Theory of Vibration - 18AE56

Old VTU Question's Answers

Module – 5

Syllabus:
Numerical Methods for Multi-Degree Freedom Systems: Introduction, Influence coefficients, Maxwell reciprocal theorem, Dunkerley's equation. Orthogonality of principal modes, Method of matrix iteration-Method of determination of all the natural frequencies using sweeping matrix and Orthogonality principle. Holzer's method, Stodola method.

Part – A Questions

1. Write a short note on influence coefficient.

An influence coefficient, denoted by a_{ij} is defined as the static deflection of the system at position 'i' due to an unit load or an unit force applied at position 'j' of the system when this unit load or force is the only load or force acting on the system.

In the case of torsional systems a_{ij} means, the angular displacement at coordinate 'i' due to an unit torque applied at coordinate 'j'.

For a 'n' degrees of freedom system, the number of influence coefficients will be equal to n^2. However only $n(n+1)/2$ will have different values since $a_{ij} = a_{ji}$.

The determination of influence coefficients are required while writing the general differential equations of motion in matrix form.

In matrix form, it is called flexibility matrix and is denoted by [A].

$$\text{For a multi degree of freedom system } [A] = \begin{bmatrix} a_{11} & a_{12} & a_{13} & \cdots\cdots & a_{1n} \\ a_{21} & a_{22} & a_{23} & \cdots\cdots & a_{2n} \\ - & - & - & - & - \\ - & - & - & - & - \\ - & - & - & - & - \\ a_{n1} & - & - & - & a_{nn} \end{bmatrix}$$

2. State and prove Maxwell's reciprocal theorem.

Maxwell's reciprocal theorem states that in a vibrating system, the deflection at a position 'i' due to an unit load or an unit force applied at position 'j' is equal to the deflection at position 'j' due to an unit load or an unit force at position i.

$$\text{i.e., } a_{ij} = a_{ji}$$

Proof :

Consider a simply supported beam carrying two concentrated loads W_1 and W_2 as shown in Fig 7.1.

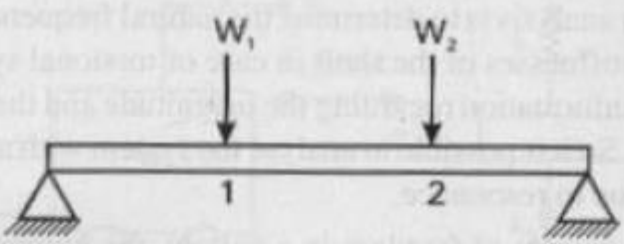


Fig. 7.1

The four influence coefficients are a_{11}, a_{12}, a_{21} and a_{22}. It is necessary to show that $a_{12} = a_{21}$ in order to prove Maxwell's reciprocal theorem. Now consider two cycles. For the first cycle apply W_1 first and then W_2. When W_1 is alone at position (1), the influence coefficients are a_{11} and a_{21}. Potential energy = ½ $W_1^2\ a_{11}$.

Now if W_2 is applied at position (2), the additional energy of the system $= \frac{1}{2} W_2^2 a_{22} + W_1 (W_2 a_{12})$

$$\therefore \text{Total energy of the system} = \frac{1}{2} W_1^2\ a_{11} + \frac{1}{2} W_2^2\ a_{22} + W_1 (W_2\ a_{12})$$

For the second cycle, apply W_2 first and then W_1. When W_2 is alone at position (2), the influence coefficients are a_{22} and a_{12}. Potential energy $= \frac{1}{2} W_2^2\ a_{22}$.

Now if W_1 is applied at position (1), the additional energy of the system $= \frac{1}{2} W_1^2\ a_{11} + W_2 (W_1\ a_{21})$

$$\therefore \text{Total energy of the system} = \frac{1}{2} W_2^2\ a_{22} + \frac{1}{2} W_1^2\ a_{11} + W_2 (W_1\ a_{21})$$

Since the condition is same at the end of both cycles, the total energy of the system must be the same. Therefore by equating the two total energies, we get $a_{12} = a_{21}$. Hence proved.

3. Determine the influence coefficients for the system shown in below Fig. Take $m_1 = m$; $m_2 = 2m$; $m_3 = 3m$; $l_1 = l_2 = l_3 = l$

[VTU Dec. 2013 / Jan. 2014]

$m_1 = m$

$m_2 = 2m$

$m_3 = 3m$

$l_1 = l_2 = l_3 = l$

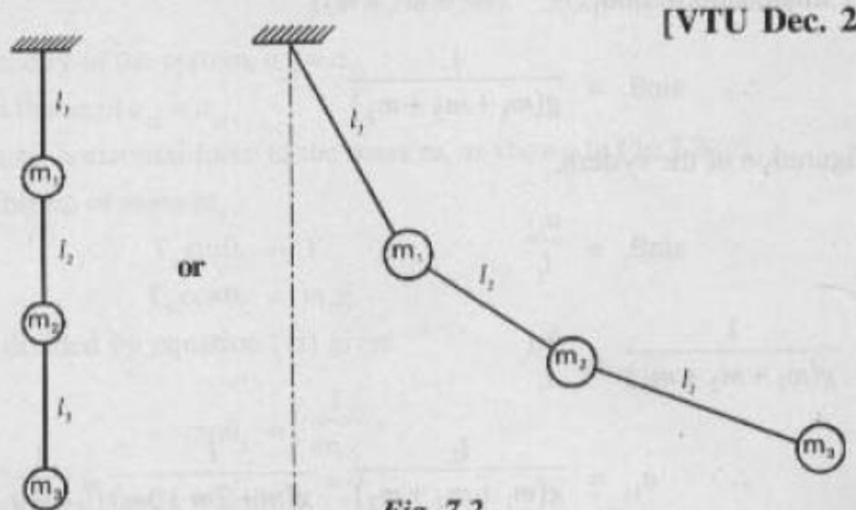


Fig. 7.2

Solution :

From definition influence coefficient a_{ij} is the deflection at the position '*i*' due to an unit force applied at position '*j*'. For a three degree of freedom system, there are nine influence coefficients. They are a_{11}, a_{12}, a_{13}, a_{21}, a_{22}, a_{23}, a_{31}, a_{32}, and a_{33}.

(a) Apply an unit horizontal force to the mass m_1 as shown in Fig 7.3 (a). For the equilibrium of mass m_1.

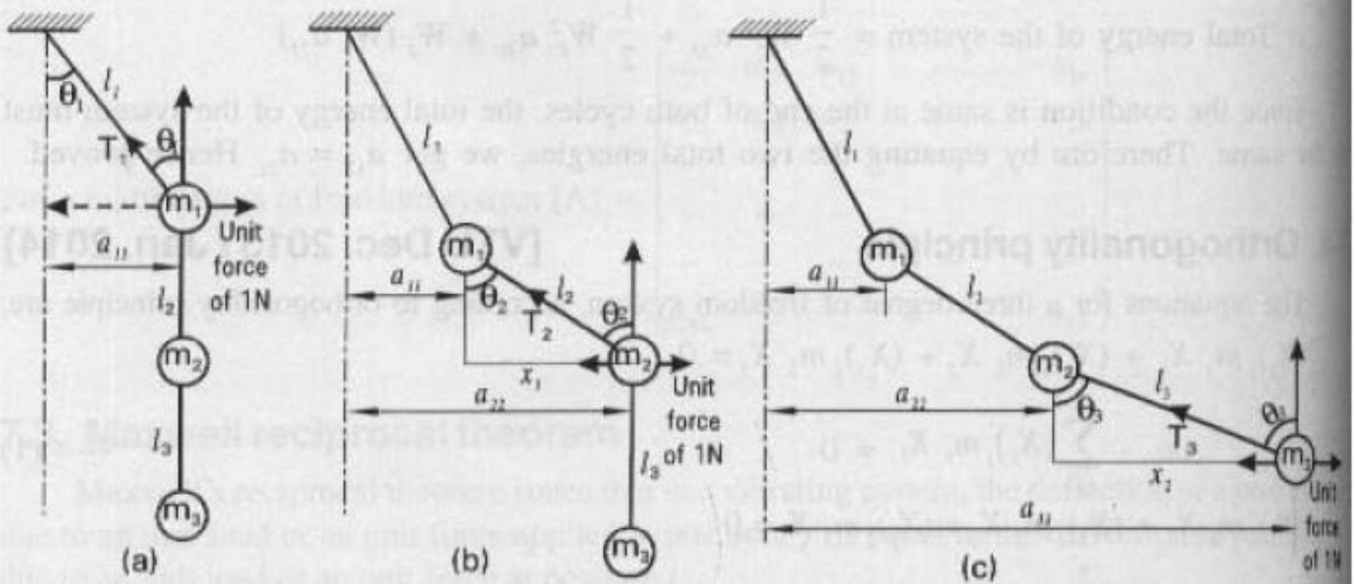


Fig. 7.3

$$T_1 \sin\theta_1 = 1 \quad \text{---(i)}$$

$$T_1 \cos\theta_1 = m_1 g + m_2 g + m_3 g = g(m_1 + m_2 + m_3) \quad \text{---(ii)}$$

Equation (i) divided by equation (ii) gives

$$\tan\theta_1 = \frac{1}{g(m_1 + m_2 + m_3)}$$

Since θ_1 is very small, $\tan\theta_1 \approx \sin\theta_1$

$$\therefore \quad \sin\theta_1 = \frac{1}{g(m_1 + m_2 + m_3)}$$

From the configuration of the system,

$$\sin\theta_1 = \frac{a_{11}}{l_1}$$

$$\therefore \quad \text{i.e.,} \quad \frac{1}{g(m_1+m_2+m_3)} = \frac{a_{11}}{l_1}$$

$$\therefore \quad a_{11} = \frac{l_1}{g(m_1+m_2+m_3)} = \frac{l}{g(m+2m+3m)} = \frac{l}{6mg}$$

From the geometry of the system, $a_{11} = a_{21} = a_{31}$

By Maxwell's reciprocal theorem, $a_{21} = a_{12}$ and $a_{31} = a_{13}$

(b) Apply an unit horizontal force to the mass m_2 as shown in Fig 7.3 (b)

