{\partial x \partial y}\right].$$

$$\frac{\partial}{\partial z}\left[\frac{\partial \gamma_{yz}}{\partial x} + \frac{\partial \gamma_{xz}}{\partial y} - \frac{\partial \gamma_{xy}}{\partial z}\right] = \frac{2\partial^3 w}{\partial x \partial y \partial z} \rightarrow (7)$$

Normal strain in Z-direction.

$$\epsilon_z = \frac{\partial w}{\partial z}$$

diff. w.r.t 'x'.

$$\frac{\partial \epsilon_z}{\partial x} = \frac{\partial^2 w}{\partial x \partial z}$$

diff. w.r.t 'y'

$$\frac{\partial^2 \epsilon_z}{\partial x \partial y} = \frac{\partial^3 w}{\partial x \partial y \partial z} \rightarrow (8)$$

Substituting Eq^n (8) in Eq^n (7)

$$\frac{\partial}{\partial z}\left[\frac{\partial \gamma_{yz}}{\partial x} + \frac{\partial \gamma_{xz}}{\partial y} - \frac{\partial \gamma_{xy}}{\partial z}\right] = \frac{2\partial^2 \epsilon_z}{\partial x \partial y} \rightarrow (D).$$

Similarly $\frac{\partial}{\partial x}\left[\frac{\partial \gamma_{xy}}{\partial z} + \frac{\partial \gamma_{xz}}{\partial y} - \frac{\partial \gamma_{yz}}{\partial x}\right] = \frac{2\partial^2 \epsilon_x}{\partial y \partial z} \rightarrow (E).$

$$\frac{\partial}{\partial y}\left[\frac{\partial \gamma_{xy}}{\partial z} + \frac{\partial \gamma_{yz}}{\partial x} - \frac{\partial \gamma_{xz}}{\partial y}\right] = \frac{2\partial^2 \epsilon_y}{\partial x \partial z} \rightarrow (F)$$

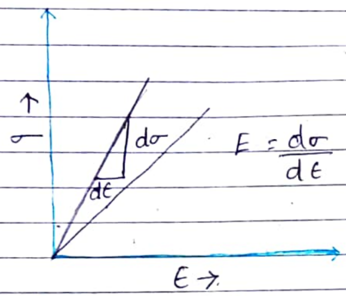
Eq^n's D, E & F are Second Set of Strain Compatability relation's.

Hooke's Law

- Hooke's Law States that stress is propotional to strain within elastic limit.
- The ratio of Stress to Strain within elastic limit is characteristic of the material.
- From Hooke's Law within elastic limit $\sigma \propto \epsilon$

  $\sigma = E\epsilon$ $\quad E \rightarrow$ propotional Constant

  known as Young's Modulus.
- Modulus of elasticity is the Slope of linear region of stress-strain diagram as shown in diagram.



$$E = \frac{d\sigma}{d\epsilon}$$

However present-day experiment's show that for mild steel Hooke's Law hold's good upto proportionality Limit which is very close to Elastic limit.

26/6/23

Stress - Strain Curve For Ductile Materials :-

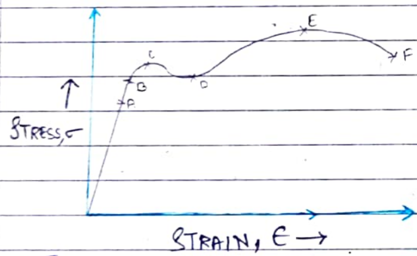


Figure show's stress-strain curve for a typical mild steel specimen. The following salient points are observed in the stress-strain curve.

(i) Proportionality Limit (A) :- It is the limiting value of stress upto which stress is propotional to strain.

(ii) Elastic Limit (B) :- It is the limitting value of stress upto which upon the release of stress the material retains it's original dimensions.

(iii) Upper Yield point (C) :- It is the stress at which the force starts reducing with increase in extension. This phenomenon is called as yielding.

→ For mild steel this stress is about 250 N/mm^2.

(iv) Lower Yield point (D) :- It is the constant value of stress at which the strain increases for sometime.

(v) Ultimate Stress (E) :- It is the maximum value of stress the material can withstand.

Necking takes place at this point, for mild steel this stress is about (370 – 400) N/mm^2.

(vi) Breaking Stress (F) :- It is the stress at which finally the specimen fails.

Stress - Strain Curve For Brittle Material :

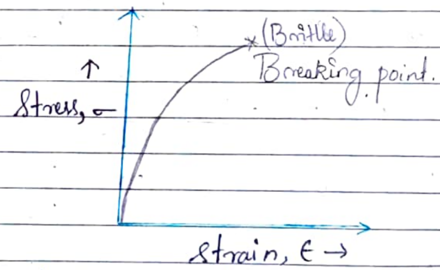


fig show's stress-strain curve for a material (Brittle) like cast iron.

The following Salient features are observed:-

i) There is no appreciable change in rate of strain

ii) There is no yield point & no necking takes place.

iii) The ultimate stress coincides with breaking stress

iv) The strain at failure is very small.

→ NUMERICAL PROBLEMS:- (ON STRAIN DISPLACEMENT RELATIONS)

Nomenclature:-

→ u, v, w → displacements in X, Y & Z directions. (respectively)

→ U → resultant displacement.

→ $\epsilon_x, \epsilon_y, \epsilon_z$ → Normal strain in X, Y & Z directions

→ $\gamma_{xy}, \gamma_{yz}, \gamma_{zx}$ → shear strain in xy, yz & zx plane respectively

★ LIST OF FORMULAE:-

vector's displacement

1) Resultant displacement $U = ui + vj + wk$

2) Magnitude of Resultant displacement $U = \sqrt{u^2+v^2+w^2}$

3) Normal strains, $\epsilon_x = \frac{\partial u}{\partial x}$, $\epsilon_y = \frac{\partial v}{\partial y}$, $\epsilon_z = \frac{\partial w}{\partial z}$

4) Shear strain's, $\gamma_{xy} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}$

$\gamma_{yz} = \frac{\partial v}{\partial z} + \frac{\partial w}{\partial y}$ $\quad \gamma_{xz} = \frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}$

5) Strain Components in matrix form:- $\epsilon_{ij} = \begin{bmatrix} \epsilon_x & \gamma_{xy} & \gamma_{xz} \\ \gamma_{xy} & \epsilon_y & \gamma_{yz} \\ \gamma_{zx} & \gamma_{yz} & \epsilon_z \end{bmatrix}$

PROBLEMS:-

1. Consider the displacement field $U = [y^2 i + 3yz j + (4+6x^2)k]10^{-2}$ → ①
Determine the strain components at the point $P(1,0,2)$

Sol^n Given displacement field. $U = [y^2 i + 3yz j + (4+6x^2)k]10^{-2}$
displacement field is generally expressed as
$U = ui + vj + wk$ → ②.

Comparing Eq's ① & ②
$u = y^2 \times 10^{-2}$, $v = 3yz \times 10^{-2}$, $w = (4+6x^2)10^{-2}$

Normal Strain → $\epsilon_x = \frac{\partial u}{\partial x} \Rightarrow \frac{\partial}{\partial x}(y^2 \times 10^{-2}) \Rightarrow \boxed{\epsilon_x = 0}$

$\epsilon_y = \frac{\partial v}{\partial y} \Rightarrow \frac{\partial}{\partial y}(3yz \times 10^{-2}) \Rightarrow \boxed{\epsilon_y = 3z \times 10^{-2}}$

$\epsilon_z = \frac{\partial w}{\partial z} \Rightarrow \frac{\partial}{\partial z}(4+6x^2)10^{-2} \Rightarrow \boxed{\epsilon_z = 0}$

Shear strain → $\gamma_{xy} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}$

→ $\gamma_{xy} = \frac{\partial}{\partial y}(y^2 \times 10^{-2}) + \frac{\partial}{\partial x}(3yz \times 10^{-2})$

$\gamma_{xy} = 2y \times 10^{-2} + 0 \Rightarrow \boxed{\gamma_{xy} = 2y \times 10^{-2}}$

→ $\gamma_{yz} = \frac{\partial v}{\partial z} + \frac{\partial w}{\partial y}$

$= \frac{\partial}{\partial z}(3yz \times 10^{-2}) + \frac{\partial}{\partial y}[(4+6x^2)10^{-2}]$

$\boxed{\gamma_{yz} = 3y \times 10^{-2}}$

→ $\gamma_{xz} = \frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}$

$= \frac{\partial}{\partial z}(y^2 \times 10^{-2}) + \frac{\partial}{\partial x}[(4+6x^2)10^{-2}]$

$= 0 + (12x)10^{-2}$

$\boxed{\gamma_{xz} = (12x)10^{-2}}$

The strain components are given by $\epsilon_x = 0$, $\epsilon_y = 3z \times 10^{-2}$,
$\epsilon_z = 0$, $\gamma_{xy} = 2y \times 10^{-2}$, $\gamma_{yz} = 3y \times 10^{-2}$, & $\gamma_{xz} = (12x) \times 10^{-2}$.

