ii) HOLLOW SHAFT.

$$J = \frac{\pi}{32}(d_o^4 - d_i^4)\ mm^4.$$

$$R = \frac{d_o}{2}\ mm. \Rightarrow Z_p = \frac{J}{R} = \frac{\frac{\pi}{32}(d_o^4 - d_i^4)}{d_o/2}.$$

$$Z_p = \frac{\pi}{16}\frac{(d_o^4 - d_i^4)}{d_o}\ mm^3.$$

POWER TRANSMITTED BY THE SHAFT:-

Consider a shaft rotating at a speed of N rpm. Subjected to torque 'T' angular velocity of the shaft.

$$\omega = \frac{2\pi N}{60}\ rad/s \rightarrow (1).$$

power transmitted $P = T\omega \rightarrow \omega \rightarrow (2)$.

From Eq (1) & (2).

$$P = \frac{2\pi NT}{60}$$

TORSIONAL RIGIDITY:-

From Torsional equation.

$$\frac{T}{J} = \frac{G\theta}{L} \Rightarrow GJ = \frac{T}{\theta/L}$$

Where GJ is called as torsional rigidity which is defined as torque required to produce unit angle of twist in unit length.

PROBLEMS ON TORSION ON CIRCULAR SHAFT.

NOMENCLATURE:-

L - Length of the shaft (mm)

D - Diameter of the solid shaft (mm).

d_o - Outer diameter of the hollow shaft. (mm)

d_i = Inner diameter of the hollow shaft (mm)

R = maximum radius of the shaft, (mm).

J = polar moment of inertia (mm⁴).

G = shear modulus / Rigidity modulus. (N/mm^2)

E = Young's modulus (N/mm^2)

(New) γ = poison's ratio.

T = Torque/torsional moment / Twisting moment. (N-mm).

N = Speed of the shaft. (rpm).

P = Power transmitted by the shaft. (W).

τ = Shear stress (N/mm^2)

θ = Angel of twist. (rad.)

θ/L = Angel of twist per unit length.

GJ = Torsional rigidity/stiffness. $\frac{N\text{-}mm^2}{rad}$.

LIST OF FORMULAE:-

1. $R = \frac{d}{2}$ mm. [for solid shaft]

* $R = \frac{d_o}{2}$ mm. [for hollow shaft]

2) $J = \frac{\pi d^4}{32}\ mm^4$. [solid shaft]

* $J = \frac{\pi}{32}(d_o^4 - d_i^4)\ mm^4$. [Hollow shaft].

3) $P = \frac{2\pi NT}{60}$ W.

4) Design based on strength $\frac{T}{J} = \frac{\tau}{R} = \frac{G\theta}{L}$

$\hookrightarrow \frac{T}{J} = \frac{\tau}{R}$.

5) Design based on Rigidity.

$$\frac{T}{J} = \frac{G\theta}{L}.$$

Problem:-

1) Calculate the maximum intensity of shear stress induced and the angle of twist produced in degrees in solid shaft of 100mm diameter, 10m long, transmitting 112.5 kW at 150 rpm. take $G = 82\ kN/mm^2$

Soln:
$\tau = ?$ $d = 100\ mm$. $P = 112.5 \times 10^3\ W$

$\theta = ?$ $L = 10{,}000\ mm$ $N = 150\ rpm$.

1) $R = \frac{d}{2} = 50\ mm$.

2) $J = \frac{\pi d^4}{32} = \frac{\pi \times 100^4}{32} = 9.81 \times 10^6\ mm^4$.

$P = \frac{2\pi NT}{60} = \frac{2\pi(150)(T)}{60}$.

$112 \times 10^3 = 15.7\,T$

$T = 7162\ N\text{-}m$

$T = 7162 \times 10^3\ N\text{-}mm$.

$\frac{T}{J} = \frac{\tau}{R} = \frac{7162 \times 10^3}{9.81 \times 10^6} = \frac{\tau}{50}$

$= 0.7300 = \frac{\tau}{50}$

$= 0.7300 \times 50 = \tau$

$\Rightarrow \tau = 36.5$

$\frac{T}{J} = \frac{G\theta}{L} = \frac{7162 \times 10^3}{9.81 \times 10^6} = \frac{82 \times 10^3(\theta)}{10{,}000}$

$= 0.73007 = 8.2(\theta)$.

$\Rightarrow \frac{0.73007}{8.2} = \theta$.

$\Rightarrow \theta = 8.9 \times 10^{-2}$ rad.

$\theta = 8.9 \times 10^{-2} \times \frac{180}{\pi}$

$\theta = 5.05° \Rightarrow \theta = 5.1°$

2) Determine a diameter of a solid shaft which will transmit 440KW at 280rpm. The angle of twist should not exceed 1 degree per meter length and the maximum torsional shear stress is to be limited to $40\ N/mm^2$. Assume $G = 84\ KN/mm^2$.

$d = ?$

$P = 440 \times 10^3\ W$

$N = 280$ rpm.

$\theta/L = 1°/m = \pi/180/10^3$ rad/mm.

$\tau = 40\ N/mm^2$.

$G = 84 \times 10^3\ N/mm^2$,

$P = \frac{2\pi NT}{60} = 440 \times 10^3 = \frac{2\pi(280)T}{60}$

$P = \frac{2\pi NT}{60}$

$\frac{T}{J} = \frac{\tau}{R}$

$440 \times 10^3 = 29.32\,T, \Rightarrow T = 15 \times 10^3\ N\text{-}m$

$15 \times 10^6\ N\text{-}mm$.

$\frac{T}{J} = \frac{\tau}{R}$

$\frac{15 \times 10^6}{\frac{\pi d^4}{32}} = \frac{40}{d/2} = \frac{32 \times 15 \times 10^6}{\pi d^4} = \frac{2 \times 40}{d}$

$= \frac{32 \times 15 \times 10^6}{80\pi} = d^3 \Rightarrow 1.9 \times 10^6 = d^3$

$\sqrt{1.9 \times 10^6} = d$.

$\therefore d = 123.85\,mm$

31/7/23 6) RELATIONSHIP B/w YOUNG'S MODULUS & SHEAR MODULUS

$E = 2G(1-\gamma)$

7) Thickness of hollow shaft $\rightarrow t = \frac{d_o - d_i}{2}$

P:-

3] The working condition to be satisfied by a shaft transmitting power are i) The shaft must not twist more than 1° in a length of 15 times diameter.

ii) The shear stress must not exceed 80 megaNewton/m² (80 MN/m²) What is the actual working stress and diameter of the shaft to transmit 736 Kw at 200rpm? Take shear modulus as $80\ GN/m^2$.

$\Rightarrow$ DATA:- i) $\theta = \frac{\pi}{180}$ rad, $L = 15d$.

ii) $\tau = 80\ MN/m^2 \Rightarrow 80\ N/mm^2$.

$\tau_{act} = ?$, $P = 736 \times 10^3\ W$

$d = ?$ $N = 200$ rpm.

$G = 80 \times 10^3\ N/mm^2$.

$\Rightarrow \frac{T}{J} = \frac{G\theta}{L}$, $P = \frac{2\pi NT}{60}$.

Power Transmitted = $\frac{2\pi NT}{60}$ W.

$736\times10^3 = 20.9439\,T$

$T = 35.14\times10^3$ N-m $\Rightarrow 35.14\times10^6$ N-mm.

$\frac{T}{J} = \frac{G\theta}{L}$.

$\frac{35.14\times10^6}{\pi d^4/32} = \frac{80\times10^3\times\pi/180}{15d}$

$\frac{35.14\times10^6}{\pi d^4/32} = \frac{1.396\times10^3}{15d}$ $\quad (35.14\times10^6)15d = (1.396\times10^3)(\pi d^4/32)$

$527.1\times10^6 d = 137.05 d^4$

$527.1\times10^6 = 137.05 d^3$

$10^6\ 3.846 = d^3$

$\frac{T}{J} = \frac{\tau}{R}$. $\quad d = 156.6 \Rightarrow 156.6$ mm

$\frac{35.4\times10^6}{\pi d^4/32} = \frac{80}{d/2}$.

$\frac{35.4\times10^6\times32}{\pi d^4} = \frac{2\times80}{d} \Rightarrow \frac{35.4\times10^6\times32}{2\times80\pi} = d^3$

$2.253\times10^6 = d^3 \Rightarrow d = 131.095$ mm.

Since $156.67 > 131.095$ the recommended diameter of the shaft is 156.6mm.

$\frac{T}{J} = \frac{\tau_{act}}{R} \Rightarrow \frac{35.14\times10^6}{59\times10^6} = \frac{\tau_{act}}{78.3}$

$J = \frac{\pi d^4}{32} = \frac{\pi(156.6)^4}{32} = 59.04\times10^6 \quad \Rightarrow 0.59559 = \frac{\tau_{act}}{78.3}$

$= 46.6$ mm.

$78.3 \Leftarrow R = \frac{156.6}{2} \Rightarrow \tau_{ad} = 46.6 < 80$ N/mm^2

Q4) During tests on a sample of steel bar 25mm in diameter, it is found that the pull of 50KN produces an extension of 0.095mm on a length of 200mm & a torque of 200Nm produces an angular twist of 0.9° on a length of 250mm. Find the Poisson's ratio.

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Ans:→ Data :- $d = 25$mm.

i) Tension Test $P \leftarrow \square \rightarrow P$ $\quad \gamma = ?$

$P = 50\times10^3$ N. $\quad L = 200$mm.

$\delta = 0.095$mm.

ii) Torsion Test.

$T = 200$ N-m $= 200\times10^3$ N-mm.

$\theta = 0.9\times\pi/180$ rad.

$L = 250$mm.

$J = \frac{\pi d^4}{32}$, $\frac{T}{J} = \frac{G\theta}{L}$, $\delta = \frac{PL}{AE}$, $A = \frac{\pi d^2}{4}$, $E = 2G(1-\gamma)$

$\delta = \frac{PL}{AE} \Rightarrow 0.095 = \frac{50\times10^3\times200}{490.87\times E} \Rightarrow 0.095 = \frac{20.371\times10^3}{E}$

$A = \frac{\pi(25)^2}{4} = 490.87$ $\quad |\quad \therefore E = \frac{20.371}{0.095} \Rightarrow 214.43\times10^3$ N/mm^2

$J = \frac{\pi d^4}{32} = \frac{\pi(25)^4}{32} = 38.34\times10^3$ mm^4

$\frac{T}{J} = \frac{G\theta}{L}$

$\frac{200\times10^3}{38.34\times10^3} = \frac{G(0.9\times\pi/180)}{250}$.

$(250)\times5.216 = (0.9\times\frac{\pi}{180})G$.

