Gopalan College of Engineering and Management

(ISO 9001:2015)

Approved by All India Council for Technical Education (AICTE), New Delhi

Affiliated to Visvesvaraya Technological University (VTU), Belagavi, Karnataka

Recognised by Govt. of Karnataka

Address: 181/1, 182/1, Sonnenahalli, Hoodi, K.R.Puram, Whitefield, Bangalore, Karnataka - 560 048

Phone No: (080) - 42229748 **Email:** gcem@gopalancolleges.com **Website:** www.gopalancolleges.com/gcem

MODULE-WISE SOLUTIONS

Year / Semester	II / IV
Course Code	21AE44
Course Name	Mechanics of Materials

Module 4: Virtual Work Principles, Energy Methods

1. **Determine the strain energy stored within an elastic bar subjected to an axial tensile force of 'P' and length 'L'. (06 Marks) Aug./Sept.2020, 18AE34**

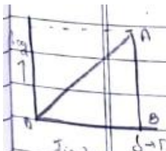


Fig 2.

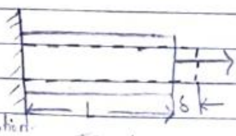


Fig. 1.

Consider a bar of length 'L' cross sectional area 'A' & Youngs Modulus 'E' subjected to an axial pull 'P' as shown in fig. 1.

Let δ be the deformation due to applied Load and σ be the Stress induced.

Fig 2 shows load deformation curve for the given bar.

Work done by the applied load is equal (work done) to area of triangle ΔOAB

$$W = \frac{1}{2} P \delta \longrightarrow (1).$$

Since strain energy stored in the bar is equal to work done by the external load. → U = W.

$$\therefore U = \frac{1}{2} P\delta \longrightarrow (2).$$

WKT. $\left.\begin{array}{l} P = \sigma A \\ \delta = \dfrac{\sigma L}{E} \end{array}\right\} \longrightarrow (3).$

Substituting (3) in (2)

$$U = \frac{1}{2}(\sigma A)\left(\frac{\sigma L}{E}\right)$$

$$U = \frac{\sigma^2 AL}{2E}$$

$$U = \frac{\sigma^2 V}{2E} \longrightarrow (4).$$

(4) is the Exp for strain energy (for) due to Axial Loading

2. Show that the strain energy within beam subjected to a pure bending moment M is $U = \int_0^L \frac{M^2 dx}{2EI}$ (08 Marks) Aug./Sept.2020, 18AE34

Consider a beam Segment of length 'L', MOI → I. & Young's Modulus - E. Subjected to Externally applied Moment 'M' as shown in fig 1. Let θ be the angle subtended by the curvature.

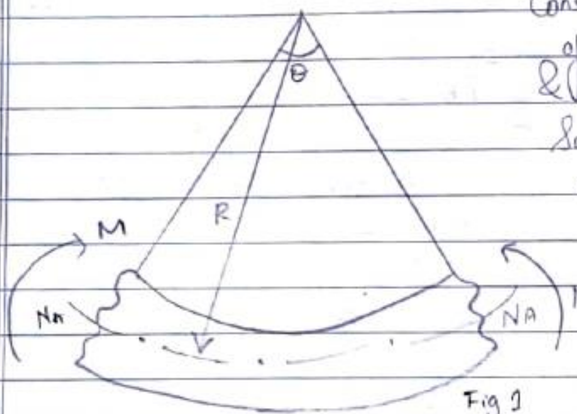
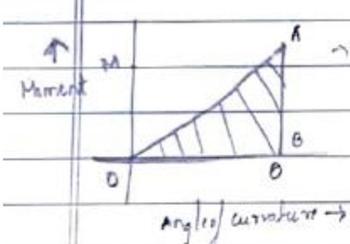


Fig 1

Fig 2 Shows Moment Vs 'θ' curve for the given beam Segment.



→ Fig 2

Work done by the applied moment is equal to area of Δ^{le} AOB $= \frac{1}{2} \times \theta \times M$. → (1)

Since strain energy stored in the beam is equal to work done by the applied moment.