The strain Components at Point (1, 0, 2) are given by.

$\epsilon_x = 0$, Unit $\quad \gamma_{xy} = 0$

$\epsilon_y = 6\times10^{-2}$ Units $\quad \gamma_{yz} = 0$

$\epsilon_z = 0$ unit. $\quad \gamma_{xz} = 12\times10^{-2}$ Units.

2) The Displacement Components in a strained body are as follows., $u = 0.01xy + 0.02y^2$

$v = 0.02x^2 + 0.01z^3y$; $w = 0.01xy^2 + 0.05z^2$.

Determine the strain Components at point (3, 2, −5)

Soln: Normal strain:→

$$\epsilon_x = \frac{\partial u}{\partial x} \Rightarrow \frac{\partial}{\partial x}(0.01xy + 0.02y^2)$$

$$\epsilon_x \Rightarrow 0.01y.$$

$$\epsilon_y = \frac{\partial v}{\partial y} \Rightarrow \frac{\partial}{\partial y}(0.02x^2 + 0.01z^3y)$$

$$\epsilon_y \Rightarrow 0.01z^3$$

$$\epsilon_z = \frac{\partial w}{\partial z} \Rightarrow \frac{\partial}{\partial z}(0.01xy^2 + 0.05z^2)$$

$$\epsilon_z = 0.05(2z) \Rightarrow \underline{0.1z}$$

$$\gamma_{xy} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \Rightarrow \frac{\partial}{\partial y}(0.01xy + 0.02y^2) + \frac{\partial}{\partial x}(0.02x^2 + 0.01z^3y)$$

$$= 0.01x + 0.02(2y) + 0.02(2x)$$

$$= 0.01x + 0.04y + 0.04x$$

$$= \underline{0.05x + 0.04y}.$$

$$\gamma_{yz} = \frac{\partial v}{\partial z} + \frac{\partial w}{\partial y} \Rightarrow \frac{\partial}{\partial z}(0.02x^2 + 0.01z^3y) + \frac{\partial}{\partial y}(0.01xy^2 + 0.05z^2)$$

$$= 0.01(3z^2)y + 0.01x(2y)$$

$$= 0.03yz^2 + 0.02xy.$$

$$\gamma_{xz} = \frac{\partial u}{\partial z} + \frac{\partial w}{\partial x} \Rightarrow \frac{\partial}{\partial z}(\cancel{0.01xy + 0.02y^2}) + \frac{\partial}{\partial x}(0.01xy^2 + 0.05z^2)$$

$$\Rightarrow \cancel{0.05(2z)} \Rightarrow \cancel{0.1z} \Rightarrow 0.01\times(y^2) \Rightarrow 0.01y^2$$

The strain Components are given by $\epsilon_x = 0.01y$, $\epsilon_y = 0.01z^3$, $\epsilon_z = 0.1z$, $\gamma_{xy} = 0.05x + 0.04y$, $\gamma_{yz} = 0.03yz^2 + 0.02xy$, $\gamma_{xz} = 0.01y^2$.

The strain Components at Point (3, 2, −5) are given by.

$\epsilon_x = 0.01(2) \Rightarrow 0.02$, $\gamma_{xy} = 0.05(3) + 0.04(2) = \underline{0.23}$

$\epsilon_y = 0.01(-5)^3 = -1.25$, $\gamma_{yz} = 0.03(2)(-5)^2 + 0.02(3)(2) = \underline{1.62}$

$\epsilon_z = 0.1(-5) = -0.5$, $\gamma_{xz} = 0.01(2)^2 = \underline{0.04}$

3) If the displacement field in the body is specified as. $u = (x^2+3)10^{-3}$, $v = (3y^2z)\,10^{-3}$, $w = (x+3z)\times10^{-3}$

Determine the strain Components at a point (1, 2, 3)

Soln: Normal strain:-

$$\epsilon_x = \frac{\partial u}{\partial x} \Rightarrow \frac{\partial}{\partial x}(x^2+3)10^{-3} \Rightarrow (2x + \cancel{3})10^{-3} \Rightarrow (2x)10^{-3}$$

$$\epsilon_y = \frac{\partial v}{\partial y} = \frac{\partial}{\partial y}(3y^2z)10^{-3} = (3(2y)z)10^{-3} \Rightarrow (6yz)10^{-3}$$

$$\epsilon_z = \frac{\partial w}{\partial z} = \frac{\partial}{\partial z}(x+3z)10^{-3} = (3)10^{-3}$$

→ $$\gamma_{xy} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \Rightarrow \frac{\partial}{\partial y}\cancel{(x^2+3)}^{0}10^{-3} + \frac{\partial}{\partial x}\cancel{(3y^2z)}^{0}10^{-3}$$

$$\gamma_{xy} = 0.$$

→ $$\gamma_{yz} = \frac{\partial v}{\partial z} + \frac{\partial w}{\partial y} \Rightarrow \frac{\partial}{\partial z}(3y^2z)10^{-3} + \frac{\partial}{\partial y}\cancel{(x+3z)}^{0}10^{-3}$$

$$\gamma_{yz} = (3y^2)10^{-3}.$$

→ $$\gamma_{xz} = \frac{\partial u}{\partial z} + \frac{\partial w}{\partial x} = \frac{\partial}{\partial z}(x^2+3)10^{-3} + \frac{\partial}{\partial x}(x+3z)\times10^{-3}$$

$$\gamma_{xz} = 1\times10^{-3}$$

SURYA Gold

Date____ Page____

$\epsilon_x = 2\times10^{-3}$ units.
$\epsilon_y = 36\times10^{-3}$ units
$\epsilon_z = 3\times10^{-3}$
$\gamma_{xy}=0,\ \gamma_{yz}=(3y^2)10^{-3}=12\times10^{-3},\ \gamma_{xz}=10^{-3}$ units.

PROBLEMS ON EQUILIBRIUM EQUATIONS:-

Nomenclature: $\sigma_x, \sigma_y, \sigma_z$ Normal stress in x, y & z.
$\tau_{xy}, \tau_{yz}, \tau_{xz}$ shear stress in xy, yz & xz plane.
f_x, f_y & f_z body forces in x, y & z direction respectively.

LIST OF FORMULAE:→
Equilibrium Equations in 3D.

* $$\frac{\partial\sigma_x}{\partial x}+\frac{\partial\tau_{xy}}{\partial y}+\frac{\partial\tau_{xz}}{\partial z}+f_x=0.$$

* $$\frac{\partial\tau_{xy}}{\partial x}+\frac{\partial\sigma_y}{\partial y}+\frac{\partial\tau_{yz}}{\partial z}+f_y=0.$$

* $$\frac{\partial\tau_{xz}}{\partial x}+\frac{\partial\tau_{yz}}{\partial y}+\frac{\partial\sigma_z}{\partial z}+f_z=0.$$

1. A state of stress at a point is given by $\sigma_x = x^3yz+xy^2$, $\sigma_y = 3y^2z+yz$, $\sigma_z = x^2y^2z^2+xz$. $\tau_{xy}=x^2yz$, $\tau_{yz}=xy^2z$, $\tau_{zx}=xyz^2$. In the absence of body forces, determine whether the equilibrium conditions are satisfied or not at point $(3,-4,2)$. If not then determine the suitable body forces required at this point so that these stress components are under equilibrium.

Soln: In Abs of f's The equillibrium eqn's are given by
So, f_x, f_y & $f_y=0$.

SURYA Gold

Date____ Page____

$$\frac{\partial\sigma_x}{\partial x}+\frac{\partial\tau_{xy}}{\partial y}+\frac{\partial\tau_{xz}}{\partial z}=0 \longrightarrow (1)$$

$$\frac{\partial\tau_{xy}}{\partial x}+\frac{\partial\sigma_y}{\partial y}+\frac{\partial\tau_{yz}}{\partial z}=0 \longrightarrow (2)$$

$$\frac{\partial\tau_{xz}}{\partial x}+\frac{\partial\tau_{yz}}{\partial y}+\frac{\partial\sigma_z}{\partial z}=0 \longrightarrow (3)$$

Diff.→

$\frac{\partial\sigma_x}{\partial x} = 3x^2yz+2xy^2$, $\frac{\partial\tau_{xy}}{\partial x} = 2xyz$.

$\frac{\partial\tau_{xy}}{\partial y} = x^2z$, $\frac{\partial\sigma_y}{\partial y} = 6yz+z$.

$\frac{\partial\tau_{xz}}{\partial z} = 2xyz$, $\frac{\partial\tau_{yz}}{\partial z} = xy^2$

$\frac{\partial\tau_{xz}}{\partial x} = yz^2$, $\frac{\partial\tau_{yz}}{\partial y} = 2xyz$.

$\frac{\partial\sigma_z}{\partial z} = 2x^2y^2z+x$.