$1304.12 = (0.01571)G. \Rightarrow \frac{1304.12}{0.01571} = G$

$G = 83.012\times10^3$ N/mm^2

$\Rightarrow E = 2G(1-\gamma)$

$214.43\times10^3 = 2[83.012\times10^3](1-\gamma)$

$214.43\times10^3 = 166.024\times10^3(1-\gamma)$

$214.43\times10^3 - 166.024\times10^3 = 166.024\times10^3\gamma$

$48.406\times10^3 = 166.024\times10^3\gamma$

$\frac{214.4\times10^3}{2(83\times10^3)} \quad 1-\gamma$

$\boxed{0.29 = -\gamma}$

$\gamma = -0.29$

5) A Hollow circular shaft of 6000mm and inner and outer diameter (it is found that the pull) of 75m & 100mm is Subjected to a torque of 10KNM. if G = 80GPa determine Maximum Shear Stress produced & total angle of twist.

Ans: $L = 6000mm$, $d_i = 75mm$, $d_o = 100mm$.

$T = 10KN\text{-}M = 10 \times 10^6 N\text{-}mm$, $G = 80 \times 10^3 N/mm^2$.

$\tau = ?$ $\theta = ?$

$R = \frac{d_o}{2} = \frac{100}{2} = 50mm$.

$J = \frac{\pi}{32}(d_o^4 - d_i^4) = 671 \times 10^3 mm^4$.

$\frac{T}{J} = \frac{\tau}{R} \Rightarrow \frac{10 \times 10^6}{671 \times 10^3} = \frac{\tau}{150} \Rightarrow \tau = 74.5 N/mm^2$

$\frac{T}{J} = \frac{G\theta}{L} \Rightarrow \frac{10 \times 10^6}{671 \times 10^3} = \frac{80 \times 10^3 \theta}{6 \times 10^3} \Rightarrow \theta = 0.11 rad.$

$= \frac{0.11 \times 180}{\pi} = 6.3^\circ$

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6) A Hollow circular shaft of 200mm external diameter and metal thickness 25mm is transmitting power at 200rpm the angle of twist over a length of 2m was found to be 0.5°. Calculate the power transmitted and the maximum shear stress induced in the section. Take Modulus of Rigidity of material as $84KN/mm^2$.

Ans: $d_o = 200mm$ $\qquad \tau = ?$

$t = 25mm.$ $\qquad G = 84 \times 10^3 N/mm^2$.

$N = 200 rpm$

$L = 2m = 2000mm$.

$\theta = 0.5^\circ = \frac{0.5\pi}{180} = 8.726 \times 10^{-3}$.

$P = ?$

$P = \frac{2\pi NT}{60} W. \Rightarrow$ $\qquad t = \frac{d_o - d_i}{2}$

$\frac{T}{J} = \frac{G\theta}{L} \Rightarrow J = \frac{\pi}{32}(d_o^4 - d_i^4)$

$2t = d_o - d_i \Rightarrow 2t - d_o = -d_i \Rightarrow 2t - d_o + d_i = 0$

$d_i = -2(25) + 200 = 150mm$ $\qquad d_i = -2t + d_o$

$J = \frac{\pi}{32}[(200)^4 - (150)^4] \Rightarrow 107.37 \times 10^6 mm^4$.

$\frac{T}{J} = \frac{G\theta}{L} \Rightarrow \frac{T}{107.37 \times 10^6} = \frac{84 \times 10^3 \times (8.726 \times 10^{-3})}{2000}$

$\Rightarrow \frac{T}{107.37 \times 10^6} = 366.492 \times 10^{-3}$ (or) (0.3664)

$T = 39.35 \times 10^6 N\text{-}mm$

$T = 393 \times 10^2 N\text{-}m$

$P = \frac{2\pi(200)(393 \times 10^2)}{60} = 823.097 \times 10^3 W.$

$= 823.097 KW$

$\frac{T}{J} = \frac{\tau}{R} = \frac{393 \times 10^2}{107.37 \times 10^6} = \frac{\tau}{100} \Rightarrow 3.93 \times 10^6 = 107.37 \times 10^6 \tau$

$\tau = 36.602 \times 10^{-3} N/mm^2$.

$R = \frac{d_o}{2} = \frac{200}{2} = 100.$

7) A Hollow propeller shaft of a steam ship is to transmit 3750KW at 240rpm. If the internal diameter is 0.8 times the external diameter and if the maximum shear stress developed is to be limited to $160N/mm^2$. Determine the size of the shaft.

Ans: $P = 3750 \times 10^3 W$ $\qquad \frac{T}{J} = \frac{\tau}{R}$

$N = 240 rpm.$

$d_i = 0.8 d_o$ $\qquad P = \frac{2\pi NT}{60}$

$\tau = 160 N/mm^2$

$d_o = ?$ $d_i = ?$ $\qquad J = \frac{\pi}{32}(d_o^4 - d_i^4)$

$J = \frac{\pi}{32}(d_o^4 - (0.8d_o)^4)$ $\qquad R = \frac{d_o}{2}$.

$J = \frac{\pi}{32}[d_o^4(1 - 0.4096)] \Rightarrow \frac{\pi}{32}(0.5904)d_o^4 = 0.0579 d_o^4$

$$P = \frac{2\pi NT}{60} = 3750\times10^3 = \frac{2\pi(240)T}{60}$$

$$= 225\times10^3 = 2\pi(240)T$$

$$225\times10^3 = 1507.96\,T.$$

$$T = 149.208$$

$$T = 149207.7\times10^3 \text{ N-m.}$$

$$\frac{T}{J} = \frac{\tau}{R} \Rightarrow \frac{149207.7\times10^3}{0.05796\,d_o^4} = \frac{160}{0.5\,d_o}$$

$$= \frac{149207.7\times10^3}{0.05796\,d_o^3} = \frac{160}{0.5}$$

$$= 74.603\times10^6 = 92.736\,d_o^3$$

$$= 8.0446\times10^6 = d_o^3$$

$$d_o = \sqrt[3]{8.0446\times10^6}$$

$$d_o = 200.37\text{mm}$$

$$d_i = 0.8(d_o) = 160.296\text{ mm}$$

8] A Shaft is required to transmit 245 KW power at 240rpm. The maximum torque may be 1.5 times the mean torque. The shear stress in the shaft should not exceed $40\,N/mm^2$. And the twist $1°/m$ length. Determine the diameter required if (a) The shaft is solid.
(b) The shaft is hollow with the external diameter twice the internal diameter.
Take modulus of rigidity equal to $80\,KN/mm^2$.

Given

$P = 245\times10^3$ W

$N = 240$ rpm

$T_{max} = 1.5\,T_m$

$\tau = 40\,N/mm^2$

$\theta/L = 1°/m = \frac{\pi/180}{10^3}$ rad/mm

(a) d ?

(b) d_o ?, d_i ?

$G = 80\times10^3\,N/mm^2$

$$P = \frac{2\pi NT}{60} = \frac{2\pi(240)T}{60}$$

$$245\times10^3 = 25.13\,T$$

$$T = 9.749\times10^3 \Rightarrow 9.74\times10^6 \text{ N-mm}$$

$$T_{max} = 1.5(9.749\times10^3)$$

$$T_{max} = 14.6235\times10^3$$

$$14.6235\times10^6 \text{ N-mm}$$

(a) Solid Shaft: Design based on strength.

$$\frac{T}{J} = \frac{\tau}{R} \Rightarrow \frac{14.6235\times10^6}{\frac{\pi}{32}(d^4)} = \frac{40}{\frac{d_o}{2}}$$

$$\Rightarrow \frac{14.6235\times10^6}{98.17\times10^3(d^4)} = \frac{40}{d/2} \Rightarrow 7311750 = 3.9268\,d^3.$$

$$1.862\times10^6 = d^3$$

$$d = 123.024\text{ mm}.$$

$$\frac{T_{max}}{J} = \frac{G\theta}{L} = \frac{14.6235\times10^6}{\frac{\pi d^4}{32}} = \frac{80\times10^3\times\pi}{180\times10^3}$$

$$\frac{14.6235\times10^6\times32}{\pi d^4} = \frac{80\times\pi}{180}$$

$$106144072 = d^4.$$

$$d = 101.57\text{mm}.$$

for solid shaft $d = 122.8$ mm is recommended.

b) Hollow Shaft:- diameter based on strength.

$$R = \frac{d_o}{2} = 0.5\,d_o, \qquad d_o = 2d_i$$

$$d_i = 0.5\,d_o$$

$$J = \frac{\pi}{32}(d_o^4 - d_i^4). \Rightarrow \frac{\pi}{32}(d_o^4 - (0.5d_o)^4)$$

$$J = 0.0920\,d_o^4$$

$$\frac{T}{J} = \frac{\tau}{R} \Rightarrow \frac{14.55\times10^6}{0.092\times d_o^4} = \frac{40}{0.5\,d_o}$$

$$d_o = 125.2\text{ mm}.$$

Based on rigidity:- $\frac{T}{J} = \frac{G\theta}{L}$

$$\frac{14.55\times10^6}{0.092\,d_o^4} = \frac{80\times10^3\times\pi}{10^3\times180}.$$

$$113268151.1 = d_o^4.$$

$$103.05\text{ mm} = d_o.$$

$$d_i = 0.5\,d_o$$

$$d_i = 68.6\text{ mm}.$$

for hollow shaft of outer diameter larger diameter is recommended.

Numerical on Comparsion b/w Solid and hallow Shaft

Note:- Let $\sqrt{}$ be the weight density of the shaft

$w = \sqrt{} A L$.

If solid and hallow shaft are made of same material

$\sqrt{}_s = \sqrt{}_h = \sqrt{}$.

$G_s = G_h = G. \Rightarrow \tau_s = \tau_h = \tau.$

$\left(\frac{\theta}{L}\right)_s = \left(\frac{\theta}{L}\right)_h = \frac{\theta}{L}.$

If Power and Speed is same for both Solid and hallow Shaft. $\Rightarrow T_s = T_h$.

1) Compared the weight of solid shaft with that of hallow one having the same length to transmit a given power at a given speed. If the material is used for the both shaft is the same. Take the inside diameter of the hallow shaft as 0.6 times the outer diameter.

$\rightarrow \frac{W_s}{W_h} = ?$ $\quad L_s = L_h = L$

$T_s = T_h = T \Rightarrow \sqrt{}_s = \sqrt{}_h = \sqrt{}$

$G_s = G_h = G \Rightarrow \tau_s = \tau_h = \tau$

$$\left(\frac{\theta}{L}\right)_s = \left(\frac{\theta}{L}\right)_h = \frac{\theta}{L}$$

$d_i = 0.6 d_o$.

Let d be the diameter of solid shaft Let d_i and d_o be the inner diameter and outer diameter of the hollow shaft respectively.

Comparision based on strength.

$\rightarrow$ For Solid shaft:-

extern radius $R = \frac{d}{2}$.

Polar moment of Inertia $J = \frac{\pi d^4}{32}$.

From torsion Equation, $\frac{T}{J} = \frac{\tau}{R}$ (or)

$$T = \tau \frac{J}{R} \Rightarrow T = \frac{\pi d^4/32}{d/2} \tau$$

$$T = \frac{\pi d^3}{16} \tau \rightarrow (1).$$

$\rightarrow$ For Hallow shaft:-

Extreme radius $R = \frac{d_o}{2}$.

