Turbomachines - 18AE46

Old VTU Question's Answers

Module – 1

Syllabus:
Introduction to turbomachines: Classification and parts of a turbo machines; comparison with positive displacement machines; dimensionless parameters and their physical significance; specific speed; illustrative examples on dimensional analysis and model studies.
Energy transfer in turbomachines: Basic Euler turbine equation and its alternate form; components of energy transfer; general expression for degree of reaction; construction of velocity triangles for different values of degree of reaction.

Part – A Questions

1. Explain the components of a turbomachine with neat sketch.
or
What is a Turbomachine? With a neat sketch, explain the principal components of turbomachine.

General definition

A turbomachine is a device in which energy transfer occurs between a flowing fluid and a rotating element due to dynamic action and results in a change in pressure and momentum of the fluid.

1.5 PARTS OF A TURBOMACHINE

A turbomachine is comprised of the following parts:

(i) Rotor or impeller or runner
(ii) Guide blade or stationary or fixed element or nozzle
(iii) Shaft
(iv) Housing or casing
(v) Diffuser
(vi) Draft tube

Rotor or impeller or runner

Rotor is the rotating element of a turbomachine. It is fixed with blades or vanes and also called the impeller or runner depending upon the particular machine. For example, the rotating member of centrifugal pumps and centrifugal compressors is called the impeller. The rotating member of radial flow hydraulic turbines and pumps is called the runner. In contrast, the rotating member of axial flow gas and steam turbines is called the rotor. Energy transfer occurs between the fluid and the rotating member due to exchange of momentum between the two.

Guide blade or stationary or fixed element or nozzle

The stationary element or guide blade is arranged depending upon the kind of flow required. The stationary element is not a compulsory part of every turbomachine. The ceiling fan is a turbomachine and there is no stationary element.

Shaft

Either input shaft or output shaft or both may be necessary depending upon the type of turbomachines. For example:

(a) Power absorbing turbomachine: only input shaft
(b) Power generating turbomachine: only output shaft
(c) Power transmitting turbomachine: both input and output shaft

Housing or casing

The housing is not a compulsory part of every turbomachine. When the housing is present, it restricts the fluid so that it flows in a given space and does not escape in directions other than those required for energy transfer. A turbomachine having housing is called enclosed machine and the one having no housing is called extended machine.

Volute: It is a type of casing where a spiral passage is used for the collection of the diffused fluid of a compressor or pump. The volute casing is used in hydraulic turbines to increase the velocity of the fluid before it enters the runner.

Diffuser

A passage with increase in cross-sectional area in the direction of fluid flow, and which converts kinetic energy into static pressure head. It is usually situated at the outlet of a compressor, for example, axial flow compressor.

Draft tube

It is a diffuser placed at the outlet of a hydraulic turbine. For example, Francis and Kaplan turbines.

2. What are turbomachines? Classify turbomachines and explain the types with Examples.
or
Define Turbomachines. Give the classifications of Turbomachines.

General definition

A turbomachine is a device in which energy transfer occurs between a flowing fluid and a rotating element due to dynamic action and results in a change in pressure and momentum of the fluid.

The turbomachines shown in Figure 1.2 are classified as follows:

1. **According to energy consideration**

   (a) Turbomachines transferring rotor energy to fluid energy, i.e. machines supply energy to the fluid as in pumps and compressors.

   (b) Machines transferring fluid energy to a rotor energy, i.e. machines that extract energy from the fluid as in turbines (steam, gas, water).

2. **According to the direction of flow**

   (a) Radial flow as in centrifugal pumps, fans, turbines and compressors
   (b) Axial flow as in axial flow pumps, compressors, fans, and turbines
   (c) Mixed flow as in Francis turbine.
   (d) Tangential flow as in Pelton wheel

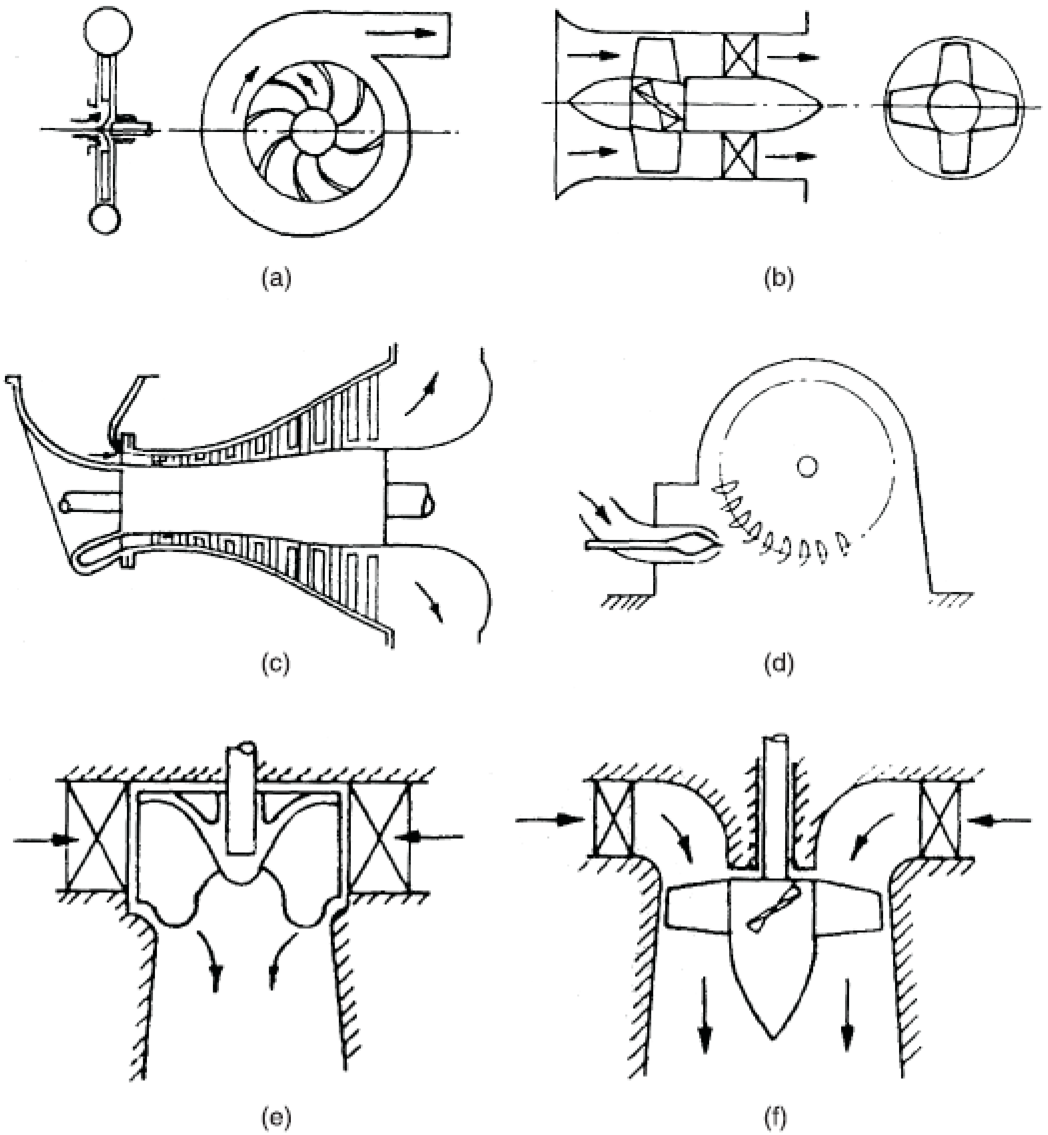

Figure 1.2 Types of turbomachines: (a) Centrifugal pump. (b) Axial flow pump. (c) Steam turbine. (d) Pelton wheel. (e) Francis turbine. (f) Kaplan turbine.