For the equilibrium of mass m_2

$$T_2 \sin\theta_2 = 1 \quad \text{------ (iii)}$$

$$T_2 \cos\theta_2 = m_2 g + m_3 g = g(m_2 + m_3) \quad \text{------ (iv)}$$

Equation (3) divided by (4) gives,

$$\tan\theta_2 = \frac{1}{g(m_2+m_3)}$$

Since θ_2 is very small, $\tan\theta_2 \approx \sin\theta_2$

$$\therefore \quad \sin\theta_2 = \frac{1}{g(m_2+m_3)}$$

Due to the unit horizontal force at mass m_2, let x_1 be the additional displacement of mass m_2. Therefore from the configuration of the system,

$$\sin\theta_2 = \frac{x_1}{l_2}$$

$$\text{i.e.,} \quad \frac{1}{g(m_2+m_3)} = \frac{x_1}{l_2}$$

$$\text{i.e.,} \quad x_l = \frac{l_2}{g(m_2+m_3)}$$

$$\therefore a_{22} = a_{11} + x_l = \frac{l_1}{g(m_1+m_2+m_3)} + \frac{l_2}{g(m_2+m_3)}$$

$$= \frac{l}{g(m+2m+3m)} + \frac{l}{g(2m+3m)} = \frac{l}{6mg} + \frac{l}{5mg} = \frac{11l}{30mg}$$

From the geometry of the system, $a_{22} = a_{32}$

By Maxwell's theorem $a_{32} = a_{23}$

(c) Apply an unit horizontal force to the mass m_3 as shown in Fig 7.3 (c)

For the equilibrium of mass m_3

$$T_3 \sin\theta_3 = 1 \quad \text{----- (v)}$$

$$T_3 \cos\theta_3 = m_3 g \quad \text{----- (vi)}$$

Equation (v) divided by equation (vi) gives

$$\tan\theta_3 = \frac{1}{m_3 g}$$

Since θ_3 is very small, $\tan\theta_3 = \sin\theta_3$

$$\therefore \quad \sin\theta_3 = \frac{1}{m_3 g}$$

Due to the unit horizontal force at m_3, let x_2 be the additional displacement of mass m_3. Hence from the configuration of the system.

$$\sin\theta_3 = \frac{x_2}{l_3}$$

$$\text{i.e.,} \quad \frac{1}{m_3 g} = \frac{x_2}{l_3}$$

$$\therefore \quad x_2 = \frac{l_3}{m_3 g}$$

$$\therefore \quad a_{33} = a_{22} + x_2 = \frac{l_1}{g(m_1 + m_2 + m_3)} + \frac{l_2}{g(m_2 + m_3)} + \frac{l_3}{m_3 g}$$

$$= \frac{l}{6mg} + \frac{l}{5mg} + \frac{l}{3mg} = \frac{21l}{30mg} = \frac{7l}{10mg}$$

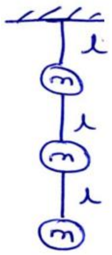
Hence the influence coefficients are,

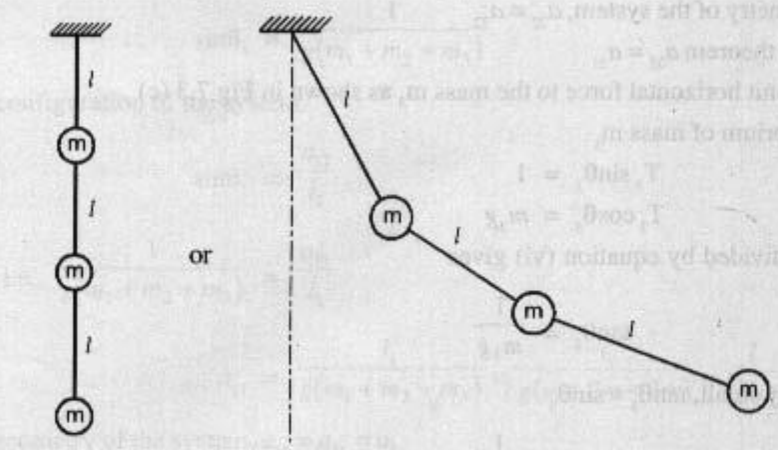
$$a_{11} = a_{12} = a_{13} = a_{21} = a_{31} = \frac{l}{6mg}$$

$$a_{22} = a_{23} = a_{32} = \frac{11l}{30mg}$$

$$a_{33} = \frac{7l}{10mg}$$

4. Determine the influence co-efficient of the triple pendulum shown in below Fig.





Solution :

Apply a unit horizontal force to the top most mass in as shown in Fig 7.5 (a)

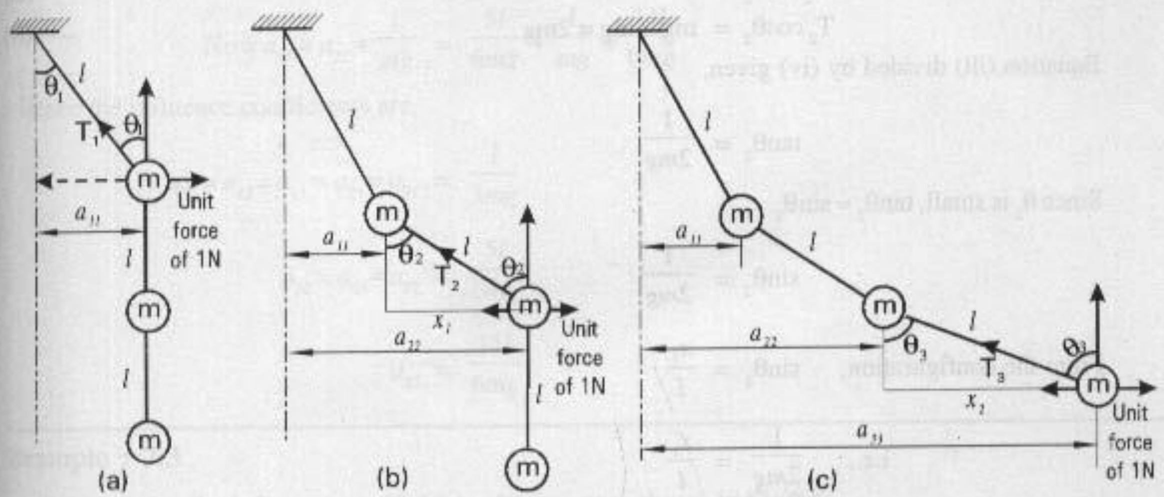


Fig. 7.5

For the equilibrium of mass m

$$T_1 \sin\theta_1 = 1 \quad \text{-----(i)}$$

$$T_1 \cos\theta_1 = mg + mg + mg \quad \text{-----(ii)}$$

Equation (i) divided by (ii) given,

$$\tan\theta_1 = \frac{1}{3mg}$$

Since θ_1 is very small, $\tan\theta_1 \approx \sin\theta_1$

$$\therefore \quad \sin\theta_1 = \frac{1}{3mg}$$

For the configuration of the system,

$$\sin\theta_1 = \frac{a_{11}}{l}$$

$$\text{i.e.,} \quad \frac{1}{3mg} = \frac{a_{11}}{l}$$

$$\therefore \quad a_{11} = \frac{l}{3mg}$$

From the geometry of the system, $a_{11} = a_{21} = a_{31}$

By Maxwell's reciprocal theorem $a_{21} = a_{12}$ and $a_{31} = a_{13}$

$$\therefore a_{11} = a_{12} = a_{13} = a_{21} = a_{31} = \frac{l}{3mg}$$

(b) Apply an unit horizontal force to the middle mass m as shown in Fig 7.5 (b). For the equilibrium of this mass,

$$T_2 \sin\theta_2 = 1 \quad \text{-----(iii)}$$

$$T_2 \cos\theta_2 = mg + mg = 2mg \quad \text{-----(iv)}$$

Equation (iii) divided by (iv) given,

$$\tan\theta_2 = \frac{1}{2mg}$$

Since θ_2 is small, $\tan\theta_2 \approx \sin\theta_2$

$$\therefore \quad \sin\theta_2 = \frac{1}{2mg}$$

From the configuration, $\sin\theta_2 = \frac{x_1}{l}$

i.e., $\frac{1}{2mg} = \frac{x_1}{l}$

$$\therefore \quad x_1 = \frac{l}{2mg}$$

$$\text{Now } a_{22} = a_{11} + x_1 = \frac{l}{3mg} + \frac{l}{2mg} = \frac{5l}{6mg}$$

From the geometry of the system $a_{22} = a_{32}$

By Maxwell's theorem $a_{32} = a_{23}$

$$\therefore \quad a_{22} = a_{23} = a_{32} = \frac{5l}{6mg}$$

(c) Apply an unit horizontal force to the lowest mass m as shown in Fig 7.5 (c). For the equilibrium of this mass,

$$T_3 \sin\theta_3 = 1 \quad \text{----(v)}$$

$$T_3 \cos\theta_3 = mg \quad \text{----(vi)}$$

Equation (v) divided by equation (vi) given,

$$\tan\theta_3 = \frac{1}{mg}$$

Since θ_3 is very small, $\tan\theta_3 \approx \sin\theta_3$

$$\therefore \quad \sin\theta_3 = \frac{1}{mg}$$

From the configuration, $\sin\theta_3 = \frac{x_2}{l}$

i.e., $\frac{1}{mg} = \frac{x_2}{l}$

$$\therefore \quad x_2 = \frac{l}{mg}$$

$$\text{Now } a_{33} = a_{22} + \frac{l}{mg} = \frac{5l}{6mg} + \frac{l}{mg} = \frac{11l}{6mg}$$