$J = \frac{\pi}{32}(d_o^4 - d_i^4)$. $\Rightarrow$ Polar moment of Inertia.

From data $d_i = 0.6 d_o$.

$$J = \frac{\pi}{32}\left\{d_o^4 - (0.6 d_o)^4\right\}.$$

$$J = \frac{\pi}{32}\left[d_o^4 - [1-0.6]^4\right]$$

$$J = 0.8704 \left(\frac{\pi}{32}\right) d_o^4.$$

From Torsion Equation:-

$$\frac{T}{J} = \frac{\tau}{R} \Rightarrow T = \frac{J}{R}\tau$$

$$T = \frac{0.8704 (\pi/32) d_o^4}{d_o/2} \tau$$

$$T = 0.8704 \left(\frac{\pi d_o^3}{16}\right) \tau \rightarrow (2).$$

$d^3 = 0.8704 d_o^3 \Rightarrow d = 0.9548 d_o$.

Weight of solid Shaft to that of hallow Shaft.

$$\frac{W_s}{W_h} = \frac{\sqrt{}_s A_s L_s}{\sqrt{}_h A_h L_h} = \frac{A_s}{A_h}$$

$$\frac{A_s}{A_h} = \frac{\pi d^2/4}{\pi/4 (d_o^2 - d_i^2)} = \frac{\pi (0.9548 d_o)^2/4}{\pi/4 (d_o^2 - (0.6 d_o)^2)}$$

$$= \frac{0.9173 d_o^2}{0.64 d_o^2} \rightarrow \frac{W_s}{W_h} = 1.43.$$

Solid shaft weight 43% more than hollow shaft.

Comparision Based On Rigidity.

→ For solid shaft:-

From torsion Equation:-

$$\frac{T}{J} = \frac{G\theta}{L} \quad (or) \quad T = \frac{G\theta}{L} J.$$

$$T = \frac{G\theta}{L} \cdot \frac{\pi d^4}{32} \longrightarrow (3).$$

→ For hollow shaft.

$$T = \frac{G\theta}{L} J \Rightarrow T = \frac{G\theta}{L}\left(0.8704 \frac{\pi}{32} d_o^4\right) \longrightarrow (4)$$

For Equations (3) & (4)

$$d^4 = 0.8704\, d_o^4.$$

$$d = 0.9659\, d_o.$$

Weight of Solid Shaft to that of hollow shaft.

$$\frac{A_s}{A_h} = \frac{\pi d^2/4}{\pi/4\,[0.64 d_o^2]}$$

$$\frac{A_s}{A_h} = \frac{0.9329 d_o^2}{0.64 d_o^2}$$

$$\frac{A_s}{A_h} = 1.45.$$

Weight of solid Shaft is 45% is more than that of hollow shaft of design is on rigidity.

from torsion equ^n. $\frac{T}{J} = \frac{\tau}{R}$.

$$T_s = \frac{J_s}{R_s} \tau_s$$

$$T_s = \frac{\pi d^4}{32} \times \frac{2}{d} \times \tau_s$$

$$T_s = \frac{\pi d^3}{16} \tau_s \longrightarrow (2).$$

For hollow shaft ⇒ $T_h = \frac{J_h}{R_h} \tau_h$

$$T_h = \frac{\frac{\pi}{32}(d_o^4 - d_i^4)}{d_o/2} \tau_h.$$

$$T_h = \frac{\pi}{16} \frac{(d_o^4 - d_i^4)}{d_o} \tau_h \longrightarrow (3).$$

$$\frac{T_h}{T_s} = \frac{\pi/16 \left(\frac{d_o^4 - d_i^4}{d_o}\right) \tau_h}{\pi/16\, d^3\, \tau_s}$$

$$\frac{T_h}{T_s} = \frac{d_o^4 - d_i^4}{d^3 d_o}$$

$$\frac{T_h}{T_s} = \frac{(d_o^2 + d_i^2)(d_o^2 - d_i^2)}{(d_o^2 - d_i^2)^{3/2}\, d_o}$$

$$\frac{T_h}{T_s} = \frac{(d_o^2 + d_i^2)}{d_o \sqrt{d_o^2 - d_i^2}}$$

$$\frac{T_h}{T_s} = \frac{d_o^2 (1 + d_i^2/d_o^2)}{d_o \sqrt{d_o^2 (1 - d_i^2/d_o^2)}}$$

$$\frac{T_h}{T_s} = \frac{d_o^2 \left\{1 + \left(\frac{d_i}{d_o}\right)^2\right\}}{d_o^2 \sqrt{1 - (d_i/d_o)^2}}$$

$$\frac{T_h}{T_s} = \frac{1 + (d_i/d_o)^2}{\sqrt{1 - (d_i/d_o)^2}}.$$

Since numerator is greater than 1 & denominator is less than one.

$$\frac{T_h}{T_s} > 1 \quad \text{or} \quad T_h > T_s$$

Hence it is proven that hollow shaft is stronger than solid shaft of same material, length & weight.

Stiffness of Solid Shaft. $(GJ)_s = G \frac{\pi d^4}{32}$

Stiffness for Hollow Shaft $(GJ)_h = G \frac{\pi}{32} (d_o^4 - d_i^4)$.

$$\frac{(GJ)_h}{(GJ)_s} = \frac{(d_o^2 + d_i^2)(d_o^2 - d_i^2)}{(d_o^2 - d_i^2)^2} \rightarrow \frac{G \frac{\pi}{32} (d_o^4 - d_i^4)}{G \frac{\pi}{32} d^4}$$

$$\frac{(GJ)_h}{(GJ)_s} = \frac{(d_o^2 + d_i^2)}{(d_o^2 - d_i^2)}$$

Since numerator is greater than denominator
$(GJ)_h > (GJ)_s$
Hence it is proven that hollow shaft is stiffer than solid shaft of same material length & weight.

1. A solid shaft transmitts 250 KW at 100 rpm. If the shear stress is not to exceed $75\ N/mm^2$, what should be the diameter of the shaft? If this shaft is to be replaced by a hollow one who's internal diameter is equal to 0.6 times the outer diameter, determine the size and the % saving in weight, the maximum shear stress being the same.

Ans:-

$P = 250 \times 10^3 W$ $\qquad d_i = 0.6 d_o$

$N = 100 rpm.$ $\qquad d_i = ? \quad d_o = ?$

$\tau = 75\ N/mm^2$ $\qquad$ % saving in weight = ?

$d = ?$

$R = \frac{d}{2}$ $\qquad P = \frac{2\pi N T}{60} \Rightarrow 250 \times 10^3 = \frac{2\pi \times 100 \times T}{60}$

$T = 23.87 \times 10^3\ N\text{-}m. \rightarrow T = 23.87 \times 10^6\ N\text{-}mm.$

$\frac{T}{J} = \frac{\tau}{R} \Rightarrow \frac{23.87 \times 10^6}{\frac{\pi}{32} d^4} = \frac{75}{\frac{d}{2}}$ $\qquad d = 117.4\ mm$

$J = \frac{\pi}{32} (d_o^4 - (0.6 d_o)^4)$

$J = 0.085 d_o^4$, $\qquad \Rightarrow d_o = 122.91\ mm$

$\frac{23.87 \times 10^6}{0.085 d_o^4} = \frac{75}{0.5 d_o}$ $\qquad d_i = 75.74\ mm.$

% Saving in weight $= \frac{W_s - W_h}{W_s} \times 100$

$= \frac{A_s - A_h}{A_s} \times 100$

$A_s = \pi d^2/4 = 10824.96\ mm$, $\quad A_h = \frac{\pi}{4}(d_o^2 - d_i^2) = 7591.92\ mm$

2. What % of strength of a solid circular steel shaft 100 mm diameter is lost by boring 50 mm axial hole in it? Compare the strength & weight ratio of the 2 cases.

Sol: Data: $\quad d = 100 mm.$

$\tau_s = \tau_h$

$T_s = \frac{J_s}{R_s} \tau_s$

$\frac{T}{J} = \frac{\tau}{R} \Rightarrow T = \frac{\tau}{R} J.$

$$T_s = \frac{\frac{\pi}{32}(100)^4}{50} \times \tau_s$$

$$T_h = \frac{J_h}{R_h} \tau_h = \frac{\frac{\pi}{32}(d_o^4 - d_i^4)}{d_o/2} \tau_h.$$

$$= \frac{\pi/32\,(100^4 - 50^4)}{50} \tau_h$$

$$\Rightarrow \frac{\frac{\frac{\pi}{32}(100)^4}{50} \times \tau_h - \frac{\frac{\pi}{32}(100^4 - 50^4)}{50} \times \tau_h}{\frac{\frac{\pi}{32}(100^4)}{50} \times \tau_h} \times 100$$

$= 6.25\%.$

$\frac{T_h}{T_s} = 0.94$, $\quad \frac{W_h}{W_s} = \frac{A_h}{A_s} = 0.75.$

A hollow circular shaft 12m long is required to transmit 100KW power when running at a speed of 300 rpm. If the maximum shearing stress allowed in the shaft is 80 N/mm² and ratio of inner diameter to the outer diameter is 0.75. Find the dimensions of the shaft & also the angle of twist of one end of the shaft relative to the other end. Modulus of rigidity of the material is 85 KN/mm²

Sol^n: Data: $L = 12m$. $\frac{d_i}{d_o} = 0.75$.

$P = 100\times10^3 W$

$N = 300 rpm$. $d_o = ?$, $d_i = ?$

$\tau = 80 N/mm^2$ $\theta = ?$

$\frac{T}{J} = \frac{\tau}{R}$ $P = \frac{2\pi NT}{60}$

$d_i = 0.75 d_o$ $T = 3.183\times10^6$.

$J = \frac{\pi}{32}(d_o^4 - d_i^4)$.

$\frac{T}{J} = \frac{\tau}{R}$.

$R = \frac{d_o}{2}$

$$\frac{3.183\times10^6}{\frac{\pi}{32}(d_o^4-d_i^4)} = \frac{80}{0.5d_o}$$

$$d_o^3 = \frac{3.183\times10^6}{\frac{\pi}{16}\times0.6836\times80} \Rightarrow d_o = 66.67.$$

$d_i = 0.75(66.67)$
$= 50.0025$

4. A solid shaft of 200 mm diameter is proposed to be replaced by the hollow shaft of external diameter D & internal diameter D/2. If the same power is to be transmitted at the speed and @ the same level of shear stress, determine the dia D.

Sol^n: $T_s = T_h$, $\frac{J_s}{R_s} = \frac{J_h}{R_h}$, $\frac{\frac{\pi}{32}(200)^4}{100} = \frac{\frac{\pi}{32}(D^4 - (\frac{D}{2})^4)}{D/2} \Rightarrow D = 204.3mm$

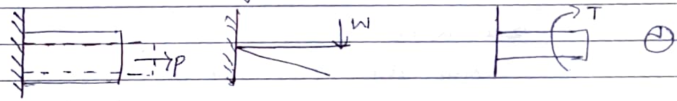
$P_s = P_h$, $N_s = N_h$, $C_s = C_h$, $\frac{T}{J} = \frac{\tau}{R}$

22/8/23

MODULE - 2.

