

Turbomachines - 18AE46

Old VTU Question's Answers

Module – 4

Syllabus:

Design and performance analysis of axial flow turbines: Turbine stage, work done, degree of reaction, losses and efficiency, flow passage; subsonic, transonic and supersonic turbines, multi-staging of turbine; exit flow conditions; turbine cooling.

Design and performance analysis of radial turbines: Thermodynamics and aerodynamics of radial turbines; radial turbine characteristics; losses and efficiency; design of radial turbine.

Part – A Questions

1. How do you differentiate between an impulse and a reaction turbine? With neat sketches explain the working of an impulse and a reaction stage.

| <i>Impulse turbine</i> | <i>Reaction turbine</i> |
|--|---|
| <ul style="list-style-type: none">• Complete expansion of the steam takes place in the nozzle, hence steam is ejected with very high kinetic energy.• Blades are symmetrical in shape.• No change in pressure between the ends of the moving blade, i.e. the pressure remains constant between the ends of the blade.• Low efficiency, i.e. part load efficiency is poor.• High speed• Less floor area for the same power generation, hence compact• Used for small power generation• Less stages for the same power generation | <ul style="list-style-type: none">• Partial expansion of the steam takes place in the nozzle (fixed blade) and further expansion takes place in the rotor blades.• Blades are non-symmetrical in shape, i.e. aerofoil section.• Pressure drops from inlet to outlet of the blade, i.e. difference in pressure exists between the ends of the moving blade.• More flattened efficiency curve, hence part load efficiency is good.• Relatively low speed• More floor area for the same power generation, hence bulky.• Used for medium and large power generation.• More stages for the same power generation. |

6.3.1 Impulse Turbine

Impulse or impetus means sudden tendency of action without reflexes. Figures 6.1(a), (b) and (c) show a single-stage impulse turbine. This turbine is called “Simple” impulse steam turbine since the expansion of the steam takes place in one set of the nozzles. A single-stage impulse turbine consists of a set of nozzles and moving blades. High pressure steam at boiler pressure (p_b) enters the nozzle and expands to low back pressure (condenser pressure) in the nozzle. Thus, the pressure energy is converted into kinetic energy increasing the velocity of steam (Figure 6.1(d)). The rapidly moving particles of steam (high velocity steam) are then directed on to a series of blades where the kinetic energy is partly absorbed and converted into an impulse force by changing the direction of flow of steam (the blades are shaped in such a way that, there

is a change in the direction of the steam flow without changing its pressure) which gives rise to a change in momentum and therefore to a force. This sets the blades in motion. The velocity of steam decreases as it flows over the blades but the pressure remains constant, i.e. the pressure

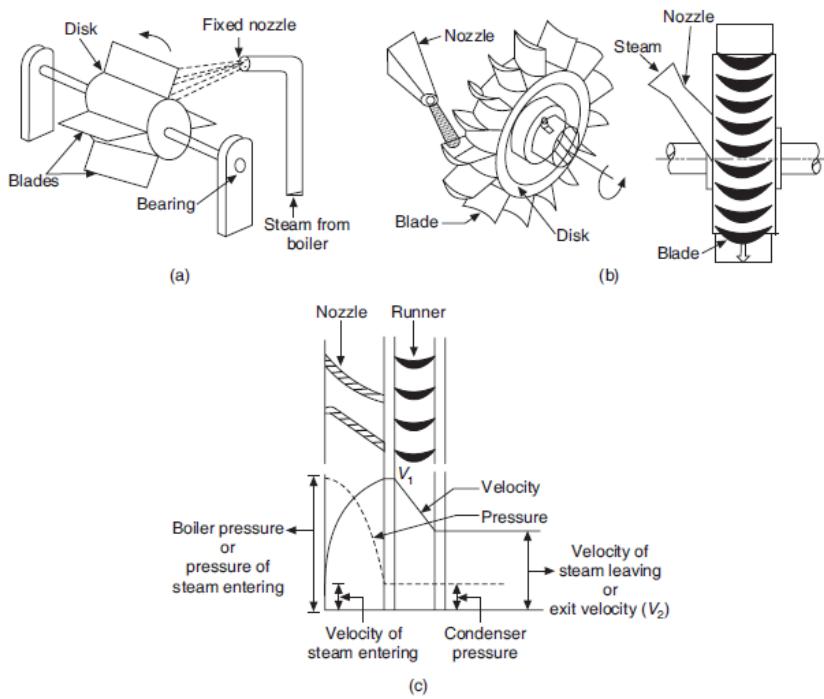


Figure 6.1 Impulse turbine.

at the outlet side of the blade is equal to that at the inlet side. Such a turbine is termed impulse turbine. The processes of expansion and direction changing may occur once, or a number of times in successions. The final velocity is much higher than the inlet velocity to the nozzles in the case of the single-stage turbine. Hence, there is considerable loss in K.E.

6.3.2 Reaction Turbine

High-pressure steam is directly passed on the blades which also act as nozzles. The pressure of steam continuously drops as it flows through the nozzles and the velocity increases. The steam leaving the blades will exert a reactive force in the backward direction of its flow. This reactive force sets the blades in motion. This is called pure reaction. This type of turbine is no longer used. See Figure 6.2.

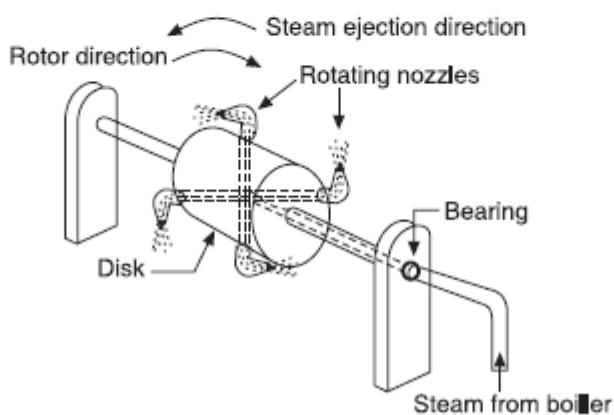


Figure 6.2 Pure reaction turbine.

2. What do you understand by velocity compounding and pressure compounding in a turbine?

6.5 METHODS OF COMPOUNDING OF STEAM TURBINE

Following are the methods of compounding of steam turbines:

1. Velocity compounding
2. Pressure compounding
3. Pressure and velocity compounding

6.5.1 Velocity Compounding

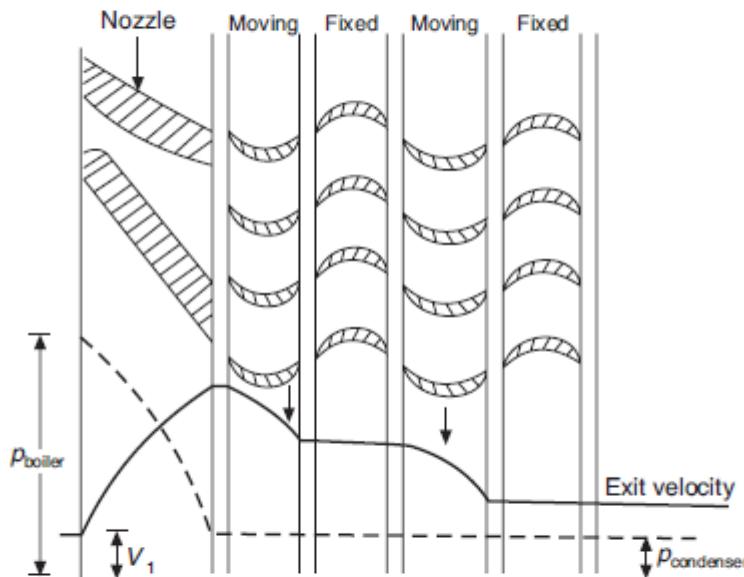


Figure 6.5 Velocity compounded impulse turbine.

