Basic Aerodynamics

Module - 2

Basic Aerodynamic Equations

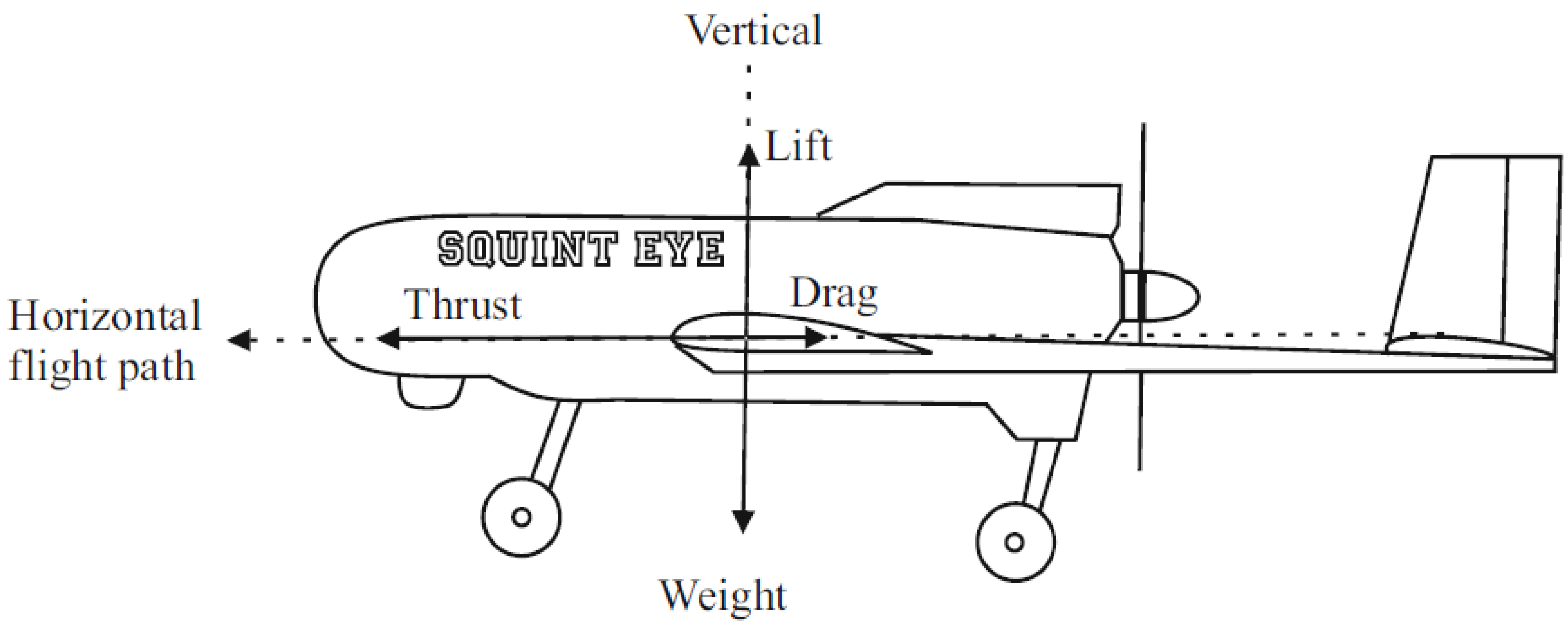


Figure 3.1 Forces on air vehicle

Basic Aerodynamic Equations

- The dynamic pressure, *q*, of a moving airstream is given by:

$$q = \frac{1}{2}\rho V^2$$

- The forces acting on an airplane wing are a function of *q*, the wing area *S*, and dimensionless coefficients (*Cl*, *Cd*, and *Cm*) that depend on Reynolds number, Mach number, and the shape of the cross-section of the wing.
- The first two forces, lift and drag, are written as follows:

$$L = C_l q S$$

$$D = C_d q S$$

Basic Aerodynamic Equations

- The third force of this aerodynamic triumvirate is pitching moment, which must include an additional term to dimensionally create a moment.
- The wing chord, c , is the usual distance chosen as the moment arm.
- Knowledge of the pitching moment is critical to the understanding of stability and control:

$$M = C_m q S c$$

Basic Aerodynamic Equations

- *Cl*, *Cd*, and *Cm* characterize the lift, drag, and moment for any airfoil cross-section, and are the aerodynamic coefficients of primary interest to the UAV designer.
- Any particular airfoil cross-sectional shape has a characteristic set of curves for the coefficients of lift, drag, and moment that depend on angle of attack and Reynolds number.
- These are determined from wind tunnel tests and are designated by lowercase subscripts.
- **Lift** is always perpendicular and drag always parallel to the relative wind.
- The moment can be taken with respect to any point, but traditionally is taken about a point 25% rearward of the wing leading edge known as the quarter chord.

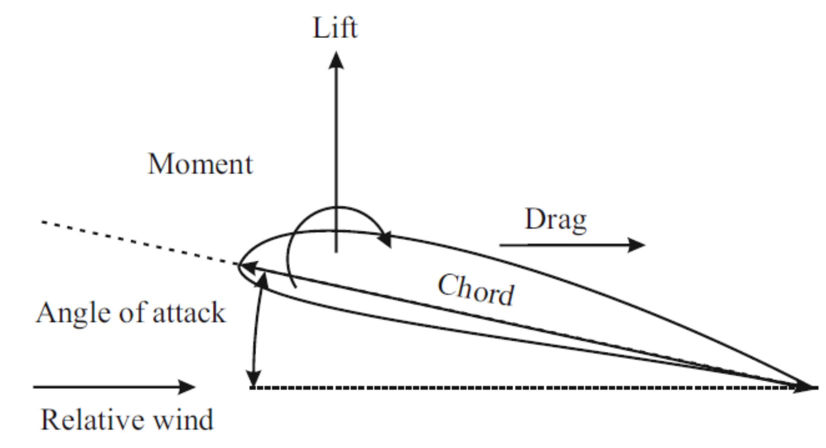


Figure 3.2 Airfoil geometry

Basic Aerodynamic Equations

- Basic aerodynamic data are usually measured from a wing that extends from wall to wall in the wind tunnel as shown in Figure 3.3.
- Extending the wing from wall to wall prevents spanwise airflow and results in a true two-dimensional pattern of air pressure.
- This concept is called the infinite-span wing because a wing with an infinite span could not have air flowing around its tips, creating spanwise flow and disturbing the two-dimensional pressure pattern that is a necessary starting point for describing the aerodynamic forces on a wing.

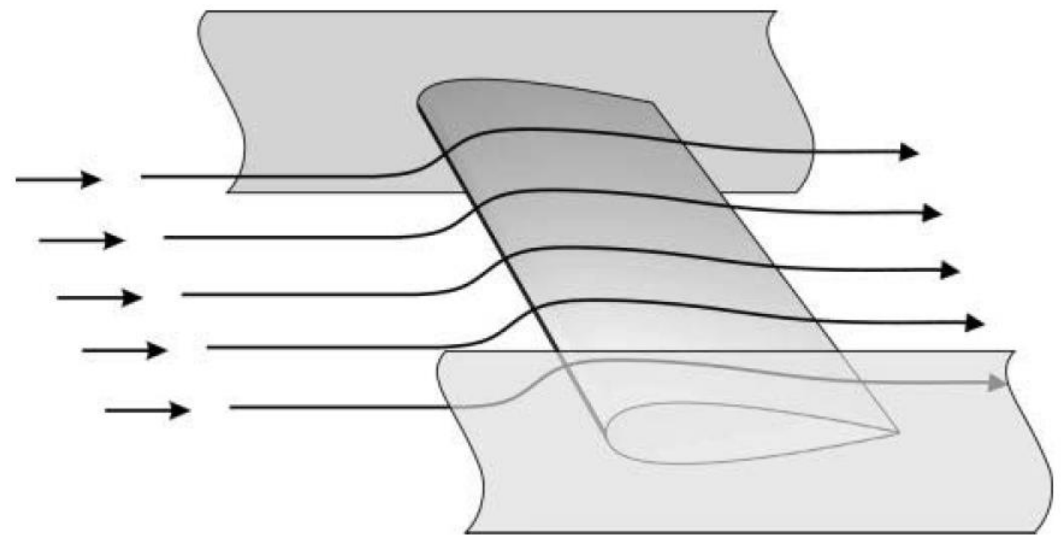


Figure 3.3 Infinite span wing

Basic Aerodynamic Equations

- Airfoil cross-sections and their two-dimensional coefficients are classified in a standard system maintained by the National Aeronautics and Space Administration (NASA) and identified by a NASA numbering system,
- NASA airfoil 23201 as an example of the information available on many airfoil designs.
- Figure 3.4 shows the profile of a cross-section of the airfoil. The *x* (horizontal) and *y* (vertical) coordinates of the surface are plotted as *x*/*c* and *y*/*c*, where *c* is the chord of the airfoil, its total length from nose to tail.