SHEAR FORCE AND BENDING MOVEMENT IN BEAM.

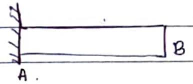
	BARS	BEAMS	SHAFTS.
LOAD	Axial force.	Lateral force/Moment	Twisting Moment
RESULT	Deformation (Change in dimension)	Deflection. (Change in position)	Distortion (Change in shape)



BEAM is a structural member that primarily resist lateral forces and moments.

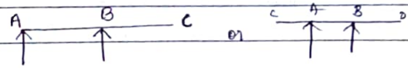
TYPES OF BEAMS:-

1. CANTILEVER BEAM:



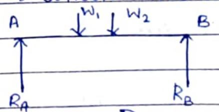
2. SIMPLY SUPPORT BEAM:

3. OVER HANGING BEAM:

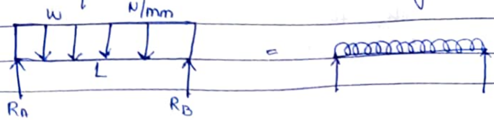


TYPES OF LOAD:-

1. POINT LOAD OR CONCENTRATED LOAD:-



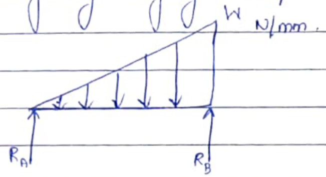
2. UDL [UNIFORMLY DISTRIBUTED LOAD] / Rectangular Load.



Total load = wL

Loads acts at a distance $L/2$ from either of the ends.

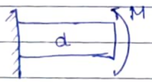
3) UVL [Uniformly Varying Load]/ Triangular Load:



Total Load = $\frac{1}{2} wL$

The Load acts at a distance of $\frac{L}{3}$ from base & $\frac{2}{3}$ from apex.

4) Externally Applied Moments:-



Shear Force.

* Shear force at any section of the beam is the force that tends to shear off the section.
* It is obtained as the algebraic Sum of all the forces including reactions acting normal to the axis of the beam either to the left or right of the section.

Bending Moment At Any Section Of The Beam:-

* It is the moment that tends to bend/deflect the section. It is obtained the algebraic Sum of moments of all forces including reactions about the axis of the beam either to the left/ right of the section.

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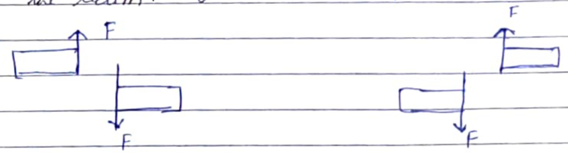
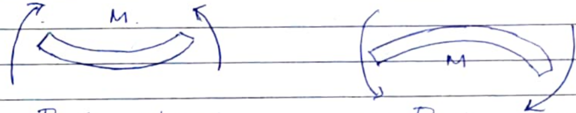
Sign Convention:-

Shear force & bending movement are vector quantities & the following sign conventions are used: a) Shear force is positive if it tends to move the left portion upward relative to the right portion.

b) Bending movement if it tends to sag the beam.

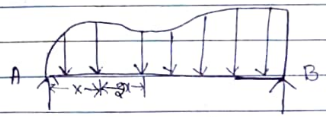


+ve shear force. −ve shear force.



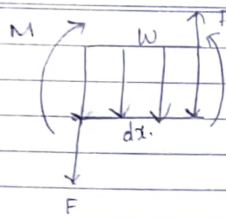
+ve Bending Moment (Sagging) -ve Bending Moment (Hogging).

Relationship B/w Load Intensity Shear Force And Bending Moment:-



This figure show a Simply Support beam subjected to general loading.

Consider an elemental length dx at a distance x from one of the Support.



Where F is the shear force, M is the bending Moment, w is the load intensity.

For the equillibrium of force,
Sum of forces = 0.

The above figure shows free body diagram of elemental length.

$$F + dF - F - wdx = 0.$$

$$dF = wdx$$

$$\frac{dF}{dx} = w \rightarrow (1)$$

For the equillibrium of moments,
Taking moment about left face of the element

$$M + dM - M + (F + dF)dx - wdx \times \frac{dx}{2} = 0 \rightarrow.$$

$$dM + Fdx + dF dx - \frac{wdx^2}{2} = 0 \quad \text{neglecting the higher order term}$$

$$dM = -Fdx$$

$$\frac{dM}{dx} = -F \rightarrow (2).$$

Eqⁿ (1) & (2) gives relation b/w load intensity, shear force

Shear Force & Bending Moment Diagrams

i] Shear Force Dia [SFD].

SFD is a graphical representation where ordinate represents shear force & abscissa represents position.

ii] BMD — It is a graphical representation where ordinate represents bending moment & abscissa represents the position.

SF & BMD reviews the following usefull information

i) Values at sailent points. i.e. the points at which values are maximum / minimum or nature of the variation changes.

ii) Nature of variation b/w 2 Salient points.

iii) Location of point of contraflexure, that is the point at which bending moment changes its sign.

★ SFD & BMD of Standard Cases:-

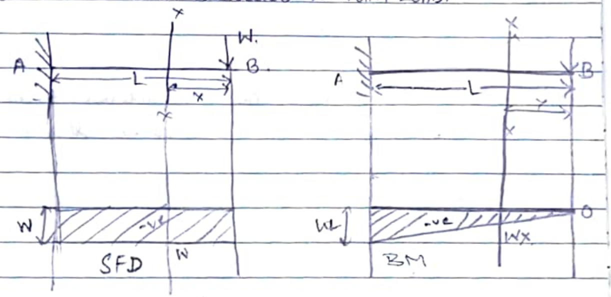
Note★:- Shear force / BM is = a (const).

SF / BM ⟶ = ax (linear)

SF / BM ⟶ = ax^2 (parabolic).

SF / BM ⟶ = ax^3 (cubic).

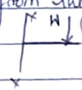
i] Cantiliaver Beam Subjected To Point Load:-



★ figure shows a cantiliaver beam of length 'L' subjected to point load at the free end.

★ Consider a section x-x at a distance x from the free end.

★ SF at Section x-x F = −W (Const)



★ At free end. (x = 0)

★ At fixed end. (x = L)

F = −W.

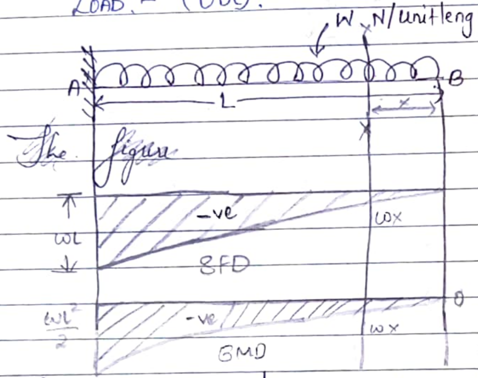
⇒ Bending Moment at Section x-x, M = −Wx (linear variation)

BM at free end. (x = 0) ⇒ M = 0.

BM at fixed end. (x = L) ⇒ [−WL = M].

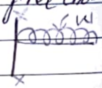
Both SF & BM are −ve.

ii) CANTILEVER BEAM SUBJECTED TO UNIFORMLY DISTRIBUTED LOAD :- (UDL).



The figure shows the Cantilever bean Subjected to UDL of intensity W as shown in the figure. Consider a Section x-x at distance x from the free end.

SF at Section x-x, $F = -Wx$ (linear variation).



At free end $(x=0)$.

$F = 0$. At fixedend $(x=L)$

$F = -\omega L$.

At fixed end $x = L$

BM at Section x-x. $M = \frac{-\omega x^2}{2}$ (Parabolic variation).

At free end $(x=0)$ = $M = 0$.

At fixed end $(x=L)$ = $M = \frac{-\omega L^2}{2}$.

iii) CANTILEVER BEAM SUBJECTED TO UNIFORMLY VARYING LOAD :- (VVL)

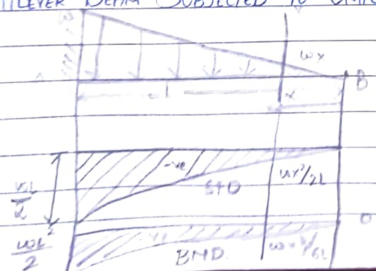
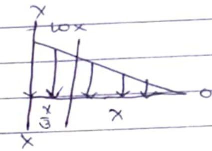


Fig shows a cantilever beam of length 'L' Subjected to UVL of maximum intensity 'w'. Consider section x-x at a distance x from the free end.



Shear force at Section x-x.

$F = \frac{-1}{2}\frac{\omega x^2}{L}$

$= \frac{-\omega x^2}{2L}$ (parabolic variation).

At free end $(x=0) \Rightarrow x = 0$.

At fixed end $(x=L) \Rightarrow F = \frac{-\omega L}{2}$

Bending moment at Section x-x

$M = -\frac{\omega x^2}{2L} \cdot \frac{x}{3} \Rightarrow \frac{-\omega x^3}{6L}$ (cubic variation).

At free end $(x=0)$ $M = 0$.

At fixed end $(x=L) = M = \frac{-\omega L^2}{6}$

iv) SIMPLY SUPPORTED BEAM SUBJECTED TO CENTRAL POINT LOAD :-

in dia

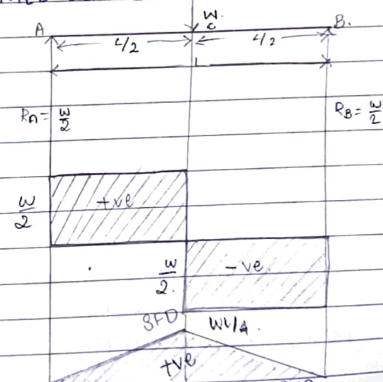


Fig shows simply supported beam of length 'L' Subjected to central point load. 'W'. Since the load is equally distributed on both the supports.

$R_A = R_B = \frac{W}{2}$. Shear force at A $= +\frac{W}{2}$

SF at C $= -\frac{W}{2}$

SF at B $= -W/2$.

Bending Moment

$M_A = M_B = 0$

$M_c = \frac{W}{2} \cdot \frac{L}{2} = +\frac{WL}{4}$

★Note:- For point load both SFD and BMD linear.

* For UDL SFD linear and BMD parabolic.
* For UVL SFD parabolic and BMD cubic.

v) Simply Supported Beam Subjected To eccentric Point Load:-

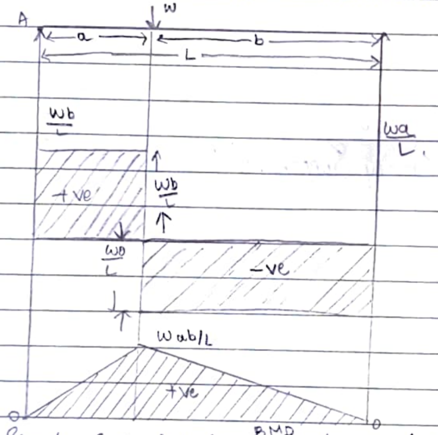


Fig. shows Simply Supported beam of length 'L' subjected to eccentric point load 'W'.

