Gopalan College of Engineering and Management

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NOTES

SUBJECT: DIGITAL SIGNAL PROCESSING

SUBJECT CODE: 21EC42

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Go Green save Mother Earth.

Module - 2.

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Applications of DFTs.

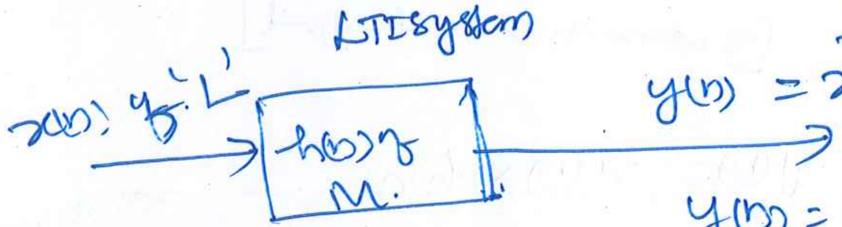
Linear filtering using ~~using~~ DFT [Linear convolutions using circular convolution.]

* Let us consider a LTI system which is having an i/p $x(n)$ of length "L" & impulse response $h(n)$ of length 'M' then the o/p response $y(n)$ is obtained by

* convolving $h(n)$ with $x(n)$ i.e.

i.e. $y(n) = x(n) * h(n)$

Linear convolution

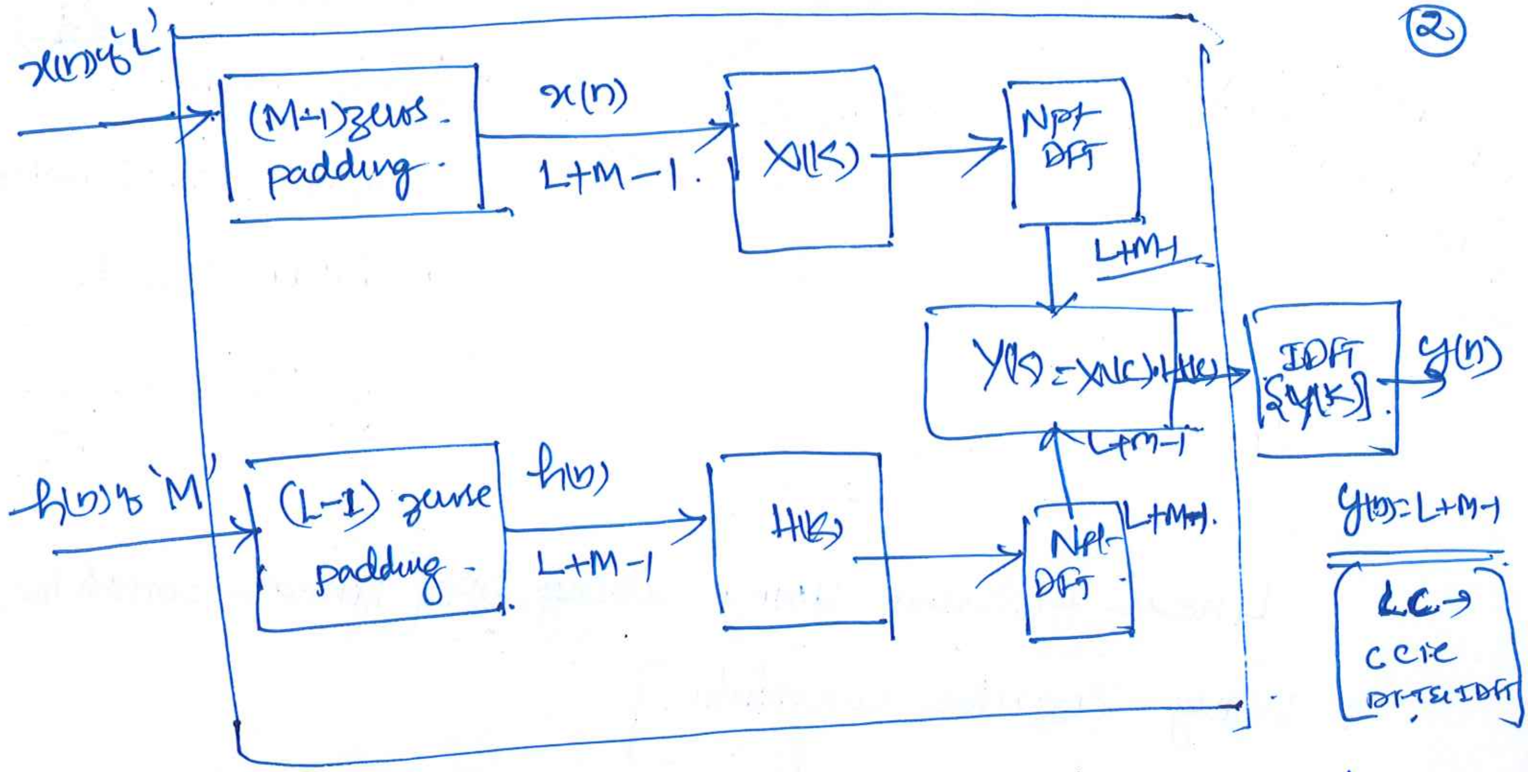


$y(n) = x(n) * h(n)$

$y(n) = L+M-1$

* Now. By using DFT [By circular convolutions] we can compute Linear convolutions efficiently.

ie.



Linear convolution by circular convolution method.

Problems

1) An FIR filter has $h(n)$ given by $\{1, 2, 3\}$. Determine the response of the filter to the i/p sequence $x(n) = \{1, 2\}$ [Linear filtering approach] ~~Use DFT & IDFT~~ & Verify results with L.convoluted o/p. [∵ whenever it is not equal]

Solⁿ LC. $y(n) = x(n) * h(n)$

	1	2
1	1	2
2	2	4
3	3	6

$y(n) = \{1, 4, 7, 6\}$ $= L+M-1 = 2+3-1 = 4$

③

DFT method

$$x(n) = \{1, 2, \underset{L}{}\underbrace{0, 0}_{}\} \quad \text{padd } M-1 \text{ zeros}$$

$$h(n) = \{1, 2, \overset{M}{3}, \underline{0}\} \quad L-1 \text{ zeros padding}$$

$$X(k) = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 1+2 \\ 1-2j \\ 1-2 \\ 1+2j \end{bmatrix} = \begin{bmatrix} 3 \\ 1-2j \\ -1 \\ 1+2j \end{bmatrix}$$

$$H(k) = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 3 \\ 0 \end{bmatrix} = \begin{bmatrix} 1+2+3 \\ 1-2j-3 \\ 1-2+3 \\ 1+2j-3 \end{bmatrix} = \begin{bmatrix} 6 \\ -2-2j \\ 2 \\ -2+2j \end{bmatrix}$$

$$Y(k) = \begin{bmatrix} 18 \\ -6+2j \\ -2 \\ -6-2j \end{bmatrix} \xrightarrow{\text{IDFT}} \boxed{\therefore y(n) = \{1, 4, 7, 6\}}$$

My. By IDFT

$$y(n) = \frac{1}{4} \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix} \begin{bmatrix} 18 \\ -6+2j \\ -2 \\ -6-2j \end{bmatrix} = \frac{1}{4} \begin{bmatrix} 4 \\ 16 \\ 28 \\ 24 \end{bmatrix} = \begin{bmatrix} 1 \\ 4 \\ 7 \\ 6 \end{bmatrix}$$

③

(HW) (2). Repeat the same for $h(n) = \{3, 2, 1\}$,

$x(n) = \{2, 1\}$.

Soln LC. $y(n) = x(n) * h(n)$

$h(n)$ \ $x(n)$	2	1
3	6	3
2	4	2
1	2	1

$y(n) = \{6, 7, 4, 1\}$. $= L+M-1 = 2+3-1 = 4$

DFT method

$x(n) = \{2, 1\} \rightarrow$ padd $M-1$ zeros.
L.