Substituting the above Expn's.
At point $(3,-4,2)$

$$\left.\begin{aligned}&\frac{\partial\sigma_x}{\partial x} = 3(3)^2(-4)(2)+2(3)(-4)^2 = -120.\\ &\frac{\partial\tau_{xy}}{\partial x} = 2(3)(-4)(2) = -48.,\ \frac{\partial\tau_{xy}}{\partial y} = (3)^2(2) = 18.\end{aligned}\right\} -150 \rightarrow (4)$$

$\frac{\partial\sigma_y}{\partial y} = 6(-4)(2)+(2) = -46$, $\frac{\partial\tau_{xz}}{\partial z} = 2(3)(-4)(2) = -48$.

$\frac{\partial\tau_{yz}}{\partial z} = (3)(-4)^2 = 48.$ $\quad -46 \rightarrow (5)$.

$\frac{\partial\tau_{xz}}{\partial x} = (-4)(2)^2 = -16$, $\frac{\partial\tau_{yz}}{\partial y} = 2(3)(-4)(2) = -48$.

$\frac{\partial\sigma_z}{\partial z} = 2(3)^2(-4)^2(2)+3. = 579.$ $\quad 515 \rightarrow (6)$.

Substituting. Eqn (4) in (1).
$-128+18-48 = \underline{-150} \neq 0.$

Sub Eq (5) in (2).

$-48 + (-46) + 48 = -46 \neq 0$.

Sub eq (6) in (3).

$-16 + (-48) + 579 = 515 \neq 0$.

$\therefore$ The equilibrium equations are not satisfied for the given stress components at point $(3,-4,2)$.

The body forces that are to be added to equilibrium equations so as to satisfied. them are

$f_x = 150$ units, $f_y = 46$ units.

$f_z = -515$ units.

2/6/23.

2. The stress components at a point in a body are given by $\sigma_x = 3xy^2z + 2x$, $\sigma_y = 5xyz + 3y$, $\sigma_z = x^2y + y^2z$, $\tau_{xy} = 0$. $\tau_{yz} = \tau_{zx} = 3xy^2z + 2xy$. In the absence of body forces, determine whether equilibrium Conditions are satisfied are not at $(1,-1,2)$. If not, then determine the suitable body forces required at this point so that these stress components are under equilibrium.

Soln: In the Absence of body forces, The equilibrium eq^{n}s are given by Eq^n (1), (2), (3).

$$\frac{\partial \sigma_x}{\partial x} + \frac{\partial \tau_{xy}}{\partial y} + \frac{\partial \tau_{zx}}{\partial z} = 0 \rightarrow (1)$$

$$\frac{\partial \tau_{xy}}{\partial x} + \frac{\partial \sigma_y}{\partial y} + \frac{\partial \tau_{yz}}{\partial z} = 0 \rightarrow (2)$$

$$\frac{\partial \tau_{zx}}{\partial x} + \frac{\partial \tau_{yz}}{\partial y} + \frac{\partial \sigma_z}{\partial z} = 0 \rightarrow (3)$$

$$\frac{\partial (3xy^2z + 2x)}{\partial x} = 3y^2z + 2.$$

$$\frac{\partial \tau_{xy}}{\partial y} = \frac{\partial (0)}{\partial y} = 0$$

$$\frac{\partial \tau_{zx}}{\partial z} = \frac{\partial}{\partial z}(3xy^2z + 2xy) = 3xy^2$$

At $(1,-1,2) \Rightarrow$

$$\left.\begin{aligned} &\frac{\partial \sigma_x}{\partial x} = 3(1)(-1)^2(2) + 2 = 3(1)(2)+2 = 8 \\ &\frac{\partial \tau_{xy}}{\partial y} = 0, \quad \frac{\partial \tau_{zx}}{\partial z} = 3(1)(-1)^2 = 3 \end{aligned}\right\} (4)$$

$8 + 0 + 3 = 0 \Rightarrow 11 \neq 0 \therefore$ The Eq^n not Satisfied.

$$\frac{\partial \tau_{xy}}{\partial x} = 0, \quad \frac{\partial \sigma_y}{\partial y} = \frac{\partial}{\partial y}(5xyz + 3y) = 5xz + 3.$$

$$\frac{\partial \tau_{yz}}{\partial z} = \frac{\partial}{\partial z}(3xy^2z + 2xy) = 3xy^2.$$

$$\left.\begin{aligned} &\text{At } (1,-1,2) \Rightarrow \frac{\partial \tau_{xy}}{\partial x} = 0, \quad \frac{\partial \sigma_y}{\partial y} = 5(1)(2) + 3 = 13 \\ &\frac{\partial \tau_{yz}}{\partial z} = 3(1)(-1)^2 = 3. \end{aligned}\right\} (5)$$

$13 + 0 + 3 = 0 \quad 16 \neq 0 \therefore$ The Eq^n not Satisfied

$$\frac{\partial \tau_{zx}}{\partial x} = \frac{\partial}{\partial x}(3xy^2z + 2xy) = 3y^2z + 2y.$$

$$\frac{\partial \tau_{yz}}{\partial y} = \frac{\partial}{\partial y}(3xy^2z + 2xy) = 3x(2y)z + 2x = 6xyz + 2x.$$

$$\frac{\partial \sigma_z}{\partial z} = \frac{\partial}{\partial z}(x^2y + y^2z) = y^2$$

$$\left.\begin{aligned} &\text{At } (1,-1,2) \Rightarrow \frac{\partial \tau_{zx}}{\partial x} = 3(-1)^2(2) + 2(-1) = 6 - 2 = 4. \\ &\frac{\partial \tau_{yz}}{\partial y} = 6(1)(-1)(2) + 2(1) = -12 + 2 = -10. \\ &\frac{\partial \sigma_z}{\partial z} = (-1)^2 = 1 \Rightarrow 4 + (-10) + 1 = -5 \quad -5 \neq 0. \end{aligned}\right\} \rightarrow (6)$$

$\therefore$ All the 3 equilibrium Eq^n not satisfied for the given stress components at point $(1,-1,2)$.

The suitable body forces required at this point so that these stress components are under equilibrium are given by $f_x = -11$ units, $f_y = -16$ units, $f_z = 5$ units.

* Previous QP Problems.

Dec 18-19 [1] Displacement field at a point on a body is given as follows, $U=(x^2+y)$, $V=(3+z)$; $w=(x^2+2y)$. Determine strain components at $(3,1,-2)$ and express them in matrix form.

Soln:

$\epsilon_x = \frac{\partial u}{\partial x} = \frac{\partial}{\partial x}(x^2+y) = 2x$, point $2(3) = 6 = \epsilon_x = 6$ units

$\epsilon_y = \frac{\partial v}{\partial y} = \frac{\partial}{\partial y}(3+z) = 0$, $\epsilon_y = 0$ units

$\epsilon_z = \frac{\partial w}{\partial z} = \frac{\partial}{\partial z}(x^2+2y) = 0$, $\epsilon_z = 0$ units

$\gamma_{xy} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} = \frac{\partial}{\partial y}(x^2+y) + \frac{\partial}{\partial x}(3+z) = 1 \Rightarrow (1) = 1$ units

$\gamma_{yz} = \frac{\partial v}{\partial z} + \frac{\partial w}{\partial y} = \frac{\partial}{\partial z}(3+z) + \frac{\partial}{\partial y}(x^2+2y) = 1+2 = 3$ units

$\gamma_{xz} = \frac{\partial u}{\partial z} + \frac{\partial w}{\partial x} = \frac{\partial}{\partial z}(x^2+y) + \frac{\partial}{\partial x}(x^2+2y) = 2x = 2(3) = 6$ units

$$\epsilon_{ij} = \begin{bmatrix} \epsilon_x & \gamma_{xy} & \gamma_{xz} \\ \gamma_{xy} & \epsilon_y & \gamma_{yz} \\ \gamma_{zx} & \gamma_{yz} & \epsilon_z \end{bmatrix} = \begin{bmatrix} 6 & 1 & 6 \\ 1 & 0 & 3 \\ 6 & 3 & 0 \end{bmatrix}$$

2. A structure is loaded and displacements associated with the deformed state are mapped. Find all the strains associated with the following observed displacement field at point $(0.6, -0.75, 1)$ m.

$u = (-x^4+3x-3y^2+8yz+5)\times 10^{-3}$ m.

$v = (-4y^4+y+8xz+1)\times 10^{-3}$ m

$w = (2z^2+3z-8xy+8)\times 10^{-3}$ m.

Soln: Normal strain: $\epsilon_x = \frac{\partial}{\partial x}(-x^4+3x-3y^2+8yz+5)\times 10^{-3}$ m

$\epsilon_x = (-4x^3+3)\times 10^{-3}$ m.

$\epsilon_y = \frac{\partial}{\partial y}(-4y^4+y+8xz+1)\times 10^{-3}$ m $= [-4(4y^3)+1]\times 10^{-3}$ m

$\epsilon_y = (-16y^3+1)10^{-3}$ m

$\epsilon_z = \frac{\partial}{\partial z}(2z^2+3z-8xy+8)\times 10^{-3}$ m $= (2(2z)+3)10^{-3}$ m

$\epsilon_z = (4z+3)10^{-3}$ m.