Taking moments about A.

$M_A = +R_B L - Wa = 0$

$R_B L = Wa$

$R_B = \frac{Wa}{L}$

$R_A + R_B = W$

$R_A + \frac{Wa}{L} = W \Rightarrow R_A = W - \frac{Wa}{L}$

$R_A = W\left(1 - \frac{a}{L}\right) = W\left(\frac{L-a}{L}\right)$

$\boxed{R_A = \frac{Wb}{L}}$ $\boxed{R_B = \frac{Wa}{L}}$

Bending moments $\Rightarrow M_A = M_B = 0$

$M_c = \frac{Wab}{L}$

vi) Simply Supported Beam Subjected To UDL.

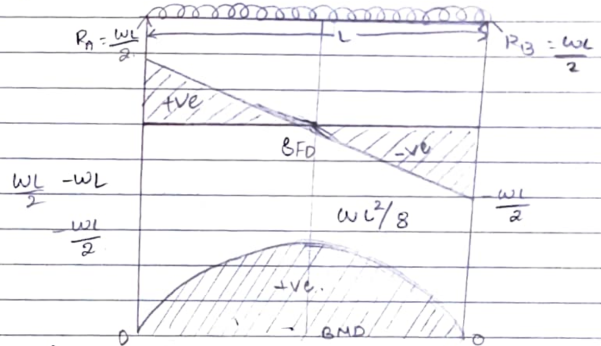


Fig shows a simply supported beam of length 'L' subjected to UDL of intensity 'w'.

$M_A = M_B = 0$.

$M_c = +\frac{WL}{2}\left(\frac{L}{2}\right) - \frac{WL}{2} \times \frac{L}{4} = \frac{WL^2}{4} - \frac{WL^2}{8} = +\frac{WL^2}{8}$

(parabolic variation)

vii) Simply Supported Beam Subjected To UVL.

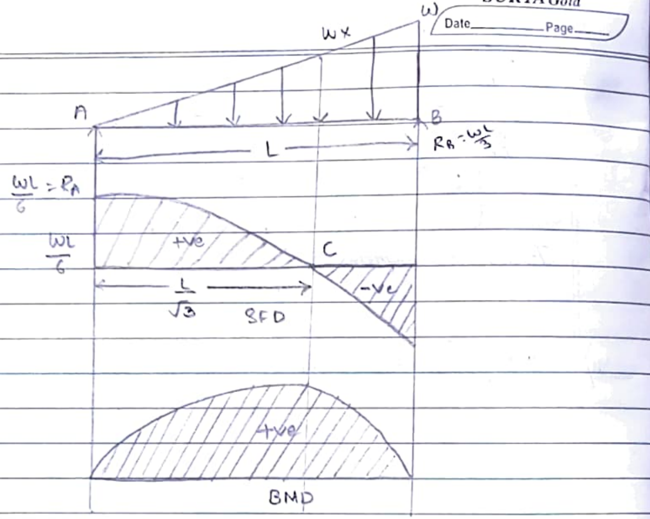
Fig shows a Simply Supported beam of length 'L' subjected to UVL of maximum intensity 'w'.

Taking moments about A.

$M_A = R_B L - \frac{1}{2} WL \times \frac{2L}{3} = 0$.

$R_B \cancel{L} = \frac{WL^2}{3}$

$R_B = \frac{WL}{3}$



$R_A + R_B = \frac{1}{2} wL$

$R_A = \frac{wL}{2} - \frac{wL}{3} = \frac{wL}{6}$

Shear force at distance x from the left support is equal to.

$\frac{W_x}{x} = \frac{w}{L}$ $\qquad w_x = \frac{wx}{L}$

$\frac{wL}{6} - \frac{1}{2}\frac{wx^2}{L} = 0$

$\frac{wx^2}{2L} = \frac{wL}{6} \Rightarrow x^2 = \frac{L^2}{3} \Rightarrow x = \frac{L}{\sqrt{3}}$

Bending moment.

$M_A = M_B = 0$

$M_C = \frac{wL}{6}(x) - \frac{1}{2}\frac{wx^2}{L}\left(\frac{x}{3}\right)$

$= \frac{wL}{6}\left(\frac{L}{\sqrt{3}}\right) - \frac{w}{2L}\left(\frac{L}{\sqrt{3}}\right)^2\left(\frac{L}{3\sqrt{3}}\right)$

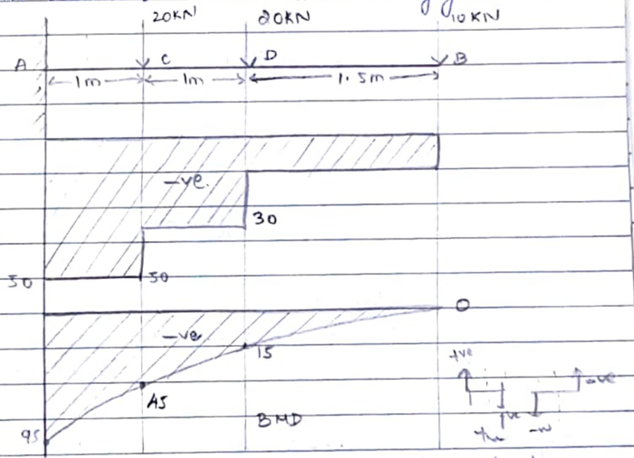
$= \frac{wL^2}{6\sqrt{3}} - \frac{wL^2}{18\sqrt{3}} \Rightarrow \frac{wL^2}{6\sqrt{3}}\left(1-\frac{1}{3}\right) = \frac{wL^2}{9\sqrt{3}}$

NUMERICALS ON SFD & BMD.

★ Note:- For cantilever beams always measure shear force and Bending moments from the free end.

* For simply supported beam draw SFD and BMD from left to right.

1. Draw the bending moment and shear force diagram for the cantilever beam shown in figure.



Shear force at A = -50KN. $\quad R_A = -10(3.5) + (-20)(2) + (-20)(1) + 10(3.5)$

SF at B = -10 KN

SF at C = -50KN

SF at D = -30KN

$M_B = 0$

$M_D = -10(1.5) = -15$ kN-m.

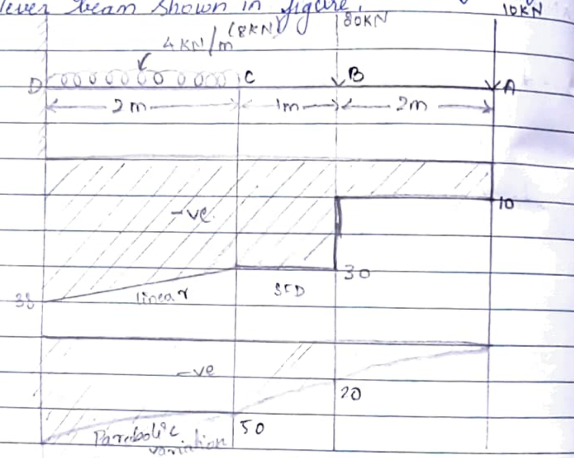
$M_C = -10(2.5) - 20(1)$

$= -45$ kN-m.

$M_A = 10(3.5) - 20(2) - 20(1)$

$= -95$ kN-m.

2. Draw shear force and bending moment diagram for Cantilever beam shown in figure.



SF at A = −10KN, SF at B = −30KN

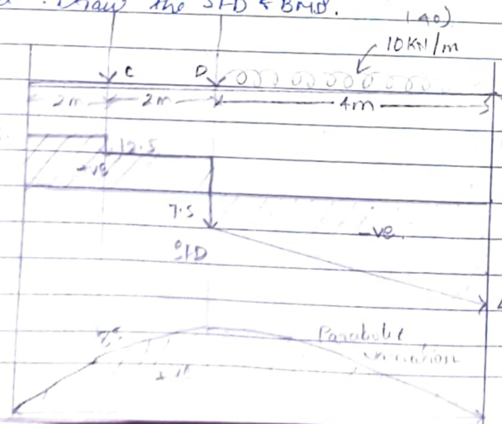
SF at C = −30KN, SF at D = −38KN.

$M_A = 0$, $M_B = -10(2) = -20$ KN-m.

$M_C = -10(3) - 20(1) = -50$ KN-m

$M_D = -10(5) - 20(3) - 8(1) = -118$ KN-m.

3. A simply supported beam shown in fig carries 2 concentrated load and a uniformly distributed load. Draw the SFD & BMD.



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Taking moments about A.

$M_A = R_B(8) - 40(6) - 20(4) - 30(2) = 0$

$R_B = 47.5$ KN.

$R_A + R_B = 30 + 20 + 40$.

$R_A = 42.5$ KN.

Shear force at A = 42.5KN, At B = −47.5KN.

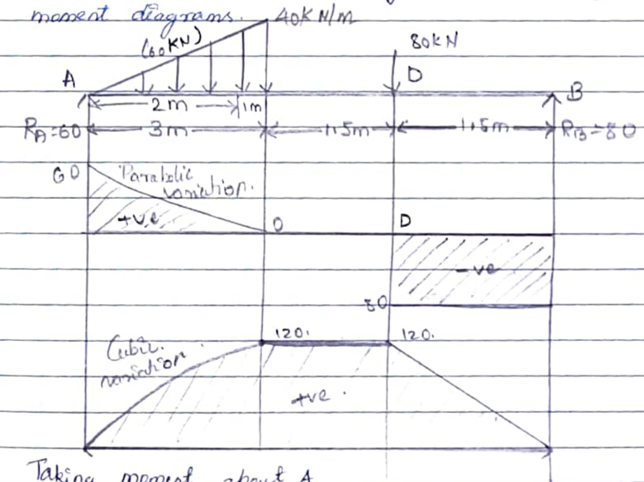
SF at C = 12.5, SF at D = −7.5.

$M_A = M_B = 0$

$M_C = 42.5(2) = 85$ KN-m

$M_D = 42.5(4) - 30(2) = 110$ KN-m.

4) A simply supported beam AB of 6m span is loaded as shown in fig. Draw the shear force and bending moment diagrams.



Taking moment about A

$M_A = R_B(6) - 80(4.5) - 60(2) = 0$

$R_B = 80$KN

$R_A + R_B = 60 + 80 = 140$

$R_A = 60$KN.

Shear force at A = 60KN. SF at C = 0

SF at B = −80KN SF at D = −80KN.