$h(n) = \{3, 2, 1\} \rightarrow$ padd $L-1$ zeros.
M

$\therefore x(n) = \{2, 1, 0, 0\}$

$h(n) = \{3, 2, 1, 0\}$.

$$\begin{bmatrix} X(0) \\ X(1) \\ X(2) \\ X(3) \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & +1 & -1 \\ 1 & j & -1 & -j \end{bmatrix} \begin{bmatrix} 2 \\ 1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 2+1 \\ 2-j \\ 2-1 \\ 2+j \end{bmatrix} = \begin{bmatrix} 3 \\ 2-j \\ 1 \\ 2+j \end{bmatrix}$$

$$\begin{bmatrix} H(0) \\ H(1) \\ H(2) \\ H(3) \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix} \begin{bmatrix} 3 \\ 2 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 6 \\ 2-2j \\ 2 \\ 2+2j \end{bmatrix}$$

$$\boxed{Y(K) = \begin{bmatrix} 18 \\ 2-6j \\ 2 \\ 2+6j \end{bmatrix}} \rightarrow \text{DFT}$$

By IDFT

$$y(n) = 1/4 \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -j \\ 1 & j & -1 & -j \end{bmatrix} \begin{bmatrix} 18 \\ 2-6j \\ 2 \\ +2+6j \end{bmatrix} \Rightarrow$$

$$y(n) = 1/4 \begin{bmatrix} 24 \\ 28 \\ 16 \\ 4 \end{bmatrix} = \begin{bmatrix} 6 \\ 7 \\ 4 \\ 1 \end{bmatrix} \Rightarrow$$

$$\boxed{y(n) = \{6, 7, 4, 1\}}$$

6M

(3) Impulse response of a system is given by $h(n) = \{3, 2\}$. Find the o/p response of a system for the i/p $x(n) = \{0, 1, 0, 1\}$ using circular conv^n

(5)

Soln. Linear conv^n

$y(n) = x(n) * h(n)$

$h(n)$ \ $x(n)$	0	1	0	1
3	0	3	0	3
2	0	2	0	2

$y(n) = \{0, 3, 2, 3, 2\}$.

CC.

$y(n) = h(n) \text{ (N) } x(n)$.

$x(n) = \{0, 1, 0, 1\}$ → M−1 zeros padding
L.

$h(n) = \{3, 2\}$. → L−1 zeros padding.
M.

∴ $x(n) = \{0, 1, 0, 1, 0\}$.

$h(n) = \{3, 2, 0, 0, 0\}$

∴ $y(n) = h(n) \text{ (5) } x(n)$

$$\begin{bmatrix} y(0) \\ y(1) \\ y(2) \\ y(3) \\ y(4) \end{bmatrix}$$

By DFT & IDFT method only.

$x(n) = \{0, 1, 0, 1, 0\}$

$X(k) = \{2\ 0, 2\ 0 .\ 3\ \_\ \_\ \}$

$h(n) = \{3, 2, 0, 0, 0\}$

$H(k) = \{\_\ \_\ \_\ \_\ \_\}$

∴ $Y(k) = X(k) \cdot H(k) \rightarrow$ DFT.

IDFT -

$$y(n) = \frac{1}{N}[W_4][X(k)]$$

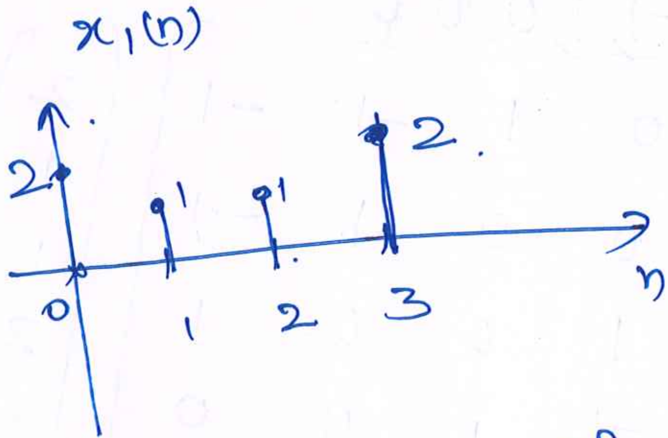
$$\boxed{y(n) = \{0, 3, 2, 3, 2\}}$$

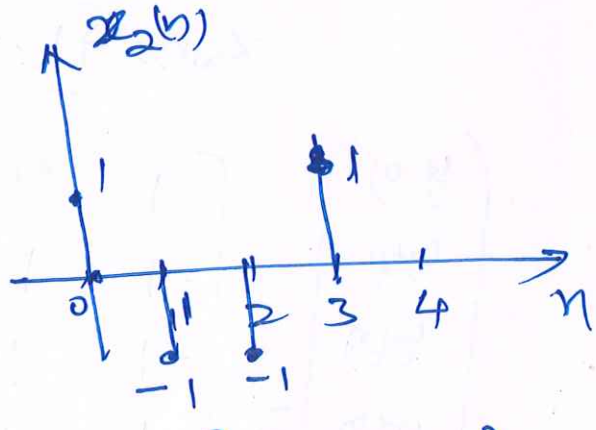
---

(5). For the sequences shown below, compute circular convolution of $x_1(n)$ & $x_2(n)$ for a) $N = 4, 5, 6, 7, 8$.

b) compute linear convolution of $x_1(n)$ & $x_2(n)$

c) what value of 'N' is necessary so that linear & circular convolution are same on the N pt interval.





Soln $x_1(n) = \{2, 1, 1, 2\}$ $x_2(n) = \{1, -1, -1, 1\}$

a) $N = 4$

$$y(n) = x_1(n) \; (4) \; x_2(n)$$

$$\begin{bmatrix} y(0) \\ y(1) \\ y(2) \\ y(3) \end{bmatrix} = \begin{bmatrix} 1 & 1 & -1 & -1 \\ -1 & 1 & +1 & -1 \\ -1 & -1 & 1 & +1 \\ 1 & -1 & -1 & +1 \end{bmatrix} \begin{bmatrix} 2 \\ 1 \\ 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 0 \\ -2 \\ 0 \\ 2 \end{bmatrix}$$

$$\boxed{y(n) = \{0, -2, 0, 2\}}$$

$N=5$

$x_1(n) = \{2, 1, 1, 2, 0\}$. $x_2(n) = \{1, -1, -1, 1, 0\}$

$\therefore y(n) = x_2(n) (5) x_1(n)$

$$\begin{bmatrix} y(0) \\ y(1) \\ y(2) \\ y(3) \\ y(4) \end{bmatrix} = \begin{bmatrix} 1 & 0 & 1 & -1 & -1 \\ -1 & 1 & 0 & 1 & -1 \\ -1 & -1 & 1 & 0 & 1 \\ 1 & -1 & -1 & 1 & 0 \\ 0 & 1 & -1 & -1 & 1 \end{bmatrix} \begin{bmatrix} 2 \\ 1 \\ 1 \\ 2 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ -2 \\ 2 \\ -2 \end{bmatrix}$$

$y(n) = \{1, 1, -2, 2, -2\}$

$N=6$

$x_1(n) = \{2, 1, 1, 2, 0, 0\}$

$x_2(n) = \{1, -1, -1, 1, 0, 0\}$.

$$\begin{bmatrix} y(0) \\ y(1) \\ y(2) \\ y(3) \\ y(4) \\ y(5) \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 1 & -1 & -1 \\ -1 & 1 & 0 & 0 & 1 & -1 \\ -1 & -1 & 1 & 0 & 0 & 1 \\ 1 & -1 & -1 & 1 & 0 & 0 \\ 0 & 1 & -1 & -1 & 1 & 0 \\ 0 & 0 & 1 & -1 & -1 & 1 \end{bmatrix} \begin{bmatrix} 2 \\ 1 \\ 1 \\ 2 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 4 \\ -1 \\ -2 \\ 2 \\ -2 \\ -1 \end{bmatrix}$$

$\therefore y(n) = \{4, -1, -2, 2, -2, -1\}$

$N=7$.

$x_1(n) = \{2, 1, 1, 2, 0, 0, 0\}$.

$x_2(n) = \{1, -1, -1, 1, 0, 0, 0\}$

$$\begin{bmatrix} y(0) \\ y(1) \\ y(2) \\ y(3) \\ y(4) \\ y(5) \\ y(6) \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 & -1 & -1 & -1 \\ -1 & +1 & 0 & 0 & 0 & -1 & -1 \\ -1 & -1 & 1 & 0 & 0 & 0 & -1 \\ -1 & -1 & -1 & 1 & 0 & 0 & 0 \\ 0 & -1 & -1 & -1 & 1 & 0 & 0 \\ 0 & 0 & -1 & -1 & -1 & 1 & 0 \\ 0 & 0 & 0 & -1 & -1 & -1 & 1 \end{bmatrix} \begin{bmatrix} 2 \\ 1 \\ 1 \\ 2 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$\therefore$ ~~y(0)~~ ~~y(1)~~ $y(n) = \{2, -1, -2, 2, -2, -1, 2\}$

* $N=8$

$x_1(n) = \{2, 1, 1, 2, 0, 0, 0, 0\}$

$x_2(n) = \{1, -1, -1, 1, 0, 0, 0, 0\}$

$$\begin{bmatrix} y(0) \\ y(1) \\ y(2) \\ y(3) \\ y(4) \\ y(5) \\ y(6) \\ y(7) \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & -1 & -1 & -1 \\ -1 & 1 & 0 & 0 & 0 & 0 & -1 & -1 \\ -1 & -1 & 1 & 0 & 0 & 0 & 0 & -1 \\ -1 & -1 & -1 & 1 & 0 & 0 & 0 & 0 \\ 0 & -1 & -1 & -1 & 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & -1 & -1 & 1 & 0 & 0 \\ 0 & 0 & 0 & -1 & -1 & -1 & 1 & 0 \\ 0 & 0 & 0 & 0 & -1 & -1 & -1 & 1 \end{bmatrix} \begin{bmatrix} 2 \\ 1 \\ 1 \\ 2 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

(10)

$y(n) = \{2, -1, -2, 2, -2, -1, 2, 0\}$

(b). Using LC.