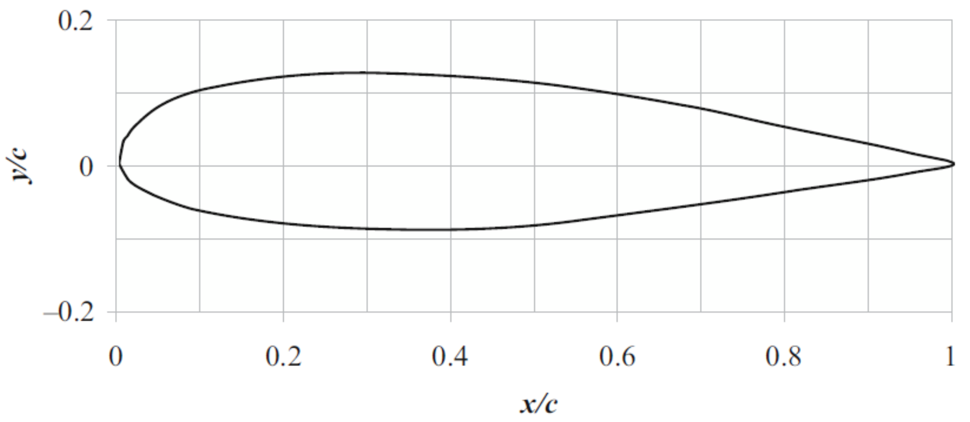


Figure 3.4 NASA 23021 airfoil profile

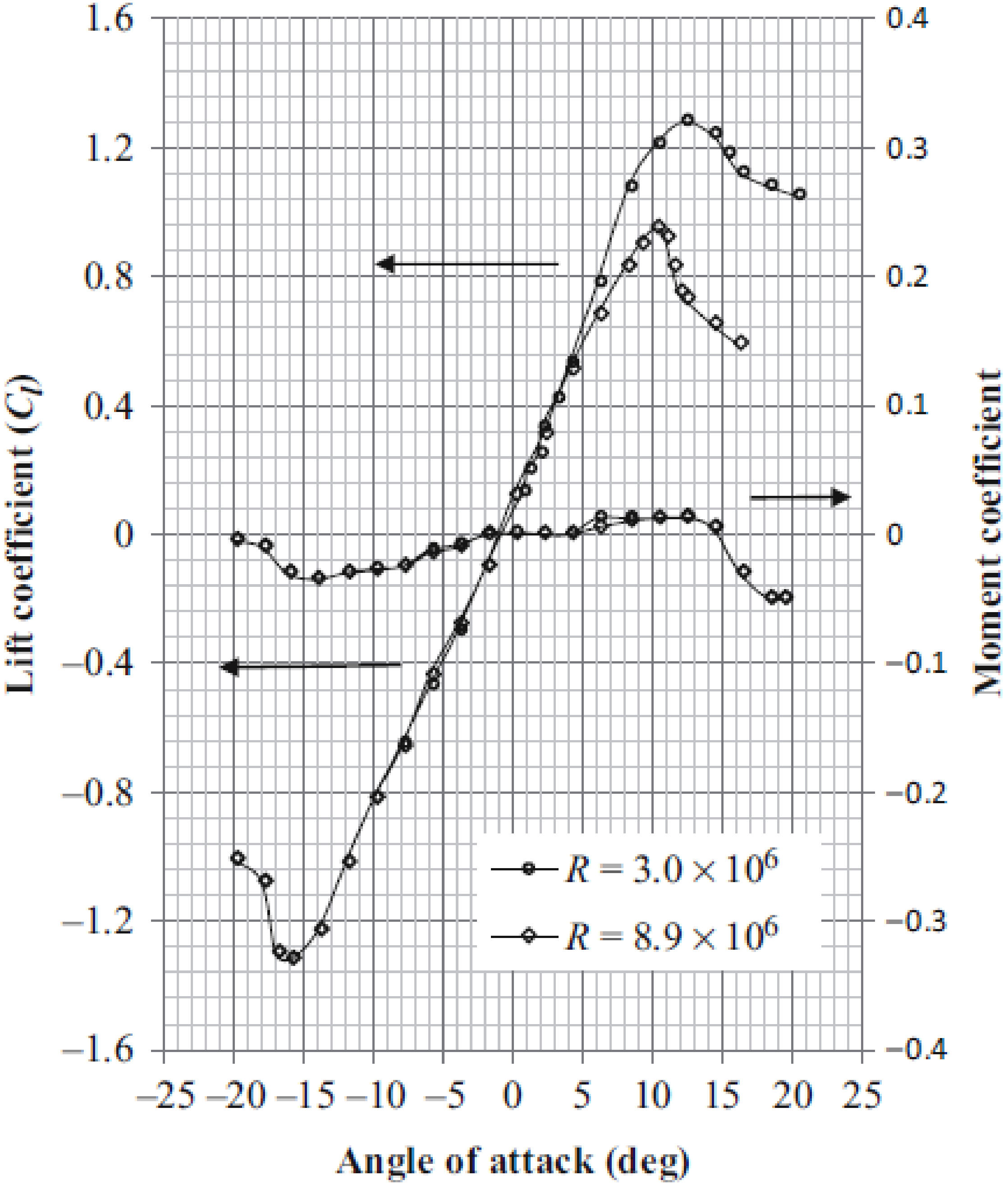


Figure 3.5 NASA 23021 airfoil coefficients versus angle of attack

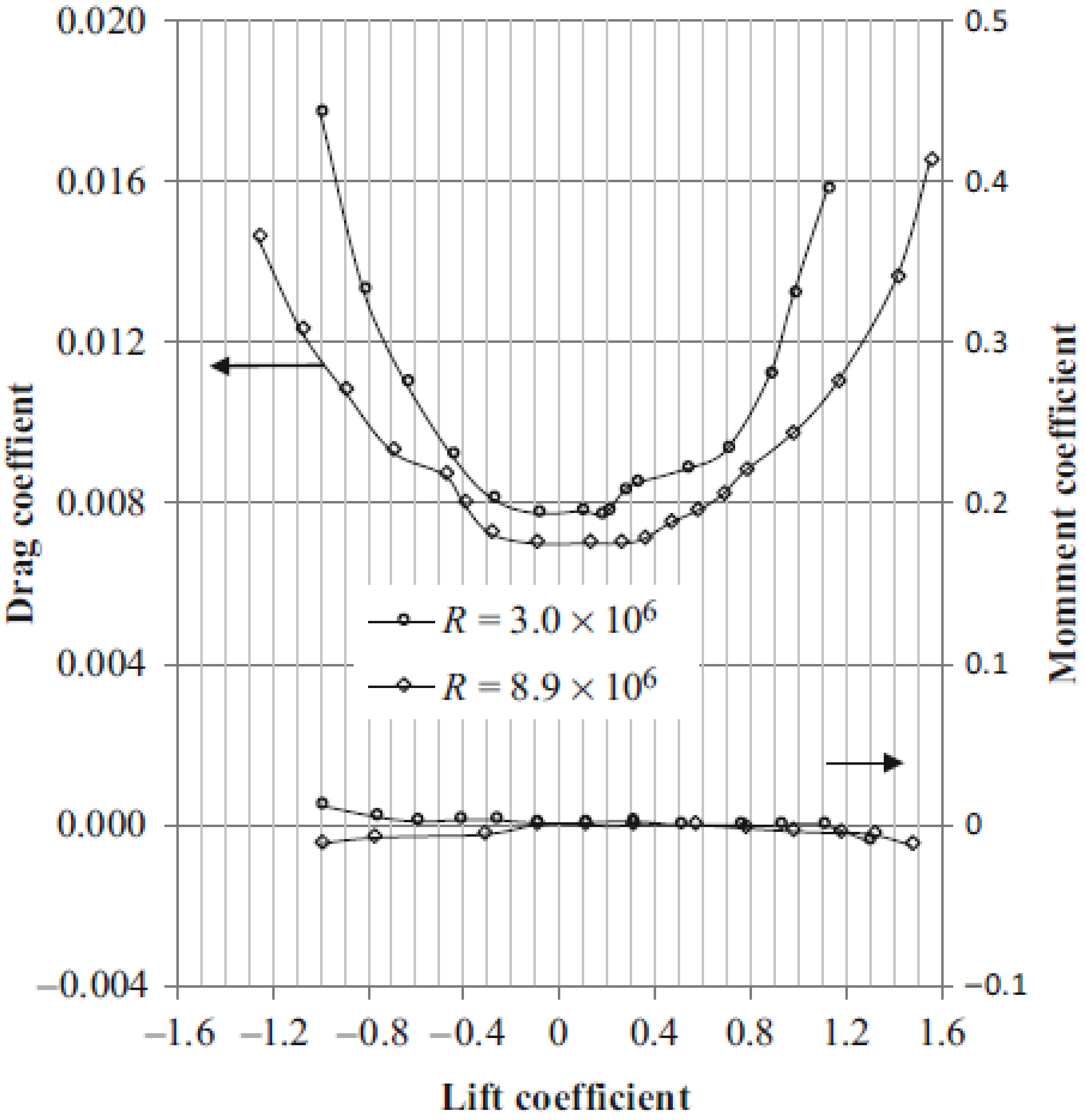


Figure 3.6 NASA 23021 airfoil coefficients versus lift coefficient

Basic Aerodynamic Equations

- A question of interest is: what is the minimum speed at which an airplane still can fly?
- This is important for understanding landing, take-off, launch from a catapult, and arrested recovery.
- To find the minimum velocity at which the airplane can fly, we set lift equal to weight in to balance the vertical forces, and solve for velocity.
- If the maximum lift coefficient, *CLM*, is known then the minimum velocity can be seen to be directly proportional to the square root of the wing loading *W/S*. Needless to say, an airplane with a large wing area and low weight can fly slower than a heavy, small-winged airplane.
- The equation for minimum velocity is:

$$V_{\min} = \sqrt{\left(\frac{W}{S}\right)\left(\frac{2}{\rho C_{LM}}\right)}$$

Aircraft Polar