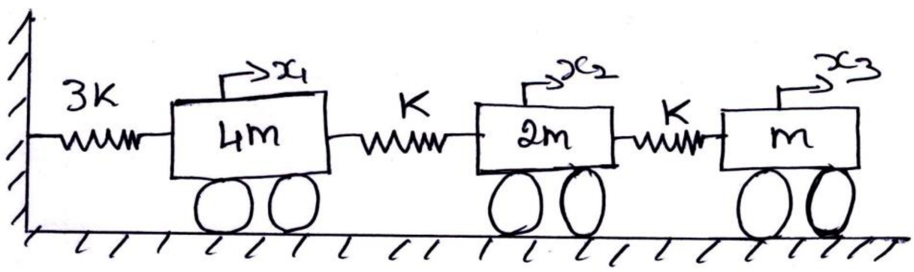
Hence the influence coefficients are,

$$a_{11} = a_{12} = a_{13} = a_{21} = a_{31} = \frac{l}{3mg}$$

$$a_{22} = a_{23} = a_{32} = \frac{5l}{6mg}$$

$$a_{33} = \frac{11l}{6mg}$$

5. For the system shown in below Fig, determine the influence coefficient



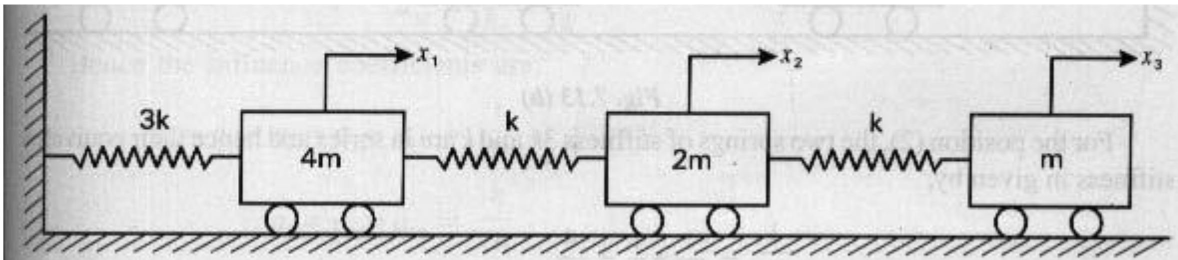


Fig. 7.12

Solution :

Assume an unit force is applied to the mass $4m$ as shown in Fig 7.13 (a).

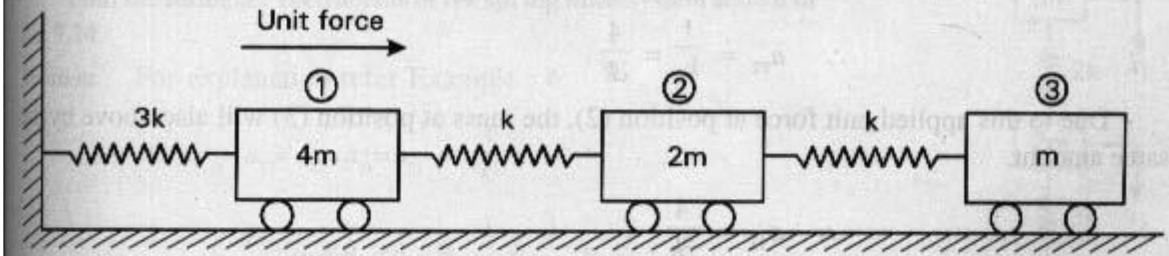


Fig. 7.13 (a)

Deflection at position (1) due to an unit force at position (1) is a_{11}

$$\therefore \quad a_{11} = \frac{1}{3k} \quad (\because \text{Deflection} = \frac{\text{Force}}{\text{Stiffness}})$$

Due to this applied unit force at position (1), the masses at position (2) and (3) will also move by the same amount.

$$\therefore \quad a_{21} = a_{31} = a_{11} = \frac{1}{3k}$$

where a_{21} = Deflection at position (2) due to the unit force at position(1)

a_{31} = Deflection at position (3) due to the unit force at position(1)

By Maxwell's reciprocal theorem,

$$a_{21} = a_{12} = \frac{1}{3k} \quad \text{and } a_{31} = a_{13} = \frac{1}{3k}$$

Now apply an unit force at mass $2m$ as shown in unit force Fig 7.13 (b)

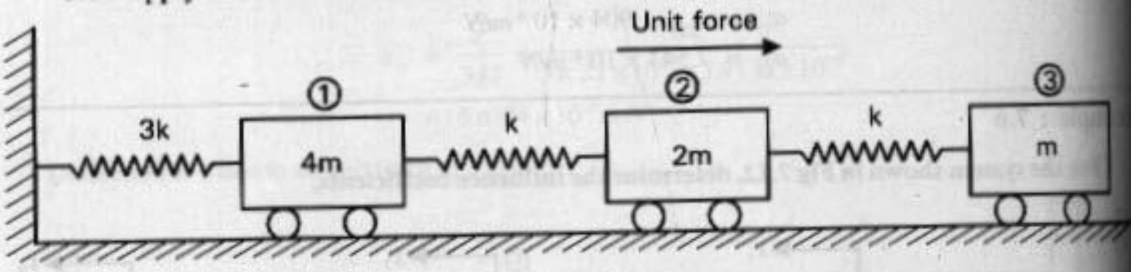


Fig. 7.13 (b)

For the position (2), the two springs of stiffness $3k$ and k are in series and hence their equivalent stiffness in given by,

$$\frac{1}{k_e} = \frac{1}{3k} + \frac{1}{k} = \frac{4}{3k}$$

$$\therefore \quad k_e = \frac{3k}{4}$$

Deflection at position (2) due to unit force at position (2) is a_{22}

$$\therefore \quad a_{22} = \frac{1}{k_e} = \frac{4}{3k}$$

Due to this applied unit force at position (2), the mass at position (3) will also above by the same amount.

$$\therefore \quad a_{32} = \frac{4}{3k}$$

By Maxwell's reciprocal theorem

$$a_{32} = a_{23} = \frac{4}{3k}$$

Apply an unit force at mass m as shown in Fig 7.13 (c),

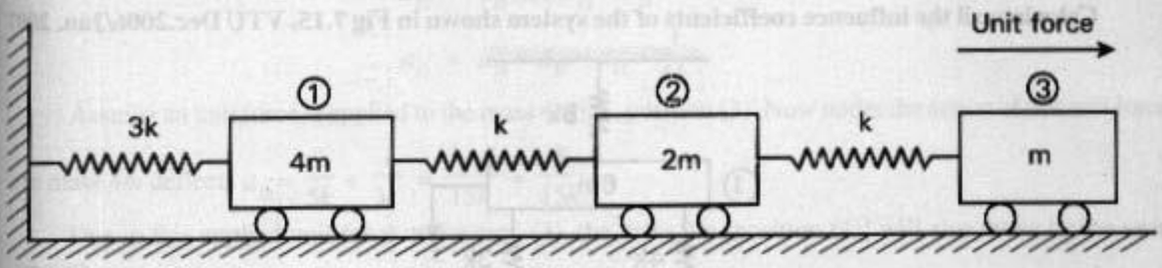


Fig. 7.13 (c)

For the position (3), the three springs of stiffness $3k$, k and k are in series and their equivalent stiffness is given by,

$$\frac{1}{k_e} = \frac{1}{3k} + \frac{1}{k} + \frac{1}{k} = \frac{7}{3k}$$

$$\therefore \quad k_e = \frac{3k}{7}$$

Deflection at position (3) due to unit force at position (3) is a_{33}

$$\therefore \quad a_{33} = \frac{1}{k_e} = \frac{7}{3k}$$

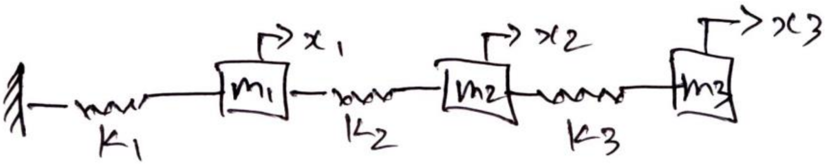
Hence the influence coefficients are,

$$a_{11} = a_{12} = a_{13} = a_{21} = a_{31} = \frac{1}{3k}$$

$$a_{22} = a_{23} = a_{32} = \frac{4}{3k}$$

$$a_{33} = \frac{7}{3k}$$

6. Find the flexibility influence coefficients for the system shown in below Fig



Solution:

Approach: Use the definition of k_{ij} and static equilibrium equations.

Let x_1, x_2, and x_3 denote the displacements of the masses m_1, m_2, and m_3, respectively. The stiffness influence coefficients k_{ij} of the system can be determined in terms of the spring stiffnesses