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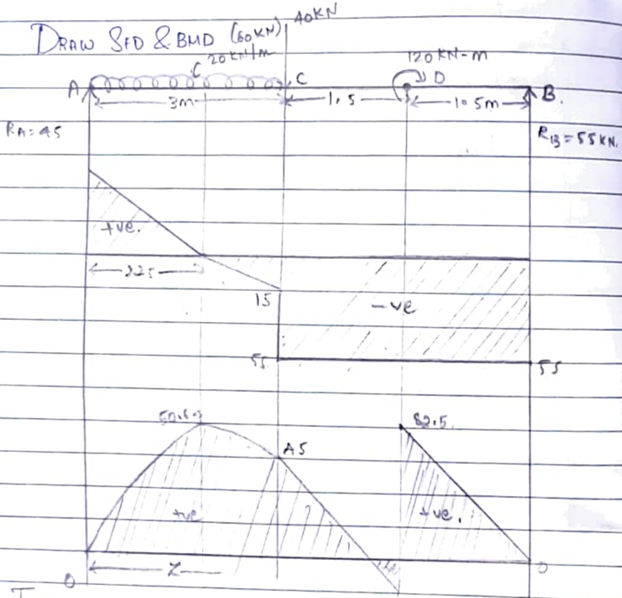
$M_A = 0$, $M_B = 0$

$M_c = 80(3) - 80(1.5) = 120 \text{ kN-m}$

$M_D = 80(1.5) = 120 \text{ kN-m}$.

5. Draw SFD & BMD



Taking moments about A

$M_A = R_B(6) - 120 - 40(3) - 60(1.5) = 0$

$R_B = 55 \text{kN}$

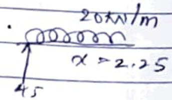
$R_A + R_B = 60 + 40 \Rightarrow R_A = 100 - 55 = R_A = 45$

S.F at A = 45 kN SF at C = − 55 kN

S.F at B = 55 kN SF at D = − 55 kN.

$SF_x = 0$.

$45 - 20x = 0$ $x = 2.25m$.



$M_A = M_B = 0$

$M_c = 45(3) = -60(1.5)$.

$= 45 \text{ kN-m}$

$M_D = 45(4.5) - 60(3) - 40(1.5) = -37.5 \text{ kN-m}$.

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$M_x = 45(2.25) - 20(2.25)(1.125)$.

$= 50.625 \text{ kN-m}$.

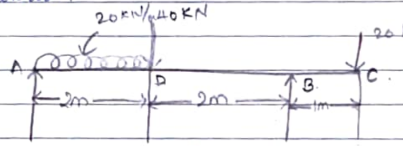
To find location of point of contraflection let x be the distance b/w left support and point of contraflexture.

Taking moment at this point $M_x = 0$.

$45x - 60(x - 1.5) - 40(x - 3) = 0$.

$x = 3.8m$

6. Draw B.M and S.F diagram for the overhanging beam. Shown in figure clearly indicate the point of contraflexture.



Taking moment about. $M_A = 0$.

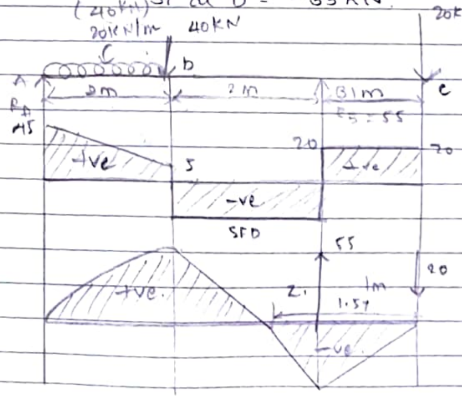
$-(20)(5) + R_B(4) - 40(2) - 40(1) = 0$.

$R_B = 55 \text{kN}$.

$R_A + R_B = 100 \text{ kN}$, $R_A = 45 \text{kN}$.

SF at A = 45 kN, SF at B = 20 kN, SF at C = 20 kN

SF at D = − 35 kN.



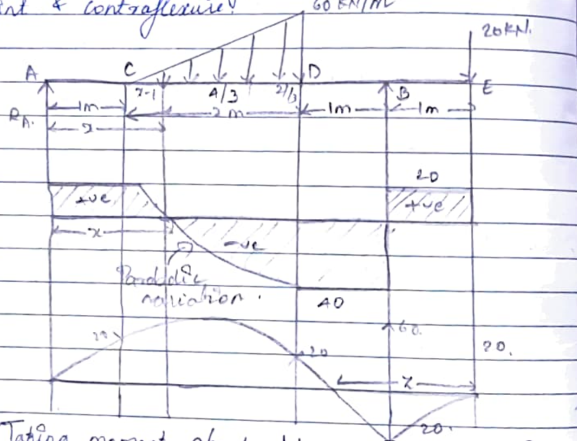
$M_A = M_C = 0$.

$M_D = 45(2) - 40(1)$

$= 50 \text{ kN-m}$.

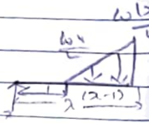
$M_B = -20 \text{kN-m}$.

7. Draw SFD & BMD for over hanging beam shown in fig. Indicate all significant values. Including the point of contraflexure.



Taking moment about $M_A = 0$, $R_B = 60\,KN$, $R_A = 20\,KN$

$$+20 - \frac{1}{2}(x-1)\left(\frac{60(x-1)}{2}\right) = 0$$



$$20 - 15(x-1)^2 = 0$$

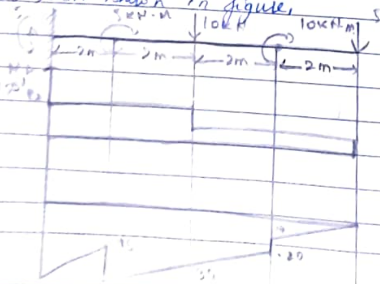
$$15(x-1)^2 = 20 \Rightarrow (x-1)^2 = 1.33$$

$$x - 1 = 1.153 \Rightarrow x = 2.153$$

$$M = 20x - \frac{1}{2} w (x-1)(x-1)\frac{(x-1)}{3}$$

$$M = 27.69$$

Find the relation at fixed end and SFD & BMD for the cantilever beam shown in figure.



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Module - 4.

Energy Methods.

Conservative Forces:- Forces that are done in moving a particle from one point to another point are called conservative forces.

Eg: Gravitational force, Magnetic force etc..

Strain Energy:- Energy absorbed by the body when it is strained within it's elastic limit.

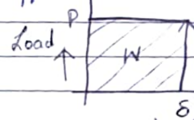
It is denoted by U. Mathematically Strain energy stored in the body is equal to workdone by the externally applied load. ⇒ U = W.

Law of Conservation of Energy.

Work done by the Externally applied load:-

a) Sudden Applied load.

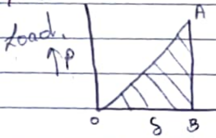
$\boxed{W = P\delta}$



b) Gradually Applied Load. (GAL)

WD by GAL = Area of Load.

$\boxed{W = \frac{1}{2} P\delta}$



c) Strain Energy Due To Axial Load.

(Strain Energy Stored in the bar).

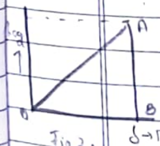


Fig 2.

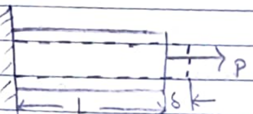


Fig. 1.

Consider a bar of length 'L' cross sectional area 'A' & Youngs Modulus 'E' Subjected to an axial pull 'P' as shown in fig. 1.

Let δ be the deformation due to applied Load and σ be the Stress induced.

Fig 2 shows load deformation curve for the given bar.

Work done by the applied load is equal (work done) to area of triangle ΔOAB

$$W = \frac{1}{2} P\delta \rightarrow ①.$$

Since strain energy stored in the bar is equal to work done by the external load. $\rightarrow U = W$.

$$\therefore U = \frac{1}{2} P\delta \rightarrow ②.$$

WK.T. $\left.\begin{array}{l} P = \sigma A \\ \delta = \dfrac{\sigma L}{E} \end{array}\right\} \rightarrow ③.$

Substituting ③ in ②

$$U = \frac{1}{2}(\sigma A)\left(\frac{\sigma L}{E}\right)$$

$$U = \frac{\sigma^2 AL}{2E}$$

$$U = \frac{\sigma^2 V}{2E} \rightarrow ④.$$

④ is the Exp for strain energy (for) due to Axial Loading

STRAIN ENERGY DUE TO TORSION (Strain E Stored in the Shaft).

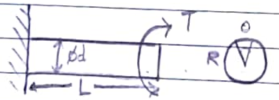


Fig. 1

Consider a shaft of length 'L' dia - 'd' & rigidity modulus 'G'. Subjected to externally applied torque - 'T' as shown in Fig 1.

Let θ be the angle of twist due to applied torque & τ be the shear stress induced.

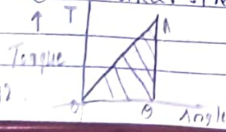


Fig 2

Fig 2 shows torque V.s Angle of twist for the given shaft.

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Work done by the applied torque is equal to Area of triangle OAB.

$$W = \frac{1}{2} T\theta \rightarrow (1).$$

Since strain energy stored in the shaft is equal to work done by the applied torque $U = W$.

$$U = \frac{1}{2} T\theta \rightarrow ②$$

From torsion Eqⁿ $T = \dfrac{\tau J}{R}$, $\theta = \dfrac{\tau L}{GR} \rightarrow ③$

Where J is polar moment of inertia.
R - maximum radius.

Substituting eq ③ in ②.

$$U = \frac{1}{2} \frac{\tau J}{R} \cdot \frac{\tau L}{GR}.$$

$$U = \frac{\tau^2 JL}{2GR^2} \rightarrow (4).$$

WKT. $\left. R = \dfrac{d}{2}, \; J = \dfrac{\pi d^4}{32} \right\} \rightarrow ⑤$

$$U = \frac{\tau^2 (\pi d^4/32) L}{2G(d/2)^2}.$$

$$U = \frac{\tau^2 L}{\cancel{2}G} \times \frac{\pi d^{\cancel{4}}}{\cancel{32}_{16}} \times \frac{\cancel{4}^{2}}{\cancel{d^2}}$$

$$U = \frac{\tau^2}{4G} \frac{\pi d^2}{4} L.$$

$$\boxed{U = \frac{\tau^2 V}{4G} \rightarrow ⑥}$$

Eqⁿ ⑥ is the exp for Strain Energy due to torsion.

STRAIN ENERGY DUE TO BENDING (STRAIN ENERGY STORED IN THE BEAM).

★ Note →:- Bending Eqⁿ is given by $\dfrac{E}{R} = \dfrac{M}{I} = \dfrac{\sigma}{y}$.

$\sigma \Rightarrow$ Bending stress.,

Where E is Young's Modulus, R is radius of curvature. M is applied moment, I is Moment of Inertia of beam cross section.
y → distance from neutral axis.

MOI → Moment of Inertia

Derivation →

Consider a beam Segment of length 'L', MOI → I. & Young's Modulus – E. Subjected to Externally applied Moment 'M' as Shown in fig 1. Let θ be the angle Subtended by the curvature.