3. **According to the action of fluid on the moving blade**

   (a) ***Impulse machines***

   Fluid energy is converted into impulsive force by changing the direction of fluid as in a steam turbine (De-Laval) and Pelton wheel.

   (b) ***Reaction machines***

   The pressure energy of fluid continuously drops as it flows over the blades and the velocity increases. The fluid leaving the blades will exert a reactive force in the backward direction of its flow. This reactive force sets the blades in motion as in a steam turbine (Parson's reaction turbine), Francis turbine, Kaplan turbine and Propeller turbine.

4. (a) Open or extended machines such as propeller, wind mill, fan.

   (b) Enclosed machines such as turbines, pumps and compressors.

5. **According to the type of fluid handled**

   (a) Water : Examples: Pumps, hydraulic turbines
   (b) Steam : Example: Steam turbines
   (c) Air or gas : Examples: Fans, compressors, blowers, turbines.

3. Differentiate between positive displacement machines and turbomachines.
or
Distinguish between a turbomachine and a positive displacement machine.

Turbomachine	*Positive displacement machine*
1. Action	
(a) Dynamic	(a) Nearly static
(b) Pressure and momentum of the fluid changes.	(b) Volume of the fluid changes.
2. Operation	
(a) Pure rotary motion of the mechanical element.	(a) Usually it is the reciprocating motion of the mechanical element but some rotary positive displacement machines are also built. *Examples:* Gear pump, vane pump.
(b) Steady flow of fluid.	(b) Unsteady flow of fluid.
(c) The fluid state will be the same as that of the surroundings when the machine is stopped.	(c) Entrapped fluid state is different from the surroundings when the machine is stopped and if heat transfers and leakage are avoided.
3. Mechanical features	
(a) Rotating masses can be completely balanced and vibrations eliminated. Hence high speeds can be adopted.	(a) Because of the reciprocating masses, vibrations are more. Hence low speeds are adopted.
(b) Light foundations suffice.	(b) Heavy foundations are required.
(c) Design is simple.	(c) Mechanical design is complex because of valves.
(d) Weight per unit output is less.	(d) Weight per unit output is more.
4. Efficiency of conversion process	
(a) Efficiency is low because of dynamic energy transfer.	(a) High efficiency because of static energy transfer.
(b) The efficiency of the compression process is low.	(b) The efficiencies of the compression and expansion processes are almost the same.
5. Volumetric efficiency	
(a) It is almost 100%.	(a) Much below that of a turbomachine because of valves.
(b) High fluid handling capacity per unit weight of machine.	(b) Low fluid handling capacity per unit weight of machine.
6. Fluid phase change and surging	
(a) Causes cavitation in pumps and turbines. (b) Erodes steam turbine blades. (c) Deteriorates performance. (d) Surging or pulsation leads to unstable flow. (e) Causes vibrations and may destroy the machine.	No serious problems are encountered.
7. Operates between a moving fluid and a rotating element, resulting in thermodynamic and dynamic action.	Operates between a near static fluid and a slow moving surface resulting in thermodynamic and mechanical action.

4. Prove that for turbo machines work produced or consumed is equal to change in stagnation enthalpy.

Considering unit mass of the working fluid and writing the general energy equation, we get from Figure 1.1

$$(u_1' + p_1 v_1) + \frac{V_1^2}{2g_c} + \frac{z_1 g}{g_c} + q = w + (u_2' + p_2 v_2) + \frac{V_2^2}{2g_c} + \frac{z_2 g}{g_c} \tag{1.1}$$

where

u_1' and u_2'	are the inlet and exit values of internal energy (kJ/kg)
$p_1 v_1$ and $p_2 v_2$	are the flow work done on or by fluid (kN-m)
v_1 and v_2	are the specific volumes (m^3/kg)
V_1 and V_2	are the inlet and exit values of velocity of the fluid (m/s)
z_1 and z_2	are the inlet and exit values of potential energy (m)
q and w	are the heat and work interactions between the surroundings and the system (control flow) (kJ/kg)
h_{01} and h_{02}	are the inlet and exit values of stagnation enthalpy (total enthalpy).

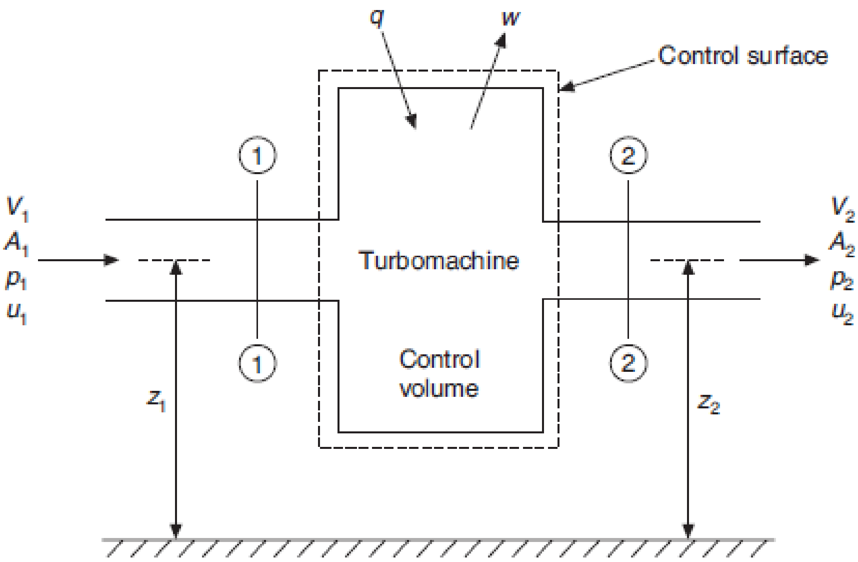

Figure 1.1 Steady flow energy process for a control volume.