- Another important concept concerning the three-dimensional air vehicle is what is known as the aircraft or drag "polar," a term introduced by Eiffel years ago, which is a curve of *CL* plotted against *CD*.
- The drag polar will later be shown to be parabolic in shape and define the minimum drag, *CD*0, or drag that is not attributable to the generation of lift.
- A line drawn from the origin and tangent to the polar gives the minimum lift-to-drag ratio that can be obtained.
- It will also be shown later that the reciprocal of this ratio is the tangent of the power-off glide angle of an air vehicle.
- The drag created by lift or induced drag is also indicated on the drag polar.

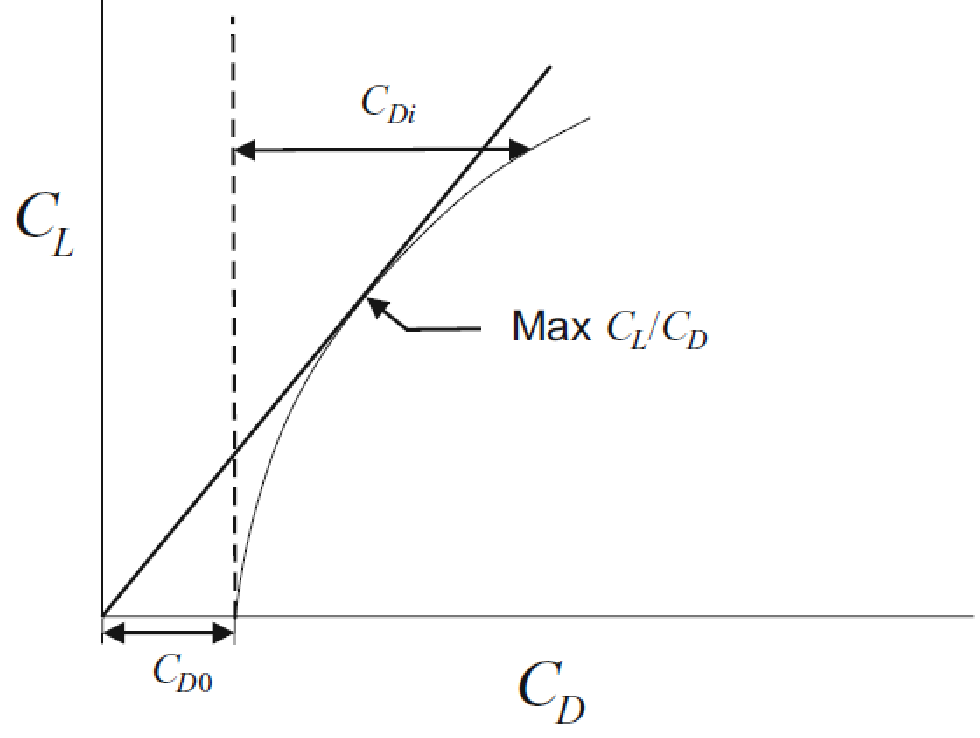


Figure 3.7 Aircraft polar

The Real Wing and Airplane

- The wing geometry has a shape, looking at it from the top, called the planform.
- It often has twist, sweepback, and dihedral (angle with the horizontal looking at it from the front) and is composed of two-dimensional airfoil sections.
- A full analysis for lift and drag must consider not only the contribution of the wing but also by the tail and fuselage and must account for varying airfoil cross-section characteristics and twist along the span.