Nozzles are fitted to the stationary casing. Each fixed blade row is fitted between the moving blades. The function of the fixed blade is to direct the steam coming from the first moving row to the next moving row without appreciable change in velocity. The whole of the pressure drop occurs in one set of stationary nozzles. The kinetic energy of steam gained in the nozzle is successively absorbed by moving rows and the steam is exited from the last row with very low velocity. The steam leaves axially from the last row. Due to this, the rotor speed decreases considerably. The turbine working on this principle is called the velocity compounded turbine (Figure 6.5). An example is that of Curtis turbine. The velocity compounded impulse turbine is also called the Curtis stage.

6.5.2 Pressure Compounding

A pressure compounded impulse turbine is shown in Figure 6.6. A number of simple impulse stages (one set of nozzles and one set of moving blades) arranged in series are known as pressure compounding. Here the turbine is provided with rows of fixed blades which act as nozzles at the entry of each row of moving blades. The total pressure drop of the steam does not take place in the first row of nozzles but is divided among all the rows of fixed blades which act as nozzles. Each of the simple impulse turbine is named "stage" of the turbine. This arrangement is equivalent to splitting up the whole pressure drop into a series of smaller pressure drops, hence the term "pressure compounded".

The steam leaving the boiler enters the first row of fixed blades, i.e. nozzles in which it is partially expanded. Steam with comparatively high velocity is then passed over the first row of moving blades where almost all its kinetic energy is absorbed. This completes the expansion of steam in one stage (work is done in one stage). Steam leaving the rotor of the first stage is still having pressure energy. Now, the steam enters the stator (fixed blade) of

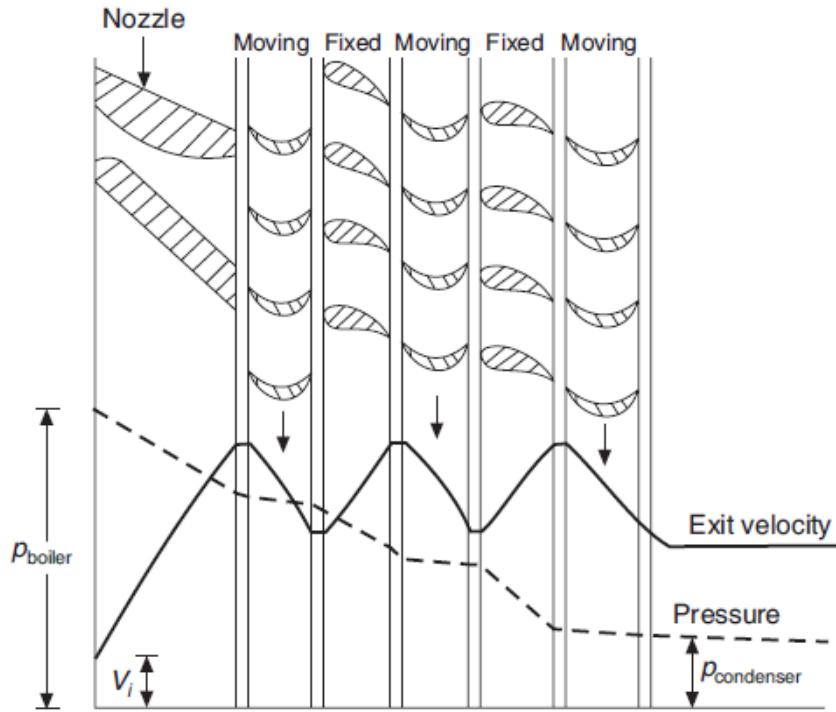


Figure 6.6 Pressure compounded impulse turbine.

the next stage. The fixed blades of pressure compounding act as nozzles. Therefore, steam partially expands in the fixed blades, kinetic energy increases and enters the moving blades. Here again the kinetic energy is absorbed. This completes expansion in two stages. This process continues until steam reaches the condenser pressure in the last stage and leaves axially from the last row.

In each stage, a small quantity of pressure energy is converted into kinetic energy and is absorbed in that stage only. Hence, the steam velocity becomes much smaller and reduces the blade (blade tip) velocities and rotational speed.

6.5.3 Pressure and Velocity Compounding

The pressure and velocity compounding turbine is shown in Figure 6.7. This arrangement is in two stages. Each stage has a two-stage velocity compounded turbine. Consider stage I and stage II individually. They are identical to velocity compounding but both the stages taken together will mean that the pressure drop occurs in the two stages. This arrangement is very popular for its simple construction, but the efficiency is low and hence seldom used.

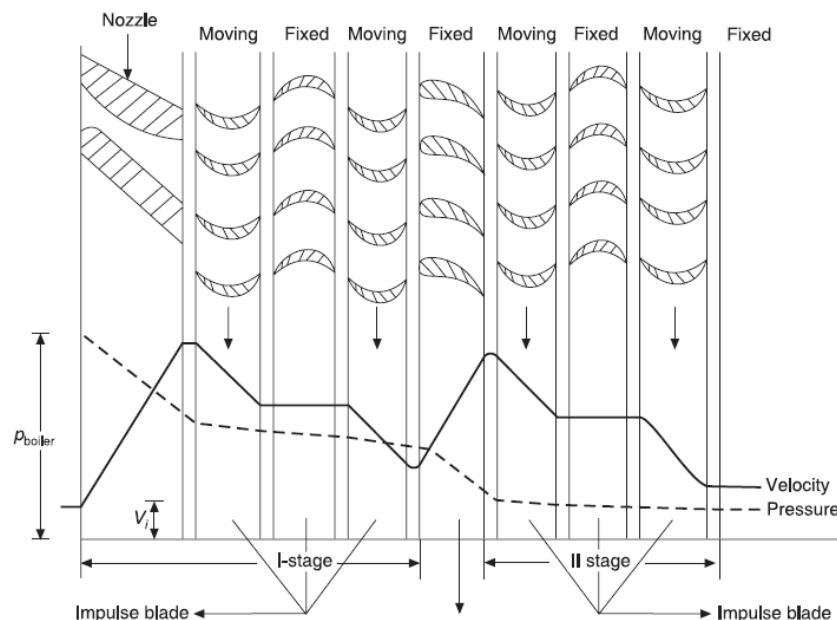


Figure 6.7 Pressure and velocity compounded impulse turbine.