In case of turbomachines, the rate of flow of working fluid is very high, the surface area available for transfer of heat is quite small and therefore, the process may be assumed to be adiabatic, i.e. $q = 0$. Equation (1.1) then becomes

$$(c_v T_1 + RT_1) + \frac{V_1^2}{2g_c} + \frac{z_1 g}{g_c} = w + (c_v T_2 + RT_2) + \frac{V_2^2}{2g_c} + \frac{z_2 g}{g_c}$$

$$T_1(c_v + R) + \frac{V_1^2}{2g_c} + \frac{z_1 g}{g_c} = w + T_2(c_v + R) + \frac{V_2^2}{2g_c} + \frac{z_2 g}{g_c}$$

$\therefore$

$$w = (c_v + R)(T_1 - T_2) + \frac{V_1^2 - V_2^2}{2g_c} + \frac{(z_1 - z_2)g}{g_c}$$

If $z_1 = z_2$,

$$w = (c_v + R)(T_1 - T_2) + \frac{V_1^2 - V_2^2}{2g_c}$$

$$= c_p(T_1 - T_2) + \frac{V_1^2 - V_2^2}{2g_c}$$

$$= (h_1 - h_2) + \frac{V_1^2 - V_2^2}{2g_c}$$

$$= \left(h_1 + \frac{V_1^2}{2g_c}\right) - \left(h_2 + \frac{V_2^2}{2g_c}\right)$$

$$w = h_{01} - h_{02} = -\Delta h_0 \quad \text{or} \quad \delta w = \delta h_0 \tag{1.2}$$

If we neglect the kinetic energy, then

$$w = -\Delta h \tag{1.3}$$

The suffix '0' represents the total head or stagnation conditions.

In power generating turbomachines, w is positive so that Δh_0 is negative, i.e. the total enthalpy of flowing fluid decreases from inlet to exit.

In power absorbing turbomachines, mechanical energy input occurs so that the stagnation enthalpy of the fluid increases from inlet to exit.

5. Define specific speed of a turbine. Write expressions for specific speed of a turbine and a pump. Mention the significance of specific speed.

or

Define specific speed of a pump and turbine. Write the equations for the same in dimensionless form.

or

Define specific speed of Turbine and specific speed of pump.

Specific speed of turbines (N_{ST})

It is the speed of a geometrically similar turbine working under unit head and developing unit power, i.e.

$$u = \frac{\pi dN}{60} \propto dN \propto V \propto \sqrt{H}$$

$$\therefore \qquad d \propto \frac{\sqrt{H}}{N}$$

$$P = \text{Power} = \frac{\rho Q g H}{\eta_0} \propto QH \propto d^2 VH \propto d^2 \sqrt{H} H \propto \frac{\sqrt{H} HH}{N^2} \propto \frac{H^{5/2}}{N^2}$$

$$\therefore \qquad N^2 \propto \frac{H^{5/2}}{P} \quad \text{or} \quad N \propto \frac{H^{5/4}}{\sqrt{P}}$$

$$\therefore \qquad N = \frac{kH^{5/4}}{\sqrt{P}}, \text{ where } k \text{ is constant of proportionality} \tag{1.40}$$

If H = unit head, P = unit power, then $N = N_{ST}$

$$\therefore \qquad N_{ST} = \frac{k 1^{5/4}}{\sqrt{1}} = k \tag{1.41}$$

Substituting $N_{ST} = k$ in Eq. (1.40),

$$N = \frac{N_{ST} H^{5/4}}{\sqrt{P}}; \quad \therefore \quad N_{ST} = \frac{N\sqrt{P}}{H^{5/4}} \tag{1.42}$$

Specific speed of pumps (N_{SP})

The specific speed of a centrifugal pump is defined as the speed of a geometrically similar pump delivering unit quantity (1 m^3/s) of liquid against a head of one metre.

$$Q = \text{area} \times \text{velocity flow} \propto \pi dbV \propto d^2V \propto d^2\sqrt{H_m}$$

$$Q = \frac{H_m}{N^2}\sqrt{H_m} \propto \frac{H_m^{3/2}}{N^2} \tag{1.43}$$

$$Q = k\frac{H_m^{3/2}}{N^2} \quad \text{where } k \text{ is the constant of proportionality}$$

If $H_m = 1$ m, $Q = 1$ m^3/s, then $N = N_{SP}$

$$\therefore \quad 1 = \frac{k \times 1^{3/2}}{N_{SP}^2}$$

$$\therefore \quad N_{SP}^2 = k$$

Substituting $N_{SP}^2 = k$ in Eq. (1.43),

$$Q = \frac{N_{SP}^2 H_m^{3/2}}{N^2}; \quad \therefore \quad N_{SP} = \sqrt{\frac{QN^2}{H_m^{3/2}}} = \frac{N\sqrt{Q}}{H_m^{3/4}} \tag{1.44}$$

6. With usual notations, using dimensional analysis derive an expression for power and capacity coefficients of a turbomachines.

and

7. With usual notations, derive expressions for unit discharge coefficient, head coefficient power coefficient, using dimensional analysis.

Incompressible flow machines

Figure 1.3 shows a control volume through which an incompressible fluid is flowing. Following are the variables considered.

Flow rate	Q	m^3/s
Speed	N	rps
Power	P	W
Head or Pressure	H	m
Fluid density	ρ	kg/m^3
Fluid viscosity	μ	$N\text{-}s/m^2$
Diameter	d	m

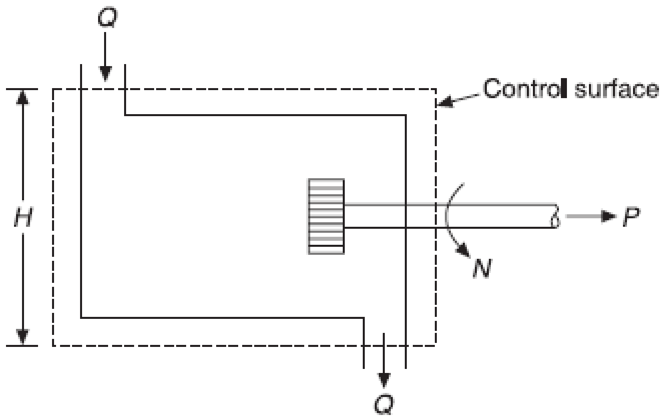

Figure 1.3 A generalized turbomachine for the purpose of dimensional analysis.

$$P = f\,[\rho\ N\ \mu\ d\ Q\ H] \qquad (1.23)$$

Select the repeating variables as $d\ N\ \rho$ $\therefore$ $m = 3$

$\therefore$ Total variables $n = 7$

$\therefore$ $(n - m) = (7 - 3) =$ Four π terms.