The Real Wing and Airplane

- Determining the three-dimensional moment coefficient also is a complex procedure that must take into account the contributions from all parts of the aircraft.
- Summing these forces about the aircraft center of gravity (CG) results in Equation (3.6):

$$M_{CG} = Lx_a + Dz_a + m_{ac} - L_t x_t + m_{act} \quad (\text{if } D_t = 0)$$

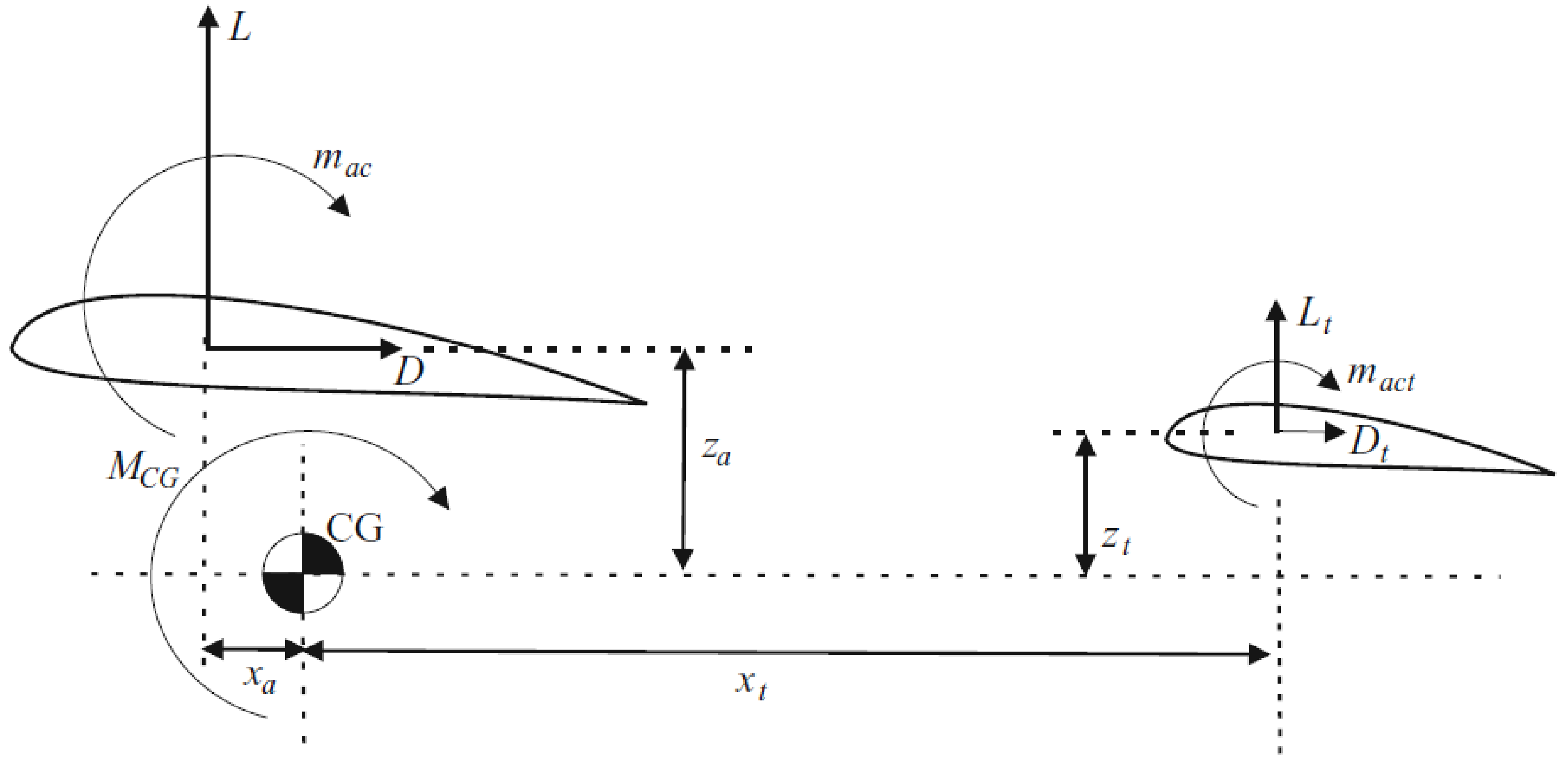


Figure 3.8 Moment balance diagram

The Real Wing and Airplane

- Dividing by *q/Sc in* $M = C_m q S c$ the three-dimensional pitching moment coefficient about the CG is obtained as,

$$C_{M_{CG}} = C_L\left(\frac{x_a}{c}\right) + C_D\left(\frac{z_a}{c}\right) + C_{mac} + C_{fus} - C_{L_t}\left(\frac{S_t}{S}\right)\left(\frac{x_t}{c}\right) + C_{mact}$$

- where *St* is the area of the tail surface and *S* the area of the wing.
- A crude estimate (given without proof) of the three-dimensional wing lift coefficient, indicated by an uppercase subscript, in terms of the "infinite wing" coefficient is:

$$C_L = \frac{C_l}{\left(1 + \frac{2}{AR}\right)}$$

Induced Drag

- Drag of the three-dimensional airplane wing plays a particularly important role in airplane design because of the influence of drag on performance and its relationship to the size and shape of the wing planform.
- The most important element of drag introduced by a wing is the "induced drag," which is drag that is inseparably related to the lift provided by the wing.

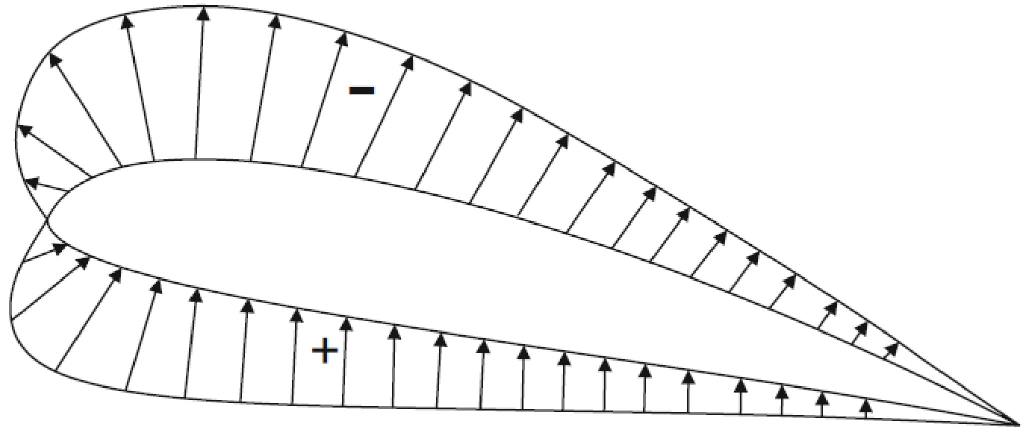


Figure 3.9 Pressure Distribution

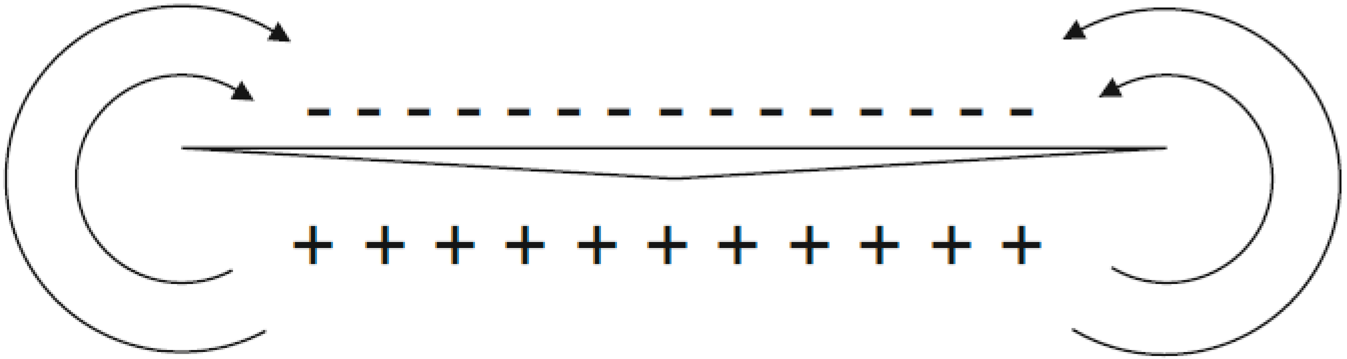


Figure 3.10 Spanwise pressure distribution

Induced Drag

- Such a condition would allow air to spill over from the higher pressure on the bottom surface to the lower pressure top causing it to swirl or form a vortex.
- The downward velocity or downwash onto the top of the wing created by the swirl would be greatest at the tips and reduced toward the wing center