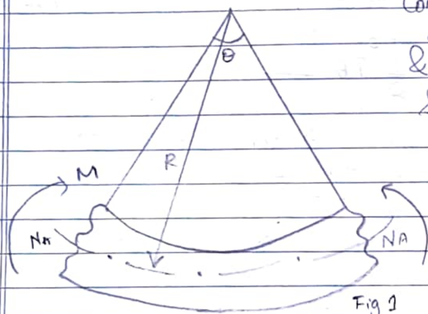
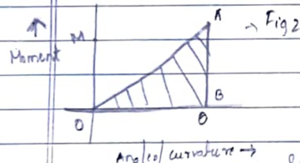


Fig 1

Fig 2 Shows Moment Vs 'θ' curve for the given beam Segment.

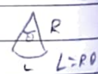


→ Fig 2

Work done by the applied moment is equal to area of Δ^{le} AOB $= W = \frac{1}{2} \times \theta \times M$ → ①

Since strain energy stored in the beam is equal to work done by the applied moment.

$U = W \Rightarrow \therefore U = \frac{1}{2} M\theta$ → ②



From Geometry of fig 1 angle of curvature $\theta = \frac{L}{R}$ → ③

Substituting ③ in ②

$U = \frac{1}{2} \times M \times \frac{L}{R} \Rightarrow U = \frac{1}{2}\frac{ML}{R}$ → ④

From Bending Eqⁿ. $\Rightarrow \frac{E}{R} = \frac{M}{I} \Rightarrow R = \frac{EI}{M}$ → ⑤

Subst ⑤ in ④

$U = \frac{1}{2}\frac{ML}{\frac{EI}{M}}$

$U = \frac{M^2 L}{2EI}$ → ⑥

Eq ⑥ th is the expression for strain energy due to bending.

If BM is not uniform throughout the beam Segment, Consider a small strip of length dx, strain energy stored in the strip. $\Rightarrow dU = \frac{M^2 dx}{2EI}$

$$U = \int_0^L \frac{M^2 dx}{2EI} \rightarrow ⑦$$

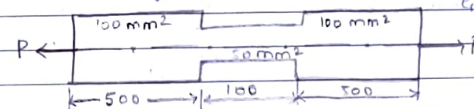
Numerical Problem on Strain Energy Due to Axial Load.

List of formulae:-

1. $U \Rightarrow \frac{\sigma^2 AL}{2E}$
2. $\sigma = \frac{P}{A}$

1.] The Max stress produced by a pull in a bar of 1100mm Length is 100 N/mm². The area of Cross Section & Length are shown in fig. Calculate the strain Energy stored in the bar. If E is equal to. 2×10^5 N/mm².

$E_1 = E_2 = E_3 = 2 \times 10^5$ N/mm²



$\sigma_2 = 100$ N/mm²

$A_1 = A_3 = 100mm^2$, $A_2 = 50mm^2$, $L_1 = L_3 = 500mm$, $L_2 = 100mm$

$\sigma_2 = \frac{P}{A_2} \Rightarrow P = 100 \times 50 \Rightarrow P = 5000N.$

$\sigma_1 = \sigma_3 = \frac{5000}{100} = 50 N/mm^2.$

$U_3 = U_1 = \frac{\sigma_1^2 A_1 L_1}{2E_1} = \frac{50^2(100)(500)}{2 \times 2 \times 10^5} = 312.5$ N-mm.

$U_2 = 125$ N-mm.

$= U_1 + U_2 + U_3 = 750$ N-mm.

$\boxed{U = 0.75J}$

2) STRAIN ENERGY DUE TO TORSION:-

Formula → $U = \frac{\tau^2 AL}{4G}$, $\tau = \frac{TR}{J}$, For solid shaft: $A = \frac{\pi d^2}{4}$, $R = \frac{d}{2}$

$J = \frac{\pi d^4}{32}$ | For hollow shaft: $A = \frac{\pi}{4}(d_o^2 - d_i^2)$, $R = \frac{d_o}{2}$

$J = \frac{\pi}{32}(d_o^4 - d_i^4)$

2) A Solid & hollow shaft of Same material at Same Max Stress are Subjected to Same Max Torque. The ratio of inner to outer diameter for the hollow shaft is 0.75. Find the ratio of Strain Energy in the two Shafts.

Data → $G_s = G_h$, $\tau_s = \tau_h$, $T_s = T_h$.

$\frac{d_i}{d_o} = 0.75$, $\Rightarrow \frac{U_s}{U_h} = ?$

Soln: Let d → diameter of Solid Shafts d_i & d_o be the inner & outer dia of hollow shaft.

Shear stress induced in the Solid shaft. $\tau_s = \left(\frac{TR}{J}\right)_s$

$\tau_s = \frac{T\,d/2}{\pi d^4/32} \Rightarrow \tau_s = \frac{16T}{\pi d^3} \to (1)$.

Shear Stress induced in the hollow Shaft

$\tau_h = \left(\frac{TR}{J}\right)_h$

$\tau_h = \frac{T\,d_o/2}{\frac{\pi}{32}(d_o^4 - d_i^4)} \Rightarrow \tau_h = \frac{16T}{\pi\left(d_o^4 - (0.75d_o)^4\right)/d_o}$

$\tau_h = \frac{16T}{0.68\pi d_o^3} \to (2)$.

from data (1) = (2).

$\tau_s = \tau_h$

$\frac{16T}{\pi d^3} = \frac{16T}{0.68\pi d_o^3} \Rightarrow d_o^3 = 1.47d^3$ $\boxed{d_o = 1.137d} \to (3)$.

Strain Energy in Solid Shaft

$U_s = \left(\frac{\tau^2 AL}{4G}\right)_s \Rightarrow U_s = \frac{\tau^2 \pi d^2 L}{16G} \to (4)$.

Strain Energy in hollow Shaft

$U_h = \left(\frac{\tau^2 AL}{4G}\right)_h \Rightarrow \frac{\tau^2\pi(d_o^2 - d_i^2)L}{16G} \Rightarrow \frac{\tau^2\pi(d_o^2 - (0.75d_o)^2)L}{16G}$.

$U_h = \frac{0.4375\,\tau^2\pi d_o^2 L}{16G} \to (5)$.

$\frac{U_s}{U_h} = \frac{\cancel{\tau^2}\cancel{\pi}d^2\cancel{L}/\cancel{16G}}{0.4375\,\tau^2\pi d_o^2 L/16G} \Rightarrow \frac{d^2}{0.4375\,(1.137d)^2}$

$\frac{U_s}{h} = 1.76$ //

3) STRAIN ENERGY DUE TO BENDING

$U = \int_0^L \frac{M^2 dx}{2EI}$, $W = \frac{1}{2}W\delta$. δ → deflection

3. A Cantilever Beam of uniform cross Section carries a point load at free end. determine i) strain Energy stored by the Cantilever beam & deflection at free end.
ii) If load W is equal to 200 KN. $E = 2\times10^8$ KN/m². $L = 3$m, $I = 10^{-3}$ m⁴. determine the above values.

⇒ $U = ?$ $\delta = ?$

$W = 200$ KN. $L = 3$m

$E = 2\times10^8$ KN/m², $I = 10^{-3}\,m^4$

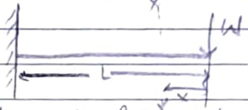


Fig Shows a Cantilever beam of length L Subjected to point load at the free end.
Consider a Section X-X at a distance x from the free end.
Moment at Section at X-X.

$M = Wx$.

Strain Energy stored in the beam $U = \int_0^L \frac{M^2 dx}{2EI}$.

$U = \int_0^L \frac{(Wx)^2 dx}{2EI} = \int_0^L \frac{W^2x^2dx}{2EI} = \frac{W^2}{2EI}\int_0^L x^2 dx$.

$U = \frac{W^2}{2EI}\left[\frac{x^3}{3}\right]_0^L \Rightarrow \boxed{U = \frac{W^2L^3}{6EI}} \to (1)$.

$W_d = \frac{1}{2}W\delta \Rightarrow \frac{1}{2}W\delta = \frac{W^2L}{6EI} \to$

$$\boxed{\delta = \frac{WL^3}{3EI}} \longrightarrow (2).$$

ii) Substitute (1) & (2)

$$U = \frac{(200\times10^3)^2(3)^3}{2\times3(2\times10^8)(10^{-3})} \qquad W = 200\times10^3\,N$$

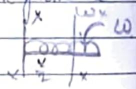
$$L = 3m$$

$$U = \frac{6\times10^8}{12\times10^{11}} \Rightarrow U = 900J. \qquad E = 2\times10^8\times10^3\,N/m^2, \quad I = 10^{-3}$$

$$\delta = 9\times10^{-3}m = 0.009m.$$

A Cantilever beam of length L carries UDL of intensity w over it's entire length as shown in fig. Determine i) U (strain) stored by Cantilever beam.
ii) If $w = 10\,KN/m$, $L = 2\,m$, $EI = 2\times10^3\,KN\text{-}m^2$ determine Strain energy stored.

Consider a Section x-x at a distance x from the free end.



U = ? Moment at Section x-x

$$M = \frac{wx^2}{2}.$$

Strain energy stored in the beam

$$U = \int_0^L \frac{M^2dx}{2EI} \Rightarrow \int_0^L \frac{w^2x^4dx}{8EI}.$$

$$U = \frac{w^2L^5}{40EI}.$$

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A simply supported beam of span L carries a point load F at mid span. Determine strain energy stored by beam and deflection at it's midspan.

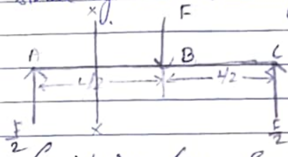


fig shows a simply supported beam of span 'L' subjected to point load F at mid span.

Considering beam segment AB. Let x-x be the section at a distance x from A.

Moment at Section x-x.

$$M = \frac{Fx}{2} \Rightarrow U_{AB} = \int_0^{L/2} \frac{M^2dx}{2EI}.$$

$$U_{AB} = \int_0^{L/2} \frac{(Fx/2)^2dx}{2EI}$$

$$U_{AB} = \int_0^{L/2} \frac{F^2x^2dx}{8EI} \Rightarrow U_{AB} = \frac{F^2}{8EI}\int_0^{L/2} x^2dx.$$

$$U_{AB} = \frac{F^2}{8EI}\left[\frac{x^3}{3}\right]_0^{L/2}$$

$$U_{AB} = \frac{F^2}{24EI}\left[\frac{L}{2}\right]^3 \Rightarrow U_{AB} = \frac{F^2L^3}{192EI}.$$

from Symmetry of the beam

$$U_{AB} = U_{BC} = \frac{F^2L^3}{192EI}.$$

Total Strain energy in the beam $U = U_{AB} + U_{BC}$.

$$\boxed{U = \frac{F^2L^3}{96EI}.}$$

Work done by the applied load, $W = \frac{1}{2}\times F\delta$.