π_1 term: $\pi_1 = d^{a_1} N^{b_1} \rho^{c_1} Q$

$$M^0L^0T^0 = [L]^{a_1} [T^{-1}]^{b_1} [ML^{-3}]^{c_1} L^3T^{-1}$$

For M : $-3c_1 = 0$

For L : $a_1 - 3c_1 + 3 = 0$

For T : $-b_1 - 1 = 0$

Solving the above equations,

$$c_1 = 0,\ b_1 = -1,\ a_1 = -3$$

Substituting for a_1, b_1 and c_1 in π_1 term,

$$\pi_1 = d^{-3} N^{-1} \rho^0 Q = \frac{Q}{Nd^3} \qquad (1.24)$$

π_2 term : $\pi_2 = d^{a_2} N^{b_2} \rho^{c_2} H$

$$M^0L^0T^0 = [L]^{a_2} [T^{-1}]^{b_2} [ML^{-3}]^{c_2} L^2T^{-2}$$

For M : $c_2 = 0$

For L : $a_2 - 3c_2 + 2 = 0$

For T : $-b_2 - 2 = 0$

Solving the above equations,

$$c_2 = 0,\ b_2 = -2,\ a_2 = -2$$

Substituting for a_2, b_2 and c_2 in π_2 term,

$$\pi_2 = d^{-2} N^{-2} \rho^0 H = \frac{H}{N^2 d^2} \qquad (1.25)$$

π_3 term :
$$\pi_3 = d^{a_3} N^{b_3} \rho^{c_3} P$$
$$M^0L^0T^0 = [L]^{a_3} [T^{-1}]^{b_3} [ML^{-3}]^{c_3} ML^2T^{-3}$$

For M : $c_3 + 1 = 0$

For L : $a_3 - 3c_3 + 2 = 0$

For T : $-b_3 - 3 = 0$

Solving the above equations,
$$c_3 = -1,\ b_3 = -3,\ a_3 = -5$$

Substituting for a_3, b_3 and c_3 in π_3 term,
$$\pi_3 = d^{-5} N^{-3} \rho^{-1} P = \frac{P}{\rho N^3 d^5} \qquad (1.26)$$

π_4 term :
$$\pi_4 = d^{a_4} N^{b_4} \rho^{c_4} \mu$$
$$M^0L^0T^0 = [L]^{a_4} [T^{-1}]^{b_4} [ML^{-3}]^{c_4} ML^{-1}T^{-1}$$

For M : $c_4 + 1 = 0$

For L : $a_4 - 3c_4 - 1 = 0$

For T : $-b_4 - 1 = 0$

Solving the above equations,
$$c_4 = -1,\ b_4 = -1,\ a_4 = -2$$

Substituting for a_4, b_4 and c_4 in π_4 term,
$$\pi_4 = d^{-2} N^{-1} \rho^{-1} \mu = \frac{\mu}{\rho N d^2} \quad \text{or} \quad \frac{\rho N d^2}{\mu} \qquad (1.27)$$

1.18.1 Capacity Coefficient or Flow Coefficient or Specific Capacity or Discharge Coefficient

From Eq. (1.24),
$$\pi_1 = \frac{Q}{Nd^3} = \text{capacity coefficient} \qquad (1.28)$$

We know that,
$$Q \propto AV \propto d^2V; \quad Nd \propto u; \quad Nd^3 = Nd(d^2) \propto ud^2$$

$$\therefore \quad \pi_1 = \frac{Q}{Nd^3} \propto \frac{d^2V}{Nd^3} \propto \frac{V}{Nd} \propto \frac{V}{u} \propto \frac{1}{\phi} \qquad (1.28a)$$

where
$$\phi = \text{speed ratio} = \frac{u}{V}$$
$$= \frac{\text{Runner tangential speed}}{\text{Theortical jet spouting speed}}$$

For a given value of $\pi_1 = \frac{Q}{Nd^3}$, it signifies that the ratio of blade velocity to jet velocity is fixed. Therefore, the shape of the velocity triangle can be determined for any given machine.

$$\therefore \quad \pi_1 = \frac{Q}{Nd^3} \propto \frac{Q}{d^2\sqrt{H}} = \text{specific discharge} \qquad (1.29)$$

1.18.2 Head Coefficient or Specific Head

From Eq. (1.25),

$$\pi_2 = \frac{H}{N^2 d^2} \propto \frac{H}{u^2} \propto \frac{H}{V^2} \propto \frac{H}{uV} \tag{1.30}$$

We know that, $Nd \propto u$

$\therefore$ $$(Nd)^2 \propto u^2 \; ; \quad V \propto u$$

Head coefficient is the ratio of the kinetic energy of the fluid (due to H) to the kinetic energy of the fluid running at the rotor tangential speed or it is the ratio of fluid head to kinetic energy of the rotor.

$$H = \frac{\Delta p}{\rho g} \propto \frac{\Delta p}{\rho} \qquad (\rho = \text{constant in incompressible fluids}) \tag{1.31}$$

$\therefore$ $\pi_2 = \dfrac{H}{N^2 d^2} = \dfrac{\Delta p}{\rho V^2}$, which is essentially a flow relationship.

1.18.3 Power Coefficient or Specific Power

From Eq. (1.26), the parameter

$\pi_3 = \dfrac{P}{\rho N^3 d^5}$ is known as power coefficient or specific power.

This parameter can also be obtained by the product of π_1 and π_2.

$\therefore$ $$\pi_3 = \pi_1 \times \pi_2 = \frac{Q}{Nd^3} \times \frac{H}{(Nd)^2} \tag{1.32}$$

Now, $$P = H \times \dot{m} = \rho Q g H$$

$\therefore$ $$\pi_3 = \frac{Q}{Nd^3} \times \frac{P}{\dot{m}(Nd)^2} \propto \frac{d^2 V}{Nd^3} \times \frac{P}{\rho A V N^2 d^2} \propto \frac{P}{\rho N^3 d^5}$$

$\therefore$ $$\pi_3 = \frac{P}{\rho N^3 d^5} \tag{1.33}$$

Now,

$$N^3 d^5 \propto u^3 d^2 \propto V^3 d^2 \propto V^2 V d^2 \propto V d^2 H \propto d^2 \sqrt{H} H$$

or $$N^3 d^5 \propto d^2 H^{3/2}$$

$\therefore$ $$\pi_3 = \frac{P}{\rho N^3 d^5} \propto \frac{P}{\rho d^2 H^{3/2}} \propto \frac{P}{d^2 H^{3/2}} \tag{1.33a}$$

Particularly in water turbines, ρ can be omitted.

The specific power of both the model and prototype will remain the same if their efficiencies are the same.

8. The resisting force F of a supersonic plane during flight can be considered as dependent upon the length of the aircraft L, velocity V, air viscosity μ, air density ρ and bulk modulus of air K. Express the functional relationship between these variables and the resisting force.