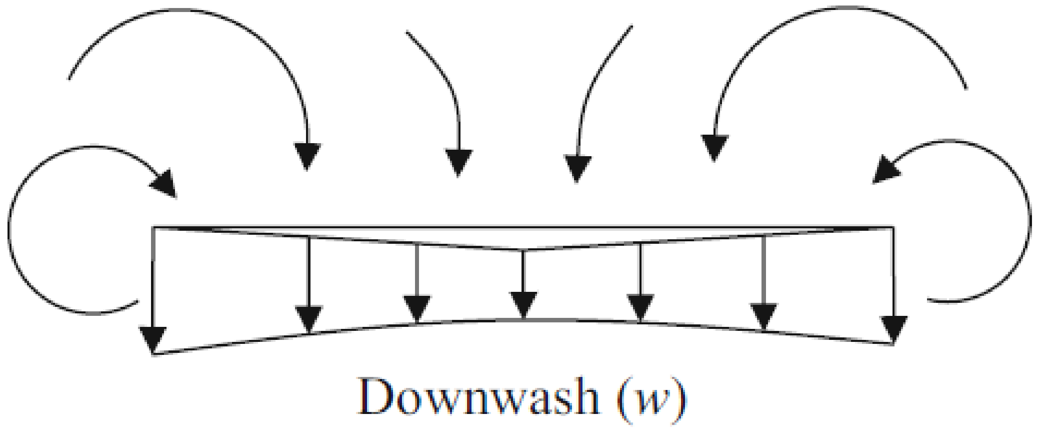


Figure 3.11 Downwash

Induced Drag

- Ludwig Prandtl has shown that a wing whose planform is elliptical would have an elliptical lift distribution and a constant downwash along the span,
- The notion of a constant downwash velocity (*w*) along the span will be the starting point for the development of the effect of three-dimensional drag.

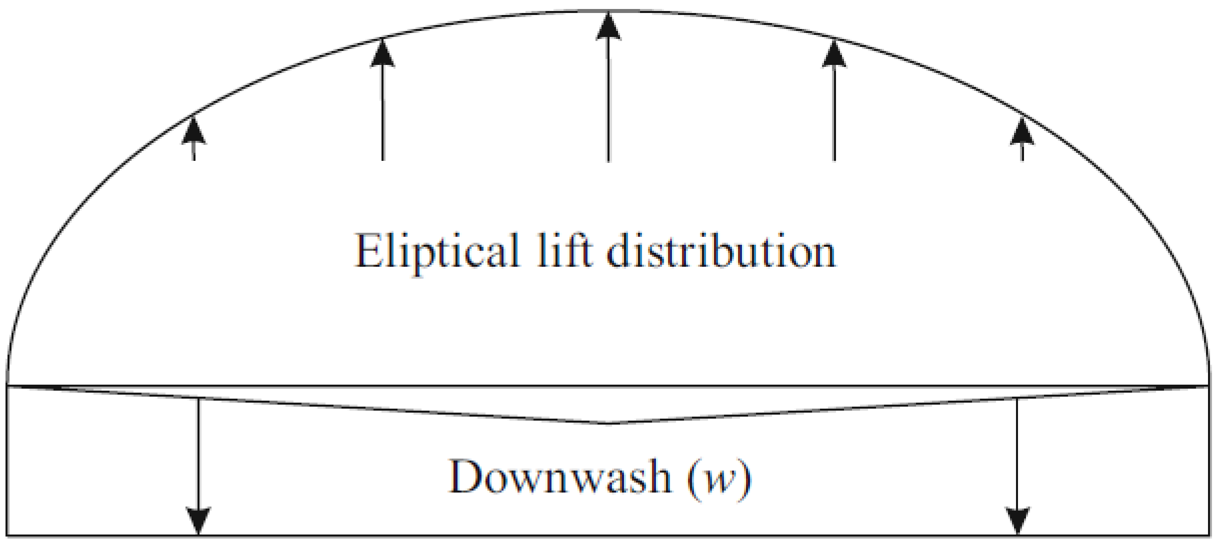


Figure 3.12 Elliptical lift distribution

Induced Drag

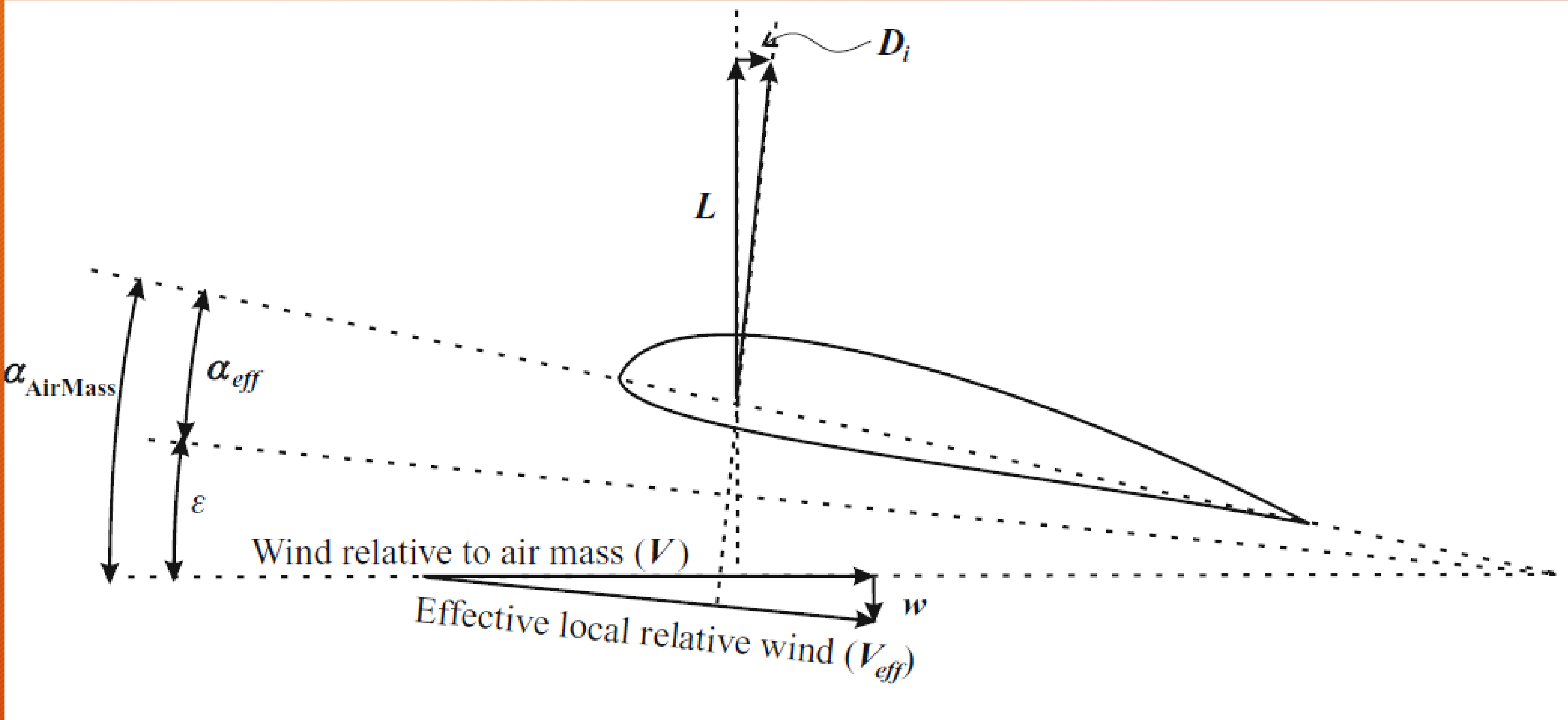


Figure 3.13 Induced drag diagram

Induced Drag

This reduction in the angle of attack is:

$$\varepsilon = \tan^{-1}\left(\frac{w}{V}\right)$$

From Figure 3.13, one can see that the velocity and force triangles are similar, so:

$$\frac{D_i}{L} = \frac{w}{V}$$

Dividing by q (see Equations (3.1) through (3.3)):

$$\frac{C_{Di}}{C_L} = \frac{w}{V}$$

For the case of an elliptical lift distribution, Ludwig Prandtl has shown that:

$$\frac{w}{V} = \frac{C_L}{\pi AR}$$

then the induced drag coefficient (C_{Di}) is given by:

$$C_{Di} = \frac{C_L^2}{\pi AR}$$

- This expression reveals to us that air vehicles with short stubby wings (small *AR*) will have relatively high-induced drag and therefore suffer in range and endurance.
- Air vehicles that are required to stay aloft for long periods of time and/or have limited power, as, for instance, most electric-motor-driven UAVs, will have long thin wings.