$\because U = W$ Since Strain Energy = Work done.

$$\frac{1}{2}F\delta = \frac{F^2L^3}{96EI} \Rightarrow \boxed{\delta = \frac{FL^3}{48EI}}$$

Energy Theorems -

Castigliano's I Theorem.→

Statement → In an elastic member, partial derivative of strain energy w.r.t to deflection is equal to load acting at that point, deflection being measured in the direction of load.

To prove → $\frac{\partial U}{\partial \delta_i} = P_i$

Consider a simply supported beam subjected to point loads P_1 & P_2 as shown in the figure. Let δ_1 & δ_2 be the corresponding deflections. Strain Energy stored in the beam,

$$U = \frac{1}{2} P_1\delta_1 + \frac{1}{2} P_2\delta_2 \longrightarrow (1)$$

Let ∂P_1 & ∂P_2 be the additional loads applied in such a proportion that additional deflection is produced at point 1 alone.

Additional strain energy due to loads ∂P_1 & ∂P_2.

$$\partial U = \frac{1}{2} \partial P_1 \partial \delta_1 + P_1 \partial \delta_1$$

Neglecting higher order terms. $\partial U = P_1 \partial \delta_1$

$$\frac{\partial U}{\partial \delta_1} = P_1 \text{ (or) in general } \frac{\partial U}{\partial \delta_i} = P_i$$

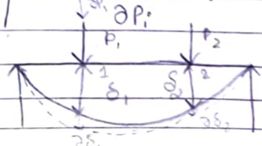
Hence Castiglian's first theorem is proved.

Castigliano's Second (II) Theorem:-

Statement:- In an elastic member, partial derivative of strain energy with respect to load at a point is equal to deflection at that point deflection being measured in direction of load.

→ To prove: $\frac{\partial U}{\partial P_i} = \delta_i$

Proof:-



Consider a simply supported beam subjected to point load P_1 & P_2 as shown in the fig.

Let δ_1 & δ_2 be the corresponding deflection. Strain Energy in the beam

$$U = \frac{1}{2} P_1\delta_1 + \frac{1}{2} P_2\delta_2 \rightarrow (1)$$

Let ∂P_1 → additional load at 1 → increments deflection $\partial\delta_1$ & $\partial\delta_2$

Additional strain energy due to ∂P_1

$$\partial U = \frac{1}{2} \partial P_1 \partial \delta_1 + P_1 \partial \delta_1 + P_2 \partial \delta_2 \rightarrow (2)$$

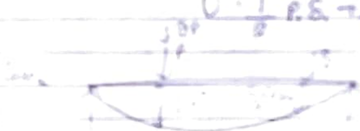
Neglecting the higher order terms

$$\partial U = P_1 \partial \delta_1 + P_2 \partial \delta_2 \rightarrow (3)$$

Total strain energy

$$U' = U + \partial U$$

$$U' = \frac{1}{2} P_1\delta_1 + \frac{1}{2} P_2\delta_2 + P_1\partial\delta_1 + P_2\partial\delta_2 \rightarrow (4)$$



Let the loads P_1, P_2 & ∂P_1 simultaneously applied on the beam. Total Strain energy U'

$$U' = \frac{1}{2}(P_1 + \partial P_1)(\delta_1 + \partial\delta_1) + \frac{1}{2} P_2(\delta_2 + \partial\delta_2)$$

$$U' = \frac{1}{2} P_1\delta_1 + \frac{1}{2} P_1\partial\delta_1 + \frac{1}{2}\partial P_1 \delta_1 + \cancel{\frac{1}{2}\partial P_1 \partial\delta_1}$$

$$+ \frac{1}{2} P_2\delta_2 + \frac{1}{2} P_2 \partial\delta_2 \rightarrow (5)$$

Equating (4) & (5)

$$P_1\partial\delta_1 + P_2\partial\delta_2 = \frac{1}{2} P_1\partial\delta_1 + \frac{1}{2} P_2\partial\delta_2 + \frac{1}{2}\partial P_1 \delta_1$$

$$\Rightarrow \frac{1}{2} P_1\partial\delta_1 + \frac{1}{2} P_2\partial\delta_2 = \frac{1}{2}\partial P_1 \delta_1 \rightarrow (6)$$

Substituting eq. (3) in (6)

$$\partial U = \partial P_1 \delta_1$$

$$\boxed{\frac{\partial U}{\partial P_1} = \delta_1}$$

Hence Castigliano's II theorem is proved.

Problems On Castigliano's Theorem.

1. $U = \int_0^L \frac{M^2 dx}{2EI}$

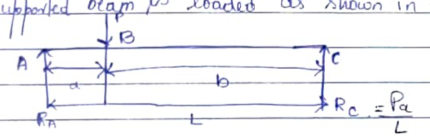
2. $\delta = \frac{\partial U}{\partial P}$.

1. Determine the deflection at the free end of cantilever beam subject to point load at it's free end. Using Castigliano's theorem.

$U = \frac{P^2 L^3}{6EI}$ $\qquad \delta = \frac{\partial U}{\partial P}$

$$\delta = \frac{\partial}{\partial P}\left(\frac{P^2L^3}{6EI}\right) = \frac{2PL^3}{6EI} \Rightarrow \boxed{\delta = \frac{PL^3}{3EI}}$$

• A simply supported beam is loaded as shown in the figure.



Moment at A.

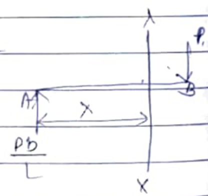
$M_A = 0$.

$R_C L - Pa = 0$.

$R_C = \frac{Pa}{L} \Rightarrow R_A + R_C = P, \Rightarrow R_A = P - \frac{Pa}{L}$

$= P\left(1-\frac{a}{L}\right) = P\left(\frac{L-a}{L}\right) = \frac{Pb}{L}$

Considering Section A-B of the beam. Let x-x be the section at a distance x from A.



Moment at section x-x

$M = \frac{Pb}{L}$

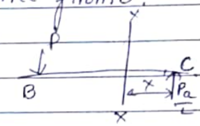
Strain energy in section AB.

$U_{AB} = \int_0^a \frac{M^2 dx}{2EI}$.

$$= \int_0^a \frac{P^2b^2x^2}{2EIL^2} = \frac{P^2b^2a^3}{6EIL^2} = \frac{P^2b^2a^3}{2EIL^2}.$$

Considering segment BC of the beam.
Let x-x be the section at a distance from C.
Moment at section x-x.

$M = \frac{Pax}{L} \Rightarrow U_{BC} = \int_0^L \frac{M^2dx}{2EI}$.



$U_{BC} = \frac{P^2a^2b^3}{6EIL^2}$

Total strain energy.

$U = U_{AB} + U_{BC}$.

$= \frac{P^2b^2a^3}{6EIL^2} + \frac{P^2a^2b^3}{6EIL^2}$

$= \frac{P^2a^2b^2}{6EIL^2}(a+b) \Rightarrow \frac{P^2a^2b^2}{6EIL^2}(L)$

$U = \frac{P^2a^2b^2}{6EIL}$.

Deflection for the given beam.

$\delta = \frac{\partial U}{\partial P}$

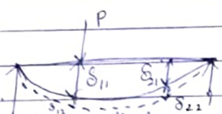
$\delta = \frac{\partial}{\partial P}\left(\frac{P^2a^2b^2}{6EIL}\right) \Rightarrow \delta = \frac{Pa^2b^2}{3EIL}$

Maxwell Reciprocal Theorem.

Statement :- In an elastic member, deflection at point i due to load applied at point j is equal to deflection at point j due to load applied at i.

To prove: $\delta_{ij} = \delta_{ji}$

Proof :- Case 1 :-



Consider a SPB subjected to load P at point 1.
Let δ_{11} & δ_{21} be the deflections at point 1 & 2 respectively.
Strain energy stored in the beam $U_1 = \frac{1}{2} P\delta_{11}$ → (1).

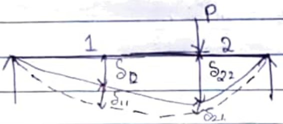
Let Load P be applied at point 2, δ_{12} & δ_{22} are the corresponding deflections at point 1 & 2 respectively. Additional strain energy due to this load.

$$U_2 = \frac{1}{2} P\delta_{22} + P\delta_{12} \rightarrow (2)$$

Total strain energy :- $U = U_1 + U_2$.

$$U = \frac{1}{2} P\delta_{11} + \frac{1}{2} P\delta_{22} + P\delta_{12} \rightarrow (3)$$

Case II :-



Consider the same beam subjected to load P at Point 2, Let δ_{12} & δ_{22} be the deflections at point 1 & 2 respectively. Strain energy stored in a beam.

$$U_2 = \frac{1}{2} P\delta_{22} \rightarrow (4).$$

Let Additional strain energy due to this load.

$$U_1 = \frac{1}{2} P\delta_{11} + P\delta_{21} \rightarrow (5).$$

Total strain energy. $\Rightarrow U = U_2 + U_1$.

$$U = \frac{1}{2} P\delta_{22} + \frac{1}{2} P\delta_{11} + P\delta_{21} \rightarrow (6).$$

Since strain energy is same in both the cases equating (3) & (6).

$$\frac{1}{2} P\delta_{11} + \frac{1}{2} P\delta_{22} + P\delta_{12} = \frac{1}{2} P\delta_{11} + \frac{1}{2} P\delta_{22} + P\delta_{21}.$$

$$P\delta_{12} = P\delta_{21}$$

$$\delta_{12} = \delta_{21}.$$

or in general. $\Rightarrow \delta_{ij} = \delta_{ji}$.

hence maxwells Reciprocal theorem is proved

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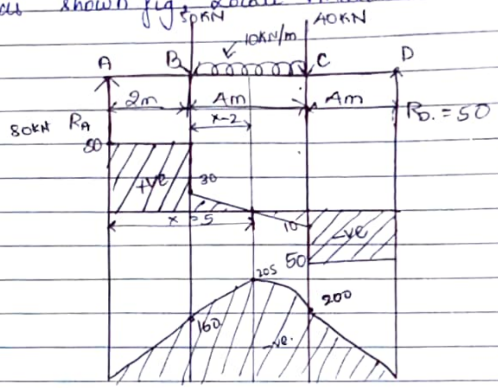
QP. Draw the SFD & BMD of the Simply Supported beam as shown fig, Locate maximum bending Moment.



$M_A = 0$

$R_D(10) - 40(6) - 40(4) - 50(2) + R_A(0) = 0.$

$R_D = 50KN.$

$R_A + R_D = 50 + 40 + 40 \Rightarrow R_A + 50 = 130 \Rightarrow R_A = 80KN$

$SF_x = +80 - 50 - 10(x-2) = 0. \Rightarrow 30 - 10x + 20 = 0$

$50 - 10x = 0 \Rightarrow 50 = 10x$

$x = 5.$

$M_A = 0 = M_D \Rightarrow M_B = 160 KN\text{-}m.$

$M_x = 80(5) - 50(3) - 10(x-2)\frac{(x-2)}{2} \quad x = 5 = 205$

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