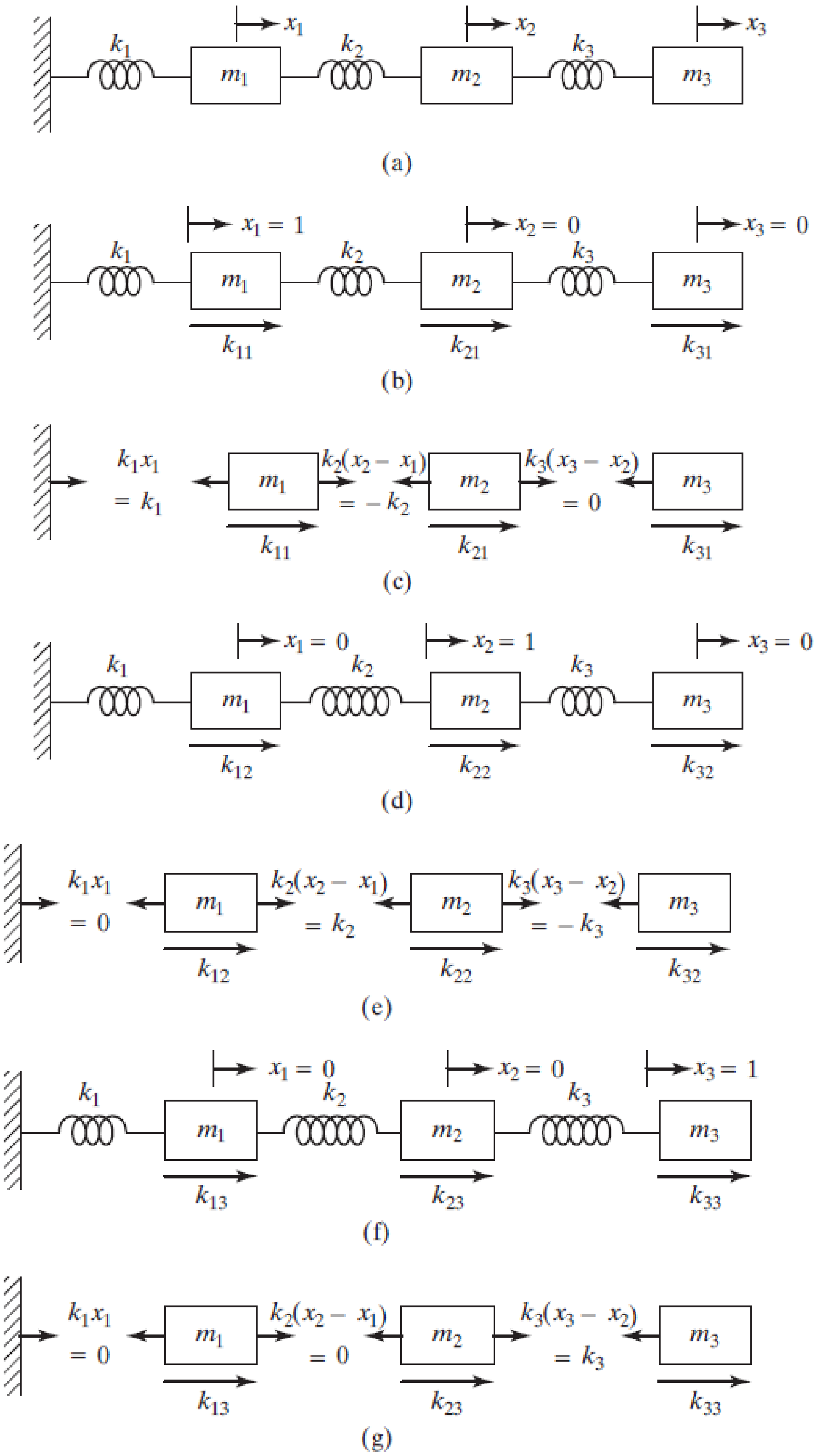


FIGURE 6.6 Determination of stiffness influence coefficients.

k_1, k_2, and k_3 as follows. First, we set the displacement of m_1 equal to one ($x_1 = 1$) and the displacements of m_2 and m_3 equal to zero ($x_2 = x_3 = 0$), as shown in Fig. 6.6(b). The set of forces k_{i1} ($i = 1, 2, 3$) is assumed to maintain the system in this configuration. The free-body diagrams of the masses corresponding to the configuration of Fig. 6.6(b) are indicated in Fig. 6.6(c). The equilibrium of forces for the masses m_1, m_2, and m_3 in the horizontal direction yields

$$\text{Mass } m_1:\ k_1 = -k_2 + k_{11} \tag{E.1}$$

$$\text{Mass } m_2:\ k_{21} = -k_2 \tag{E.2}$$

$$\text{Mass } m_3:\ k_{31} = 0 \tag{E.3}$$

The solution of Eqs. (E.1) to (E.3) gives

$$k_{11} = k_1 + k_2, \qquad k_{21} = -k_2, \qquad k_{31} = 0 \tag{E.4}$$

Next the displacements of the masses are assumed as $x_1 = 0$, $x_2 = 1$, and $x_3 = 0$, as shown in Fig. 6.6(d). Since the forces $k_{i2}\,(i = 1, 2, 3)$ are assumed to maintain the system in this configuration, the free-body diagrams of the masses can be developed as indicated in Fig. 6.6(e). The force equilibrium equations of the masses are:

$$\text{Mass } m_1:\ k_{12} + k_2 = 0 \tag{E.5}$$

$$\text{Mass } m_2:\ k_{22} - k_3 = k_2 \tag{E.6}$$

$$\text{Mass } m_3:\ k_{32} = -k_3 \tag{E.7}$$

The solution of Eqs. (E.5) to (E.7) yields

$$k_{12} = -k_2, \qquad k_{22} = k_2 + k_3, \qquad k_{32} = -k_3 \tag{E.8}$$

Finally the set of forces $k_{i3}\,(i = 1, 2, 3)$ is assumed to maintain the system with $x_1 = 0$, $x_2 = 0$, and $x_3 = 1$ (Fig. 6.6(f)). The free-body diagrams of the various masses in this configuration are shown in Fig. 6.6(g), and the force equilibrium equations lead to

$$\text{Mass } m_1:\ k_{13} = 0 \tag{E.9}$$

$$\text{Mass } m_2:\ k_{23} + k_3 = 0 \tag{E.10}$$

$$\text{Mass } m_3:\ k_{33} = k_3 \tag{E.11}$$

The solution of Eqs. (E.9) to (E.11) yields

$$k_{13} = 0, \qquad k_{23} = -k_3, \qquad k_{33} = k_3 \tag{E.12}$$

Thus the stiffness matrix of the system is given by

$$[k] = \begin{bmatrix} (k_1 + k_2) & -k_2 & 0 \\ -k_2 & (k_2 + k_3) & -k_3 \\ 0 & -k_3 & k_3 \end{bmatrix} \tag{E.13}$$

7. Explain Dunkerley's method.

This method is semi-empirical which gives approximate results and it is used when the diameter of the shaft is uniform. According to Dunkerley's method a shaft subjected to number of point loads and *udl* is,

$$\frac{1}{f_n^2} = \frac{1}{f_{n_1}^2} + \frac{1}{f_{n_2}^2} + \frac{1}{f_{n_3}^2} + \text{------} + \frac{1}{f_{n_s}^2} \qquad \text{-----}\ (7.6.1)$$

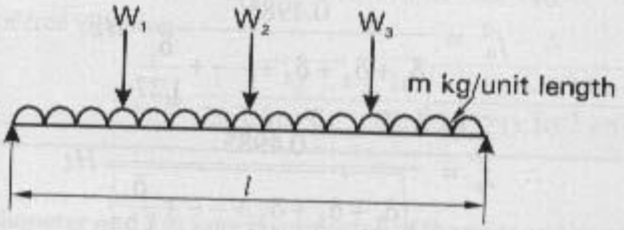


Fig. 7.26

Let W_1, W_2, W_3, etc., = Concentrated loads at different points along the shaft.

m_1, m_2, m_3, etc., = Corresponding masses of loads W_1, W_2, W_3, etc.

f_n = Natural frequency of the shaft subjected to number of point loads and *u.d.l.*

f_{n_1} = Natural frequency of the shaft due to load W_1 alone.

f_{n_2} = Natural frequency of the shaft due to load W_2 alone.

f_{n_3} = Natural frequency of the shaft due to load W_3 alone.

f_{n_s} = Natural frequency of the shaft due to *u.d.l.* alone.

δ_1 = Deflection of the shaft under the load W_1 due to W_1 alone.

δ_2 = Deflection of the shaft under the load W_2 due to W_2 alone

δ_3 = Deflection of the shaft under the load W_3 due to W_3 alone

δ_s = Deflection due to *udl* alone.

$$\therefore \quad f_{n_1} = \frac{0.4985}{\sqrt{\delta_1}}\,Hz \quad \text{or} \quad f_{n_1} = \frac{1}{2\pi}\sqrt{\frac{g}{\delta_1}}\,Hz$$

$$f_{n_2} = \frac{0.4985}{\sqrt{\delta_2}}\,Hz \quad \text{or} \quad f_{n_2} = \frac{1}{2\pi}\sqrt{\frac{g}{\delta_2}}\,Hz$$

$$f_{n_3} = \frac{0.4985}{\sqrt{\delta_3}}\,Hz \quad \text{or} \quad f_{n_3} = \frac{1}{2\pi}\sqrt{\frac{g}{\delta_3}}\,Hz$$

$$f_{n_s} = \frac{0.5614}{\sqrt{\delta_s}}\,Hz \quad \text{or} \quad f_{n_s} = \frac{\pi}{2}\sqrt{\frac{EI}{ml^4}} = \frac{\pi}{2}\sqrt{\frac{5}{384}\times\frac{g}{\delta_s}}\quad Hz$$

$$\left[\because \quad \delta_s = \frac{5}{384}\,\frac{mgl^4}{EI}\ ;\quad \therefore \quad \frac{EI}{ml^4} = \frac{5}{384}\times\frac{g}{\delta_s}\right]$$

Substituting these values in Dunkerley's equation

$$\frac{1}{f_n^2} = \frac{\delta_1}{0.4985^2} + \frac{\delta_2}{0.4985^2} + \frac{\delta_3}{0.4985^2} + - - - - - + \frac{\delta_s}{0.5614^2}$$

$$= \frac{1}{0.4985^2}\left[\delta_1 + \delta_2 + \delta_3 + - - - - - - + \frac{\delta_s}{1.27}\right]$$

$$\therefore \quad f_n^2 = \frac{0.4985^2}{\delta_1 + \delta_2 + \delta_3 + - - + \frac{\delta_s}{1.27}}\,Hz$$

$$\therefore \quad f_n = \frac{0.4985}{\sqrt{\delta_1 + \delta_2 + \delta_3 + - - + \frac{\delta_s}{1.27}}}\,Hz \qquad \text{----- (7.6.2)}$$

or

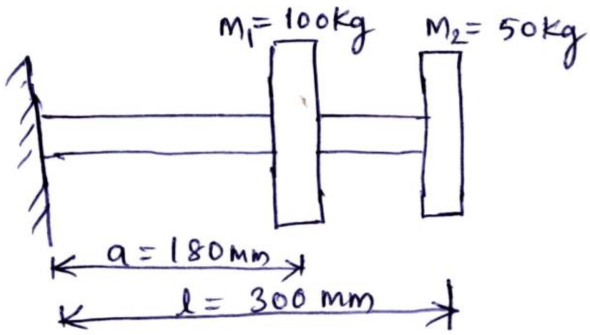
$$f_n = \frac{1}{2\pi}\sqrt{\frac{g}{\delta_1 + \delta_2 + \delta_3 + \ldots\ldots\ldots + \frac{\delta_s}{1.27}}}\quad Hz$$