The Boundary Layer

- A fundamental axiom of fluid dynamics is the notion that a fluid flowing over a surface has a very thin layer adjacent to the surface that sticks to it and therefore has a zero velocity.
- The next layer (or lamina) adjacent to the first has a very small velocity differential, relative to the first layer, whose magnitude depends on the viscosity of the fluid.
- The more viscous the fluid, the lower the velocity differential between each succeeding layer.
- At some distance δ, measured perpendicular to the surface, the velocity is equal to the free-stream velocity of the fluid.
- The distance δ is defined as the thickness of the boundary layer.

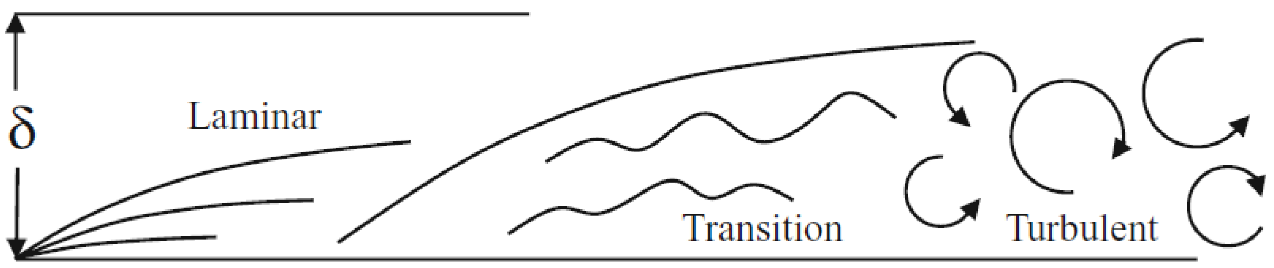


Figure 3.14 Typical boundary layer

The Boundary Layer

The boundary is composed of three regions beginning at the leading edge of a surface:

(1) **the laminar region** where each layer or lamina slips over the adjacent layer in an orderly manner creating a well-defined shear force in the fluid,

(2) **a transition region**, and

(3) **a turbulent region** where the particles of fluid mix with each other in a random way creating turbulence and eddies.

- The shear force in the laminar region and the swirls and eddies in the turbulent region both create drag, but with different physical processes.

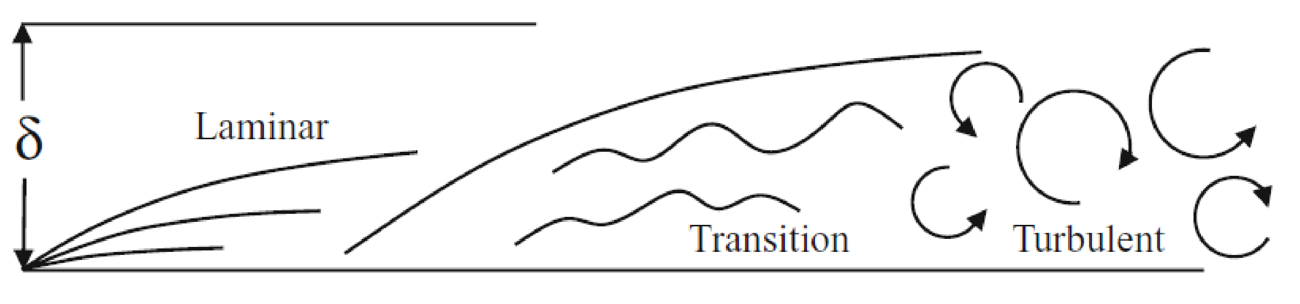


Figure 3.14 Typical boundary layer

The Boundary Layer

- The shearing stress that the fluid exerts on the surface is called skin friction and is an important component of the overall drag.
- The two distinct regions in the boundary layer (laminar and turbulent) depend on the velocity of the fluid, the surface roughness, the fluid density, and the fluid viscosity.

Reynolds number, which mathematically is expressed as:

$$R = \rho V \left(\frac{l}{\mu} \right)$$

- Laminar flow creates considerably less drag than turbulent but nevertheless causes difficulties with small surfaces.
- Typical Reynolds numbers are:
  - General Aviation Aircraft 5,000,000
  - Small UAVs 400,000
  - A Seagull 100,000
  - A Gliding Butterfly 7,000

The Boundary Layer

- Laminar flow causes drag by virtue of the friction between layers and is particularly sensitive to the surface condition. Normally, laminar flow results in less drag and is desirable.
- The drag of the turbulent boundary layer is caused by a completely different mechanism that depends on knowledge of Bernoulli's theorem.
- its velocity increases (because of the law of conservation of mass) and, as a consequence of Bernoulli's theorem, its pressure decreases, causing what is known as a favorable pressure gradient. because it helps push the fluid in the boundary layer on its way.
- After reaching a maximum velocity, the fluid begins to slow and consequently forms an unfavorable pressure gradient (i.e., hinders the boundary layer flow) as seen by the velocity profiles

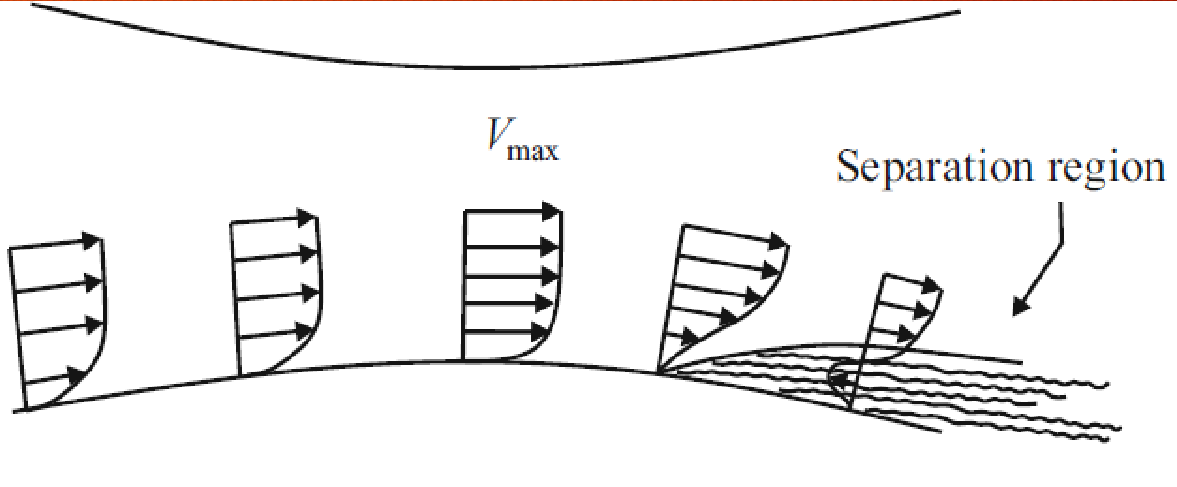
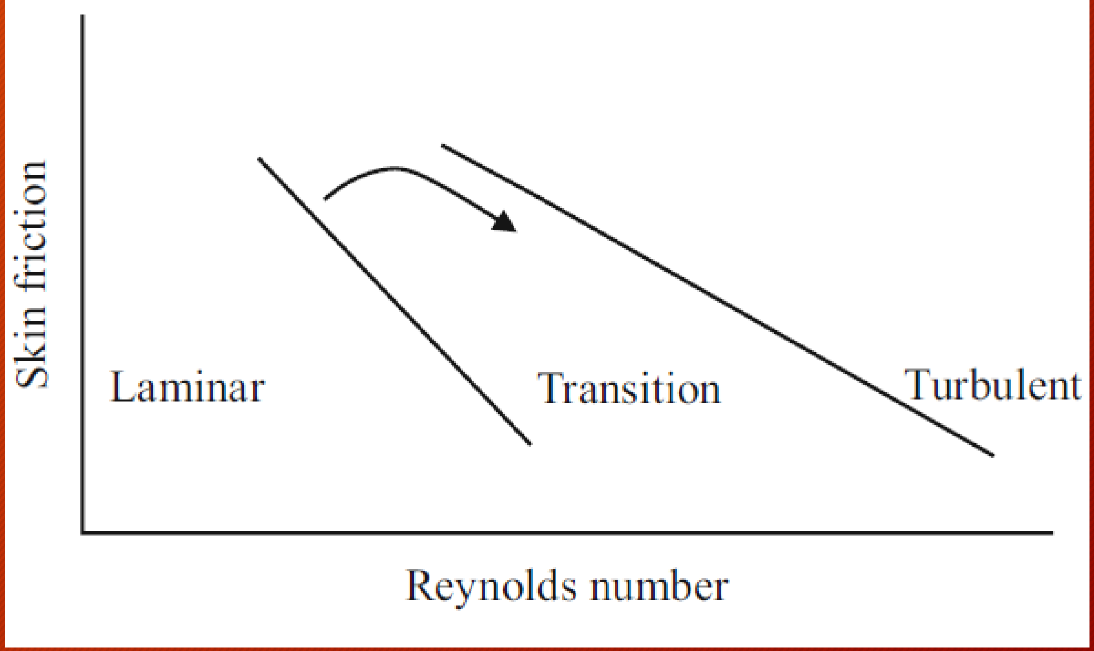