$x_1(n) = \{2, 1, 1, 2\}$

$x_2(n) = \{1, -1, -1, 1\}$

$x_2(n)$ \ $x_1(n)$	2	1	1	2
1	2	1	1	2
-1	-2	-1	-1	-2
-1	-2	-1	-1	-2
1	2	1	1	2

$y(n) = \{2, -1, -2, 2, -2, -1, 2\}$

c) For $N = 7$, LC & CC are same

(5). Repeat the same for $x_1(n) = \{1, 2, 2, 1\}$, $x_2(n) = \{-1, 1, 1, -1\}$

$N=4$. $y_1(n) = \{0, -2, 0, 2\}$

$N=5$ $y_2(n) = \{-2, -2, 1, 2, 1\}$

$N=6$ $y_3(n) = \{-2, -1, 1, 2, 1, -1\}$

$N=7$) $y_4(n) = \{-2, -1, 1, 2, 1, -1, -1\}$

$N=8$. $y(n) = \{-1, -1, 1, 2, 1, -1, -1, 0\}$ $\cdot x$

$\sum_{\alpha}^{\alpha=7} \sum_{+N}^{c=4} = \left(\frac{N}{56}\right) X$

(10)

(2)

Linear filtering of Long data sequences.

[Signal segmentation]

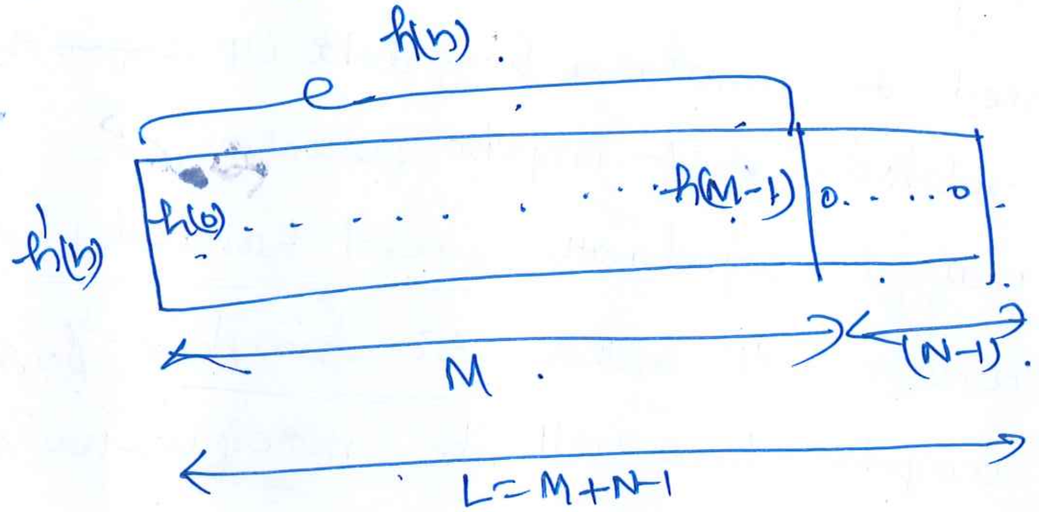
* Till now in our discussions, it is assumed that $x(n)$ & $h(n)$ consists of roughly same no of samples. otherwise they are zero filled upto the same length of transforms.
* Actually in practice we encounter a situation wherein we need to convolve a long data i/p ~~sequen~~ signals with relatively short impulse responses. i.e we may get signals with several hundreds of samples to convolve with less than 50 samples. In such cases computation time will be more if we use same length of transform.
* In real applications, use of very long data transform lengths for $x(n)$ gives rise to unacceptable delay.
* This type of problem can be overcome by segmenting i/p signal into sections of manageable lengths & then performing fast convolution on each sections.
* 2 common fast convolution techniques are

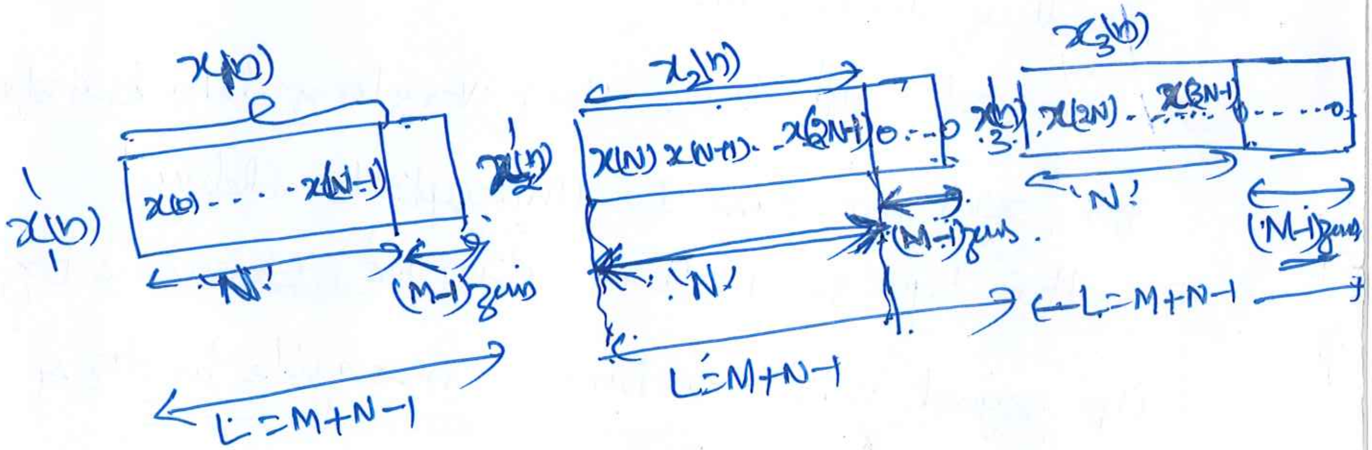
1) overlap add method of convolution.
2) overlap save method of convolution.

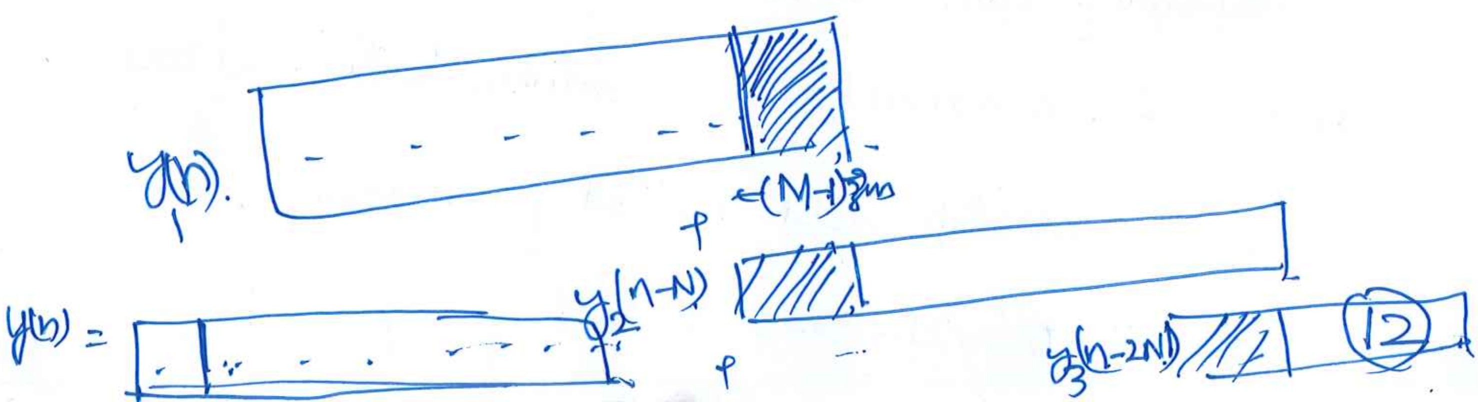
~~ove~~ overlap-add method of convolution

* Let us consider 2 sequences $x(n)$ & $h(n)$ having lengths $K \times N$ & M respectively with K being very large integer.

* ~~Actually~~, in
* $x(n)$ is subdivided into ~~overlapping~~ several sections each of length 'N'.