If there is no uniformly distributed load,

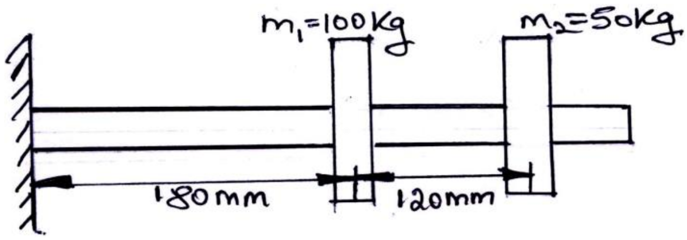
$$f_n = \frac{0.4985}{\sqrt{\delta_1 + \delta_2 + \delta_3 + - - -}} Hz \qquad \text{----- (7.6.3)}$$

$$= \frac{1}{2\pi}\sqrt{\frac{g}{\delta_1 + \delta_2 + \delta_3 + ...}}$$

8. Find the natural frequency of the system shown in below Fig by Dunkerly's method. Take E = 1.96×10^{11} N/m2, I = 4×10^{-7} m^4.



or



Solution :

By Dunkerley's method

Static deflection due to mass 100 kg

$$\delta_1 = \frac{m_1 g a^3}{3EI} = \frac{100 \times 9.81 \times 0.18^3}{3 \times 1.96 \times 10^{11} \times 4 \times 10^{-7}} = 2.4325 \times 10^{-5}\, m$$

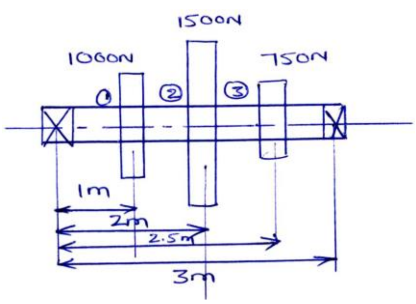
Static deflection due to mass 50 kg

$$\delta_2 = \frac{m_2 g l^3}{3EI} = \frac{50 \times 9.81 \times 0.3^3}{3 \times 1.96 \times 10^{11} \times 4 \times 10^{-7}} = 5.631 \times 10^{-5}\, m$$

Natural frequency of free vibration

$$f_{n_1} = \frac{1}{2\pi}\sqrt{\frac{g}{\delta_1 + \delta_2}} = \frac{1}{2\pi}\sqrt{\frac{9.81}{2.4325 \times 10^{-5} + 5.631 \times 10^{-5}}} = 55.5\, Hz$$

9. A shaft shown in below Fig. of 50 mm diameter and 3m long is supported at the end and carries three weight of 1000 N, 1500 N and 750 N at 1m, 2m and 2.5m from the left support. Taking E = 200 GPa, find the frequency of transverse vibration.



Solution :

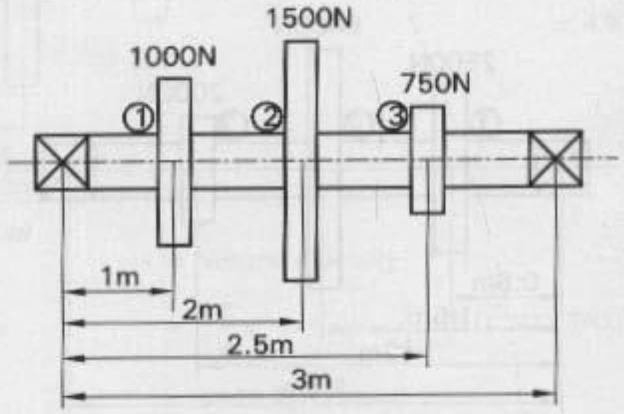


Fig. 7.28

$$d = 50 \text{ mm} = 0.05 \text{ m}$$

$$E = 200 \text{ GPa} = 200 \times 10^9 \text{ N/m}^2$$

$$I = \frac{\pi}{64} d^4 = \frac{\pi}{64} \times (0.05)^4 = 0.3068 \times 10^{-6} \text{ m}^4$$

By Dunkerley's method

Static deflection due to weight 1000 N

$$\delta_1 = W_1 a_{11} = \frac{W_1 l_1^2 l_2^2}{3EIl} = \frac{1000 \times 1^2 \times 2^2}{3 \times 200 \times 10^9 \times 0.3068 \times 10^{-6} \times 3} = 7.243 \times 10^{-3} \text{ m}$$

Static deflection due to weight 1500 N

$$\delta_2 = W_2 a_{22} = \frac{W_2 l_1^2 l_2^2}{3EIl} = \frac{1500 \times 2^2 \times 1^2}{3 \times 200 \times 10^9 \times 0.3068 \times 10^{-6} \times 3} = 10.864 \times 10^{-3} \text{ N}$$

Static deflection due to weight 750 N

$$\delta_3 = W_3 a_{33} = \frac{W_3 l_1^2 l_2^2}{3EIl} = \frac{750 \times 2.5^2 \times 0.5^2}{3 \times 200 \times 10^9 \times 0.3068 \times 10^{-6} \times 3} = 2.122 \times 10^{-3} \text{ m}$$

Natural frequency of free transverse vibration

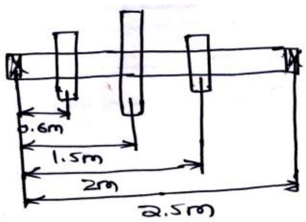
$$f_{n_1} = \frac{1}{2\pi} \sqrt{\frac{g}{\delta_1 + \delta_2 + \delta_3}} \; Hz$$

$$= \frac{1}{2\pi} \sqrt{\frac{9.81}{7.243 \times 10^{-3} + 10.864 \times 10^{-3} + 2.122 \times 10^{-3}}} = 3.5 \text{ Hz.}$$

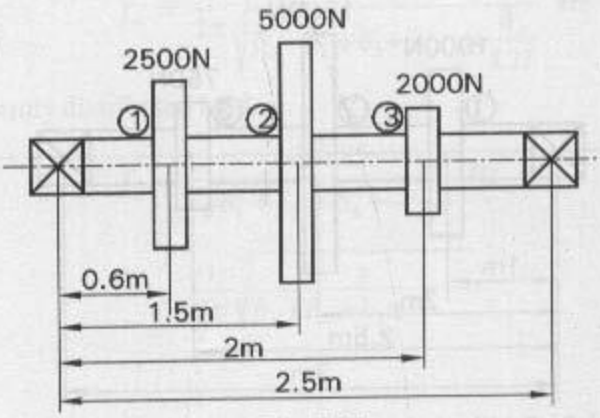
10. A shaft 180mm diameter is supported in two bearing 2.5m apart. It carries 3 discs of weight 2500N, 5000N and 2000N at 0.6m, 1.5m and 2m from left end. Assume the shaft weight to be 1900N/m length. Determine the natural frequency of transverse vibration by Dunkerley's method. Take E = 200GPa

Or/ Similar

A shaft 180mm dia is supported at 2.5m apart. It carries three discs of weight 2500N, 500N and 2000N at 0.6m, 1.5m and 2m from left end. Assume shaft weight to be 1900N/m and E = 200GPa. Determine the natural frequency of transverse vibration.



Solution :



$$I = \frac{\pi}{64}d^4 = \frac{\pi}{64}\times 0.18^4 = 5.15\times10^{-5}\,\text{m}^4$$

Fig. 7.29

Static deflection due to load 2500 N $\left.\right\}$ $\delta_1 = W_1a_{11} = \frac{W_1 l_1^2 l_2^2}{3EIl} = \frac{2500\times0.6^2\times1.9^2}{3\times200\times50^9\times5.15\times10^{-5}\times2.5} = 4.2\times10^{-5}\text{m}$

Static deflection due to load 5000 N $\left.\right\}$ $\delta_2 = W_2a_{22} = \frac{W_2 l_1^2 l_2^2}{3EIl} = \frac{5000\times1.5^2\times1^2}{3\times200\times10^9\times5.15\times10^{-5}\times2.5} = 1.46\times10^{-4}\text{m}$

Static deflection due to load 2000 N $\left.\right\}$ $\delta_3 = W_3a_{33} = \frac{W_3 l_1^2 l_2^2}{3EIl} = \frac{2000\times2^2\times0.5^2}{3\times200\times10^9\times5.15\times10^{-5}\times2.5} = 2.6\times10^{-5}\text{m}$

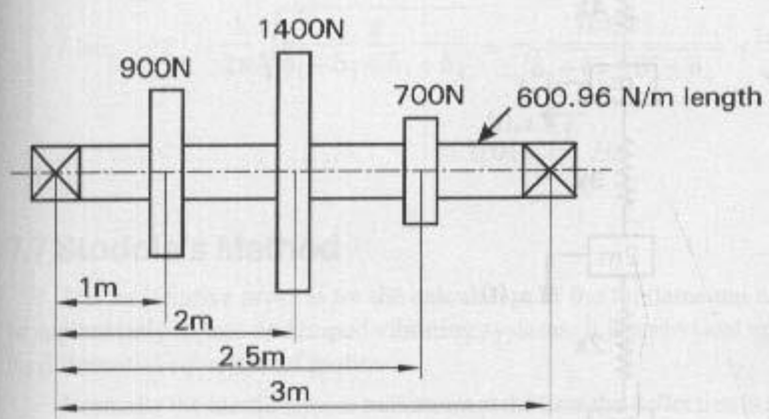
Static deflection due to *udl* $\left.\right\}$ $\delta_s = \frac{5}{384}\frac{wl^4}{EI} = \frac{5}{384}\times\frac{1900\times2.5^4}{200\times10^9\times5.15\times10^{-5}} = 9.3\times10^{-5}\text{m}$

Natural frequency of free transverse vibration by Dunkerley's method $\left.\right\}$ $\therefore f_{n_1} = \frac{1}{2\pi}\sqrt{\frac{g}{\delta_1+\delta_2+\delta_3+\frac{\delta_s}{1.27}}}$

$$= \frac{1}{2\pi}\sqrt{\frac{9.81}{4.2\times10^{-5}+1.46\times10^{-4}+2.6\times10^{-5}+\frac{9.3\times10^{-5}}{1.27}}} = \mathbf{29.38\ Hz.}$$

11. A shaft 100mm diameter is supported in short bearing 3m apart and carries 3 discs weighing 900N, 1400N, 700N situated in 1m, 2m and 2.5m from one of the bearings respectively. Assuming E = 200GPa and density of shaft material = 7800 Kg/m3, calculate the frequency of transverse vibration, by Dunkerley's method.