Steam leaving the boiler with high pressure enters a set of nozzles of the first stage. The pressure drops partially in the nozzles, and steam enters the first set of rotors of first stage with very high velocity. In the first set of rotors, kinetic energy is absorbed partially with no change in pressure energy. Then, steam leaves the rotors and enters the stators (fixed blades). Here, theoretically, pressure energy and kinetic energy will remain constant. Then, steam with comparatively high kinetic energy enters the second set of rotors of first stage. Here, the remaining kinetic energy is absorbed. This completes expansion in one stage.

Now, steam with a comparatively high pressure energy enters the fixed blades of the second stage which act as nozzles. Here the pressure drops once again, and is converted into kinetic energy. This kinetic energy is absorbed in the next two stages of the velocity compounded turbine. Finally, the steam will leave the last stage axially at condenser pressure with low kinetic energy. An example of this type of turbine is the Curtis turbine.

3. Define degree of reaction (R) and utilization factor (ϵ). For an axial flow turbine show that the

$$\epsilon = \frac{V_1^2 - V_2^2}{V_1^2 - RV_2^2}$$

utilization factor is given by,

or

Define degree of reaction (R) and utilization factor (ϵ). For axial flow turbines obtain relation between R and ϵ .

Degree of Reaction (R)

A parameter is used to classify the turbomachines based on the relative proportions of dynamic & static pressure changes. This parameter is known as D.O.R.

$$R = \frac{\text{Energy Transfer due to change of static pressure in the Rotor}}{\text{Total energy transfer in the rotor}}$$

Utilization factor (ϵ)

$$\epsilon = \frac{\text{energy utilized}}{\text{energy available to the rotor.}}$$

$$\epsilon = \frac{\text{work developed by the rotor}}{\text{Total energy available for conversion into work}}$$

$$\text{W.D.} = \text{Energy utilized} = \frac{(V_1^2 - V_2^2) + (u_1^2 - u_2^2) + (V_{r2}^2 - V_{r1}^2)}{2g_c}$$

$$\text{Energy available to the rotor} = \frac{V_1^2 + (u_1^2 - u_2^2) + (V_{r2}^2 - V_{r1}^2)}{2g_c}$$

$$= \frac{(V_1^2 - V_2^2) + (u_1^2 - u_2^2) + (V_{r2}^2 - V_{r1}^2)}{2g_c} + \frac{V_2^2}{2g_c}$$

$$= \text{W.D.} + V_2^2 / 2g_c = \text{W.D.} + \text{losses}$$

$$\begin{aligned} \epsilon &= \frac{\text{Ideal work}}{\text{Energy supplied}} = \frac{\text{Energy utilized}}{\text{Energy available to the rotor}} & (2.38) \\ &= \frac{\text{Ideal work output}}{\text{Ideal energy available for conversion into work}} \\ &= \frac{\text{Work developed by the rotor}}{\text{Ideal energy available for conversion into work}} \end{aligned}$$

Since exit absolute K.E. is lost, i.e. is not available for work development,

$$\begin{aligned} \epsilon &= \frac{\frac{(V_1^2 - V_2^2) + (u_1^2 - u_2^2) + (V_{r2}^2 - V_{r1}^2)}{2g_c}}{\frac{V_1^2 + (u_1^2 - u_2^2) + (V_{r2}^2 - V_{r1}^2)}{2g_c}} \\ &= \frac{(V_1^2 - V_2^2) + (u_1^2 - u_2^2) + (V_{r2}^2 - V_{r1}^2)}{V_1^2 + (u_1^2 - u_2^2) + (V_{r2}^2 - V_{r1}^2)} & (2.39a) \end{aligned}$$

$$= \frac{\text{W.D.}}{\text{W.D.} + V_2^2 / 2g_c} = \frac{\text{W.D.}}{\text{W.D.} + \text{losses}} = \frac{\text{E}_{\text{ideal}}}{\text{E}_{\text{ideal}} + \text{losses}} & (2.39b)$$

The only loss in the absence of fluid friction is that due to the K.E. at the exit $V_2^2 / 2g_c$, this energy represents the energy not utilized by the rotor.

$$\therefore \epsilon = \frac{(V_{w1}u_1 - V_{w2}u_2) / g_c}{[V_1^2 + (u_1^2 - u_2^2) + (V_{r2}^2 - V_{r1}^2)] / 2g_c} = \frac{2 \times (V_{w1}u_1 - V_{w2}u_2)}{V_1^2 + (u_1^2 - u_2^2) + (V_{r2}^2 - V_{r1}^2)} & (2.39c)$$

Equation (2.13) can be re-arranged as

$$\underbrace{(u_1^2 - u_2^2) + (V_{r2}^2 - V_{r1}^2)}_x = R(V_1^2 - V_2^2) + \underbrace{[(u_1^2 - u_2^2) + (V_{r2}^2 - V_{r1}^2)]}_x R$$

or

$$x - Rx = R(V_1^2 - V_2^2); x(1 - R) = R(V_1^2 - V_2^2)$$

or

$$(u_1^2 - u_2^2) + (V_{r2}^2 - V_{r1}^2) = R(V_1^2 - V_2^2) / (1 - R) & (2.39d)$$

Substituting Eq. (2.39d) in (2.39a),

$$\varepsilon = \frac{\frac{(V_1^2 - V_2^2) + \frac{R(V_1^2 - V_2^2)}{(1-R)}}{V_1^2 + \frac{R(V_1^2 - V_2^2)}{(1-R)}}}{V_1^2(1-R) + R(V_1^2 - V_2^2)} = \frac{(1-R)(V_1^2 - V_2^2) + R(V_1^2 - V_2^2)}{V_1^2(1-R) + R(V_1^2 - V_2^2)} = \frac{V_1^2 - V_2^2}{V_1^2 - RV_2^2} \quad (2.39e)$$

Equation (2.39e) holds good for a single rotor of any turbine, under the conditions where the Euler turbine equations are expected to hold.

4. What is turbine cooling? Explain different methods utilized to cool the turbine.

or

Justify why cooling is necessary for turbines and explain methods of cooling used for turbine blades.

It has always been the practice to pass a quantity of cooling air over the turbine disc and blade roots. When speaking of the *cooled turbine*, however, we mean the application of a substantial quantity of coolant to the nozzle and rotor blades themselves. Chapters 2 and 3 should have left the reader in no doubt as to the benefits in reduced *SFC* and increased specific power output (or increased specific thrust in the case of aircraft propulsion units) which follow from an increase in permissible turbine inlet temperature. The benefits are still substantial even when the additional losses introduced by the cooling system are taken into account.