Figure 3.15 Boundary layer velocity profile

The Boundary Layer

- The sum of the pressure drag and skin friction (friction drag—primarily due to laminar flow) on a wing is called profile drag.
- It would seem that laminar flow is always desired (for less pressure drag), and usually it is, but it can become a problem when dealing with very small UAVs that fly at low speeds.
- Small characteristic lengths and low speeds result in low Reynolds numbers and consequently laminar flow, which is normally a favorable condition.

- **laminar** when $Re < 2300$
- **transient** when $2300 < Re < 4000$
- **turbulent** when $4000 < Re$



Flapping Wings

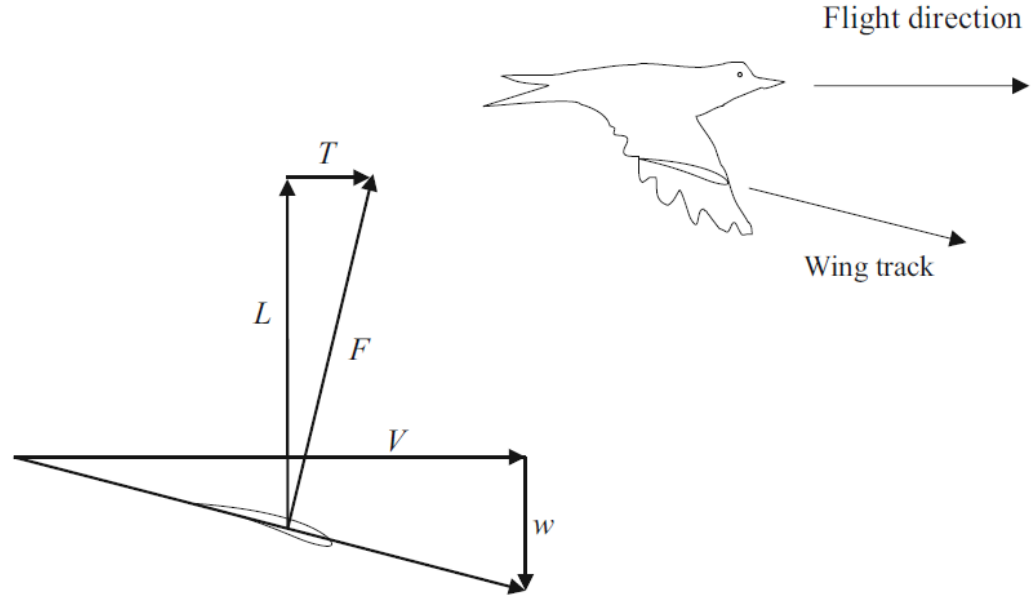


Figure 3.17 Wing flapping diagram

- The net velocity of the wing through the air mass is the sum of the forward velocity of the bird's body (*V*) and the downward velocity of the wing, driven by the muscles of the bird (*w*), which varies over the length of the wing, being greatest at the wing tip.
- The resulting total velocity through the air mass is forward and down, which means that the relative wind over the wing is to the rear and up.
- The net aerodynamic force generated by that relative wind (*F*) is perpendicular to the relative wind and can be resolved into two components, lift (*L*) upward and thrust (*T*) forward.

Flapping Wings

- The velocity and force triangles vary along the length of the wing because *w* is approximately zero at the root of the wing, where it joins the body of the bird
- and has a maximum value at the tip of the wing, so that the net force, *F*, is nearly vertical at the root of the wing and tilted furthest forward at the tip.
- As a result, it sometimes is said that the root of the bird’s wing produces mostly lift and the tip produces mostly thrust.
- It is also possible for the bird to introduce a variable twist in the wing over its length, which could maintain the same angle of attack as *w* increases and the relative wind becomes tilted more upward near the tip.
- This twist can also be used to create an optimum angle of attack that varies over the length of the wing. This can be used to increase the thrust available from the wing tip.

Flapping Wings

- To maximize the average lift and thrust, the angle of attack is “selected” by the bird to be large during the down stroke, which creates a large net aerodynamic force. This results in a large lift and large positive thrust.
- During the up stroke, the angle of attack is reduced, leading to a smaller net aerodynamic force. This means that even though the thrust is now negative, the average thrust over a complete cycle is positive. The lift remains positive, although smaller than during the up stroke