Solution: Let the functional relationship be

$$F = f[L, V, \mu, \rho, K]$$

The above equation in its general form may be written as

$$f_1[F, L, V, \mu, \rho, K] = 0$$

Here $n = 6$, $m = 3$, $(n - m) = 6 - 3$ = three π terms; $f_1(\pi_1, \pi_2, \pi_3) = 0$ (i)

Let L, V, and ρ be the repeating variables. Then, we get

$$\pi_1 = [L]^{a_1}[V]^{b_1}[\rho]^{c_1} F$$

Hence: $$M^0L^0T^0 = (L)^{a_1}(LT^{-1})^{b_1}(ML^{-3})^{c_1}(MLT)^{-2}$$

M : $c_1 + 1 = 0$ $\therefore$ $c_1 = -1$

L : $a_1 + b_1 - 3c_1 + 1 = 0$ $\therefore$ $a_1 = -2$

T : $-b_1 - 2 = 0$ $\therefore$ $b_1 = -2$

Substituting a_1, b_1 and c_1 in the π_1 term,

$$\pi_1 = L^{-2}V^{-2}\rho^{-1}F = \frac{F}{L^2V^2\rho} \qquad \text{(ii)}$$

Now, $$\pi_2 = (L)^{a_2}(V)^{b_2}(\rho)^{c_2}\mu$$

Hence: $$M^0L^0T^0 = (L)^{a_2}(LT^{-1})^{b_2}(ML^{-3})^{c_2}(ML^{-1}T)^{-1}$$

M : $c_2 + 1 = 0$ $\therefore$ $c_2 = -1$

L : $a_2 + b_2 - 3c_2 - 1 = 0$ $\therefore$ $a_2 = -1$

T : $-b_2 - 1 = 0$ $\therefore$ $b_2 = -1$

Substituting a_2, b_2 and c_2 in the π_2 term,

$$\pi_2 = L^{-1}V^{-1}\rho^{-1}\mu = \frac{\mu}{LV\rho} \qquad \text{(iii)}$$

Again, $$\pi_3 = (L)^{a_3}(V)^{b_3}(\rho)^{c_3}K$$

Hence: $$M^0L^0T^0 = (L)^{a_3}(LT^{-1})^{b_3}(ML^{-3})^{c_3}(ML^{-1}T)^{-2}$$

M : $c_3 + 1 = 0$ $\therefore$ $c_3 = -1$

L : $a_3 + b_3 - 3c_3 - 1 = 0$ $\therefore$ $a_3 = 0$

T : $-b_3 - 2 = 0$ $\therefore$ $b_3 = -2$

Substituting a_3, b_3 and c_3 in the π_3 term,

$$\pi_3 = L^0V^{-2}\rho^{-1}K = \frac{K}{V^2\rho} \qquad \text{(iv)}$$

Substituting π_1, π_2, π_3 in Eq. (i),

$$f_1\left[\frac{F}{L^2V^2\rho}, \frac{\mu}{LV\rho}, \frac{K}{V^2\rho}\right] = 0$$

$$\therefore \quad F = [L^2V^2\rho]\,\phi\left[\frac{\mu}{LV\rho}, \frac{K}{V^2\rho}\right] \qquad \textbf{Ans.}$$

9. Show that the discharge of a centrifugal pump is given by $Q = ND^3 f\left[\frac{gH}{N^2D^2}, \frac{\mu}{ND^2\rho}\right]$ where N is the speed of the pump in rpm D the diameter of the impeller, g the acceleration due to gravity, H the mano metric head, m viscosity fluid and ρ the density of the fluid.

Solution: Let the functional relationship be

$$Q = f\,[N, D, g, H, \mu, \rho] \qquad \text{(i)}$$

The general form of the above relation can be written as

$$f_1\,[Q, N, D, g, H, \mu, \rho] = 0 \qquad \text{(ii)}$$

$n = 7$, $m = 3$, and let N, D, ρ be the repeating variables.

$$\therefore \quad (n - m) = (7 - 3) = 4\pi \text{ terms}$$

Equation (ii) can be written as $f_1(\pi_1, \pi_2, \pi_3, \pi_4) = 0$ (iii)

Here, $\pi_1 = N^{a_1} D^{b_1} \rho^{c_1} Q$

$$M^0L^0T^0 = (T^{-1})^{a_1} (L)^{b_1} (ML^{-3})^{c_1} (L^3T)^{-1}$$

Hence:

M : $c_1 = 0$ $\quad\therefore\quad c_1 = 0$

L : $b_1 - 3c_1 + 3 = 0$ $\quad\therefore\quad b_1 = -3$

T : $-a_1 - 1 = 0$ $\quad\therefore\quad a_1 = -1$

$$\therefore \quad \pi_1 = N^{-1}D^{-3}\rho^0 Q = \frac{Q}{ND^3}$$

Now, $\pi_2 = N^{a_2} D^{b_2} \rho^{c_2} g$

$$M^0L^0T^0 = (T^{-1})^{a_2} (L)^{b_2} (ML^{-3})^{c_2} (LT)^{-2}$$

Hence:

M : $c_2 = 0$ $\quad\therefore\quad c_2 = 0$

L : $b_2 - 3c_2 + 1 = 0$ $\quad\therefore\quad b_2 = -1$

T : $-a_2 - 2 = 0$ $\quad\therefore\quad a_2 = -2$

$$\pi_2 = N^{-2} D^{-1} e^0 g = \frac{g}{N^2D}$$

Also, $\pi_3 = N^{a_3} D^{b_3} \rho^{c_3} H$

$$M^0L^0T^0 = (T^{-1})^{a_1} (L)^{b_1} (ML^{-3})^{c_1} (L)$$

Hence:

M : $c_3 = 0$ $\quad\therefore\quad c_3 = 0$

L : $b_3 - 3c_3 + 1 = 0$ $\quad\therefore\quad b_3 = -1$

T : $-a_3 = 0$ $\quad\therefore\quad a_3 = 0$

$$\therefore \quad \pi_3 = N^0 D^{-1} \rho^0 H = \frac{H}{D}$$

Again, $\pi_4 = N^{a_4} D^{b_4} \rho^{c_4} \mu$

$$M^0L^0T^0 = (L)^{a_4} (ML^{-3})^{b_4} (ML^{-3})^{c_4} (ML^{-1}T)^{-1}$$

Hence:

M : $c_4 + 1 = 0 \quad \therefore \quad c_4 = -1$

L : $b_4 - 3c_4 - 1 = 0 \quad \therefore \quad b_4 = -2$

T : $-a_4 - 1 = 0 \quad \therefore \quad a_4 = -1$

$$\therefore \quad \pi_4 = N^{-1} D^{-2} \rho^{-1} \mu = \frac{\mu}{ND^2 \rho}$$

Substituting the values of all πs in Eq. (iii), we get

$$\frac{Q}{ND^3} = \phi\left[\frac{g}{N^2 D}, \frac{H}{D}, \frac{\mu}{ND^2 \rho}\right]$$

$$\therefore \quad Q = ND^3 \phi\left[\frac{gH}{N^2 D^2}, \frac{\mu}{ND^2 \rho}\right] \qquad \textbf{Ans.}$$

10. Performance of lubricating oil ring in a turbomachine depends on the diameter (D), shaft speed (N), discharge of oil (Q), density (ρ), viscosity (μ), surface tension (σ) and specific weight (W) of oil. Find the relation for discharge of an oil in terms of non-dimensional terms.