Figure 7.29 illustrates the methods of blade cooling that have received serious attention and research effort. Apart from the use of spray cooling for thrust boosting in turbojet engines, the liquid systems have not proved to be practicable. There are difficulties associated with channelling the liquid to and from the blades—whether as primary coolant for forced convection or free convection open thermosyphon systems, or as secondary coolant for closed thermosyphon systems. It is impossible to eliminate corrosion or the formation of deposits in open systems, and very difficult to provide adequate secondary surface cooling area at the base of the blades for closed systems. The only method used successfully in production engines has been internal, forced convection, air cooling. With 1.5–2 per cent of the air mass flow used for cooling per blade row, the blade temperature can be reduced by between 200 and 300 °C. Using current alloys, this permits turbine inlet temperatures of more than 1650 K to be used. The blades are either cast, using cores to form the cooling passages, or forged with holes of any desired shape produced by electrochemical or laser drilling. Figure 7.30 shows the type of turbine rotor blade introduced in the 1980s. The

next step forward is likely to be achieved by transpiration cooling, where the cooling air is forced through a porous blade wall. This method is by far the most economical in cooling air, because not only does it remove heat from the wall more uniformly, but the effusing layer of air insulates the outer surface from the hot gas stream and so reduces the rate of heat transfer to the blade. Successful application awaits further development of suitable porous materials and techniques of blade manufacture.

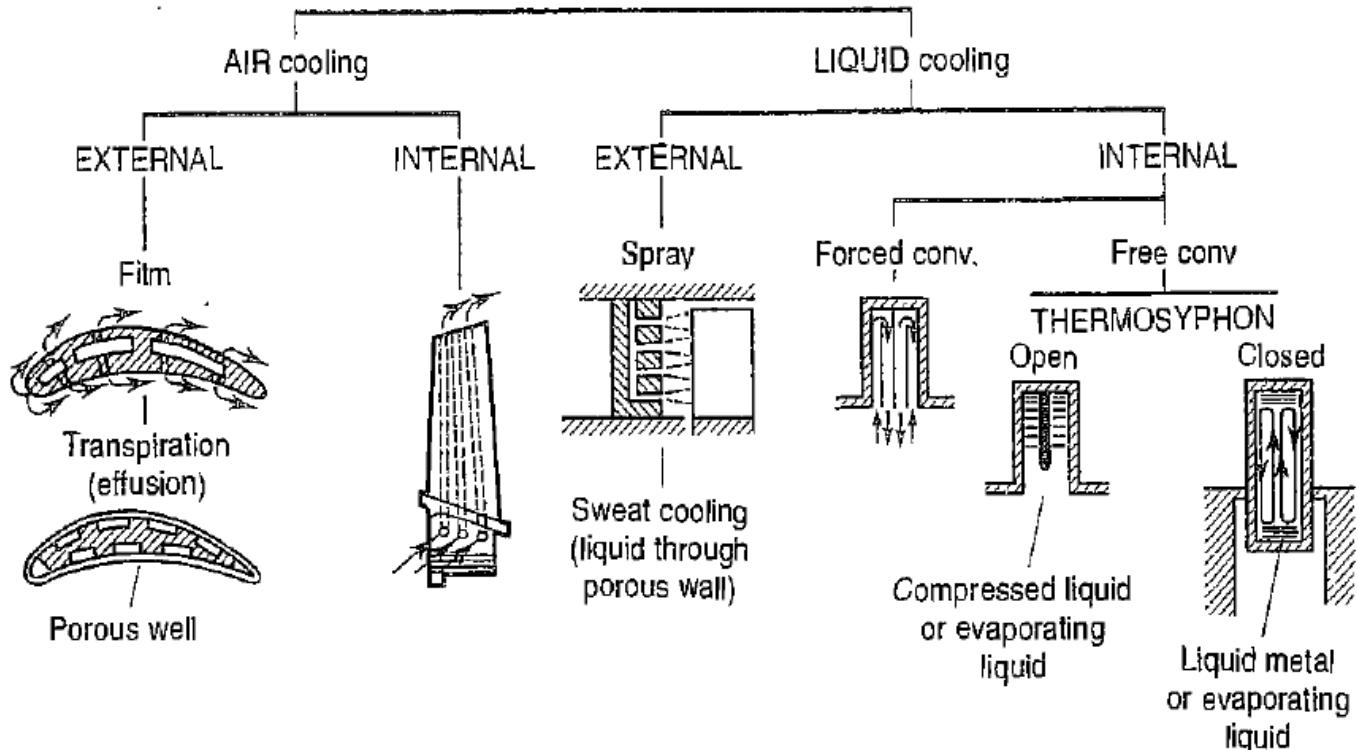


FIG. 7.29 Methods of blade cooling

5. Show that the overall isentropic turbine efficiency is greater than the stage efficiency for an expansion process.

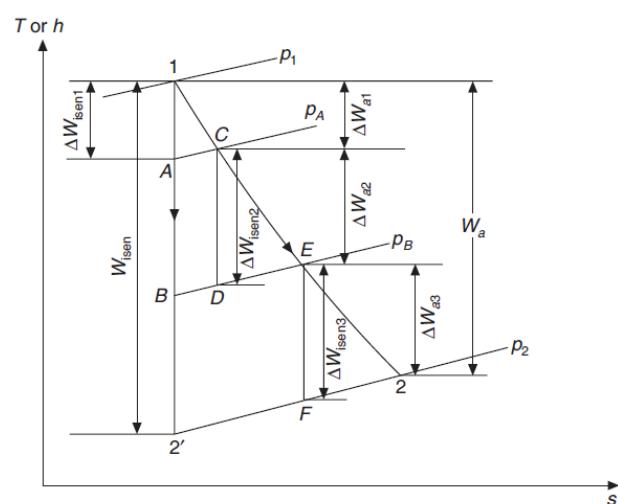


Figure 3.11 Effect of reheating on expansion.

Referring to Figure 3.11, we have

Process 1-2' = isentropic expansion from p_1 to p_2

Process 1-2 = isentropic expansion from p_1 to p_2

The working medium leaves the 1st stage and enters the 2nd stage at *C*. Isentropic expansion in the 2nd stage is represented by *CD* and actual expansion by *CE*. The temperature difference between the ideal process ($T_C - T_D$) of 2nd stage is greater than, if the compression would have taken place as a single stage between P_A to P_B , i.e. process between *A* and *B*.

∴

$$(T_C - T_D) < (T_A - T_B)$$

Hence, the isentropic work of the 2nd stage is less. This is due to the inefficiency of the previous stage. This effect is called 'Reheat'. Hence the isentropic work as a single stage (between p_1 to p_2) is greater than the sum of isentropic work of individual stages (p_1 to p_A , p_A to p_B , p_B to p_2).

Hence, we can say that "in a multistage turbine each succeeding stage is benefitted by the inefficiency of the previous stage."

The multistage compressor as shown in Figure 3.11 is operating between p_1 and p_2 . This is split into three stages of equal pressure ratios.

Let

$\eta_{ts-s} = \eta_{to}$ = overall isentropic expansion efficiency between pressure limits of p_1 to p_2

η_s = stage efficiency (isentropic efficiency) (a total of three stages)

$$= \eta_{s1} = \eta_{s2} = \eta_{s3}$$

p_s = stage pressure ratio

$$= p_{s1}(p_1 \text{ to } p_A) = p_{s2}(p_A \text{ to } p_B) = p_{s3}(p_B \text{ to } p_2)$$

$$= \frac{p_1}{p_A} = \frac{p_A}{p_B} = \frac{p_B}{p_2}$$

p_R = static-to-static overall pressure ratio between p_1 to p_2
(considered as a single stage)

$$= p_2/p_1$$

p_{R0} = total-to-total (stagnation) overall pressure ratio between p_{01} to p_{02}
(considered as a single stage)

$$= p_{01}/p_{02}$$

ΔW_{isen} = isentropic work in each stage (considered as a multistage)

$$= \Delta W_{isen1} = \Delta W_{isen2} = \Delta W_{isen3}$$

W_a = actual work developed considered as a single stage (between p_1 to p_2)

W_{isen} = isentropic work developed (considered as a single stage)

$W_{a1} = W_{a2} = W_{a3}$ = actual work developed in each stage
(considered as a multistage expansion)

$$= \eta_s \Delta W_{isen}$$

Let us consider only one stage (i.e. 1st stage), i.e. pressure limit between p_1 to p_A .