Step1 → . * ~~the 'M' length of the sequence~~. Consider the length of impulse response $h(n)$ has 'M' length sequence the sequence $h(n)$ is zero padded so that last $(N-1)$ values are zeros.

choose $L = 2^M$ as ~~so that~~ total length L. ~~we will get~~.

* Step2 → . i/p sequence $x(n)$ is subdivided into $x_1(n)$ & $x_2(n)$ subblocks of length 'N' such that

$L = M+N-1$.

* the last $(M-1)$ values of $x_1'(n)$ & $x_2'(n)$ are padded with zeros.

Step3 → compute the complex product

$$Y_1'(K) = H'(K) \cdot X_1'(K)$$

where $X_1'(K)$ is zeropadded sequence $x_1'(n)$

Step4 → Take IDFT of $Y_1'(K)$ to give $y_1'(n)$.

ie. $y_1'(n) = y(n) = \text{IDFT}\{Y_1'(K)\}$.

$0 \leq n \leq L-1$

Step5 → Similarly replace $X_1'(K)$ by $X_2'(K)$, $X_3'(K)$ . . .

carry out step ③ & ④ to get $y_2(n)$ & $y_3(n)$

Step-6 → shift $y_2(n)$ to produce $y_2(n-N)$

shift $y_3(n)$ to produce $y_3(n-2N)$ and so on.

* Similarly shift $y_3(n)$ to produce the first $(M-1)$ points of the last $(M-1)$ points of $y_1(n)$ & the first $(M-1)$ points of

* $y_2(n-N)$ are overlapped & added.

Step 7 → Repeat step (5) & (6) till the end of i/p data is reached.

ie. $y(n) = y_1(n) + y_2(n-N) + y_3(n-2N) + \ldots$

Problems

1) compute $x(n) * h(n)$ for the sequences $x(n)$ and $h(n)$ given below, using overlap add fast convolution techniques.

~~Source~~ Sequence.

$h(n) = \{1, 1, 1\}$

$x(n) = \{\underbrace{1, 2, 0, -3, 4, 2}_{x_1(n)} \mid \underbrace{-1, 1, -2, 3, 2, 1}_{x_2(n)} \mid -3\}$

Soln $M = 3$.

$L = 2^M = 2^3 = 8$

$L = M + N - 1$

$8 = 3 + N - 1$

$N = 6$.

$y_1'(n) = y_1(n)$

$x_1'(n) = \{1, 2, 0, -3, 4, 2, \underbrace{0\ \ 0}_{(M-1)\text{ zeros}}\}$ → padding (M−1) zeros.

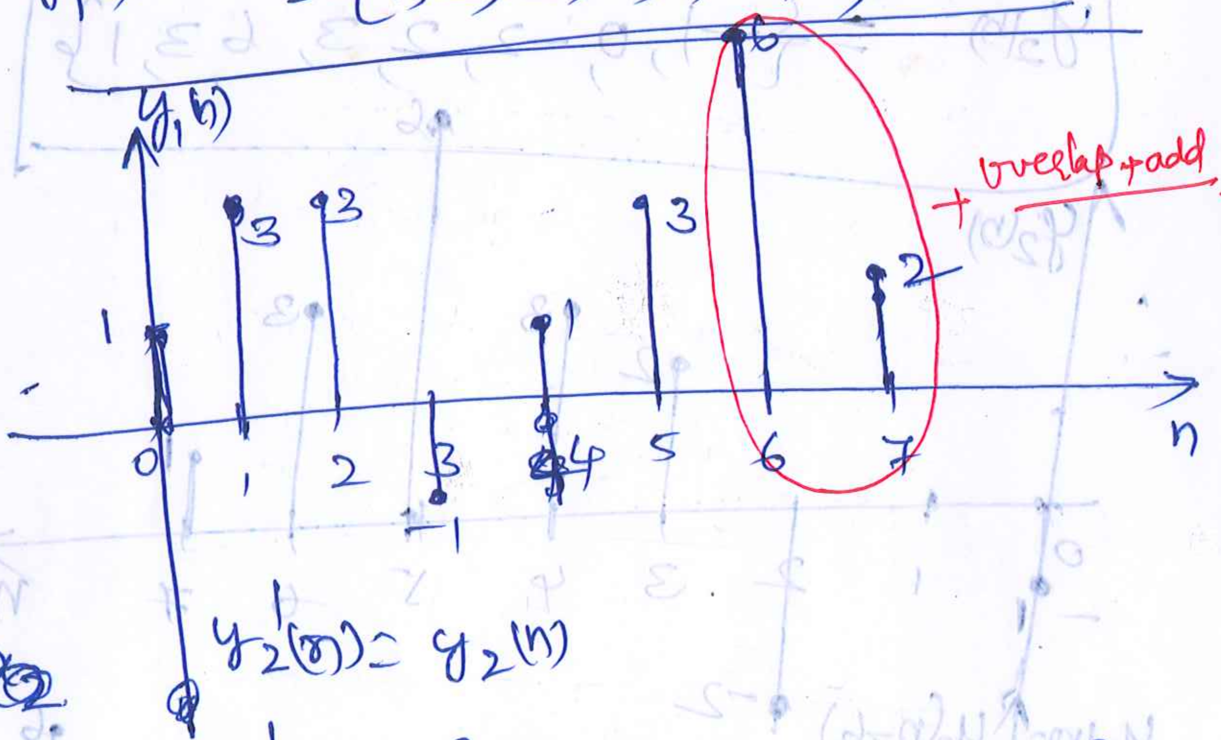
$h'(n) = \{1, 1, 1, 0, 0, 0, 0, 0\}$ padd (N−1) zeros.

$y_1'(n) = y_1(n) = x_1'(n) \circledast h'(n)$

15

$$\begin{bmatrix} y_1(0) \\ y_1(1) \\ y_1(2) \\ y_1(3) \\ y_1(4) \\ y_1(5) \\ y_1(6) \\ y_1(7) \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 0 \\ -3 \\ 4 \\ 2 \\ 0 \\ 0 \end{bmatrix}$$

$$y_1(n) = \{1, 3, 3, -1, 1, 3, 6, 2\}$$



$$y_2'(n) = y_2(n)$$

$$x_2'(n) = \{-1, 1, -2, 3, 2, 1, 0, 0\}$$

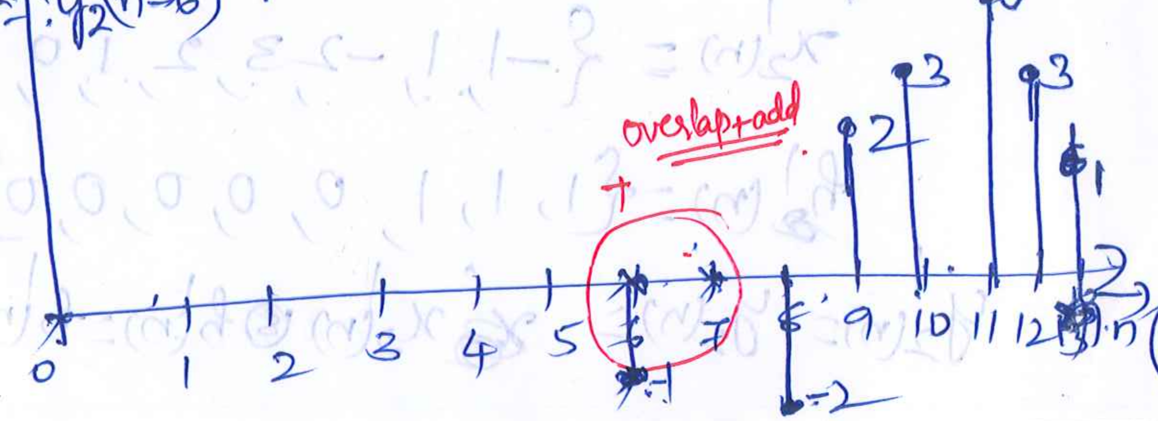
$$h_2'(n) = \{1, 1, 1, 0, 0, 0, 0, 0\}$$

$$y_2'(n) = y_2(n) = x_2(n) \circledast h(n) = h(n) \circledast x_2(n)$$

$$\begin{bmatrix} y_2(0) \\ y_2(1) \\ y_2(2) \\ y_2(3) \\ y_2(4) \\ y_2(5) \\ y_2(6) \\ y_2(7) \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} -1 \\ 1 \\ -2 \\ 3 \\ 2 \\ 1 \\ 0 \\ 0 \end{bmatrix}$$

$$\boxed{y_2(n) = \{-1, 0, -2, 2, 3, 6, 3, 1\}}$$

$y_2(n) = y_2(n-6)$



Resultant linear o/p is given by.