$\gamma_{xy} = \frac{\partial}{\partial y}(-x^4+3x-3y^2+8yz+5)\times 10^{-3}\text{ m} + \frac{\partial}{\partial x}(-4y^4+y+8xz+1)\times 10^{-3}$ m

$= (-3(2y)+8z)10^{-3}\text{ m} + (8z)\times 10^{-3}\text{ m} \Rightarrow (-6y+8z)10^{-3} + 8z\times 10^{-3}$ m

$= (-6y+16z)\times 10^{-3}$ m.

$\gamma_{yz} = \frac{\partial}{\partial z}(-4y^4+y+8xz+1)\times 10^{-3}\text{ m} + \frac{\partial}{\partial y}(2z^2+3z-8xy+8)\times 10^{-3}$ m.

$\gamma_{yz} = (8x)\times 10^{-3}\text{ m} + (-8x)\times 10^{-3}\text{ m} = 0$

$\gamma_{xz} = \frac{\partial}{\partial z}(-x^4+3x-3y^2+8yz+5)\times 10^{-3}\text{ m} + \frac{\partial}{\partial x}(2z^2+3z-8xy+8)\times 10^{-3}$ m

$\gamma_{xz} = (8y)\times 10^{-3}\text{ m} + (-8y)\times 10^{-3}\text{ m} = 0.$

At point → $\epsilon_x = (-4(0.6)^3+3)\times 10^{-3}\text{ m} = 2.136\times 10^{-3}$ units

$\epsilon_y = (-16y^3+1)10^{-3}\text{ m} \Rightarrow (-16(-0.75)^3+1)10^{-3}\text{ m} = 7.75\times 10^{-3}$ units

$\epsilon_z = (4z+3)10^{-3}\text{ m} \Rightarrow (4(1)+3)10^{-3}\text{ m} \Rightarrow 7\times 10^{-3}$ units.

$\gamma_{xy} = [(-6)(-0.75)+16(1)]\times 10^{-3}\text{ m} = 20.5\times 10^{-3}$ units.

$\gamma_{yz} = \gamma_{xz} = 0$ units.

9m

*3 Displacement field at a point on a body is given as follows $u=(x^2yz+z^2)$, $v=(xy^2z+y^2)$, $w=(xyz^2+x^2)$ determine strain components at $(2,1,2)$ and express them in matrix form.

$\epsilon_x = \frac{\partial u}{\partial x} = \frac{\partial}{\partial x}(x^2yz+z^2) = 2xyz$, $\gamma_{xy} = x^2z+y^2z$.

$\epsilon_y = \frac{\partial v}{\partial y} = 2xyz+2y$, $\gamma_{yz} = xy^2+xz^2$

$\epsilon_z = \frac{\partial w}{\partial z} = 2xyz$, $\gamma_{xz} = x^2y+2z+yz^2+2x$.

at point $(2,1,2)$.

$\epsilon_x = 8$, $\gamma_{xy} = 10$

$\epsilon_y = 10$, $\gamma_{yz} = 10$

$\epsilon_z = 8$, $\gamma_{xz} = 16$

The matrix form.

$$= \begin{bmatrix} 8 & 10 & 16 \\ 10 & 10 & 10 \\ 16 & 10 & 8 \end{bmatrix}$$

6/7/23

SURYA Gold

Date______ Page______

Allowable Stress.

* The stress to be taken into Consideration while designing any engineering member is much lesser than the ultimate/yield stress. this stress is called as allowable stress.
* The ratio of ultimate or yield stress to allowable stress is called factor of safety.
* For ductile materials factor of safety

$$FOS = \frac{\text{Yield stress}}{\text{Allowable stress}} = \frac{\sigma_y}{\sigma_a}$$

* For Brittle materials.

$$FOS = \frac{\text{Ultimate Stress}}{\text{Allowable stress}} = \frac{\sigma_u}{\sigma_a}.$$

* Part - 2.

Extension / Shortening Of A Bar:-

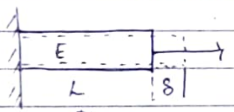


Fig 1.

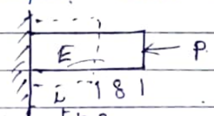


Fig 2.

* Fig 1 & Fig 2 show's a uniform bar of length (l). Young's modulus 'E' Subjected to tensile & Compressive force respectively.
* Let 'δ' be the change in length.
* Stress in the material $\sigma = \frac{P}{A} \rightarrow (1)$
* Strain in the material $\epsilon = \frac{\delta}{L} \rightarrow (2).$

From Hooke's law Young's Modulus $E = \frac{\sigma}{\epsilon} \rightarrow (3)$

Substitute Eqn (1) & (2) in (3).

$$E = \frac{P/A}{\delta/L} \Rightarrow \frac{PL}{A\delta}$$

Eqn (4) is the eqn for Ex/Sh of A Bar. $\Rightarrow \delta = \frac{PL}{AE} \rightarrow (4).$

SURYA Gold

Date______ Page______

Numericals :-

Problems On extension or shortening of a Bar.

Nomenclature:-

$d \rightarrow$ diameter of the bar, mm.

$L \rightarrow$ Length of the bar, mm.

$A \rightarrow$ area of cross-section of bar mm^2.

$d_o \rightarrow$ Outer diameter of Hallow bar.

$d_i \rightarrow$ Inner diameter of Hallow bar.

$t \rightarrow$ thickness of the Hallow bar, mm.

$E \rightarrow$ Young's Modulus of the material $[N/mm^2]$.

$P \rightarrow$ Applied load Newton.

$\delta \rightarrow$ Change in length (Extention for Tension ↑) (Shortening ↓ for Compression), mm

$\sigma \rightarrow$ (Sigma) stress., N/mm^2.

$\epsilon \rightarrow$ strain.

$\sigma_y \rightarrow$ Yield stress, N/mm^2

$\sigma_u \rightarrow$ ultimate stress, N/mm^2.

$Fos \rightarrow$ Factor of Safety.

$\sigma_a \rightarrow$ Allowable stress, N/mm^2.

List Of Formula:-

1. For solid bar, $A = \frac{\pi d^2}{4} mm^2$.
2. For hallow bar, $A = \frac{\pi}{4}(d_o^2 - d_i^2) mm^2$.

   Thickness, $\rightarrow t = \frac{d_o - d_i}{2}$. $[t = R_o - R_i]$
3. $\delta = \frac{PL}{AE}$
4. $\sigma = \frac{P}{A}$
5. $\epsilon = \frac{\delta}{L}$
6. $E = \frac{\sigma}{\epsilon}$ N/mm^2.
7. $FOS = \frac{\sigma_y}{\sigma_a}$ (ductile)
8. $Fos = \frac{\sigma_u}{\sigma_a}$ (brittle)
8) % elongation $\rightarrow \frac{L_f - L_i}{L_i} \times 100.$
9) % reduction in Area $\Rightarrow \frac{A_i - A_f}{A_i} \times 100$

A circular Rod of diameter 20mm & 500 mm long is Subjected to A tensile force 45 KN. The modulus of elasticity for steel maybe taken as 200 KN/mm². find stress, strain & elongation of the bar due to applied load.

⇒ Data: $d = 20mm$

$L = 500mm$

$P = +45\times10^3 N$

$E = 200\times10^3 N/mm^2$

$\sigma = ?$

$\epsilon = ?$

$\delta = ?$

⇒ $\sigma = \frac{P}{A}$ ⇒ ① $A = \frac{\pi d^2}{4}$

$A = \frac{\pi d^2}{4} = \frac{\pi (20)^2}{4} = 314.15 mm^2$

$\sigma = \frac{P}{A} = \frac{45\times10^3}{314.15} = 143.24 N/mm^2$ (tensile)

→ $E = \frac{\sigma}{\epsilon}$ $\epsilon = \frac{\sigma}{E}$

$\epsilon = \frac{143.24}{200\times10^3}$ — $\epsilon = 716.218$

→ $\delta = \epsilon L$

$\delta = \frac{PL}{AE}$

$\delta = \frac{(45\times10^3)(500)}{(314.15)(200\times10^3)}$

$\delta = 0.35 mm$

2. A Surveyor's steel tape 30m long has a cross-section of 15mm (by) x 0.75mm. With this, line AB is measured 150 m. If the force applied during measurement is 100N more than the force applied at the time of calibration. What is the actual length of the line? Take modulus of Elasticity for steel as 200 KN/mm².

→ Data:- $L = 30m = 30\times10^3 mm$.

$A = 15mm \times 0.75mm = 11.25 mm^2$.

$P = 100N$.

$E = 200\times10^3 N/mm^2$.

$\delta = \frac{PL}{AE}$ → $\frac{100\times30\times10^3}{11.25\times200\times10^3}$ ⇒ $1.33 mm = 1.33\times10^{-3} m$.