Let us consider only one stage (i.e. 1st stage), i.e. pressure limit between p_1 to p_A .

∴

$$\eta_{s1} = \eta_{t1} = \frac{\Delta W_{a1}}{\Delta W_{isen1}} = \frac{\text{Process 1-C}}{\text{Process 1-A}}$$

ΔW_a = actual work required for one stage

Similarly,

$$\begin{aligned}\eta_{s2} = \eta_{t2} &= \frac{\Delta W_{a2}}{\Delta W_{isen2}} = \frac{\text{Process } C-E}{\text{Process } C-D} \\ \eta_{s3} = \eta_{t3} &= \frac{\Delta W_{a3}}{\Delta W_{isen3}} = \frac{\text{Process } E-2}{\text{Process } E-F} \\ \eta_s = \sum_{i=1}^3 \eta_{si} &= \frac{\Delta W_{a1}}{\Delta W_{isen1}} + \frac{\Delta W_{a2}}{\Delta W_{isen2}} + \frac{\Delta W_{a3}}{\Delta W_{isen3}} = \text{Sum of efficiencies of three stages} \\ &= \frac{\Sigma \Delta W_a}{\Sigma \Delta W_{isen}} = \frac{W_a}{\Sigma \Delta W_{isen}} = \frac{\text{Process } 1-2}{\Sigma \Delta W_{isen}}\end{aligned}\tag{3.53}$$

Now consider a single stage instead of three stages (i.e. 1-2 and 1-2')

$$\eta_{to} = \frac{W_a}{W_{isen}} = \frac{\text{Process } 1-2}{\text{Process } 1-2'}\tag{3.54}$$

From Eqs. (3.53) and (3.54), we have

$$W_a = \eta_s \Sigma \Delta W_{isen}\tag{3.55}$$

and

$$W_a = \eta_{to} W_{isen}\tag{3.56}$$

Equating Eqs. (3.55) and (3.56),

$$\eta_{to} W_{isen} = \eta_s \Sigma \Delta W_{isen}$$

$$\text{or } \frac{\eta_{to}}{\eta_s} = \frac{\Sigma \Delta W_{isen}}{W_{isen}}\tag{3.57}$$

We know that

$$\Sigma \Delta W_{isen} > W_{isen}, \text{ therefore, } \eta_{to} > \eta_s\tag{3.58}$$

Thus, for the expansion process the overall isentropic turbine efficiency is greater than the stage efficiency.

6. Explain the following briefly, Loading co-efficient (ψ) Vs Flow co-efficient (Φ) graph.

- **Flow coefficient, Φ** for an axial flow compressor can be defined as the ratio of axial velocity of flow (C_f) to the peripheral velocity of the blades (C_b).

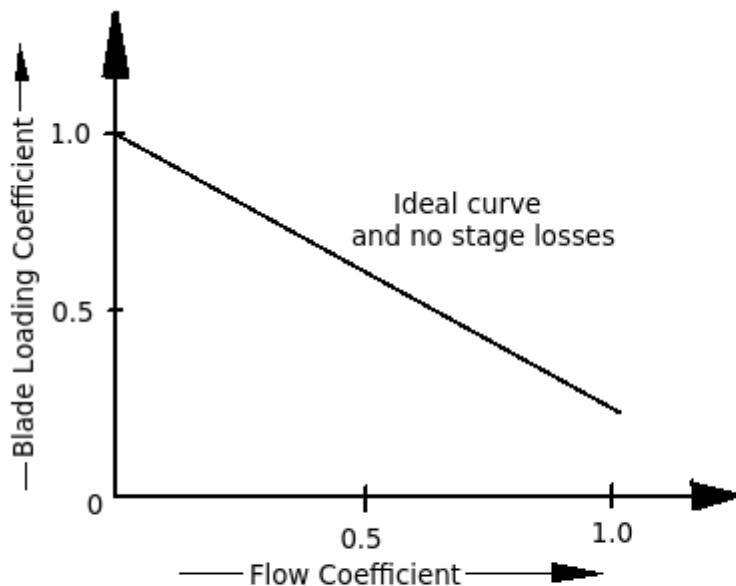
$\therefore \Phi = (\text{Axial velocity of flow } C_f) / (\text{Peripheral velocity of the blades } C_b)$

- **Blade loading coefficient, Ψ** is given by the formula,

$$\Psi = \text{Workdone} / (\text{Blade Velocity})^2$$

$$\Psi = W / (C_b)^2$$

- Blade loading coefficient for an axial flow compressor decreases with increase in the flow coefficient for the same compressor as shown in diagram.



The performance of 50% reaction axial flow compressor can be predicted by using blade loading coefficient and flow coefficient.

7. Construct the velocity triangles for an axial flow turbine for the following conditions: i) $R = 0$ ii) $R = 0.5$ iii) $R = 1$. Comment on how energy transfer takes place in each of the above cases.

Axial flow machines are those in which the fluid enters and leaves the rotor at the same radius. Hence $U_1 = U_2$ for axial flow machines. In this kind of machines, the flow velocity (V_f) is assumed to be constant from inlet to outlet. Axial flow turbines comprise, the familiar steam turbines, gas turbines etc.

2.3.5.1 Velocity Diagrams for Different Values of R :

With $U_1 = U_2$ condition for axial flow turbines, degree of reaction (R) eqn.(2.12) becomes

$$R = \frac{V_{r2}^2 - V_{r1}^2}{(V_1^2 - V_2^2) + V_{r2}^2 - V_{r1}^2} = \frac{(V_{r2}^2 - V_{r1}^2)/2g_c}{E} \quad (2.23)$$

and energy transfer eqn. (2.17) becomes,

$$E = \frac{1}{2g_c} \{(V_1^2 - V_2^2) + (V_{r2}^2 - V_{r1}^2)\} \quad (2.24)$$

From equation (2.23), the different values of R can be obtained depending on magnitudes of velocity components. Thus, for different values of R , different velocity triangles can be drawn as shown in Fig. 2.7.

(i) When $R < 0$ (i.e., R is negative)

If R to be negative, in eqn. (2.23), V_{r1} should be greater than V_{r2} , i.e., $V_{r1} > V_{r2}$. The corresponding velocity diagram is as shown in Fig. 2.7(a). In this case, even though the R is negative, the energy transfer E is positive.