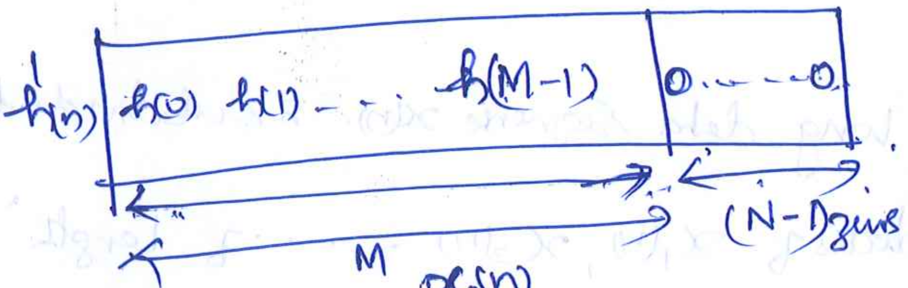
$$y(n) = \{1, 3, 3, -1, 1, 3, 5, 2, -2, 2, 3, 6, 3, 1\}$$

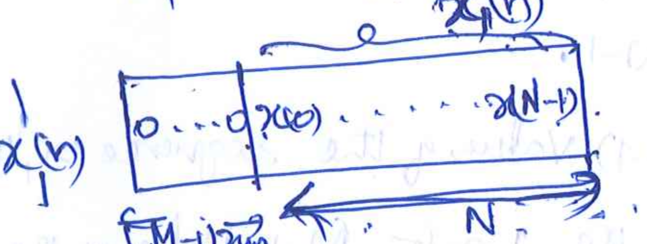
HW) ** 2) Compute $(x(n) * h(n))$ (fast convⁿ) for the sequence $x(n)$ & $h(n)$ using overlap-add fast convⁿ techniques.

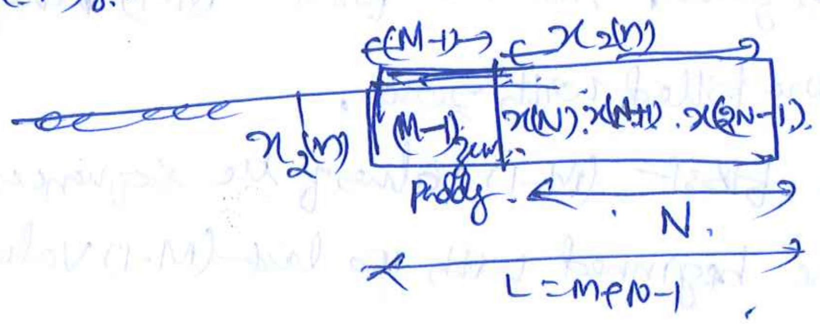
$h(n) = \{1, 0, 1\}$

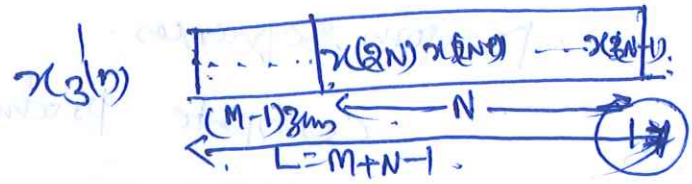
$x(n) = \{2, 1, 0, 4, -3, 2, -1, 1, -2, 3, 1, 2, -3\}$

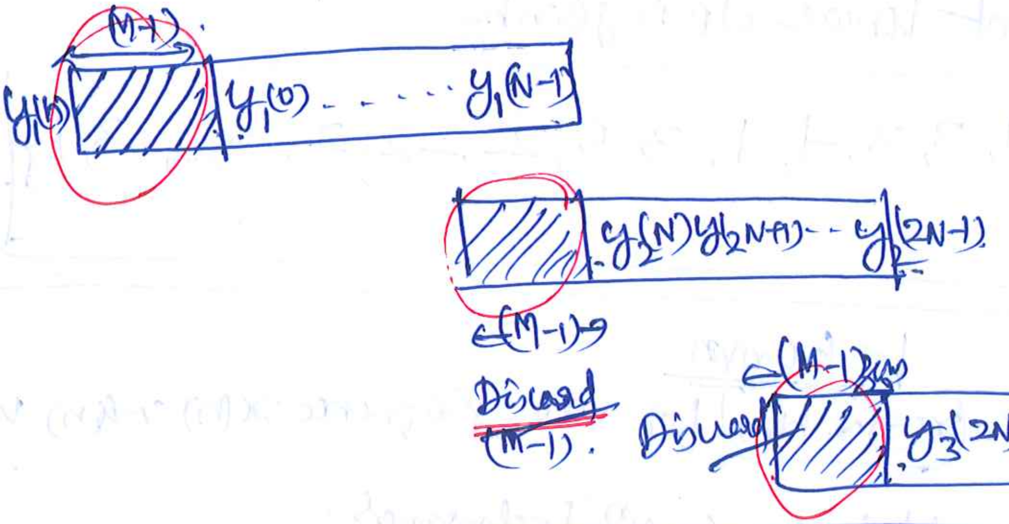
Overlap-save method of convolution











$$\boxed{y(n) = y_1(0) \ldots\ldots y_1(N-1), y_2 N, \ldots\ldots y_2(2N-1), y_3(2N) \ldots y_3(3N-1)}$$

Step 1 → M samples of $h(n)$ is known so that we can decide 'L', ie $L = 2^M$. Then zero padd the sequence $h(n)$ such that last $(N-1)$ values are filled with zeros.

Step 2. Long data sequence $x(n)$ is subdivided into separate blocks of $x_1(n), x_2(n) \ldots$ of length 'N'. Such that $L = M+N-1$.

* Here the first $(M-1)$ values of the sequence $x_1'(n)$ are filled with zeros. Also the first $(M-1)$ values of the sequence $x_1'(n)$ are filled with zeros.

* Also the first $(M-1)$ values of the sequences $x_2'(n), x_3'(n) \ldots$ are beginned with the last $(M-1)$ values of the previous sequences.

Step 3 compute product $Y_1'(K) = X_1'(K) \cdot H'(K)$

Step 4. Take IDFT of $Y_1'(k)$,

i.e. $y_1'(n) = y_1(n) = \text{IDFT}\{Y_1'(k)\}$

Step 5. Replace $X_1'(k)$ by $X_2'(k), X_3'(k) \ldots$ repeat the steps ③ & ④ to get $y_2(n), y_3(n) \ldots$ etc.

Step 6 → The first $(M-1)$ samples of $y_1(n), y_2(n), y_3(n) \ldots$ are discarded & the o/p is obtained by writing.

$$\boxed{y(n) = [y_1(n), y_2(n), y_3(n) \ldots]}$$

problems.

1) perform $x(n) * h(n)$ for the sequences $x(n)$ & $h(n)$

$h(n) = \{1, 1, 1\}$. & $x(n) = \{\underbrace{1, 2, 0, -3, 4, 2}_{x_1(n)}, \underbrace{-1, 1, -2, 3, 1}_{x_2(n)}, \underbrace{-3}_{}\}$

using overlap save method.

Soln

$L = 2^M = 2^3 = 8$. $[\because M = 3]$

WKT

$\therefore L = M + N - 1$

$8 = 3 + N - 1 \quad \therefore N = 6$

$y_1'(n) = y_1(n) = h'(n) \; ⑧ \; x_1'(n)$

$x_1'(n) = \{\overset{(M-1)\text{ zeros}}{0, 0}, 1, 2, 0, -3, 4, 2\}$

$h'(n) = \{1, 1, 1, 0, 0, 0, 0, 0\}$.

$$\begin{bmatrix} y_1(0) \\ y_1(1) \\ y_1(2) \\ y_1(3) \\ y_1(4) \\ y_1(5) \\ y_1(6) \\ y_1(7) \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \\ 2 \\ 0 \\ -3 \\ 4 \\ 2 \end{bmatrix}$$

$$= \{\overbrace{\cancel{6, 2}}^{\text{Discard } (M-1)}, 1, 3, 3, -1, 1, 3\}$$

$$y_1(n) = \{1, 3, 3, -1, 1, 3\}$$

$$y'_2(n) = y_2(n) = h'_1(n) ⊛ x'_2(n)$$

$$x'_2(n) = \{\overbrace{4, 2}^{\text{previous } (M-1) \text{ bits}}, -1, 1, -2, 3, 2, 1\}$$

$$h'(n) = \{1, 1, 1, 0, 0, 0, 0, 0\}$$

$$\begin{bmatrix} y_2(0) \\ y_2(1) \\ y_2(2) \\ y_2(3) \\ y_2(4) \\ y_2(5) \\ y_2(6) \\ y_2(7) \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 4 \\ 2 \\ -1 \\ 1 \\ -2 \\ 3 \\ 2 \\ 1 \end{bmatrix} = \begin{bmatrix} 7 \\ 7 \\ 5 \\ 2 \\ -2 \\ 2 \\ 3 \\ 6 \end{bmatrix}$$

$y_2^1(n) = \{\cancel{7}, 7.5, 2, -2, 2, 3, 6\}$ Discard

$y_2(n) = \{5, 2, -2, 2, 3, 6\}$

$\therefore y(n) = \{1, 3, 3, -1, 1, 3, 5, 2, -2, 2, 3, 6\}$.