Solution: The functional relationship can be written as

$$P = f\,[D, N, Q, \rho, \mu, \sigma, w] \qquad \text{(i)}$$

The above equation can be written in general form as

$$f'[P, D, N, Q, \rho, \mu, \sigma, w] = 0 \qquad \text{(ii)}$$

$\therefore \quad n = 8,\ m = 3, \quad \therefore$ There are five π terms

$$\therefore \quad f'\,[\pi_1, \pi_2, \pi_3, \pi_4, \pi_5] = 0$$

Let D, N, ρ be the repeating variables.

Now, $\pi_1 = D^{a_1} N^{b_1} \rho^{c_1} P$

$$M^0L^0T^0 = L^{a_1} (T^{-1})^{b_1} (ML^{-3})^{c_1} (MLT^{-2})$$

Hence:

M : $c_1 + 1 = 0 \quad \therefore \quad c_1 = -1$

L : $a_1 - 3c_1 + 1 = 0 \quad \therefore \quad a_1 = -4$

T : $-b_1 - 2 = 0 \quad \therefore \quad b_1 = -2$

$$\therefore \quad \pi_1 = D^{-4} N^{-2} \rho^{-1} P = \frac{P}{\rho N^2 D^4}$$

Now, $\pi_2 = D^{a_2} N^{b_2} \rho^{c_2} Q$

$$M^0L^0T^0 = (L)^{a_2} (T^{-1})^{b_2} (ML^{-3})^{c_2} (L^3 T^{-1})$$

Hence:

M :	$c_2 = 0$	$\therefore \quad c_2 = 0$
L :	$a_2 - 3c_2 + 3 = 0$	$\therefore \quad a_2 = -3$
T :	$-b_2 - 1 = 0$	$\therefore \quad b_2 = -1$

$$\therefore \quad \pi_2 = D^{-3} N^{-1} \rho^0 Q = \frac{Q}{ND^3}$$

Also, $\pi_3 = D^{a_3} N^{b_3} \rho^{c_3} \mu$

$$M^0L^0T^0 = L^{a_3} (T^{-1})^{b_3} (ML^{-3})^{c_3} (ML^{-1}T^{-1})$$

Hence:

M :	$c_3 + 1 = 0$	$\therefore \quad c_3 = -1$
L :	$a_3 - 3c_3 - 1 = 0$	$\therefore \quad a_3 = -2$
T :	$-b_3 - 1 = 0$	$\therefore \quad b_3 = -1$

$$\therefore \quad \pi_3 = D^{-2} N^{-1} \rho^{-1} \mu = \frac{\mu}{\rho N D^2}$$

Again, $\pi_4 = D^{a_4} N^{b_4} \rho^{c_4} \sigma$

$$M^0L^0T^0 = L^{a_4} (T^{-1})^{b_4} (ML^{-3})^{c_3} (MT^{-2})$$

Hence:

M :	$c_4 + 1 = 0$	$\therefore \quad c_4 = -1$
L :	$a_4 - 3c_4 = 0$	$\therefore \quad a_4 = -3$
T :	$-b_4 - 2 = 0$	$\therefore \quad b_4 = -2$

$$\therefore \quad \pi_4 = D^{-3} N^{-2} \rho^{-1} \sigma = \frac{\sigma}{\rho N^2 D^3}$$

Again, $\pi_5 = D^{a_5} N^{b_5} \rho^{c_5} w$

$$M^0L^0T^0 = L^{a_5} (T^{-1})^{b_5} (ML^{-3})^{c_5} (ML^{-2}T^{-2})$$

Hence:

M :	$c_5 + 1 = 0$	$\therefore \quad c_5 = -1$
L :	$a_5 - 3c_5 - 2 = 0$	$\therefore \quad a_5 = -1$
T :	$-b_5 - 2 = 0$	$\therefore \quad b_5 = -2$

$$\therefore \quad \pi_5 = D^{-1} N^{-2} \rho^{-1} w = \frac{w}{\rho N^2 D}$$

Substituting the values of all πs in Eq. (iii),

$$f'\left[\frac{P}{D^4N^2\rho}, \frac{Q}{ND^3}, \frac{\mu}{\rho ND^2}, \frac{\sigma}{\rho N^2 D^3}, \frac{w}{\rho N^2 D}\right] = 0$$

$$\therefore \quad P = D^4 N^2 \rho \phi \left[\frac{Q}{ND^3}, \frac{\mu}{\rho ND^2}, \frac{\sigma}{\rho N^2 D^3}, \frac{w}{\rho N^2 D}\right]$$ **Ans.**

11. A single stage centrifugal pump with 300mm impeller diameter rotates at 2000rpm supplying 3 m^3/s to a height of 30m with an efficiency of 75%. Find the number of stages and diameter of each impeller of a similar multistage pump to lift 5 m^3/s of water to a height of 200m when running at 1500 rpm.

Solution:

(a) **Number of stages (N_s):**

We have the specific speed relation as

$$\left[\frac{N\sqrt{Q}}{H^{3/4}}\right]_1 = \left[\frac{N\sqrt{Q}}{H^{3/4}}\right]_2$$

i.e.
$$\left[\frac{2000\sqrt{3}}{30^{3/4}}\right]_1 = \left[\frac{1500\sqrt{5}}{H^{3/4}}\right]_2$$

∴
$$H_2 = 28.73 \text{ m}$$

Hence
$$N_s = \frac{H_t}{H} = \frac{200}{28.73} = 6.96 \approx 7$$
Ans.

(b) **Diameter of each impeller (D_2):**

From Eq. (1.30),

$$\left[\frac{H}{(ND)^2}\right]_1 = \left[\frac{H}{(ND)^2}\right]_2$$

or
$$\left[\frac{30}{(2000 \times 300)^2}\right]_1 = \left[\frac{28.73}{(1500 \times D)^2}\right]_2$$

∴
$$D_2 = 39 \text{ m}$$
Ans.

12. The thrust (T) of a propeller is assumed to depend on the axial velocity of the fluid V, the density ρ and viscosity μ of fluid, the speed N in RPM and the diameter D. Find the relationship of T by dimensional analysis.

→ $f(T, V, \rho, \mu, N, D)$.