$U = W \Rightarrow \therefore U = \frac{1}{2} M\theta \rightarrow (2)$

$L = R\theta$

From Geometry of fig 1 angle of curvature $\theta = \frac{L}{R}$ → (3)

Substituting (3) in (2)

$U = \frac{1}{2} \times M \times \frac{L}{R} \Rightarrow U = \frac{1}{2}\frac{ML}{R} \rightarrow (4)$

From Bending Eqn. → $\frac{E}{R} = \frac{M}{I} \Rightarrow R = \frac{EI}{M} \rightarrow (5)$

Subst (5) in (4)

$U = \frac{1}{2} \frac{ML}{\frac{EI}{M}}$

$U = \frac{M^2 L}{2EI} \rightarrow (6)$

i.e. (6)th is the expression for Strain Energy due to bending.

If BM is not uniform throughout the beam Segment, Consider a small strip of length dx, Strain Energy stored in the strip ⇒ $dU = \frac{M^2 dx}{2EI}$.

$$U = \int_0^L \frac{M^2 dx}{2EI}$$

3. **A simply supported beam of span 'l' carries a point load 'P' at mid-span. Determine the strain energy stored by the beam. Also find the deflection at mid-span. (06 Marks) Aug./Sept.2020, 18AE34**

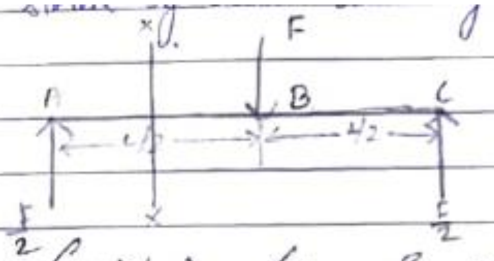


fig shows a Simply supported beam of span 'L' Subjected to point load F at mid span.

Considering beam Segment AB. let x-x be the Section at a distance at a distance x from A.

Moment at Section x-x.

$$M = \frac{Fx}{2} \Rightarrow U_{AB} = \int_0^{L/2} \frac{M^2 dx}{2EI}.$$

$$U_{AB} = \int_0^{L/2} \frac{(Fx/2)^2 dx}{2EI}$$

$$U_{AB} = \int_0^{L/2} \frac{F^2 x^2 dx}{8EI} \Rightarrow U_{AB} = \frac{F^2}{8EI} \int_0^{L/2} x^2 dx.$$

$$U_{AB} = \frac{F^2}{8EI} \left[\frac{x^3}{3} \right]_0^{L/2}$$

$$U_{AB} = \frac{F^2}{24EI} \left[\frac{L}{2} \right]^3 \Rightarrow U_{AB} = \frac{F^2 L^3}{192EI}.$$

from Symmetry of the beam . .

$$U_{AB} = U_{BC} = \frac{F^2 L^3}{192EI}.$$

Total Strain energy in the beam $U = U_{AB} + U_{BC}$.

$$\boxed{U = \frac{F^2 L^3}{96EI}.}$$

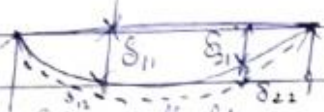
Work done by the applied load, $W = \frac{1}{2} \times F\delta$.

$\because U = W$ Since Strain Energy = Work done.

$$\frac{1}{2} F\delta = \frac{F^2 L^3}{96EI} \Rightarrow \boxed{\delta = \frac{FL^3}{48EI}}$$

4. **State and explain Castigliano's I and II theorem. (10 Marks) Jan./Feb. 2021, 18AE34 (Refer Class Notes)**
5. **State and derive Maxwell's reciprocal theorem. (10 Marks) Jan./Feb. 2021, 18AE34**

Proof :-> Case I :-



Consider a SPB Subjected to load P at point 1.

Let δ_{11} & δ_{21} be the deflections at point 1 & 2 respectively.

Strain energy stored in the beam $U_1 = \frac{1}{2} P\delta_{11} \rightarrow (1)$.

Let Load P be applied at point 2., δ_{12} & δ_{22} are the corresponding deflections at point 1 & 2 respectively.

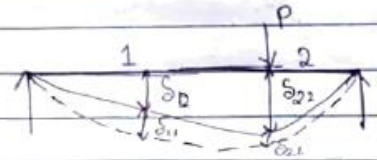
Additional strain energy due to this load.

$$U_2 = \frac{1}{2} P\delta_{22} + P\delta_{12} \rightarrow (2)$$

Total strain energy is $U = U_1 + U_2$.

$$U = \frac{1}{2} P\delta_{11} + \frac{1}{2} P\delta_{22} + P\delta_{12} \rightarrow (3)$$

Case II :-



Consider the same beam subjected to load P at Point 2,

Let δ_{12} & δ_{22} be the deflections at point 1 & 2 respectively.