Actual length of the tape during measurement is Equal to. $30 + (1.33\times10^{-3})$

$= 30.0013 m$.

∴ The actual length of measured line AB.

$= \frac{150\times30.0013}{30}$

$= 150.0065 m$.

3] A Hallow steel tube is to be used for carrying an Axial Compressive load of 140 KN. The yield stress for steel is 250 N/mm². A factor of safety of 1.75 is to be used in the design. The following 3 classes of tubes of external diameter, 101.6mm are available.

CLASS	Thickness.
Light	3.65mm
Medium	4.05mm
Heavy	4.85mm

Which section do u recommend.

Soln: $P = -140\times10^3 N$. $t = \frac{d_o - d_i}{2}$, $A = \frac{\pi}{4}(d_o^2 - d_i^2)$.

$\sigma_y = 250 N/mm^2$

$FOS = 1.75$. $\sigma_a = \frac{P}{A}$, $FOS = \frac{\sigma_y}{\sigma_a}$.

$d_o = 101.6mm$.

$t = ?$

Allowable stress for given material.

$$\sigma_a = \frac{\sigma_y}{FOS}$$

$$\sigma_a = \frac{250}{1.75} = 142.85\ N/mm^2$$

Req. Gross Sectional area.

$$A = \frac{P}{\sigma_a} = \frac{140\times10^3}{142.85} \quad A = 980.04\ mm^2$$

$$A = \frac{\pi}{4}(d_o^2 - d_i^2)$$

$3920.160 = \pi(101.06^2 - d_i^2)$

$-d_i^2 = \frac{4A}{\pi} - d_o^2$

$3920.160 - 32085.47\times\pi(-d_i^2)$

$32085.47\times\pi\cdot d_i^2 = 3920.160$

$-d_i^2 = \frac{4(980.04)}{\pi} - (101.06)^2$

$-d_i^2 = -9074.73$

$\sqrt{9074.73}$

$d_i = 95.26$

$\Rightarrow 980.04 = \frac{\pi}{4}\left[(101.06)^2 - d_i^2\right]$

$\Rightarrow 980.04 = \frac{\pi}{4}\left[10212.123 - d_i^2\right]$

$\Rightarrow 980.04 = 8021.36 - \frac{\pi}{4}(d_i^2)$

$8021.36 - 980.04 = \frac{\pi}{4}(d_i^2)$

$\Rightarrow 7041.32 = \frac{\pi}{4}(d_i^2)$

$\Rightarrow 28165.28 = \pi(d_i)^2$

$\Rightarrow 9074.73 = d_i^2$

$\Rightarrow 95.26 = d_i$

$$t = \frac{d_o - d_i}{2} = 3.17\ mm.$$

Since required thickness = 3.17 mm light glass of 3.65 mm thickness is recommended.

4. A Specimen of steel 25 mm in diameter with a gauge length 200 mm is tested to destruction. It has an extension of 0.16 mm under a load of 80 KN and the loaded at elastic limit is 160 KN. The maximum load is 56 mm and diameter at the neck is 18 mm. Find.

i) The stress at elastic limit.
ii) Youngs Modulus.
iii) Percentage Elongation.
iv) Percentage reduction in area.
v) Ultimate tensile stress.

Soln: Data:- $d = 25mm$, $L = 200mm$, $\delta = 0.16mm$.
$P = 80\times10^3 N$, $P_e = 160\times10^3 N$, $P_u = 180\times10^3 N$.

i) $\sigma_e = ?$ ii) $E = ?$ iii) % Elongation iv) $\sigma_u = ?$

i) Cross-sectional area of the specimen.

$$A = \frac{\pi d^2}{4} = \frac{\pi(25)^2}{4} = 490.8\ mm^2$$

Stress at elastic limit.

$$\sigma_e = \frac{P_e}{A} = \frac{160\times10^3}{490.8}$$

$$\sigma_e = 325.9\ N/mm^2.$$

ii) $\sigma = \frac{P}{A} = \frac{80\times10^3}{490.8} \Rightarrow \sigma = 162.9\ N/mm^2$.

$$\epsilon = \frac{\delta}{L} = \frac{0.16}{200} \Rightarrow \epsilon = 8\times10^{-4}.$$

Young's Modulus.

$$E = \frac{\sigma}{\epsilon} = \frac{162.9}{8\times10^{-4}}$$

$$E = 203.625\times10^3\ N/mm^2.$$

iii) % Elongation. $= \frac{L_f - L_i}{L_i}\times100 \Rightarrow \frac{56}{200}\times100 = 28\%$.

iv) % reduction in Area. $= \frac{A_i - A_f}{A_i}\times100$.

$$A_f = \frac{\pi d_f}{A} \Rightarrow \frac{\pi(18)^2}{4} = 254.46\ mm^2 = 48\%.$$

iv) Ultimate Stress $\Rightarrow \sigma_u = \frac{P_u}{A} = \frac{180\times10^3}{490.8}$

$$= 366.7\ N/mm^2.$$

11/7/23

Bar's Of Cross-Section Varying In Steps:-

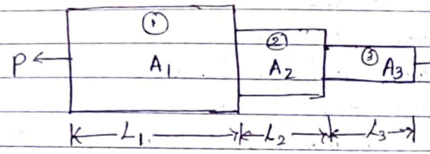


Fig. 1.

Figure 1 shows the bar of cross section varying in steps subjected to axial load P.
Let L_1, L_2, L_3 & A_1, A_2, A_3 be the length & cross-section area of the 3-portions respectively.
Let E be the young's modulus of the material.

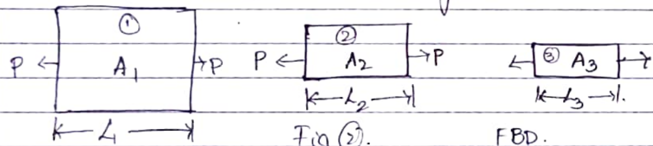


Fig (2). FBD.

Fig 2 show's FBD of each of the 3 portions.

Portion	Area of c/s	Length	Extension	Stress	strain
1	A_1	L_1	$\delta_1 = \frac{PL_1}{A_1E}$	$\sigma_1 = \frac{P}{A_1}$	$\epsilon_1 = \frac{\delta_1}{L_1} = \frac{\sigma_1}{E}$
2	A_2	L_2	$\delta_2 = \frac{PL_2}{A_2E}$	$\sigma_2 = \frac{P}{A_2}$	$\epsilon_2 = \frac{\delta_2}{L_2} = \frac{\sigma_2}{E}$
3	A_3	L_3	$\delta_3 = \frac{PL_3}{A_3E}$	$\sigma_3 = \frac{P}{A_3}$	$\epsilon_3 = \frac{\delta_3}{L_3} = \frac{\sigma_3}{E}$

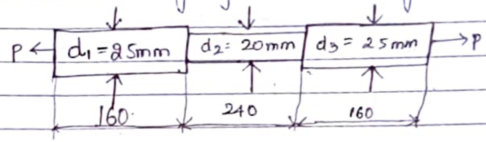
Total extension / Change in length $\delta = \delta_1 + \delta_2 + \delta_3$

$$\delta = \frac{PL_1}{A_1E} + \frac{PL_2}{A_2E} + \frac{PL_3}{A_3E}$$

$$\delta = \frac{P}{E}\left(\frac{L_1}{A_1} + \frac{L_2}{A_2} + \frac{L_3}{A_3}\right)$$

Numerical:-

1. The bar shown in figure is tested in universal testing machine. It is observed that at the load of 40KN, the total extension of the bar is 0.285mm. Determine the young's modulus of the material.



Data:- $P = 40\times10^3 N$, $\delta = 0.285mm$.
$d_1 = d_3 = 25mm$, $d_2 = 20mm$
$L_1 = L_3 = 160mm$, $L_2 = 240mm$.
$E = ?$ $A_3 = \frac{\pi d_3^2}{4}$, $A_2 = \frac{\pi d_2^2}{4}$, $A_1 = \frac{\pi d_1^2}{4}$.

Cross-sectional area of 3 portions.
$A_1 = A_3 = \frac{\pi d_1^2}{4}$, $A_1 = A_3 = \frac{\pi(25)^2}{4} = 490.8mm^2$

$A_2 = \frac{\pi d_2^2}{4} = \frac{\pi(20)^2}{4} = 314.15mm^2$

Extension of first portion.
$\delta_1 = \frac{PL_1}{A_1E} = \frac{40\times10^3\times160}{490.8E} \Rightarrow \delta_1 = \frac{13.039\times10^3}{E} = \delta_3$

$\delta_2 = \frac{PL_2}{A_2E} = \frac{30.55\times10^3}{E}$

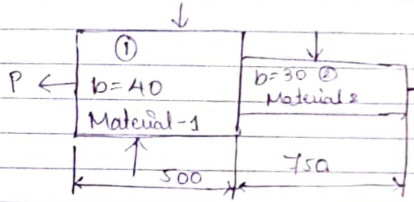
The Total Extension $\delta = \delta_1 + \delta_2 + \delta_3$.
$\delta = \frac{13.039\times10^3}{E} + \frac{30.55\times10^3}{E} + \frac{13.039\times10^3}{E}$

$\delta = \frac{1}{E}(56.628\times10^3) \Rightarrow 0.285 = \frac{56.2628\times10^3}{E}$

$$\boxed{E = 198.69\times10^3\ N/mm^2}$$

2) The step bar shown in figure is made up of two different materials. The material 1 bar Young's modulus $= 2\times10^5 N/mm^2$,

while that of material 2 is $E = 1\times10^5\ N/mm^2$. Find the extension of the bar under a pull of 25 kN. If both the portions are 20 mm in thickness.