(ii) When $R=0$ (i.e., Impulse type)

$R=0$, when $V_{r1} = V_{r2}$, see eqn. (2.23), and hence $\beta_1 = \beta_2$. This is the characteristic of impulse turbine. This also implies that there is no static pressure change across the rotor. The corresponding velocity diagram is as shown in Fig.2.7(b). In this case, energy E is positive. Hence energy transformation occurs purely due to the change in absolute kinetic

2-10

$$\text{energy i.e., } E = \frac{1}{2g_c} (V_1^2 - V_2^2)$$

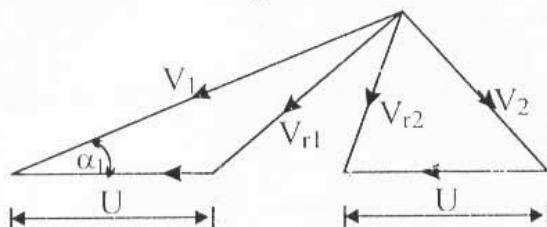


Fig. 2.7 (a). Velocity diagram for $R<0$

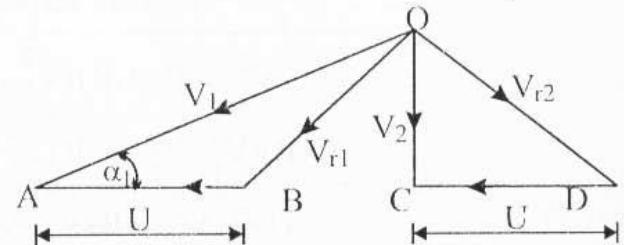


Fig. 2.7 (b). Velocity diagram for $R=0$

(iii) When $R=0.5$ (i.e., 50% Reaction)

In order to get $R=0.5$, the condition will be: $V_1^2 - V_2^2 = V_{r1}^2 - V_{r2}^2$ i.e., $V_1 = V_{r2}$ and $V_2 = V_{r1}$. In this case energy transformation occurs, initially by impulse action and then by reaction. Velocity triangles are shown in Fig.2.7(c).

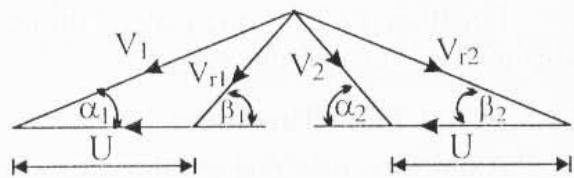


Fig. 2.7(c). Velocity diagram for $R=0.5$

(iv) When $R=1$ (Fully reaction)

This is possible when $V_1 = V_2$, see eqn. (2.23). In this case, i.e., when $R = 1$, energy transformation (E) occurs purely due to change in relative K.E. of fluid.

$$\text{i.e. } E = \frac{1}{2g_c} (V_{r2}^2 - V_{r1}^2)$$

Velocity triangles are shown in Fig.2.7(d).

(v) When $R>1$

This is possible when $V_2 > V_1$. Then the energy transfer (E) may be negative or positive. Velocity triangles are shown in Fig.2.7(e).

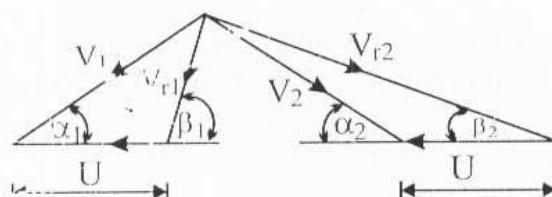


Fig. 2.7(d). Velocity diagram for $R=1$

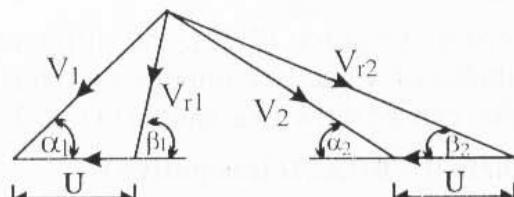
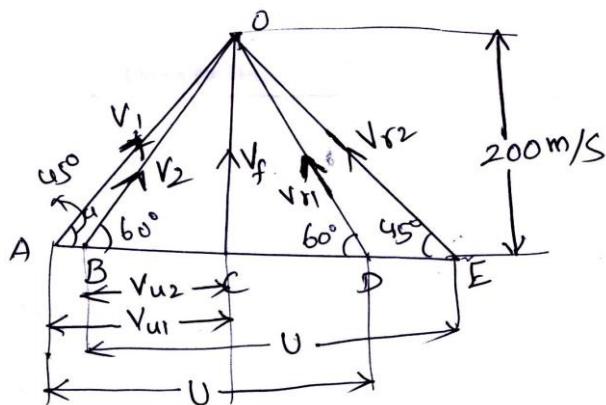


Fig. 2.7(e). Velocity diagram for $R>1$

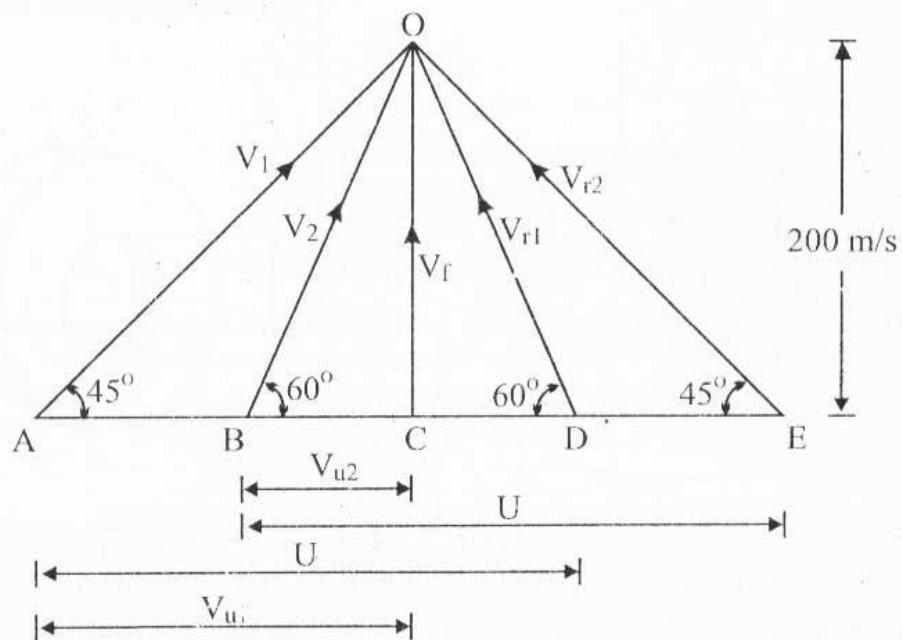
8. Air flows through one stage of turbomachine with a velocity diagram shown in Fig. Justify and find:

- (i) Is this power absorbing or power generating machine
- (ii) Change in total enthalphy
- (iii) Degree of reaction
- (iv) Utilization factor



Example 2.2 : A fluid flows through one stage of a turbo machine. The velocity diagram is as shown in Fig. (a) Is this a power generating or absorbing m/c (b) What is the change in total enthalpy of fluid across the stage ? (c) value of utilization factor ? [Oct. 1995 KU]