Strain energy stored in a beam.

$$U_2 = \frac{1}{2} P\delta_{22} \rightarrow (4).$$

Let Additional strain energy due to this load.

$$U_1 = \frac{1}{2} P\delta_{11} + P\delta_{21} \rightarrow (5).$$

Total strain energy. $\Rightarrow U = U_2 + U_1$.

$$U = \frac{1}{2} P\delta_{22} + \frac{1}{2} P\delta_{11} + P\delta_{21} \rightarrow (6).$$

Since strain energy is same in both the cases equating (3) & (6).

$$\frac{1}{2} P\delta_{11} + \frac{1}{2} P\delta_{22} + P\delta_{12} = \frac{1}{2} P\delta_{11} + \frac{1}{2} P\delta_{22} + P\delta_{21}.$$

$$P\delta_{12} = P\delta_{21}$$

$$\delta_{12} = \delta_{21}.$$

or in general $\Rightarrow \delta_{ij} = \delta_{ji}$.

hence maxwells Reciprocal theorem is proved

6. **Define: (i) Flexural Rigidity (ii) Proof Resilience (04 Marks) Feb./Mar. 2022, 18AE34**

   Flexural rigidity, denoted as "EI," is a property of a structural element like a beam that measures its resistance to bending when subjected to an applied load or moment. It is calculated by multiplying the modulus of elasticity (E) of the material by the moment of inertia (I) of the beam. Flexural rigidity quantifies a beam's stiffness and its ability to resist bending.

   Proof resilience is a material property that measures its ability to absorb energy without permanent deformation. It is calculated as $U = \frac{\sigma^2}{2E}$, where σ is the yield stress and E is the Young's modulus.

7. **Consider a beam of length 2 m and diameter 100 mm is applied with point load at the end 2000 N and other end is fixed as cantilever. Determine the strain energy of the beam. Take E = 200 GPa. (08 Marks) June/July 2019, 17AE34**

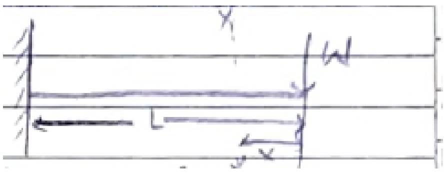


Fig shows a cantilever beam of length L subjected to point load at the free end.
Consider a section x-x at a distance x from the free end.
Moment at section at x-x.

$$M = Wx.$$

Strain Energy stored in the beam $U = \int_0^L \frac{M^2 dx}{2EI}$.

$$U = \int_0^L \frac{(Wx)^2 dx}{2EI} = \int_0^L \frac{W^2 x^2 dx}{2EI} = \frac{W^2}{2EI}\int_0^L x^2 dx.$$

$$U = \frac{W^2}{2EI}\left[\frac{x^3}{3}\right]_0^L \Rightarrow \boxed{U = \frac{W^2 L^3}{6EI}} \rightarrow (1).$$

$$U = \frac{2000^2 \times 2^3 \times 64}{6 \times 200 \times 10^9 \times \pi \times 0.1^4}$$

$$\boldsymbol{U = 5.43\ J}$$

8. **Derive expressions for slope and deflection at the free end of a cantilever beam of length L carrying point load W at its free and using Castiglione's theorem. (10 Marks) Dec.2018/Jan.2019, 15AE34**

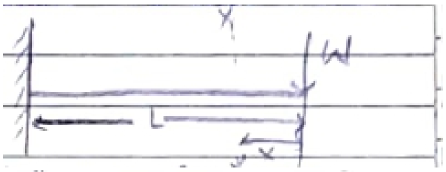


Fig shows a Cantilever beam of length L subjected to point load at the free end.
Consider a section x-x at a distance x from the free end
Moment at section at x-x.

$$M = Wx.$$

Strain Energy stored in the beam $U = \int_0^L \frac{M^2 dx}{2EI}.$

$$U = \int_0^L \frac{(Wx)^2 dx}{2EI} = \int_0^L \frac{W^2 x^2 dx}{2EI} = \frac{W^2}{2EI} \int_0^L x^2 dx.$$

$$U = \frac{W^2}{2EI} \left[\frac{x^3}{3} \right]_0^L \Rightarrow \boxed{U = \frac{W^2 L^3}{6EI}} \rightarrow (1).$$