HW 2) perform fast circular convolution for sequences of $x(n)$ & $h(n)$ shown below

$h(n) = \{1, 0, 1\}$

$x(n) = \{2, 1, 0, 4, -3, 2, 4, 1, -1, 3, -2, 1, 2, -3\}$

Soln

$y_1^1(n) = \{\cancel{-3}, \cancel{2}, 2, 1, 2, 5, -3, 6\}$

$y_1(n) = \{2, 1, 2, 5, -3, 6\}$

$y_2^1(n) = \{\cancel{-5}, \cancel{3}, 1, 3, 3, 4, -3, 4\}$

$y_2(n) = \{1, 3, 3, 4, -3, 4\}$

$\therefore y_3^1(n) = \{ \cancel{\ } \}$.

$y_3(n) = \{ \}$

$\boxed{y(n) = \{ \}.}$

(3)

$y_2(n) = \{5, 2, -2, 2, 3, 6\}$

$$\boxed{y(n) = \{1, 3, 3, -1, 1, 3, 5, 2, -2, 2, 3, 6\}}$$

(2) perform fast convolution for the following sequences. If $x(n)$ & $h(n)$ given by

$\cancel{x(n)}\ h(n) = \{1, 0, 1\}$

$\cancel{h(n)}\ x(n) = \{\cancel{1}, \cancel{2}, \cancel{0}, \cancel{-3}, \cancel{4}, 2, \cancel{-1}, 1, \cancel{-2}, 3, \cancel{2}, 1, \cancel{-3}\}$

$\{\underbrace{2, 1, 0, 4, -3, 2}_{x_1(n)}, \underbrace{4, 1, -1, 3, -2, 1}_{x_2(n)}, \underbrace{2, -3}_{x_3(n)}\}$

by using overlap-save method

Soln

$L = 2^M$. $\quad M = 3$

$L = 2^3 = 8$.

WKT $\quad L = M + N - 1$

$8 = 3 + N - 1$

$\therefore N = 6$

$y_1'(n) = y_1(n) = h'(n) \circledast_8 x_1'(n)$

$x_1'(n) = \{\underbrace{0, 0}_{(M-1)\text{ zeros}}, 2, 1, 0, 4, -3, 2\}$ $\quad \cancel{1, 2, 0, -3, 4, 2}$

$h'(n) = \{1, 0, 1, 0, 0, 0, 0, 0\}$

$\therefore y_2(n) = ?$

$$\begin{bmatrix} y_2(0) \\ y_2(1) \\ y_2(2) \\ y_2(3) \\ y_2(4) \\ y_2(5) \\ y_2(6) \\ y_2(7) \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 2 \\ 1 \\ 0 \\ 4 \\ -3 \\ 2 \end{bmatrix}$$

$$y_1'(n) = \{\cancel{-3}, \cancel{2}, 2, 1, 1, 2, 5, -3, 6\}$$

$$\boxed{y_1(n) = \{2, 1, 2, 5, -3, 6\}}$$

$$y_2'(n). = y_2(n) = h_2(n). \circledast x_2'(n)$$

$$x_2'(n) = \{-3, 2, 4, 1, -1, 3, -2, 1\}$$

$$h'(n) = \{1, 0, 1, 0, 0, 0, 0, 0\}$$

$$\begin{bmatrix} y_2(0) \\ y_2(1) \\ y_2(2) \\ y_2(3) \\ y_2(4) \\ y_2(5) \\ y_2(6) \\ y_2(7) \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} -3 \\ 2 \\ 4 \\ 1 \\ -1 \\ 3 \\ -2 \\ 1 \end{bmatrix}$$

$y_2'(n) = \{ \underbrace{\cancel{5}, \cancel{3}}_{\text{Discard}}, 1, 3, 3, 4, -3, 4 \}$.

$y_2(n)\ \{1, 3, 3, 4, -3, 4\}$.

$\therefore y(n) = \{2, 1, 2, 5, -3, 6, 1, 3, 3, 4, -3, 4\}$

(12). Find the o/p $y(n)$ of a filter, whose impulse response is $\{1, 2\}$. & i/p signal to the filter is

$x(n) = \{\underbrace{1, 2, -1}_{x_1(n)} | \underbrace{2, 3, -2}_{x_2(n)} | \underbrace{-3, -1, 1}_{x_3(n)} | \underbrace{1, 2, -1}_{x_4(n)}\}$

using overlap-save method

Soln

$L = 2^M = 2^2 = \underline{4}$

$M = 2$

$L = M + N - 1$

$4 = 2 + N - 1$

$\therefore \boxed{N = 3}$

$\therefore y_1'(n) = y_1(n) = h_1(n) \circledast x_1'(n)$

$\therefore x_1'(n) = \{0, 1, 2, -1\}$

$h_1'(n) = \{1, 2, 0, 0\}$.

$$\begin{bmatrix} y_1(0) \\ y_1(1) \\ y_1(2) \\ y_1(3) \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 2 \\ 2 & 1 & 0 & 0 \\ 0 & 2 & 1 & 0 \\ 0 & 0 & 2 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 2 \\ -1 \end{bmatrix} = \begin{bmatrix} \cancel{-2} \\ 1 \\ 4 \\ 3 \end{bmatrix}$$

Discard

$\therefore y_1(n) = \{1, 4, 3\}$.

$$y_2'(n) = y_2(n) = x_2'(n) \,④\, h'(n) = h_2'(n) \,④\, x_2'(n).$$

$x_2'(n) = \{-1, 2, 3, -2\}$

$h_2'(n) = \{1, 2, 0, 0\}$.

$$\begin{bmatrix} y_2(0) \\ y_2(1) \\ y_2(2) \\ y_2(3) \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 2 \\ 2 & 1 & 0 & 0 \\ 0 & 2 & 1 & 0 \\ 0 & 0 & 2 & 1 \end{bmatrix} \begin{bmatrix} -1 \\ 2 \\ 3 \\ -2 \end{bmatrix} = \begin{bmatrix} \cancel{-5} \\ 0 \\ 7 \\ 4 \end{bmatrix}$$

Discard

$$\therefore y_2(n) = \{0, 7, 4\}$$

$$y_3'(n) = y_3(n) = h'(n) \,④\, x_3'(n)$$

$\because x_3'(n) = \{-2, -3, -1, 1\}$

$h'(n) = \{1, 2, 0, 0\}$

$$\begin{bmatrix} y_3(0) \\ y_3(1) \\ y_3(2) \\ y_3(3) \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 2 \\ 2 & 1 & 0 & 0 \\ 0 & 2 & 1 & 0 \\ 0 & 0 & 2 & 1 \end{bmatrix} \begin{bmatrix} -2 \\ -3 \\ -1 \\ 1 \end{bmatrix} = \begin{bmatrix} \cancel{0} \\ -7 \\ -7 \\ -1 \end{bmatrix}$$

Discard

$$\therefore y_3(n) = \{-7, -7, -1\}$$

$$y_4'(n) = y_4(n) = h'(n) \,④\, x_4'(n)$$

$x_4'(n) = \{\underline{1}, 1, 2, -1\}$

$h'(n) = \{1, 2, 0, 0\}$

$$\therefore \begin{bmatrix} y_4(0) \\ y_4(1) \\ y_4(2) \\ y_4(3) \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 2 \\ 2 & 1 & 0 & 0 \\ 0 & 2 & 1 & 0 \\ 0 & 0 & 2 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 2 \\ -1 \end{bmatrix} = \begin{bmatrix} \cancel{-1} \\ 3 \\ 4 \\ 3 \end{bmatrix}$$

Discard

$$y_4(n) = \{3, 4, 3\}$$

$$\therefore \boxed{y(n) = \{1, 4, 3, 0, 7, 4, -7, -7, -1, 3, 4, 3\}}$$

(5) Consider an FIR filter with $h(n) = \{3, 2, 1, 1\}$. (25)

If the i/p is $x(n) = \{\underbrace{1, 2, 3, 3}_{x_1(n)} \mid \underbrace{2, 1, -1, -2}_{x_2(n)} \mid \underbrace{-3, 5, 6, -1}_{x_3(n)} \mid \underbrace{2, 0, 2, 1}_{x_4(n)}\}$

Find the o/p $y(n)$ using overlap-add method.

Assuming block of 7 length.

Soln.

$L = 7$, (Given).