Solution :



$$E = 200\times10^9\,\text{N/m}^2$$

$$I = \frac{\pi}{64}d^4 = \frac{\pi}{64}\times(0.1)^4 = 4.908\times10^{-6}\,\text{m}^4$$

Fig. 7.30

$$m = \text{Volume}\times\text{Density}$$

$$= \left(\frac{\pi}{4}d^2\right)\times l\times\rho = \frac{\pi}{4}\times(0.1)^2\times1\times7800$$

$$= 61.26\ \text{kg/m length}$$

$$\therefore \quad w = mg = 61.26 \times 9.81 = 600.96 \text{ N/m length}$$

$$\delta_1 = W_1 a_{11} = \frac{W_1 l_1^2 l_2^2}{3EIl} = \frac{900 \times 1^2 \times 2^2}{3 \times 200 \times 10^9 \times 4.908 \times 10^{-6} \times 3} = 4.07 \times 10^{-4} \text{m}$$

$$\delta_2 = W_2 a_{22} = \frac{W_2 l_1^2 l_2^2}{3EIl} = \frac{1400 \times 2^2 \times 1^2}{3 \times 200 \times 10^9 \times 4.908 \times 10^{-6} \times 3} = 6.33 \times 10^{-4} \text{m}$$

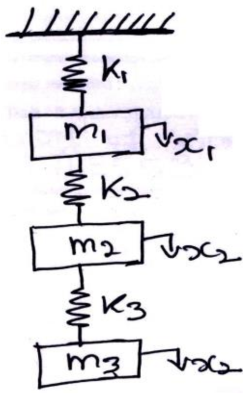
$$\delta_3 = W_3 a_{33} = \frac{W_3 l_1^2 l_2^2}{3EIl} = \frac{700 \times 2.5^2 \times 0.5^2}{3 \times 200 \times 10^9 \times 4.908 \times 10^{-6} \times 3} = 1.24 \times 10^{-4} \text{m}$$

$$\delta_s = \frac{5}{384}\frac{wl^4}{EI} = \frac{5}{384} \times \frac{600.96 \times 3^4}{200 \times 10^9 \times 4.908 \times 10^{-6}} = 6.46 \times 10^{-4} \text{m}$$

$$\therefore \quad f_{n_1} = \frac{1}{2\pi}\sqrt{\frac{g}{\delta_1 + \delta_2 + \delta_3 + \frac{\delta_s}{1.27}}} \; Hz$$

$$= \frac{1}{2\pi}\sqrt{\frac{9.81}{4.07 \times 10^{-4} + 6.33 \times 10^{-4} + 1.24 \times 10^{-4} + \frac{6.46 \times 10^{-4}}{1.27}}} = \mathbf{12.19\ Hz.}$$

12. Using Stodola's method find the fundamental mode of vibration and its natural frequency of spring mass system shown in below Fig. Given K1 = K2 = K3 = 1 N/m, m1 = m2 = m3 = 1 kg.



Solution :

Procedure : (Formulation of Tabular column)

Step 1 :

Assume any arbitrary set of values for deflection (unity for simplicity) to represent the fundamental mode of vibration

i.e., $x_1 = 1$

$\therefore \quad x_1 = 1, x_2 = 1$ and $x_3 = 1$

Enter the assumed diflection of each mass in the first row under mass (m) column.

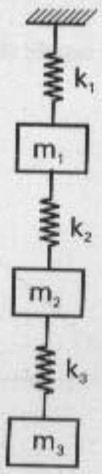


Fig. 7.32

Step 2 :

Determine the inertia force on each mass as below:

Let $x_i = A \sin \omega t \quad \dot{x}_i = A\omega \cos \omega t$

$\ddot{x}_i = -A\omega^2 \sin \omega t \quad \therefore \quad \ddot{x}_i = -\omega^2 x_i$ (maximum)

$$\therefore \quad F_i = -m_i \ddot{x}_i$$

$$= -m_i(-\omega^2 x_i) = m_i \omega^2 x_i$$

i.e., $F_1 = m_1\omega^2 x_1 ; F_2 = m_2\omega^2 x_2 ; F_3 = m_3\omega^2 x_3$

$F_1 = F_2 = F_3 = \omega^2 \; (\because m_1 = m_2 = m_3 = 1; x_1 = x_2 = x_3 = 1)$

Enter the inertia forces of each mass in the 2nd row under mass column.

Step 3 :

Spring force

It is the total inertia force acting on each spring. These must be entered in the third row under stiffness (k) column.

Step 4 :

Spring deflection

Each term in the the third row divided bythe respective spring stiffness is the spring deflection and enter these values in the fourth row under stiffness (k) column.

Step 5 :

Calculated deflection

These are the total deflection of each spring and are obtained by adding the deflection due to each spring. with the mass near the fixed end having the least deflection and so on. These are entered in the fifth row under mass (m) column.

Step 6 :

These are the normalised values of fifth row. The entries in this step (6) are compared with the assumed deflection of step 1. The process is continued untill the calculated deflections are equal or proportional to the asumed deflections. When this is achieved the assumed deflection will represent the fundamental mode of vibration.

$$\text{Inertia force} = m_i x_i \omega^2$$

$$\text{But } m_1 = m_2 = m_3 = 1 \text{ and } x_1 = x_2 = x_3 = 1 \text{ (given)}$$

$$\therefore \text{ Inertia force} = \omega^2$$

$$\text{Spring stiffness} = \frac{\text{Force}}{\text{Deflection}}$$

$$\therefore \text{ Spring deflection} = \frac{\text{Force}}{\text{Spring stiffness}}$$

$$k_1 = k_2 = k_3 = 1 \text{ (given)}$$

	k_1=1	m_1=1	k_2=1	m_2=1	k_3=1	m_3=1
Trial I						
1. Assumed deflection (x_i)		1		1		1
2. Inertia force		ω^2		ω^2		ω^2
3. Spring force	$3\omega^2$		$2\omega^2$		ω^2	
4. Spring deflection	$3\omega^2$		$2\omega^2$		ω^2	
5. Calculated deflection		$3\omega^2$		$5\omega^2$		$6\omega^2$
		1		1.67		2
Trial II						
1. Assumed deflection		1		1.67		2
2. Inertia force		ω^2		$1.67\omega^2$		$2\omega^2$
3. Spring force	$4.67\omega^2$		$3.67\omega^2$		$2\omega^2$	
4. Spring deflection	$4.67\omega^2$		$3.67\omega^2$		$2\omega^2$	
5. Calculated deflection		$4.67\omega^2$		$8.34\omega^2$		$10.34\omega^2$
		1		1.79		2.21

	k_1	m_1	k_2	m_2	k_3	m_3
Trial III						
1. Assumed deflection		1		1.79		2.21
2. Inertia force		ω^2		$1.79\,\omega^2$		$2.21\,\omega^2$
3. Spring force	$5\,\omega^2$		$4\,\omega^2$		$2.21\,\omega^2$	
4. Spring deflection	$5\,\omega^2$		$4\,\omega^2$		$2.21\omega^2$	
5. Calculated deflection		$5\,\omega^2$		$9\,\omega^2$		$11.21\,\omega^2$
		1		1.8		2.24

The calculated deflection in trial III are very close to the assumed values. Hence the fundamental or principle mode of vibration is given by $\begin{bmatrix} 1.0 \\ 1.8 \\ 2.24 \end{bmatrix}$ and fundamental natural frequency is obtained by equating the sum of the calculated deflection to the sum of the assumed deflections

i.e., $(5+9+11.21)\,\omega^2 = (1+1.8+2.24)$

$\therefore\ \omega_{n_1} = 0.447$ rad/sec.

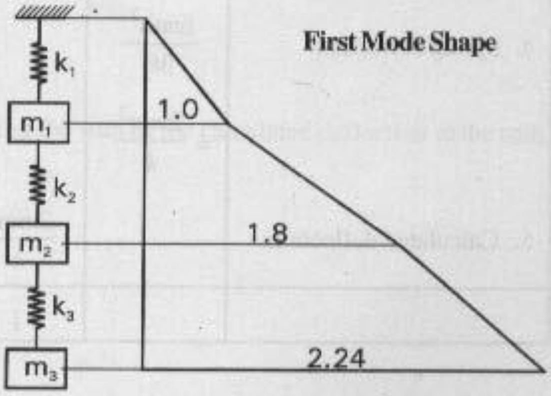
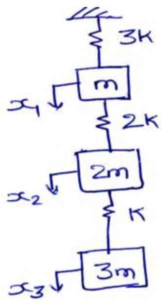


Fig. 7.33

i.e., Fundamental natural frequency $f_{n_1} = \dfrac{1}{2\pi}\omega_{n_1} = \dfrac{1}{2\pi}\times 0.447 = 0.071\,Hz$

13. Using Stodola's method, determine the fundamental mode of vibration and its natural frequency of the spring mass system shown in below Fig.