Total no. of variable = 6 (n)

FV => 3. (m)

no. of π term 6 − 3 = 3

$f(\pi_1, \pi_2, \pi_3) = \text{const}$

(or),

$\pi_1 = f(\pi_2, \pi_3)$

RV : D N ρ

$D = L$ $T = MLT^{-2}$
$N = T^{-1}$ $V = LT^{-1}$
$\rho = ML^{-3}$ $\mu = ML^{-1}T^{-1}$

$\boxed{\pi_1} = D^{a_1} N^{b_1} \rho^{c_1} T$

$M^{0}L^{0}T^{0} = L^{a_1} \times (T^{-1})^{b_1} \cdot (ML^{-3})^{c_1} \cdot (MLT^{-2})$

M: $0 = c_1 + 1$ L: $0 = a_1 - 3c_1 + 1$ T: $0 = -b_1 - 2$

$\boxed{c_1 = -1}$ $\boxed{a_1 = -4}$ $\boxed{b_1 = -2}$

$$\pi_1 = D^{-4} N^{-2} \rho^{-1} T$$

$$\boxed{\pi_1 = \frac{T}{D^4 N^2 \rho}}$$

$\boxed{\pi_2} = D^{a_2} N^{b_2} \rho^{c_2} V$

$M^{0}L^{0}T^{0} = L^{a_2} \times (T^{-1})^{b_2} (ML^{-3})^{c_2} (LT^{-1})$

M: $\boxed{0 = c_2}$ L: $0 = a_2 - 3c_2 + 1$ T: $0 = -b_2 - 1$

$\boxed{a_2 = -1}$ $\boxed{b_2 = -1}$

$$\pi_2 = D^{-1} N^{-1} \rho^{0} V$$

$$\boxed{\pi_2 = \frac{V}{DN}}$$

$\boxed{\pi_3} = D^{a_3} N^{b_3} \rho^{c_3} \mu$

$M^{0}L^{0}T^{0} = L^{a_3} (T^{-1})^{b_3} (ML^{-3})^{c_3} (ML^{-1}T^{-1})$

M: $0 = c_3 + 1$ L: $0 = a_3 - 3c_3 - 1$ T: $0 = -b_3 - 1$

$\boxed{c_3 = -1}$ $\boxed{a_3 = -2}$ $\boxed{b_3 = -1}$

$$\pi_3 = D^{-2} N^{-1} \rho^{-1} \mu$$

$$\boxed{\pi_3 = \frac{\mu}{D N \rho}}$$

$$f\left(\frac{T}{D^4N^2\rho}, \frac{V}{DN}, \frac{\mu}{DN\rho}\right) = \text{constant.}$$

$$\Rightarrow \left[\frac{T}{D^4N^2\rho} = f'\left(\frac{V}{DN}, \frac{\mu}{DN\rho}\right)\right]$$

13. From the performance curve of a turbine, it is seen that a turbine of 1m diameter acting under a head of 1m develops a speed of 25RPM. Determine the diameter of the prototype, if it develops 10,000kW, working under a head of 200m with a specific speed of 150RPM.

→ Model:

$D_m = 1m$

$N_m = 25 RPM$

$H_m = 1m.$

prototype:

$P_p = 10,000$ kW

$H_p = 200m.$

$N_s = 150$

$D_p = ?$

Sp. speed of turbine (prototype) :- $N_s = \frac{N_p (P_p)^{1/2}}{(H_p)^{5/4}}$

$$150 = \frac{N_p (10000)^{1/2}}{(200)^{5/4}}$$

$N_p = 1128.1809$ RPM

$$N_p = \frac{1128.1809}{60}$$

$$\boxed{N_p = 18.8030 \text{ RPS}}$$

head coefficient for model & prototype is given by

$$\frac{gH_m}{N_m^2 \times D_m^2} = \frac{gH_p}{N_p^2 D_p^2}$$

$$\frac{1}{(0.4167)^2 \times 1^2} = \frac{200}{18.8030^2 \times D_p^2}$$

$$D_p = \sqrt{\frac{200 \times 0.4167^2 \times 1^2}{18.8030^2}}$$

$$\boxed{D_p = 0.3134 \text{ m}}$$

14. A centrifugal pump delivers 1 m^3/s against a pressure of 40 m of H2O at a speed of 1200 rpm. Calculate: (i) Specific speed of pump (ii) Power of pump if it requires 50% more discharge. Take diameter of impeller 0.5 m and η_0 = 33%.

or

A centrifugal pump delivers, m^3/s against a pressure of 40m of water at 1200rpm. Calculate the dimensionless specific speed of the machine. Further, find the speed of rotation and power of the motor (prototype) for delivering 50% more discharge. Take the impeller diameter as 0.5m and overall efficiency of 33%. Assume (D1 = D2).

$$\rightarrow \quad N_s = \frac{N_1 \sqrt{Q}}{H^{3/4}} \Rightarrow \frac{1200 \times \sqrt{1}}{(40)^{3/4}}$$

$$\boxed{N_s = 75.446 \text{ rpm}}$$

$$P \Rightarrow \frac{\rho g Q H}{\eta}$$

$$\Rightarrow \frac{1000 \times 9.8 \times 1.5 \times 40}{0.33}$$

$$\boxed{P = 1783.6364 \text{ kW}}$$

15. Test on a turbine runner 1.25m in diameter at 30m head gave the following results, power developed 736kW, speed is 180rpm and the discharge is 2.7m^3/sec. Find the diameter, speed and discharge of a similar runner to operate at 45m head and give 1472kW at the same efficiency. What is the specific speed of both the turbines?

→ turbines.

$D_1 = 1.25$ m	$D_2 = ?$
$H_{m_1} = 30$ m	$N_2 = ?$
$P_1 = 736$ kW.	$Q_2 = ?$ $\boxed{3.6}$
$N_1 = 180$ rpm	$H_{m_2} = 45$ m.
$Q_1 = 2.7$ m^3/sec	$P_2 = 1472$ kW

$$\eta_1 = \eta_2$$

D_2,

discharge coefficient for turbine $= \frac{Q}{ND^3}$

$$\frac{Q_1}{N_1 D_1^3} = \frac{Q_2}{N_2 D_3^3}$$

$$\eta_T = \frac{wQH}{P}$$

$$\frac{P_1}{wQ_1 H_{m_1}} = \frac{P_2}{wQ_2 H_{m_2}}$$

$$\frac{736}{2.7 \times 30} = \frac{1472}{Q_2 \times 45}$$

$$\boxed{Q_2 = 3.6} \; m^3/sec$$

$$\frac{N_1 \sqrt{P_1}}{H_m^{5/4}} = \frac{N_2 \sqrt{P_2}}{H_m^{5/4}}$$

$$\frac{180\sqrt{736}}{30^{5/4}} = \frac{N_2 \sqrt{1472}}{45^{5/4}}$$

$$\boxed{N_2 = 211.286 \text{ rpm}}$$

$$\frac{Q_1}{N_1 D_1^3} = \frac{Q_2}{N_2 D_2^3}$$

$$\frac{2.7 \times 211.286}{180 \times 1.25^3 \times 3.6} = \frac{1}{D_2^3}$$

$$D_2^3 = 2.2185$$

$$\boxed{D_2 = 1.304 \text{ m}}$$

specific speed

$$\frac{N_1 \sqrt{Q_1}}{Hm_1^{3/4}} = \frac{N_2 \sqrt{Q_2}}{Hm_2^{3/4}}$$

$$\frac{180 \sqrt{2.7}}{30^{3/4}} = \frac{211.286 \times \sqrt{3.6}}{Hm_2^{3/4}}$$