$L = M + N - 1$.

$7 = 4 + N - 1$ $\quad [\because M = 4]$

$\therefore N = 7 - 3 = 4$

$\therefore y_1'(n) = y_1(n) = x_1'(n) \circledast h'(n) = h'(n) \circledast x_1'(n)$

where $x_1'(n) = \{1, 2, 3, 3, 0, 0, 0\}$

$h'(n) = \{3, 2, 1, 1, 0, 0, 0\}$.

$$\begin{bmatrix} y_1(0) \\ y_1(1) \\ y_1(2) \\ y_1(3) \\ y_1(4) \\ y_1(5) \\ y_1(6) \end{bmatrix} = \begin{bmatrix} 3 & 0 & 0 & 0 & 1 & 1 & 2 \\ 2 & 3 & 0 & 0 & 0 & 1 & 1 \\ 1 & 2 & 3 & 0 & 0 & 0 & 1 \\ 1 & 1 & 2 & 3 & 0 & 0 & 0 \\ 0 & 1 & 1 & 2 & 3 & 0 & 0 \\ 0 & 0 & 1 & 1 & 2 & 3 & 0 \\ 0 & 0 & 0 & 1 & 1 & 2 & 3 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 3 \\ 3 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$y_1(n) = \{3, 8, 14, 18, 11, 6, 3\}$

(25)

$$y_2'(n) = x_2'(n) \overset{LCC}{\underset{7}{\circledast}} h'(n) = h'(n) \overset{LCC}{\underset{7}{\circledast}} x_2'(n)$$

$$x_2'(n) = \{2, 1, -1, -2, 0, 0, 0\}$$

$$h'(n) = \{3, 2, 1, 1, 0, 0, 0\}.$$

$$\therefore \begin{bmatrix} y_2(0) \\ y_2(1) \\ y_2(2) \\ y_2(3) \\ y_2(4) \\ y_2(5) \\ y_2(6) \end{bmatrix} = \begin{bmatrix} 3 & 0 & 0 & 0 & 1 & 1 \\ 2 & 3 & 0 & 0 & 0 & 1 \\ 1 & 2 & 3 & 0 & 0 & 0 \\ 1 & 1 & 2 & 3 & 0 & 0 \\ 0 & 1 & 1 & 2 & 3 & 0 \\ 0 & 0 & 1 & 1 & 2 & 3 \\ 0 & 0 & 0 & 1 & 1 & 2 \end{bmatrix} \begin{bmatrix} 2 \\ 1 \\ -1 \\ -2 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$y_2(n) = \{6, 7, 1, -5, -4, -3, -2\}.$$

$$y_3'(n) = y_3(n) = x_3'(n) \overset{LC}{\underset{7}{\circledast}} h_3'(n) = h'(n) \overset{LC}{\underset{7}{\circledast}} x_3'(n)$$

$$h'(n) = \{3, 2, 1, 1, 0, 0, 0\}.$$

$$x_3'(n) = \{-3, 5, 6, -1\}.0, 0, 0\}.$$

$$\therefore \begin{bmatrix} y_3(0) \\ y_3(1) \\ y_3(2) \\ y_3(3) \\ y_3(4) \\ y_3(5) \\ y_3(6) \end{bmatrix} = \begin{bmatrix} 3 & 0 & 0 & 0 & 1 & 1 & 2 & 3 \\ 2 & 3 & 0 & 0 & 0 & 1 & 1 & 2 \\ 1 & 2 & 3 & 0 & 0 & 0 & 1 & 1 \\ 1 & 1 & 2 & 3 & 0 & 0 & 0 & 1 \\ 0 & 1 & 1 & 2 & 3 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 2 & 3 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 2 & 3 & 0 \end{bmatrix} \begin{bmatrix} -3 \\ 5 \\ 6 \\ -1 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} y_3(0) \\ y_3(1) \\ y_3(2) \\ y_3(3) \\ y_3(4) \\ y_3(5) \\ y_3(6) \end{bmatrix} = \begin{bmatrix} \quad \end{bmatrix} \begin{bmatrix} \quad \end{bmatrix}$$

$$y_3(n) = \{9, 9, 25, 11, 9, 5, -1\}$$

$\therefore y_4'(n) = y_4(n) = x_4'(n) \; ⑦ \; h_4'(n)$

$$x_4'(n) = \{2, 0, 2, 1, 0, 0, 0\}$$

$$h_4'(n) = \{3, 2, 1, 1, 0, 0, 0\}.$$

$$\begin{bmatrix} y_4(0) \\ y_4(1) \\ y_4(2) \\ y_4(3) \\ y_4(4) \\ y_4(5) \\ y_4(6) \end{bmatrix} = \begin{bmatrix} 3 & 0 & 0 & 0 & 1 & 1 & 2 & 3 \\ 2 & 3 & 0 & 0 & 0 & 1 & 1 & 2 \\ 1 & 2 & 3 & 0 & 0 & 0 & 1 & 1 \\ 1 & 1 & 2 & 3 & 0 & 0 & 0 & 1 \\ 0 & 1 & 1 & 2 & 3 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 2 & 3 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 2 & 3 & 0 \end{bmatrix} \begin{bmatrix} 2 \\ 0 \\ 2 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$\therefore y_4(n) = \{6, 4, 8, 9, 4, 3, 1\}$

$\therefore$ Resultant Linear O/P's $y(n) = \{3, 8, 14, 18, \cancel{17}, 13, 4, -5, -13, 6, 2, 3$

15, 9, 7, 9, 4, 3, 1}.

(6). Repeat for.

$x(n) = \{2, 3, 4, 1, 1, 1, 2, -2, -3, 5, 6, -1, 2, 1, 2\}$

$h(n) = \{2, 1, 1, 1\}$.

Assuming block of length 7 using overlap add method.

(7). Find the o/p y(n) of a filter whose impulse response is

$h(n) = \{1, 2, 3, 4\}$. i/p $x(n) = \{\underbrace{1, 2, 1}_{x_1(n)} \mid \underbrace{1, 3, 0}_{x_2(n)} \mid \underbrace{5, 6, 2}_{x_3(n)} \mid \underbrace{-2, -5, 6}_{x_4(n)} \mid \underbrace{7, 1, 2}_{x_5(n)} \mid \underbrace{0, 1}_{x_6(n)}\}$. using overlap add method

Use 6 point circular convolution.

soln

$L = 6$

$L = M + N - 1$ $\qquad M = 4$

$6 = 4 + N - 1$

$\therefore \boxed{N = 6 - 3 = 3}$

$\therefore y_1'(n) = y(n) = x_1'(n) \circledast h'(n)$

$x_1'(n) = \{1, 2, 1, 0, 0, 0\}$

$h'(n) = \{1, 2, 3, 4, 0, 0\}$

$$\begin{bmatrix} y_1(0) \\ y_1(1) \\ y_1(2) \\ y_1(3) \\ y_1(4) \\ y_1(5) \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 4 & 3 & 2 \\ 2 & 1 & 0 & 0 & 4 & 3 \\ 3 & 2 & 1 & 0 & 0 & 4 \\ 4 & 3 & 2 & 1 & 0 & 0 \\ 0 & 4 & 3 & 2 & 1 & 0 \\ 0 & 0 & 4 & 3 & 2 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 4 \\ 4 \\ 8 \\ 12 \\ 11 \\ 4 \end{bmatrix}$$

$y_2'(n) = y_2(n) = x_2'(n) \circledast_6 h_2'(n)$

$x_2'(n) = \{-1, 3, 0, 0, 0, 0\}$

$h'(n) = \{1, 2, 3, 4, 0, 0\}$

$$\begin{bmatrix} y_2(0) \\ y_2(1) \\ y_2(2) \\ y_2(3) \\ y_2(4) \\ y_2(5) \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 4 & 3 & 2 \\ 2 & 1 & 0 & 0 & 4 & 3 \\ 3 & 2 & 1 & 0 & 0 & 4 \\ 4 & 3 & 2 & 1 & 0 & 0 \\ 0 & 4 & 3 & 2 & 1 & 0 \\ 0 & 0 & 4 & 3 & 2 & 1 \end{bmatrix} \begin{bmatrix} -1 \\ 3 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$y_2(n) = \{-1, 1, 3, 5, 12, 0\}$

$y_3'(n) = y_3(n) = x_3'(n) \circledast_6 h_3'(n)$

$x_3'(n) = \{5, 6, 2, 0, 0, 0.\}$

$h_3'(n) = \{1, 2, 3, 4, 0, 0\}$

$$\begin{bmatrix} y_3(0) \\ y_3(1) \\ y_3(2) \\ y_3(2) \\ y_3(4) \\ y_3(5) \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 4 & 3 & 2 \\ 2 & 1 & 0 & 0 & 4 & 3 \\ 3 & 2 & 1 & 0 & 0 & 4 \\ 4 & 3 & 2 & 1 & 0 & 0 \\ 0 & 4 & 3 & 2 & 1 & 0 \\ 0 & 0 & 4 & 3. & 2 & 1 \end{bmatrix} \begin{bmatrix} 5 \\ 6 \\ 2 \\ 0 \\ 0 \\ 0. \end{bmatrix}$$