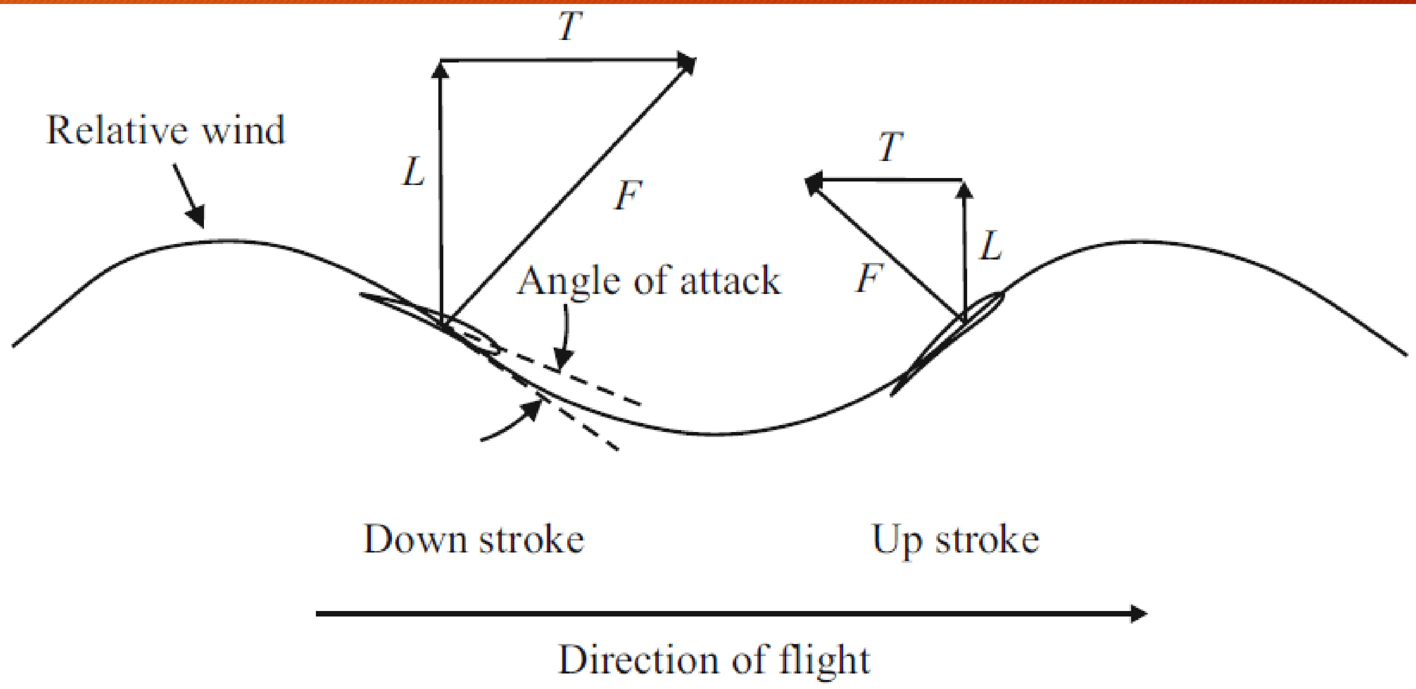


Figure 3.18 Flight of a bird

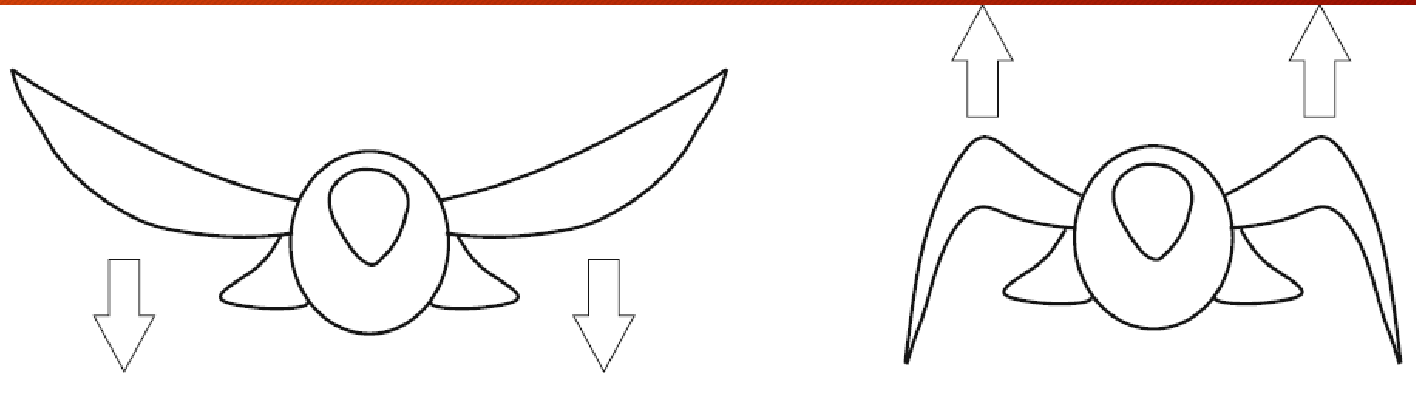


Figure 3.19 Wing articulation

Flapping Wings

- There are some significant differences between how birds fly and how insects fly, and not all birds fly in exactly the same way.
- In the early days of heavier-than-air flight, there were many attempts to use flapping wings to lift a human passenger. All were unsuccessful.
- As interest has increased in recent years in small, even tiny, UAVs, the biomechanics of bird and insect flight are being closely reexamined and recently have been successfully emulated by machines.

Total Air-Vehicle Drag

- The total resistance to the motion of an air-vehicle wing is made up of two components: the drag due to lift (induced drag), and the profile drag, which in turn is composed of the friction drag and the pressure drag.
- For the overall air vehicle, the drag of all the non-wing parts are lumped together and called parasite (or parasitic) drag.
- If the various drag components are expressed in terms of drag coefficients, then simply multiplying their sum by the dynamic pressure q and a characteristic area (usually the wing, S) results in the total drag:
- where *CD*0 is the sum of all the profile drag coefficients and *CDi* is the wing-induced drag coefficient,

$$D = \frac{1}{2}\left(C_{D0} + C_{Di}\right)\rho V^2 S$$

Performance

Climbing Flight

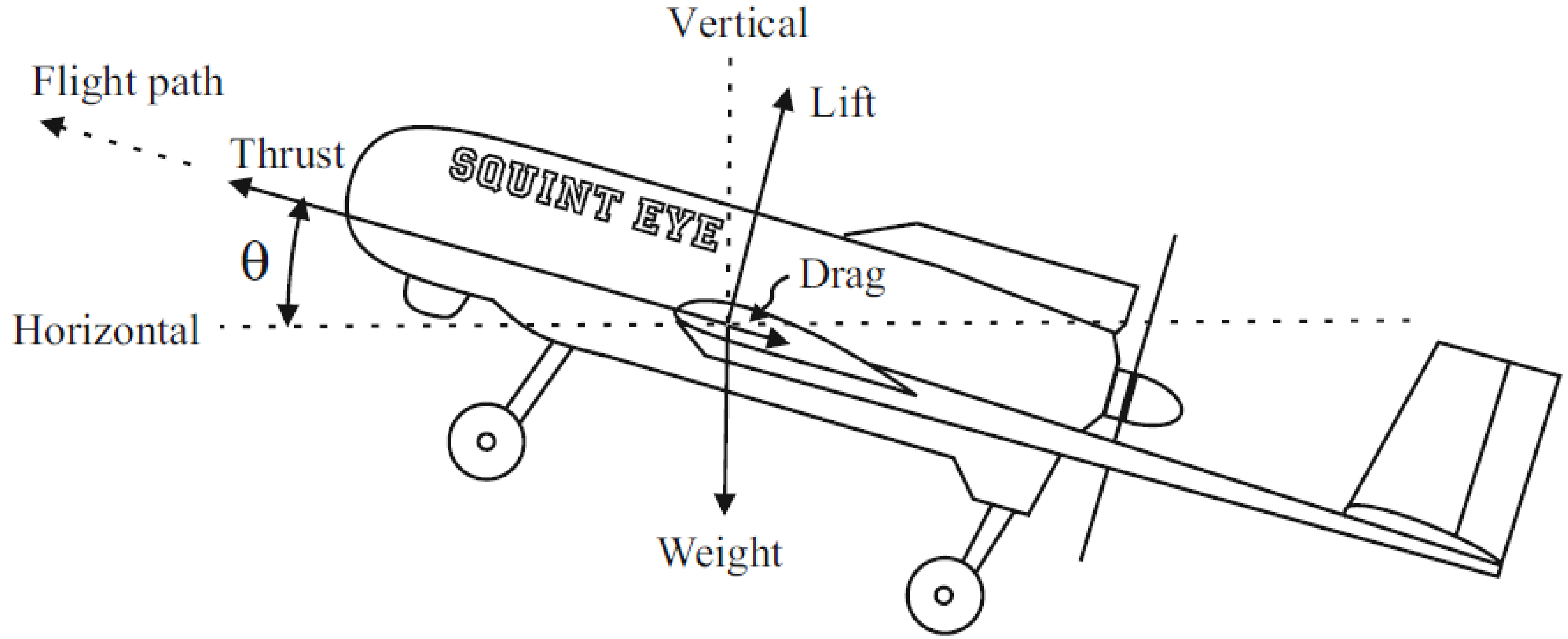


Figure 4.1 Force diagram