Data:-

$E_1 = 2\times10^5\ N/mm^2$.

$E_2 = 10^5\ N/mm^2$, $b_1 = 40mm$, $b_2 = 30mm$.

$P = +25\times10^3 N$, $t_1 = t_2 = 20mm$.

$L_1 = 500mm$, $L_2 = 750mm$.

$\delta = ?$

Cross-Sectional area of portion 1. $A_1 = b_1 t_1$

$A_1 = 40\times20 = 800mm^2$

$A_2 = b_2 t_2 = 30\times20 = 600mm^2$

Extension of first portion.

$$\delta_1 = \frac{PL_1}{A_1E_1} \Rightarrow \frac{25\times10^3(500)}{800(2\times10^5)}$$

$$\delta_1 = 78.13\times10^{-3} mm.$$

$$\delta_2 = \frac{PL_2}{A_2E_2} = \frac{25\times10^3(750)}{600(10^5)} \Rightarrow 312.5\times10^{-3} mm.$$

$$\delta = \delta_1 + \delta_2$$

$$\delta = 390.63\times10^{-3} mm$$

2/7/23

3) A bar of length 1000mm and diameter 30mm is centrally bored for 400mm, the bore diameter being 10mm as shown in figure. Under a load of 25kN, if the extension of the bar is 0.185 mm. What is the modulus of elasticity of the bar

$L_1 = 600m$

L_2 400m

$d_1 = 30mm$.

d_{2o} 30mm, d_{2i} 10mm

$P = 25\times10^3 N$, $\delta = 0.185mm$

$E = ?$

$A_1 = \frac{\pi d_1^2}{4} = 706.8\ mm^2$

$A_2 = \frac{\pi}{4}(d_{2o}^2 - d_{2i}^2)$ = 628.31 mm²

$\delta_1 = \frac{PL_1}{A_1 E}$, $\delta_2 = \frac{PL_2}{A_2 E}$

$$\delta = \delta_1 + \delta_2.$$

$$\delta_1 = \frac{25\times10^3(600)}{706.8\times E}$$ $$\delta_2 = \frac{15.91\times10^3}{E}$$

$$\delta_1 = \frac{21.2\times10^3}{E}$$

$$0.185 = \frac{21.2\times10^3}{E} + \frac{15.91\times10^3}{E}$$

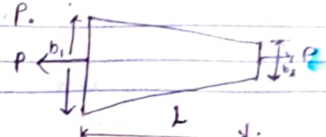
$$0.185 = \frac{1}{E}(21.2\times10^3 + 15.91\times10^3)$$

$$0.185 = \frac{37110.00}{E} \Rightarrow 0.185 = \frac{37.11\times10^3}{E}$$

$$E = 200.59\times10^3\ N/mm^2$$

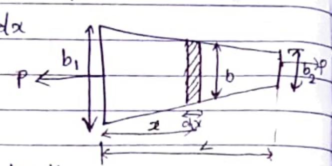
BARS OF CROSS SECTIONAL AREA CONTINUOUSLY VARYING (TAPERED BARS)

1) A bar of uniform thickness t tapers uniformly from a width of b_1 at one end to b_2 in a length l as shown in figure. Find the exp. for change in length of the bar when subjected to axial force P.



Rate of change of width $K = \frac{b_1 - b_2}{L}$ → (1).

Consider an element of length dx at a distance x from one of the ends.



Width at this section $b = b_1 - Kx$

Cross-sectional area of the element $A = (b_1 - Kx)t$.

Extention of the element → $d\delta = \frac{P\,dx}{(b_1 - Kx)tE}$

Total extension of the bar

$$\delta = \frac{P}{tE}\int_0^L \frac{dx}{(b_1 - Kx)}$$

$$\left[\int \frac{1}{(a+bx)}\,dx = \frac{1}{b}\log_e(a+bx).\right] \rightarrow \text{formula}.$$

$$\delta = \frac{P}{tE}\left(\frac{-1}{K}\right)\left[\log_e(b_1 - Kx)\right]_0^L$$

$$\delta = \frac{-P}{tEK}\left[\log_e b_2 - \log_e b_1\right]$$

$$\delta = \frac{P}{tEK}\left[\log_e b_1 - \log_e b_2\right]$$

$$\delta = \frac{P}{tEK}\log_e\left(\frac{b_1}{b_2}\right) \rightarrow (2).$$

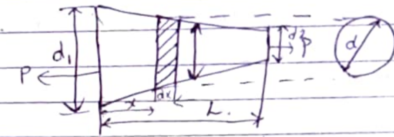
$$\delta = \frac{PL}{(b_1 - b_2)tE}\log_e\left(\frac{b_1}{b_2}\right) \rightarrow (3)$$

This is the expression for extension of tapered bar for rectangular cross-section.

2) A Tapering rod has diameter d_1 At one end and it's tapers uniformly to a diameter d_2 at the other end in a length L as shown in figure. If modulus of the elasticity of the material is E, find it's change in length when subjected to axial force P.

Rate of change of diameter.

$K = \frac{d_1 - d_2}{L}$ → (1).



Consider an element of length dx at a distance x from one of the ends.

Diameter at this section $d = d_1 - Kx$.

Cross-Sectional area of the element $A = \frac{\pi}{4}(d_1 - Kx)^2$.

Extention of the element → $d\delta = \frac{P\,dx}{\frac{\pi}{4}(d_1 - Kx)^2 E}$

Extension of the bar, $\delta = \frac{4P}{\pi E}\int_0^L \frac{dx}{(d_1 - Kx)^2}$

$$\int \frac{1}{(a+bx)^2}\,dx = \frac{-1}{b(a+bx)}.$$

$$\delta = \frac{4P}{\pi E K}\left[\frac{1}{(d_1 - Kx)}\right]_0^L$$

$$\delta = \frac{4P}{\pi E K}\left[\frac{1}{d_2} - \frac{1}{d_1}\right] \rightarrow (2).$$

Substitute eq (1) in (2).

$$\delta = \frac{4PL}{\pi E(d_1 - d_2)}\left[\frac{d_1 - d_2}{d_1 d_2}\right].$$

$$\delta = \frac{4PL}{\pi d_1 d_2 E}, \quad \delta = \frac{PL}{\frac{\pi d_1 d_2 E}{4}} \rightarrow (3).$$

Eqn (3) is the expression for extention of taper bar of circular Cross Section.

PROBLEM:-

1) A steel flat of thickness 10mm taper's uniformly from 60mm at one end to 40mm at the other end in a length of 600mm. If the bar is subjected to a load of 60KN, find it's extention. Take E=2×

W't is the % error if Avg Area is used for calculating the Extension.

Solⁿ: Data :→ $t = 10mm$, $b_1 = 60mm$, $b_2 = 40mm$,
$L = 600mm$, $P = 60 \times 10^3 N$, $\delta = ?$, $E = 2 \times 10^5 N/mm^2$.

$\delta = 0.3649 mm.$

$$\delta = \frac{PL}{(b_1 - b_2) t E} \log_e \left(\frac{b_1}{b_2}\right)$$

$$\delta = \frac{60 \times 10^3 (600)}{(60-40)(10)(2 \times 10^5)} \log_e \left(\frac{60}{40}\right)$$

$\delta = 0.90000 \times 0.4054$.

$\delta = 0.36492$

Average area $= A_{av}$

$$A_{av} = \frac{(60 \times 10) + (40 \times 10)}{2}$$

$A_{av} = 500 mm^2$

Extension using Average area.

$$\delta = \frac{PL}{A_{av} E} = \frac{60 \times 10^3 \times 600}{500 \times 2 \times 10^5}$$

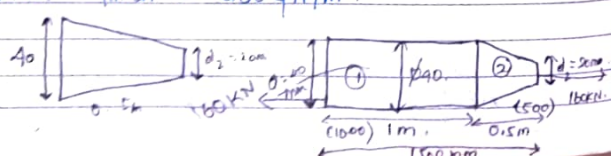
$\delta = 0.36 mm$

% error.

$$= \frac{0.36 \sim 0.3649}{0.3649} \times 100.$$

$= 1.34\%$

2. A 1.5m long steel bar is having uniform diameter 40mm for a length of 1m. In the next 0.5m it's diameter gradually reduces from 40mm to 20mm as shown in figure. Determine the elongation of this bar when subjected to an axial tensile load of 160 KN. Given $E = 200 GN/m^2$.



$$\delta_1 = \frac{PL_1}{A_1 E} = \frac{160 \times 10^3 \times 1000}{\frac{\pi (40)^2}{4} \times 200 \times 10^3}$$

$$\delta_2 = \frac{4PL}{\pi d_1 d_2 E} = \frac{4 \times 160 \times 10^3 \times 500}{\pi \times 40 \times 20 \times 200 \times 10^3} = 0.6366.$$

$\delta_1 + \delta_2 = \delta.$

$\delta = 1.2732 mm.$

17/7/23.