Solution :- From triangle OAC,



$$V_{u1} = F_C = \frac{OC}{\tan 45^\circ} = \frac{200}{\tan 45^\circ} = 200 \text{ m/s}$$

$$V_1 = \frac{V_f}{\sin 45^\circ} = \frac{200}{\sin 45^\circ} = 282.8 \text{ m/s} = V_{r2}$$

$$V_2 = \frac{V_f}{\sin 60^\circ} = \frac{200}{\sin 60^\circ} = 230.9 \text{ m/s} = V_{rl}$$

$$V_{u2} = V_2 \cos 60^\circ = 230.9 \cos 60^\circ = 115.45 \text{ m/s}$$

and $U = AC + CD$

$$U = V_{1,1} + V_{1,2} = 200 + 115.45 = 315.45 \text{ m/s}$$

(a) **Power generating or power absorbing:**

Here, $U_1 = U_2 = U$ As $V_{u1} > V_{u2}$, it is power generating m/c

(b) **Change in total enthalpy (Δh_0) .**

$$\Delta h_0 = U(V_{u1} - V_{u2})/g_c \quad (\text{As } V_{u1} \text{ & } V_{u2} \text{ are in the same direction})$$

$$\Delta h_0 = 315.45 (200 - 115.45)/1000$$

$$\Delta h_0 = 26.67 \text{ kJ/kg}$$

(c) **Degree of reaction (R) :**

By observing the values, we can write that

$$V_1 = V_{r2} \text{ and } V_2 = V_{r1}, \text{ also } \alpha_1 = \beta_2 \text{ and } \alpha_2 = \beta_1$$

Hence $R = 0.5$ ie. 50% Reaction

Alternative :

$$\frac{V_1^2 - V_2^2}{2g_c} = \frac{282.8^2 - 230.9^2}{2000} = 13.33 \text{ kJ/kg}$$

$$R = \frac{\Delta h_0 - (V_1^2 - V_2^2)/2g_c}{\Delta h_0} = \frac{26.67 - 13.33}{26.67} = 0.5$$

(d) **Utilization factor (ϵ) :-**

$$\epsilon = \frac{V_1^2 - V_2^2}{V_1^2 - RV_2^2} = \frac{282.8^2 - 230.9^2}{282.8^2 - 0.5 \times 230.9^2}$$

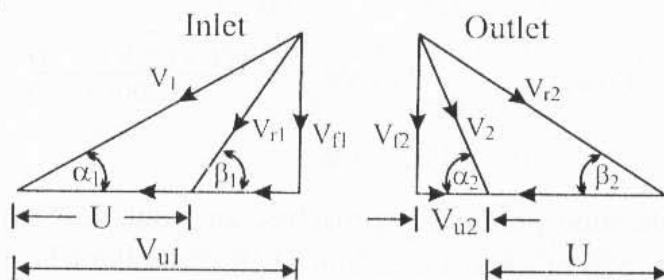
$$\epsilon = 0.5$$

9. Air enters in an axial flow turbine with a tangential component of the absolute velocity equal to 600m/sec in the direction of rotation. At the rotor exit, the tangential component of the absolute velocity is 100m/sec in a direction opposite to that of rotational speed. The tangential blade speed is 250m/sec. Evaluate: i) The change in total enthalpy of air between the inlet and outlet of the rotor ii) The power in KW if the mass flow rate is 10kg/sec iii) The change in total temperature across the rotor.

With reference to flow passage write a brief description of subsonic, transonic and supersonic turbines.

Solution : Given : $V_{u1} = 600 \text{ m/s}$, $V_{u2} = 100 \text{ m/s}$, $\dot{m} = 10 \text{ kg/s}$.

To Find : Δh_0 and P



(i) **Total enthalpy change (Δh_0)**

$$\Delta h_0 = U(V_{u1} + V_{u2})/g_c \quad (\text{As } V_{u2} \text{ is opposite to } U \text{ or } V_{u1})$$

$$= \frac{250(600 + 100)}{1000} \text{ kJ/kg}$$

$$\Delta h_0 = 175 \text{ kJ/kg}$$

(ii) Power output (P) :-

$$\text{Power (P)} = \frac{\dot{m}}{g_c} U(V_{u1} + V_{u2}) = \dot{m} \Delta h_0 = 10 \times 175 \text{ kW}$$

$$P = 1750 \text{ kW}$$

(iii) Change in total temperature (ΔT_0) across the rotor

We know that the total enthalpy change

$$\Delta h_0 = C_p \Delta T_0$$

$$175 = 1.005 \Delta T_0$$

$$\Delta T_0 = 174^0 \text{ K}$$

As the Temperature drops in turbines, $\Delta T_0 = -174^0 \text{ K}$

10. At a 50% reaction stage axial flow turbine, the mean blade diameter is 60cm. The maximum utilization factor is 0.9. Steam flow rate is 10kg/s. Calculate the inlet and outlet absolute velocities and power developed if the speed is 2000rpm.

Solution : $R = 50\%$, $D = 0.6 \text{ m}$, $\epsilon_{\text{max}} = 0.9$, $\dot{m} = 10 \text{ kg/s}$, $N = 2000 \text{ RPM}$.

To Find : V_1 & V_2 , P.

(i) Tangential speed of rotor :

$$U = (\pi DN) / 60 = (\pi \times 0.6 \times 2000) / 60 = 62.8 \text{ m/s} = V_{u1}$$

$$\epsilon_{\text{max}} = \frac{V_1^2 - V_2^2}{V_1^2 - RV_2^2}$$

$$0.9 = \frac{V_1^2 - V_2^2}{V_1^2 - 0.5V_2^2}$$

$$V_1^2 = 5.5V_2^2$$

$$\text{From I/L Vel } \Delta V: V_1^2 = V_2^2 + U^2 = V_2^2 + 62.8^2$$

$$\text{In Eqn. (1). } V_2^2 + 62.8^2 = 5.5V_2^2$$

$$V_2 = 29.6 \text{ m/s}$$

$$\therefore V_1 = \sqrt{5.5 \times 29.6^2} = 69.43 \text{ m/s}$$

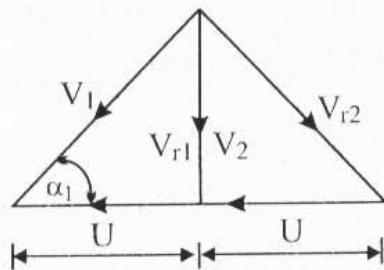
(ii) Power output (P) :

$$P = (\dot{m} UV_{u1}) / g_c \quad (\because V_{u2} = 0)$$

$$P = (\dot{m} U^2) / g_c = 10 \times (62.8)^2 / 1 \times 1000 \quad (\because U = V_{u1})$$

$$P = 39.44 \text{ kW}$$

combined velocity triangle



11. Show that ϵ_{\max} of an axial flow turbine with degree of reaction = $\frac{1}{4}$, the relationship of blade speed

$$\frac{U}{V_1} = \frac{2}{3} \cos \alpha_1$$

U to absolute velocity at rotor inlet V_1 , should be $\frac{2}{3} \cos \alpha_1$, where α_1 = nozzle angle at inlet. Assume flow velocity is constant from inlet to outlet.