For the formation of Tabular column refer Example 7.39

	$k_1=3k$	$m_1=m$	$k_2=2k$	$m_2=2m$	$k_3=k$	$m_3=3m$
Trial I						
1. Assumed deflection		1		1		1
2. Inertia force		$m\omega^2$		$2\,m\omega^2$		$3\,m\omega^2$
3. Spring force	$6\,m\omega^2$		$5\,m\omega^2$		$3\,m\omega^2$	
4. Spring deflection	$\dfrac{6m\omega^2}{3k} = \dfrac{2m\omega^2}{k}$		$\dfrac{5m\omega^2}{2k} = \dfrac{2.5m\omega^2}{k}$		$\dfrac{3m\omega^2}{k}$	
5. Calculated deflection		$\dfrac{2m\omega^2}{k}$		$\dfrac{4.5m\omega^2}{k}$		$\dfrac{7.5m\omega^2}{k}$
		1		2.25		3.75

Trial II						
1. Assumed deflection		1		2.25		3.75
2. Inertia force		$m\omega^2$		$4.5m\omega^2$		$11.25m\omega^2$
3. Spring force	$16.75m\omega^2$		$15.75m\omega^2$		$11.25m\omega^2$	
4. Spring deflection	$\frac{16.75m\omega^2}{3k} = \frac{5.58m\omega^2}{k}$		$\frac{15.75m\omega^2}{2k} = \frac{7.87m\omega^2}{k}$		$\frac{11.25m\omega^2}{k}$	
5. Calculated deflection		$\frac{5.58m\omega^2}{k}$		$\frac{13.45m\omega^2}{k}$		$\frac{24.70m\omega^2}{k}$
		1.0		2.41		4.42

Trial III						
1. Assumed deflection		1.0		2.41		4.42
2. Inertia force		$m\omega^2$		$4.82m\omega^2$		$13.26m\omega^2$
3. Spring force	$19.08m\omega^2$		$17.08m\omega^2$		$13.26m\omega^2$	
4. Spring deflection	$\frac{19.08m\omega^2}{3k} = \frac{6.36m\omega^2}{k}$		$\frac{18.08m\omega^2}{2k} = \frac{9.04m\omega^2}{k}$		$\frac{13.26m\omega^2}{k}$	
5. Calculated deflection		$\frac{6.36m\omega^2}{k}$		$\frac{15.4m\omega^2}{k}$		$\frac{28.66m\omega^2}{k}$
		1.0		2.42		4.5

As the assumed deflections are almost equal to the calculated deflections. The principal mode of vibration may be taken as $\begin{Bmatrix} 1.0 \\ 2.42 \\ 4.5 \end{Bmatrix}$

Fundamental natural frequency is obtained by equating the sum of the calculated deflection to the sum of the assumed deflection.

$$\frac{6.36m\omega^2}{k} + \frac{15.4m\omega^2}{k} + \frac{28.66m\omega^2}{k} = 1 + 2.42 + 4.45$$

$$\frac{50.42m\omega^2}{k} = 7.87$$

$$\therefore \quad \frac{m\omega^2}{k} = 0.156 \text{ is } \omega^2 = 0.156\,\frac{k}{m}$$

$$\therefore \quad \omega_{n_1} = 0.395\sqrt{\frac{k}{m}} \text{ rad/sec}$$

i.e., Fundamental natural frequency $f_{n_1} = \frac{1}{2\pi}\omega_{n_1} = \frac{1}{2\pi} \times 0.395\sqrt{\frac{k}{m}} = 0.063\sqrt{\frac{k}{m}}\,Hz$

First Mode Shape

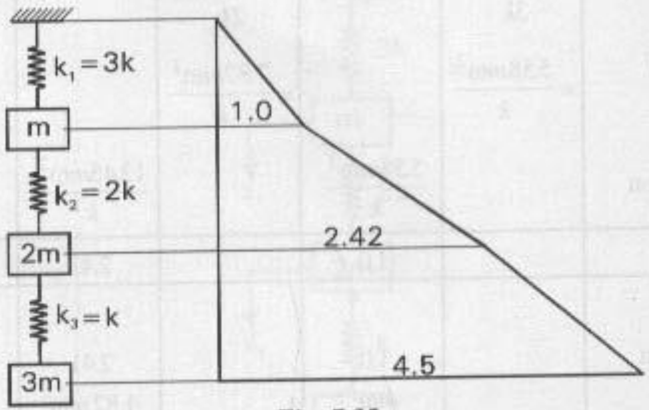
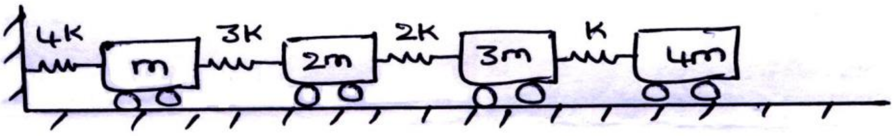


Fig. 7.35

14. Using Stodola's method, determine the lower natural frequency of the system shown in below Fig



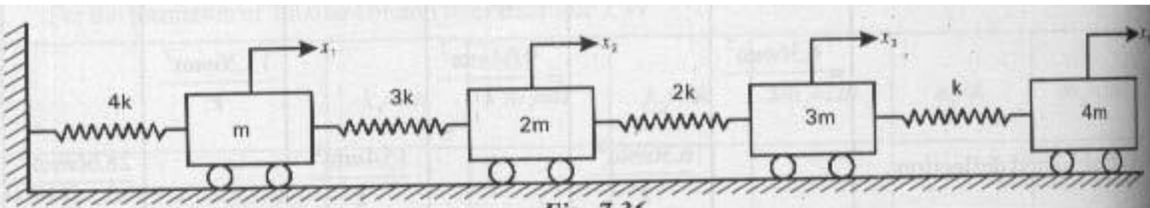


Fig. 7.36

Solution : *[VTU, Dec'07/Jan' 08, June/July ' 08, Dec. 2012, Dec. 2013 / Jan. 2014]*

For the formation of Tabular column refer Example 7.21

	$k_1=4k$	$m_1=m$	$k_2=3k$	$m_2=2m$	$K_3=2k$	$m_3=3m$	$k_4=k$	$m_4=4m$
Trial I								
1. Assumed deflection (x_i)		1		1		1		1
2. Inertia force $F_i = m_i\omega^2 x_i$		$m\omega^2$		$2m\omega^2$		$3m\omega^2$		$4m\omega^2$
3. Spring force	$10m\omega^2$		$9m\omega^2$		$7m\omega^2$		$4m\omega^2$	
4. Spring deflection	$\frac{10m\omega^2}{4k} = \frac{2.5m\omega^2}{k}$		$\frac{9m\omega^2}{3k} = \frac{3m\omega^2}{k}$		$\frac{7m\omega^2}{2k} = \frac{3.5m\omega^2}{k}$		$\frac{4m\omega^2}{k}$	
5. Calculated deflection		$\frac{2.5m\omega^2}{k}$		$\frac{5.5m\omega^2}{k}$		$\frac{9m\omega^2}{k}$		$\frac{13m\omega^2}{k}$
		1		2.2		3.6		5.2

Trial II								
1) Assumed deflection		1		2.2		3.6		5.2
2) Inertia force $F = m_i\omega^2 x_i$		$m\omega^2$		$4.4m\omega^2$		$10.8m\omega^2$		$20.8m\omega^2$
3) Spring force	$37m\omega^2$		$36m\omega^2$		$31.6m\omega^2$		$20.8m\omega^2$	
4) Spring deflection	$\frac{37m\omega^2}{4k} = \frac{9.25m\omega^2}{k}$		$\frac{36m\omega^2}{3k} = \frac{12m\omega^2}{k}$		$\frac{31.6m\omega^2}{2k} = \frac{15.8m\omega^2}{k}$		$\frac{20.8m\omega^2}{k}$	
5) Calculated deflection		$\frac{9.25m\omega^2}{k}$		$\frac{21.25m\omega^2}{k}$		$\frac{37.05m\omega^2}{k}$		$\frac{57.85m\omega^2}{k}$
		1.00		2.3		4		6.3
Trial III								
1) Assumed deflection		1.00		2.3		4		6.3
2) Inertia force $F = m\omega^2 x$		$m\omega^2$		$4.6m\omega^2$		$12m\omega^2$		$25.2m\omega^2$
3) Spring force	$42.8m\omega^2$		$41.8m\omega^2$		$37.2m\omega^2$		$25.2m\omega^2$	
4) Spring deflection	$\frac{42.7m\omega^2}{4k}$		$\frac{41.8m\omega^2}{3k}$		$\frac{37.2m\omega^2}{2k}$		$\frac{25.2m\omega^2}{k}$	
5) Calculated deflection		$\frac{10.7m\omega^2}{k}$		$\frac{24.6m\omega^2}{k}$		$\frac{43.2m\omega^2}{k}$		$\frac{68.4m\omega^2}{k}$
		1.0		2.3		4		6.39

Assumed deflection $\begin{Bmatrix} 1 \\ 2.3 \\ 4 \\ 6.3 \end{Bmatrix} = \begin{Bmatrix} 1 \\ 2.3 \\ 4 \\ 6.39 \end{Bmatrix}$

1st Mode converge $\begin{Bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{Bmatrix} = \begin{Bmatrix} 1 \\ 2.3 \\ 4 \\ 6.39 \end{Bmatrix}$

First mode shape

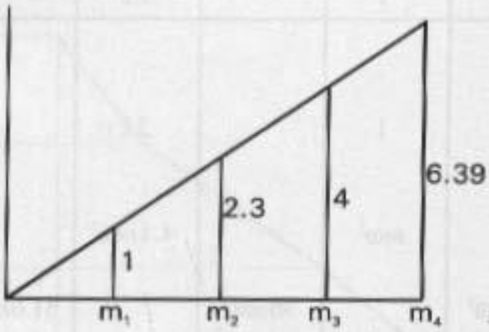


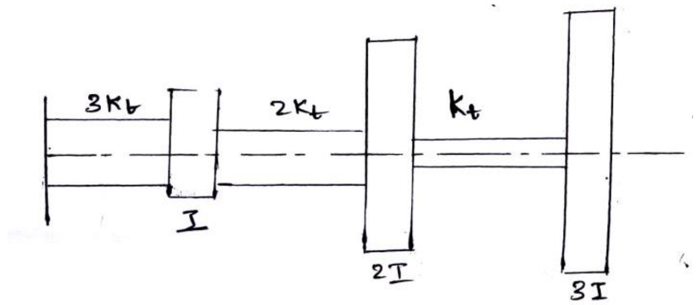
Fig. 7.37

$$[1+2.3+4.0+6.39] = \frac{m\omega^2}{k}[10.7+24.6+43.2+68.4]$$

$$\boxed{\omega_{n_1} = 0.305\sqrt{\frac{k}{m}} \text{ rad/sec}}$$

$$\text{i.e., } f_{n_1} = \frac{1}{2\pi}\omega_{n_1} = \frac{1}{2\pi}\times 0.305\sqrt{\frac{k}{m}} = 0.0485\sqrt{\frac{k}{m}}, Hz$$