$$Hm_2^{3/4} = 17.3743$$

$$Hm_2 = (17.3743)^{4/3}$$

$$\boxed{Hm_2 = 44.99 \text{ m}}$$

16. A hydraulic turbine has a scale ratio of 1:10. Following data refers to model and prototype, model: P = 25kW, N = 500rpm, H = 10m, η_0 = 0.8. Prototype : H = 130m. Determine the discharge speed, power and overall efficiency of the prototype.

→ efficiency eq^n for turbine,

$$\eta_0 = \frac{P}{wQ\,Hm}$$

$$\frac{P_1}{Q_1 Hm_1} = \frac{P_2}{Q_2 Hm_2}.$$

specific speed of turbine.

$$\frac{N_1 \sqrt{P_1}}{Hm_1^{5/4}} = \frac{N_2 \sqrt{P_2}}{Hm_2^{5/4}}$$

head coefficient :

$$\frac{H_{m_1}}{N_1^2 D_1^2} = \frac{H_{m_2}}{N_2^2 D_2^2}$$

$$N_2 = \sqrt{\frac{H_{m_2} \times N_1^2 \times D_1^2}{D_2^2 \times H_{m_1}}}$$

$$= \sqrt{\frac{130 \times 500^2 \times 1}{10 \times 100}}$$

$$N_2 = 570.0877 \text{ rpm}$$

$$\sqrt{P_2} = \frac{H_{m_2}^{5/4} \times N_1 \times \sqrt{P_1}}{H_{m_1}^{5/4} \times N_2}$$

$$P_2 = \left(\frac{H_{m_2}^{5/4} \times N_1 \times \sqrt{P_1}}{H_{m_1}^{5/4} \times N_2}\right)^2$$

$$= \left(\frac{(130)^{5/4} \times 500 \times \sqrt{25}}{(10)^{5/4} \times 570.0877}\right)^2$$

$$P_2 = 11.7180 \text{ KW}$$

efficiency to find Q_2 :-

$$\frac{P_1}{\underbrace{Q_1 H_{m_1}}_{0.8}} = \frac{P_2}{Q_2 H_{m_2}}$$

$$0.8 = \frac{11.7180}{Q_2 \times 130}$$

$$Q_2 = \frac{11.7180}{130 \times 0.8}$$

$$\boxed{Q_2 = 0.1127 \ m^3/sec}$$

Part – B Questions

1. Derive Euler's energy equation for turbomachines using theory of conservation of momentum.
or
Derive the basic Euler energy equation for a turbomachine

2. Derive alternate form of Euler's energy equation for turbo machines.
or
Starting from the fundamentals arrive at the alternate form of Euler turbine equation.
or
Derive an alternate from of Euler turbine equation and explain significance of each energy components.

3. Explain degree of reaction and how it is used to classify turbomachines

4. Define degree of reaction, derive a general expression for degree of reaction of turbomachine.

5. Define degree of reaction and derive relation between degree of reaction and utilization factor.
or
Define utilization factor. Obtain a relation between degree of reaction and utilization factor.

6. What is degree of reaction? Briefly explain the significance when degree of reaction is 0.5, less than 0.5 and greater than 0.5 with the velocity diagrams.
or
Define degree of reaction (R). Construct the velocity triangles for i) R < 0 ii) R = 0 iii) R = 0.5 iv) R = 1

7. Define degree of reaction and explain how static and dynamic pressure heads influence it. Why degree of reaction for an impulse turbine is zero?

8. Define degree of reaction of a turbomachine. For an impulse type of turbine, why degree of reaction R is zero? Supplement your answer with the definition.

9. The following data refers to a turbomachine. Inlet velocity of whirl=16m/s, velocity of flow=10m/s, blade speed=33m/s, outlet blade speed=8m/s. Discharge is radial with an absolute velocity of 16m/s. If water is the working fluid flowing at the rate of 1m^3/s, calculate (a) the power in kW, (b) the change in total pressure in kN/m^2, (c) the degree of reaction.

10. The following data refers to an axial flow compressor.
Machine: Axial flow compressor
Degree of reaction: R=0.5
Inlet blade angle: β_1=45°
Axial flow is constant: V_{f1}=V_{f2}=100 m/s

Speed of blade: n=6000 rpm
Diameter of the blade: d=0.5 m
Blade speed: $u_1=u_2$
Mass of air: m=2 kg/s
Calculate (a) the fluid angles at inlet and outlet and (b) the power required.

11. A model of Francis turbine of 1:5 scale ratio is tested under a head 1.5m. It develops 3kW at 360RPM. Determine the speed and power developed under a head of 6m.

12. An output of 10kW was recorded on a turbine, 0.5m diameter, revolving at a speed of 800RPM, under a head of 20m. Determine the diameter and output of another turbine which works under a head of 180m at a speed of 200RPM, when their efficiencies are same.

13. Combustion products from combustion chamber in a turbojet engine approaches an axial flow turbine rotor with an absolute velocity of 550 m/s at 18^0 angle from wheel tangent the mass flow rate is 60 kg/s. If the axial velocity is constant at inlet and outlet of turbine find the power output and degree of reaction when blade speed is 300 m/s. Also the absolute velocity at rotor exit is in axial direction.

14. Find the degree of reaction for a sprinkler through which water leaves the jet with an absolute velocity of 3 m/s. The sprinkler arms are 0.2m in length and it rotates at 140rpm.

15. An inward flow radial turbine has the following data: Power = 180kW; speed = 34000rpm, outer diameter of impeller = 0.20m; inner diameter of impeller = 0.08m; Absolute fluid inlet velocity = 293m/s (radial). The fluid enters the impeller axially. Determine the percentage of energy transfer due to change of radius.

16. In a mixed flow turbomachine the fluid enters such that the absolute velocity is axial at the inlet and at outlet relative velocity is radial. What is the degree of reaction and energy input to the fluid, if relative velocity at outlet is same as tangential blade speed at inlet? The following data may be used,
i) Inlet diameter = 16cm,
ii) Exit diameter = 50cm,
iii) Speed = 3000rpm,
iv) Blade angle at inlet = 45^0.