Solution: Given : $R = 1/4$, $U_1 = U_2 = U$,
 V_1 is axial for ϵ_{\max} , i.e., $V_{u2} = 0$

From triangle ADE :

$$V_{r2}^2 = U^2 + V_2^2 = U^2 + V_1^2 \sin^2 \alpha_1$$

From triangle ABD

$$BD = V_1 \cos \alpha_1 \quad \& \quad BC + CD = V_1 \cos \alpha_1$$

$$U + X = V_1 \cos \alpha_1 \quad \& \quad X = V_1 \cos \alpha_1 - U$$

From triangle ACD.

$$V_{rl}^2 = AD^2 + CD^2 = V_2^2 + (V_1 \cos \alpha_1 - U)^2$$

$$= V_1^2 \sin^2 \alpha_1 + (V_1 \cos \alpha_1 - U)^2 = V_1^2 \sin^2 \alpha_1 + V_1^2 \cos^2 \alpha_1 + U^2 - 2V_1 U \cos \alpha_1 \quad (b)$$

$$R = \frac{(V_{r2}^2 - V_{rl}^2)}{(V_1^2 - V_2^2) + (V_{r2}^2 - V_{rl}^2)} = \frac{1}{4}$$

$$4V_{r2}^2 - 4V_{rl}^2 = V_1^2 - V_2^2 + V_{r2}^2 - V_{rl}^2$$

$$\text{or } 3V_{r2}^2 - 3V_{rl}^2 = V_1^2 - V_2^2 \quad (c)$$

Substituting eqn. (a) & (b) into eqn. (c), we get.

$$3[U^2 + V_1^2 \sin^2 \alpha_1 - V_1^2 \sin^2 \alpha_1 - V_1^2 \cos^2 \alpha_1 - U^2 + 2UV_1 \cos \alpha_1] = V_1^2 - V_2^2$$

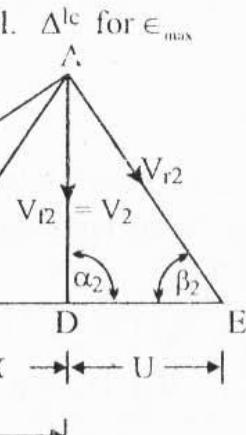
$$= V_1^2 - V_1^2 \sin^2 \alpha_1 \quad (\because V_2 = V_1 \sin \alpha_1)$$

$$= V_1^2 (1 - \sin^2 \alpha_1)$$

$$\text{or } V_1^2 \cos^2 \alpha_1 = 6UV_1 \cos \alpha_1 - 3V_1^2 \cos^2 \alpha_1$$

$$4V_1^2 \cos^2 \alpha_1 = 6VU_1 \cos \alpha_1$$

$$\text{or } \frac{U}{V_1} = \frac{2}{3} \cos \alpha_1$$



Combined Vel. Δ^{lc} for ϵ_{\max}

(a)

V_{r2}

V_{rl}

V_{r1}

V_2

V_1

U

X

U

V_{u1}

α_1

β_1

α_2

β_2

Proved

Part - B Questions

1. Draw Enthalpy-Entropy diagram for a radial turbine and explain the same.

or

Draw and explain the components of radial turbine and operational principle.

2. Mention different types of losses in a radial flow turbine and define nozzle loss co-efficient or

Describe the various stage losses occurring in a radial turbines.

or

Define 'nozzle loss' and 'rotor loss' coefficients writing suitable expressions.

3. Draw and explain Blade-to-gas speed ratio (σ) (V_s) Stage efficiency (η_s) graph for a radial turbine

4. A radial outward flow machine has no inlet whirl. The blade speed at the exit is twice that at inlet.

Radial velocity is constant taking inlet blade angle 45° . Show that

$$R = \frac{2 + \cot \beta_2}{4}$$

5. Sketch and explain the working of a 90° inward radial flow turbine (IFR).

6. In a multi stage Inward Flow Radial turbine (IFR), what are the aerodynamic losses occur in the stage. Discuss

or

Describe briefly the various aerodynamic losses occurring in an inward flow radial turbine stage.

7. Explain briefly: i) Nozzle loss coefficient ii) Rotor loss coefficient writing suitable expressions for a radial gas turbine.

8. An inward flow reaction turbine has outer and inner diameter of the wheel as 1m and 0.5 m respectively, the vanes are radial at inlet and discharge is radial at outlet. Water enters the vanes at an angle of 10° . Assuming velocity of flow to be constant and equal to 3 m/s find: i) The speed of the wheel ii) The vane angle at outlet iii) The degree of reaction.

9. The output of a 3 stage gas turbine is 30MW at the shaft coupling at an entry temperature of 1500K. The overall pressure ratio across the turbine is 11 and efficiency 88%. If the pressure ratio of each stage is same, determine, i) Pressure ratio of each stage, ii) Polytropic efficiency. Assume $\gamma = 1.4$, $c_p = 1.005 \text{ kJ/kg.K}$. Assume an efficiency of 91% to take into account shaft losses due to disc and bearing friction.

10. A hydraulic reaction turbine of the radial inward flow types works under a head of 160 m of water. At the point of fluid entry the rotor blade angle is 119° and the diameter of the runner is 3.65 m. At the exit the runner diameter is 2.45 m. If the absolute velocity at the wheel outlet is radially directed with a magnitude of 15.5 m/s and the radial component of velocity at the inlet is 10.3 m/s. Determine (i) Power developed for flow rate of $110 \text{ m}^3/\text{s}$. (ii) Degree of reaction (iii) Utilization factor.

11. An inward flow radial reaction turbine has axial discharge at outlet with outlet blade angle of 45° . The radial velocity of flow is constant. The blade speed at the inlet is twice that of exit. Express energy transfer per unit mass and degree of reaction in terms of nozzle angle α_1 . Assume $V_m = (29c)^{1/2}$.

12. A single stage 90° IFR fitted with an exhaust diffuser has the following data: Overall stage pressure ratio = 4, Temperature at entry = 557 K, Diffuser exit pressure = 1 bar, Mass flow rate = 6.5 kg/s, Flow co-efficient = 0.3, Speed = 18000 rpm, Rotor tip dia = 42 cm. Enthalpy losses in nozzle and rest of the stages are equal. Assuming negligible velocities at the nozzle entry and diffuser exit. Find (i) The nozzle exit air angle (ii) Power developed.

13. A 90° IFR turbine stage has the following data: Total-to-static pressure ratio $P_{01}/P_3 = 3.5$, exit pressure = 1 bar, stagnation temperature at entry = 650°C , blade-to-isentropic speed ratio $\sigma = 0.66$, rotor diameter ratio $d_3/d_2 = 0.45$, rotor speed $N = 16000\text{rpm}$, nozzle exit air angle = 20° , nozzle efficiency = 0.95, rotor width at entry $b_2 = 5\text{cm}$. Assuming constant meridional velocity, axial exit and that the properties of the working fluid are the same as those of air, determine: the rotor diameter, rotor blade exit air angle, mass flow rate, hub and tip diameter at the rotor exit, power developed, total-to-static efficiency of the stage.