15. Using Stodala's method, determine the lowest natural frequency of the torsional system shown in below Fig



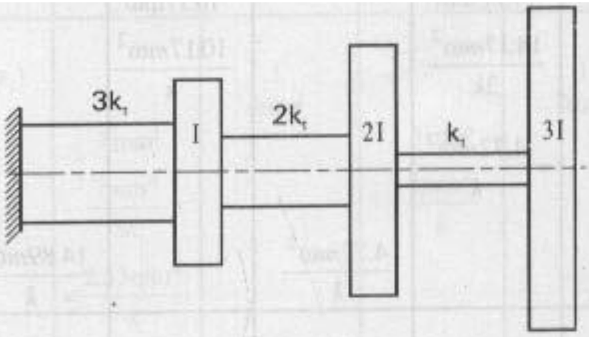


Fig: 7.41

Solution :

For the formation of Tabular column refer Example : 7.24

	$k_{t_1} = 3k_t$	I_1 or $J_1 = I$	$k_{t_2} = 2k_t$	$I_2 = 2I$	$k_{t_3} = k_t$	$I_3 = 3I$
Trial : I						
1 Assumed deflection (θ_i)	1		1		1	
2 Inertia torque (T_i)		$I\omega^2$		$2I\omega^2$		$3I\omega^2$
3 Shaft torque	$6I\omega^2$		$5I\omega^2$		$3I\omega^2$	
4 Shaft twist	$\frac{6I\omega^2}{3k_t} = \frac{2I\omega^2}{k_t}$		$\frac{5I\omega^2}{2k_t} = \frac{2.5I\omega^2}{k_t}$		$\frac{3I\omega^2}{k_t}$	
5 Calculated deflection		$\frac{2I\omega^2}{k_t}$		$\frac{4.5I\omega^2}{k_t}$		$\frac{7.5I\omega^2}{k_t}$
		1		2.25		3.75
Trial : II						
1 Assumed deflection		1		2.25		3.75
2 Inertia torque		$I\omega^2$		$4.5I\omega^2$		$11.25I\omega^2$
3 Shaft torque	$16.75I\omega^2$		$15.75I\omega^2$		$11.25I\omega^2$	
4 Shaft twist	$\frac{16.75I\omega^2}{3k_t}$		$\frac{15.75I\omega^2}{2k_t}$		$\frac{11.25I\omega^2}{k_t}$	

	$= \frac{5.58 I\omega^2}{k_t}$		$= \frac{7.875 I\omega^2}{k_t}$			
5 Calculated deflection		$\frac{5.58 I\omega^2}{k_t}$		$\frac{13.455 I\omega^2}{k_t}$		$\frac{24.705 I\omega^2}{k_t}$
		1		2.41		4.43
Trial : III						
1 Assumed deflection		1		2.41		4.43
2 Inertia torque		$I\omega^2$		$4.82 I\omega^2$		$13.29 I\omega^2$
3 Shaft torque	$19.11 I\omega^2$		$18.11 I\omega^2$		$13.29 I\omega^2$	
4 Shaft twist	$\frac{19.11 I\omega^2}{k_t}$ $= \frac{6.37 I\omega^2}{k_t}$		$\frac{18.11 I\omega^2}{2k_t}$ $= \frac{9.055 I\omega^2}{k_t}$		$\frac{13.29 I\omega^2}{k_t}$	
5 Calculated deflection		$= \frac{6.37 I\omega^2}{k_t}$		$\frac{15.425 I\omega^2}{k_t}$		$\frac{28.715 I\omega^2}{k_t}$
		1		2.42		4.50
Trial : IV						
1 Assumed deflection		1		2.42		4.5
2 Inertia torque		$I\omega^2$		$4.84 I\omega^2$		$13.5 I\omega^2$
3 Shaft torque	$19.34 I\omega^2$		$18.34 I\omega^2$		$13.5 I\omega^2$	
4 Shaft twist	$\frac{19.34 I\omega^2}{3k_t}$ $= \frac{6.45 I\omega^2}{k_t}$		$\frac{18.34 I\omega^2}{2k_t}$ $= \frac{9.17 I\omega^2}{k_t}$		$\frac{13.5 I\omega^2}{k_t}$	
5 Calculated deflection		$\frac{6.45 I\omega^2}{k_t}$		$\frac{15.62 I\omega^2}{k_t}$		$\frac{29.12 I\omega^2}{k_t}$
		1		2.42		4.51

The calculated deflections in trial IV are very close to the assumed values. Hence the fundamental mode of vibration is given by $\begin{bmatrix} 1 \\ 2.42 \\ 4.51 \end{bmatrix}$

The fundamental or least natural frequency is $\frac{6.45 I\omega^2}{k_t} + \frac{15.62 I\omega^2}{k_t} + \frac{29.12 I\omega^2}{k_t} = 1 + 2.42 + 4.51$

$$\text{i.e., } 51.19 \frac{I\omega^2}{k_t} = 7.93$$

$$\therefore \omega_{n_1} = 0.394 \sqrt{\frac{k_t}{I}} \frac{\text{rad}}{\text{sec}} \qquad \text{i.e., } f_{n_1} = \frac{1}{2\pi}\omega_{n_1} = \frac{1}{2\pi} \times 0.394 \sqrt{\frac{k_t}{m}} = 0.063 \sqrt{\frac{k_t}{m}} Hz$$

$$\text{OR} \quad \frac{6.45 I\omega^2}{k_t} = 1$$

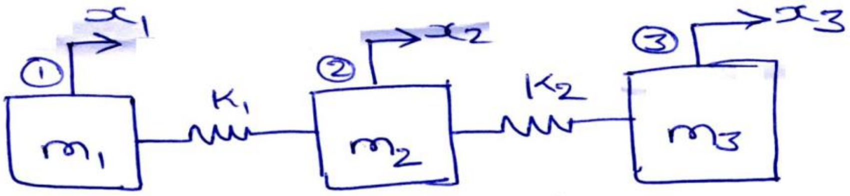
$$\text{i.e., } \omega^2 = \frac{1}{6.45} \frac{k_t}{I}$$

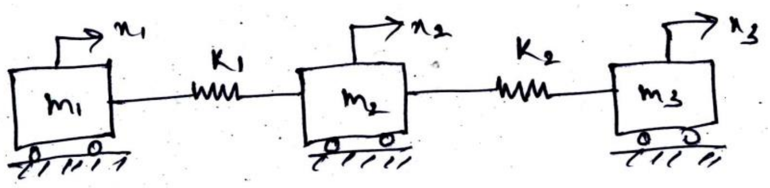
$$\therefore \omega_{n_1} = 0.394 \sqrt{\frac{k_t}{I}} \ \frac{\text{rad}}{\text{sec}}$$

First mode shape diagram similar to Fig 7.39

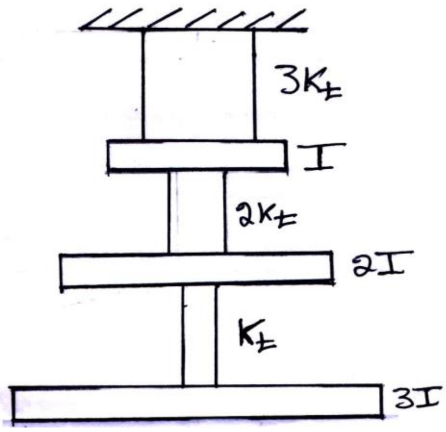
Part – B Questions

1. Determine the natural frequency and the mode shape of the system shown in below Fig by Holzer's method. m1 = 2 kg, m2 = 4 kg, m3 = 2 kg, k1 = 5 N/m, k2 = 10 N/m

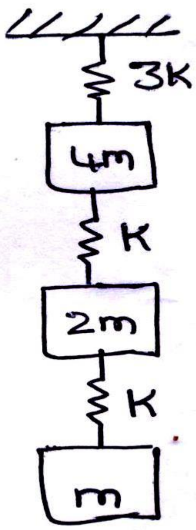




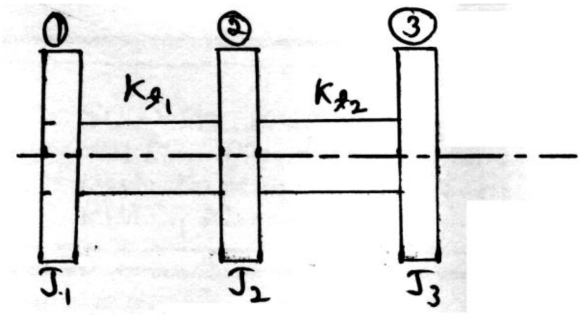
2. Using Holzer's method, determine the first two natural frequencies of the system shown in below Fig.



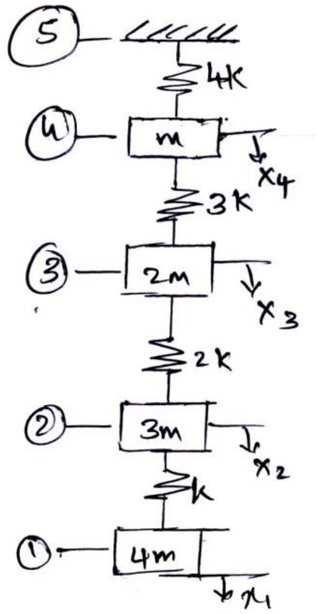
3. By Holzer's method, find the natural frequencies of the system in Fig Q10. Assume K = 1N/m, m= 1Kg



4. Determine the natural frequency of the system shown Fig 9(b) by Holzer method. Given J1 = J2 = J3 = 1Kg m2, K K 1N.m/ rad



5. Using Holzer's method find the natural frequencies of the four mass system as shown in Fig. Q10, if K = 1 N/m and m = 1 kg



6. Find the natural frequency of the system shown in Fig. Q8 by Holzer's method. Assume m1= m2= m3= 1 kg and k1= k2= k3= 1 N/m