3) The steel flat shown in fig. has uniform thickness of 20mm. Under an axial load of 80KN, the extension is found to be 0.17mm. Determine the Young's Modulus of the material.

Solⁿ:

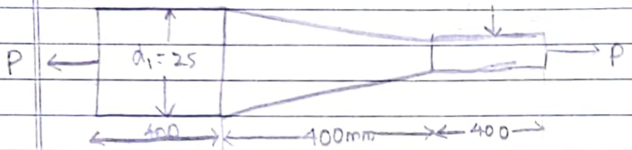
$\delta = 0.17mm.$ $b_1 = 80mm.$
$t = 20mm.$ $b_2 = 40mm.$
$P = 80KN : 80 \times 10^3 N.$

$$\delta = \frac{PL}{(b_1 - b_2) t E} \log_e \left(\frac{b_1}{b_2}\right).$$

$$0.17 = \frac{(80 \times 10^3)(500)}{(80-40)(20) E} \ln \left(\frac{80}{40}\right).$$

$= 203.8 \times 10^3 N/mm^2$

4) Find the extension of the bar shown in fig. under an axial load of 20KN. Take $E = 200\,GN/m^2$.



Soln: Data :- $P = 20\times10^3 N$

$E = 200\,GN/m^2 = 200\times10^3$

$$\delta_1 = \frac{PL_1}{A_1E}$$

$$\delta_1 = \frac{20\times10^3\times400}{\frac{\pi(25)^2}{4}\times200\times10^3} = 490.873 = 0.0815$$

$$\delta_2 = \frac{4PL}{\pi d_1 d_2 E} = \frac{4(20\times10^3)(400)}{\pi(25)(15)\times200\times10^3}$$

$$\delta_2 = 0.135$$

$$\delta_3 = \frac{PL_3}{A_3E} = \frac{(20\times10^3)\times400}{\frac{\pi(15)^2}{4}\times200\times10^3}$$

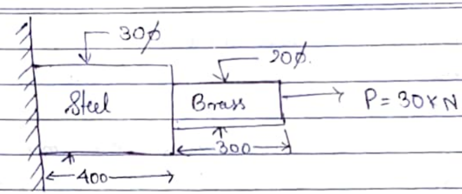
$$= 0.2263$$

$$\delta = \delta_1 + \delta_2 + \delta_3$$

$$= 0.0815 + 0.135 + 0.2263$$

$$\delta = 0.4428$$

5. A Composit bar shown in fig is subjected to a tensile force of 30KN. The extension observed is 0.372mm. Find the young's modulus of brass, if young's modulus of brass, if young's modulus of steel is $2\times10^5 N/mm^2$.



Data :- $P = 30\times10^3 N$.

$\delta = 0.372mm$

$E_B = ?$

$E_S = 2\times10^5 N/mm^2$

$d_1 = 30$

$d_2 = 20$.

$$\delta = \frac{PL}{A_1E} = \frac{30\times10^3\times400\times4}{\pi(30)^2\times200\times10^3}$$

$$= 0.0849$$

$$\delta_2 = \frac{PL_2}{A_2E} = \frac{30\times10^3\times300}{\cancel{\pi(30)^2\times200\times10^3}} \quad 314.15\times E_2$$

$$\delta_2 = \frac{28648.73}{E_2}$$

$$\delta = \delta_1 + \delta_2.$$

$$0.372 = 0.0848 + \frac{28648.73}{E_2}$$

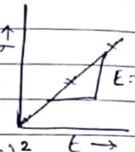
$$0.2872 = \frac{28648.73}{E_2}$$

$$E_2 = 99751.8\,N/mm^2$$

6) Tension test was conducted on a Specimen and the following readings recorded diameter equal to 22mm. Gauge length of Extensometer = 200mm. Least count of extensometer = 0.001mm At a load of 22KN, extensometer reading = 60. At a load of 36KN, extensometer reading = 94. Yield load = 95KN = 95×10^3 N. Maximum load = 157 KN = 157×10^3 N. Diameter at Neck = 15mm Final length over 110 mm Original length = 132mm. Find Young's modulus, yield stress, ultimate stress, Percentage elongation and Percentage reduction in Area.

Sol^n:

$\sigma = 57.8$
$E = 3 \times 10^{-4}$



$P = 95KN$, 60, $P = 22 \times 10^3 N$

$\delta = 60 \times 0.001$ mm

$\delta = 0.06$ mm

$\sigma = \frac{P}{A}$

$\sigma = \frac{95 \times 10^3}{380.132}$

$\sigma = 249.9\ N/mm^2$

$A_1 = \frac{\pi d^2}{4}$

$A_1 = \frac{\pi (22)^2}{4}$

$A_1 = 380.132$

Maximum load = 157KN = 157×10^3

Ultimate stress = $\frac{157 \times 10^3}{380.132} = 413.01\ N/mm^2$

For a load of (36−22) KN = 14KN. $\delta = \frac{PL}{AE}$

Extension is (94−60)m = 34×0.001 mm.

$34 \times 0.001 = \frac{14 \times 10^3 \times 200}{380.132 \times E} \Rightarrow E = 2.16643 \times 10^5\ N/mm^2$.

% Elongation = $\frac{\text{increase in length}}{\text{original length}} \times 100$.

$= \frac{132-110}{110} \times 100 \Rightarrow 20\%$.

% reduction in area = $\frac{\frac{\pi}{4}(22)^2 - \frac{\pi}{4}(15)^2}{\frac{\pi}{4}(22)^2} \times 100$.

$= 53.51\%$

2A

Bars Subjected To Varying Loads:-

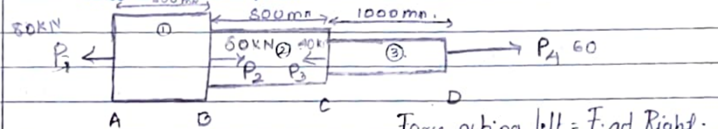
Problems:-

1. The following details refer to the bar shown in fig.

Portion	Length	Cross-Section.
AB	600 mm	40mm x 40mm
BC	800 mm	30mm x 30mm
CD	1000 mm	20mm x 20mm

If the loads are $P_1 = 80KN$, $P_2 = 60KN$ & $P_3 = 40KN$ find the extension of the bar.

Take $E = 2 \times 10^5\ N/mm^2$.

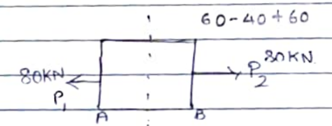


Force acting left = F. act Right.

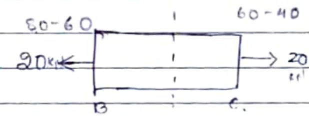
$P_1 + P_3 = P_2 + P_4$.

$80 + 40 = 60 + P_4$

$P_4 = 60KN$

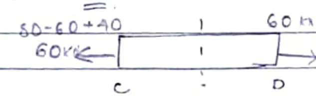


$\delta_1 = \frac{PL_1}{A_1 E} = \frac{80 \times 10^3 \times 600}{40 \times 40 \times 2 \times 10^5} = 0.15\ mm$



$\delta_2 = \frac{PL_2}{A_2 E} = \frac{20 \times 10^3 \times 800}{30 \times 30 \times 2 \times 10^5}$

$\delta_2 = 0.08889\ mm \approx 0.088mm$



$\delta_3 = \frac{PL_3}{A_3 E} = \frac{60 \times 10^3 \times 1000mm}{20 \times 20 \times 2 \times 10^5}$

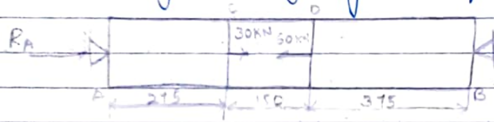
$\delta_3 = 0.75\ mm$

$\delta = \delta_1 + \delta_2 + \delta_3$

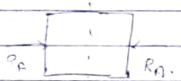
$\delta = 0.15 + 0.08889 + 0.75$

$\delta = 0.98890mm$.

2. A bar of 800 mm length is attached rigidly at A & B has shown in fig. forces of 30 kN & 60 kN act as shown on the bar if $E = 200$ gpa determine the reactions at the two ends. if the bar diameter is 25mm find the stresses & change in length of each portion.



The extension of first portion $\delta = \frac{-R_A \, 275}{A \times 200 \times 10^3}$



$\delta_1 = -\frac{R_A 275}{490.87}$

$\delta_1 = -2.8 \times 10^{-6} R_A$ mm

$\delta_2 = -\frac{(30+R_A)150}{490.87 \times 200 \times 10^3} \Rightarrow \delta_2 = -1.52 \times 10^{-6}(30+R_A)$

$\delta_3 = (30-R_A)\, 3.81 \times 10^{-6}$ mm.

$\delta_1 + \delta_2 + \delta_3 = 0$.

$-2.8 \times 10^{-6} R_A + \left[-1.52 \times 10^{-6}(30+R_A)\right] + (30-R_A)\, 3.81 \times 10^{-6}$.

$-(R_A \times 275) - (30+R_A)150 + (30-R_A)375 = 0$.

$-275 R_A - 4500 - 150 R_A + 11250 - 375 R_A = 0$.

$R_A(-275 - 4500 - 150 + 11250 R_A - 375) = 0$

$R_A = 8.45$

Reaction at support B. $R_B = 30 - R_A$.

$R_B = 30 - 8.45$

$= 21.55$

$\sigma_1 = \frac{-8.45 \times 10^3}{490.87} = 17.21$ N/mm² (compression)

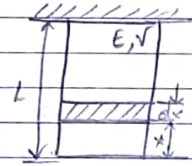
$\sigma_2 = \frac{-38.45 \times 10^3}{490.87}$

$= -78.33$ N/mm² (compression)

$\sigma_3 = \frac{21.55 \times 10^3}{490.87} = 43.9$ N/mm² (Tensile)

Elongation due to self weight:-

D. A bar of uniform cross-section A & length l is suspended from top. find the expression for extention of the bar due to self wait only if young's modulus is E & unit weight of the material is γ.



Consider an elemental length dx at a distance x from the free end.

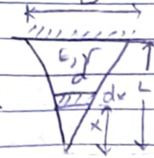
Extension of this element

$d\delta = \frac{P dx}{AE} = \frac{\gamma A x \, dx}{AE}$

$\delta = \int_0^l \frac{\gamma x \, dx}{E} = \frac{\gamma}{E}\left[\frac{x^2}{2}\right]_0^l \Rightarrow \delta = \frac{\gamma l^2}{2E}$.

Problem:-

1. A solid conical bar of uniformly varying diameter has diameter D at one end & zero at the other End. if the length of the bar is l, modulus of Elasticity is E & unit weight gamma find the extension of the bar due to self weight only.



Consider an elemental length dx at a distance x from the free end.

Let d be the diameter of this element.

Extension of the element $d\delta = \frac{PL}{AE}$

$d\delta = \frac{A dx}{AE}$.

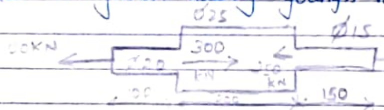
$\gamma = \frac{P}{\frac{1}{3} \times \frac{\pi d^2 x}{4}}$

$$d\delta = \frac{\sqrt{/3}\ \frac{\pi d^2}{4}\ x\,dx}{\frac{\pi d^2}{4} E} = \frac{\sqrt{x}}{3E}\,dx.$$

$$\delta = \frac{\sqrt{}}{3E}\int_0^L x\,dx.$$

$$\delta = \frac{\sqrt{} L^2}{6E}$$

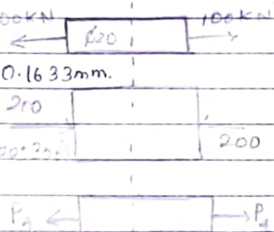
P.EQ 1) Determine the stresses in various segments of the circular bar shown in figure. Complete the total elongation taking young's modulus to be 195gpa.



$$A_1 = \frac{\pi d_1^2}{4} = \frac{\pi (20)^2}{4} = 314.15 \quad A_2 = \frac{\pi d_2^2}{4} = \frac{\pi (25)^2}{4} = 490.8$$

$$A_3 = \frac{\pi d_3^2}{4} = \frac{\pi (15)^2}{4} = 176.71.$$

For AB



$$\delta_1 = \frac{PL_1}{A_1 E} = \frac{(100)(100)\times 10^3}{314.15(195\times 10^3)} = 0.1633\text{mm}.$$

For BC. –

$= 0.1633\times 10^{-4}$ mm

$$\delta_2 = \frac{PL_2}{A_2 E} = \frac{-200(200)\times 10^3}{490.625\times 195\times 10^3}$$

$= -0.417$mm.

For CD –

$$\delta_3 = \frac{PL_3}{A_3 E} = \frac{50\times 10^3\times 150}{176.625\times 195\times 10^3}$$

$= 0.2177$mm

$\delta = \delta_1 - \delta_2 + \delta_3$

$\delta = 0.1633 - 0.417 + 0.2177.$

$\delta = -0.032$mm

26/7/23

MODULE - 3

Torsion Of Circular Shafts.

* **Torsion:–** A member is said to be in torsion when it is subjected to moment about its axis.

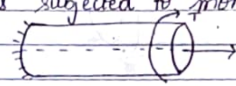


Fig: Shows shaft in torsion.

The effect of the torsional moment on a member is to twist its cross-sections and hence torsional moment is also called as twisting moment.

* **Pure Torsion:–**

A member is said to be in pure torsion when it is subjected to only torsional moments and not accompany it by axial forces or bending moments.

* **Assumptions In The Theory Of Pure Torsion:–**

(same properties)

1. The material is Homogeneous and isotropic.
2. Cross-sections which are plane before applying twisting moment remains plane even after applying twisting moment.
3. The stresses are within elastic limit i.e shear stress is proportional to shear strain.
4. Radial lines remain radial even after applying twisting moment.
5. Torsion along the shaft is uniform.

Shear Modulus / Rigidity Modulus :-

Shear stress is proportional to shear strain within elastic limit.

Mathematically $\tau \propto \gamma$ or $\tau = G\gamma$ where G is the constant to propotionality called as shear modulus or rigidity modulus.

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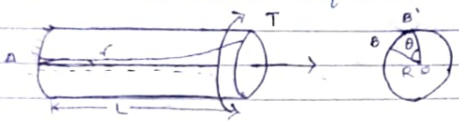
Date______ Page______

Derivation of Torsion Equation.



Consider a shaft of length 'L' and radius 'R' subjected to twisting moment 'T' as shown in the fig.
Let 'O' be the center of circular section and 'B' be any point on the surface.
Let AB is a line parallel to axis of the shaft
From geometry of the fig.

$$BB' = R\theta = L\gamma \rightarrow (1)$$

From the defination of shear modulus, shear strain

$$\gamma = \frac{\tau}{G} \rightarrow (2).$$

where τ is the shear stress at radius 'R' & G is the shear modulus.
Substitute Eqn (2) in (1).

$$R\theta = L\frac{\tau}{G}$$

$$\frac{\tau}{R} = \frac{G\theta}{L} \rightarrow (3)$$

If point 'B' is taken a any radius 'r' it can be shown that.

$$\frac{\tau_r}{r} = \frac{G\theta}{L} \rightarrow (4)$$

from Eqn (3) & (4)

$$\frac{\tau_r}{r} = \frac{\tau}{R}$$

$$\tau_r = \frac{\tau r}{R} \rightarrow (5).$$

Consider an elemental area dA at a distance r from the center as shown in fig 2.

Fig. 2.

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Date______ Page______

Torque applied. $\rightarrow dT = dF \times r.$

Tangential Force $\rightarrow dF = \tau_r\, dA.$

$$dT = \tau_r\, r\, dA \rightarrow (6)$$

Substitute Eq (5) in (6).

$$dT = \frac{\tau}{R} r^2\, dA.$$

$$T = \frac{\tau}{R}\int r^2 dA.$$

From defination polar moment of inertia

$$J = \int r^2 dA.$$

$$T = \frac{\tau J}{R}$$

$$\frac{\tau}{R} = \frac{T}{J} \rightarrow (7).$$

From Eqn (4) & (7).

$$\frac{T}{J} = \frac{\tau}{R} = \frac{G\theta}{L} \rightarrow (8)$$

Eqn (8) is the torsional equation for circular shaft.

* **Polar Modulus**

For torsion eqn $\rightarrow \frac{T}{J} = \frac{\tau}{R}$

$$T = \frac{J}{R}\tau \Rightarrow T = Z_p \tau$$

where, $Z_p = \frac{J}{R}$ and is defined as ratio of polar moment of intertia to the radial distance from the center.

i) **Solid Shaft:-**

For solid shaft $\rightarrow J = \frac{\pi d^4}{32}$ mm^4.

$$R = \frac{d}{2} \text{ mm}$$

polar modulus $\Rightarrow Z_p = \frac{J}{R} = \frac{\pi d^4/32}{d/2}$

$$Z_p = \frac{\pi d^3}{16} \text{ mm}^3$$