$\boxed{y_3(n) = \{5, 16, 29, 42, 30, 8\}}$

$y_4'(n) = y_4(n) = x_4'(n) ⑥ h_4'(n).$

$x_4'(n) = \{-2, 5, 6, 0, 0, 0\}.$

$h_4'(n) = \{1, 2, 3, 4, 0, 0\}.$

$$\begin{bmatrix} y_4(0) \\ y_4(1) \\ y_4(2) \\ y_4(3) \\ y_4(4) \\ y_4(5) \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 4 & 3 & 2 \\ 2 & 1 & 0 & 0 & 4 & 3 \\ 3 & 2 & 1 & 0 & 0 & 4 \\ 4 & 3 & 2 & 1 & 0 & 0 \\ 0 & 4 & 3 & 2 & 1 & 0 \\ 0 & 0 & 4 & 3 & 2 & 1 \end{bmatrix} \begin{bmatrix} -2 \\ 5 \\ 6 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$y_4(n) = \{\underset{\uparrow n=0}{-2}, -9, -10, -11, -2, 24\}$$

$y_5'(n) = x_5'(n) ⑥ h_5'(n).$

$x_5'(n) = \{7, 1, 2, 0, 0, 0\}.$

$h'(n) = \{1, 2, 3, 4, 0, 0\}.$

$$\begin{bmatrix} y_5(0) \\ y_5(1) \\ y_5(2) \\ y_5(3) \\ y_5(4) \\ y_5(5) \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 4 & 3 & 2 \\ 2 & 1 & 0 & 0 & 4 & 3 \\ 3 & 2 & 1 & 0 & 0 & 4 \\ 4 & 3 & 2 & 1 & 0 & 0 \\ 0 & 4 & 3 & 2 & 1 & 0 \\ 0 & 0 & 4 & 3 & 2 & 1 \end{bmatrix} \begin{bmatrix} 7 \\ 1 \\ 2 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

FFT Algorithms. [Fast Fourier Transform]

* It is an algorithm, using this we can compute DFT in a numerically efficient manner. Basically there exist 2 main types.

1) DIT-FFT Algorithm [Decimation in time FFT]

2) DIF-FFT Algorithm [Decimation in frequency FFT]

DIT-FFT Algorithm

[Ex. 8pt DFT. N=8
CM → $8^2 = 64 \doteq N^2$ CM
CA → $N^2 - N \doteq 64 - 8 = 56$ CA
$X(k) = \sum_{n=0}^{7} x(n) W_8^{kn}$]

DIT-FFT Algorithm uses Divide & Conquer approach.

In this approach N-pt ~~DFT~~ DFT is broken into 2 N/2 pt transforms, then each N/2 pt DFT is broken into 2 N/4 pt transforms & ~~then~~ this process is continued till 2 pt DFTs are formed.

* In other words. N pt DFT is performed as several 2pt DFT-s.

10M

** * Derivation.

* . DFT of N pt is given by

$$X(K) = \sum_{n=0}^{N-1} x(n) W_N^{kn} \qquad K = 0, 1, 2 \ldots N-1$$

* Decompose the sum into 2 parts one for even & one for odd indexed values.

i.e $X(K) = \sum_{n=0 \text{ even}}^{N-2} x(n) W_N^{kn} + \sum_{n=1 \text{ odd}}^{N-1} x(n) W_N^{kn}$. (32)

$y_5(n) = \{7, 15, 25, 35, 10, 8\}$

$y_6(n) = x'_6(n) ⑥ h'(n) = h(n) ⑥ x'_6(n)$

$x'_6(n) = \{0, 1, 0, 0, 0, 0\}$

$h'(n) = \{1, 2, 3, 4, 0, 0\}$

$$\begin{bmatrix} y_6(0) \\ y_6(1) \\ y_6(2) \\ y_6(3) \\ y_6(4) \\ y_6(5) \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 4 & 3 & 2 \\ 2 & 1 & 0 & 3 & 4 & 3 \\ 3 & 2 & 1 & 0 & 0 & 4 \\ 4 & 3 & 2 & 1 & 0 & 0 \\ 0 & 4 & 3 & 2 & 1 & 0 \\ 0 & 0 & 4 & 3 & 2 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$\boxed{y_6(n) = \{ \quad \}}$

$y_4(n) = \{1, 4, 8, 11, 15, 7, 10, 28, 29, 40, 21, -2, -4, 13, 49, 35, 11\}$.

HW

(8). Find the o/p $y(n)$ of a filter whose impulse response is $h(n) = \{2, 3, 1, 4\}$ & i/p $x(n) = \{3, 1, -1 \mid 2, -1, 3 \mid 0, 5, -1 \mid -4, 3, 2 \mid 2, -5, -2 \mid 3, -2\}$ using overlap ~~save~~ add | save method. use 6 point circular convn.

Computational complexity of Direct DFT eqn & Need for FFT.

* WKT

$$X(K) = \sum_{n=0}^{N-1} x(n)\, W_N^{kn}$$

$K = 0, 1, 2 \ldots N-1$

here $x(n)$ is nothing but real or complex no.

$W_N \rightarrow$ another complex no, i.e Twiddle factor or phase factor..

* computation of $X(K)$ means repeated multiplication & addition of CNos.

i.e $X(K) = x(0)W_N^{0} + x(1)W_N^{1K} + x(2) \cdot W_N^{2K} + \ldots \ldots + x(N-1) W_N^{(N-1)K}$

1st CM. 2nd CM. Nth CM. 1st CA. 2nd CA. (N-1) CA.

* For 1 value of K, we need $\left\{ \begin{array}{l} \rightarrow N\ CM. \\ (N-1)\ CA. \end{array} \right.$

* For all value of K we need $\left\{ \begin{array}{l} N \times N = N^2\ CM \\ N(N-1) = (N^2 - N)\ CA. \end{array} \right.$

1) for Example 8pt DFT :→ complexity.

$$CM = N^2 = 8^2 = 64\,CM.$$

$$CA = (N^2 - N)CA = 8^2 - 8 = 56\,CA.$$

2) compute 1024, 64 pt DFT CM & CA.

$$CM = \overset{N=1024}{N^2} = (1024)^2 \cong (10^3)^2 = 10^6\,CM.$$

$$CA = (N^2 - N) = (1024^2 - 1024) \cong 10^6 - 1000 \cong 10^6\,CA$$

. This is very very time consuming to get DFT to a processor that is why we are going for FFT algorithm, ~~which~~ is very very faster, where the same o/p we get by doing with less no of CM & CA. that is why we are going for FFT, & we always do DFT using FFT method only.

let us sub $n = 2r$ into first summ^n & $n = 2r+1$ into 2^{nd} summ^n, we get

$$X(K) = \sum_{r=0}^{N/2-1} x(2r)\, W_N^{2rK} + \sum_{r=0}^{N/2-1} x(2r+1)\, W_N^{K(2r+1)}$$

[$\because n = 2r+1$,

$\because$ when $n = 0$. $n = 2r$. $2r = 0$ $\therefore r = 0$

$n = N-2$. $2r = N-2$. $r = N/2 - 1$

1st Term $\therefore \sum_{r=0}^{N/2-1}$.

ily $n = 1$. $2r+1 = 1$. $2r = 0$. $r = 0$.

$n = N-1$. $2r+1 = N-1$. $2r = N-2$. $r = N/2 - 1$

2nd term $\therefore \sum_{r=0}^{N/2-1}$]

$$\therefore X(K) = \sum_{r=0}^{N/2-1} x(2r)\, W_{N/2}^{rK} + \sum_{r=0}^{N/2-1} x(2r+1)\, W_{N/2}^{rK} (W_N^{K})$$

[$\because W_N^{2rK} = \left[e^{-j2\pi/N}\right]^{2rK} = e^{-j\frac{4\pi rK}{N}}$

$W_{N/2}^{rK} = \left[e^{-j2\pi/N/2}\right]^{rK} = e^{-j\frac{4\pi rK}{N}}$]

$$\therefore X(K) = \sum_{r=0}^{N/2-1} x(2r)\, W_{N/2}^{rK} + W_N^{K} \sum_{r=0}^{N/2-1} x(2r+1)\, W_{N/2}^{rK}$$

$$\therefore X(K) = G(K) + W_N^K H(K) \longrightarrow (1)$$

$0 \le K \le N/2 - 1$

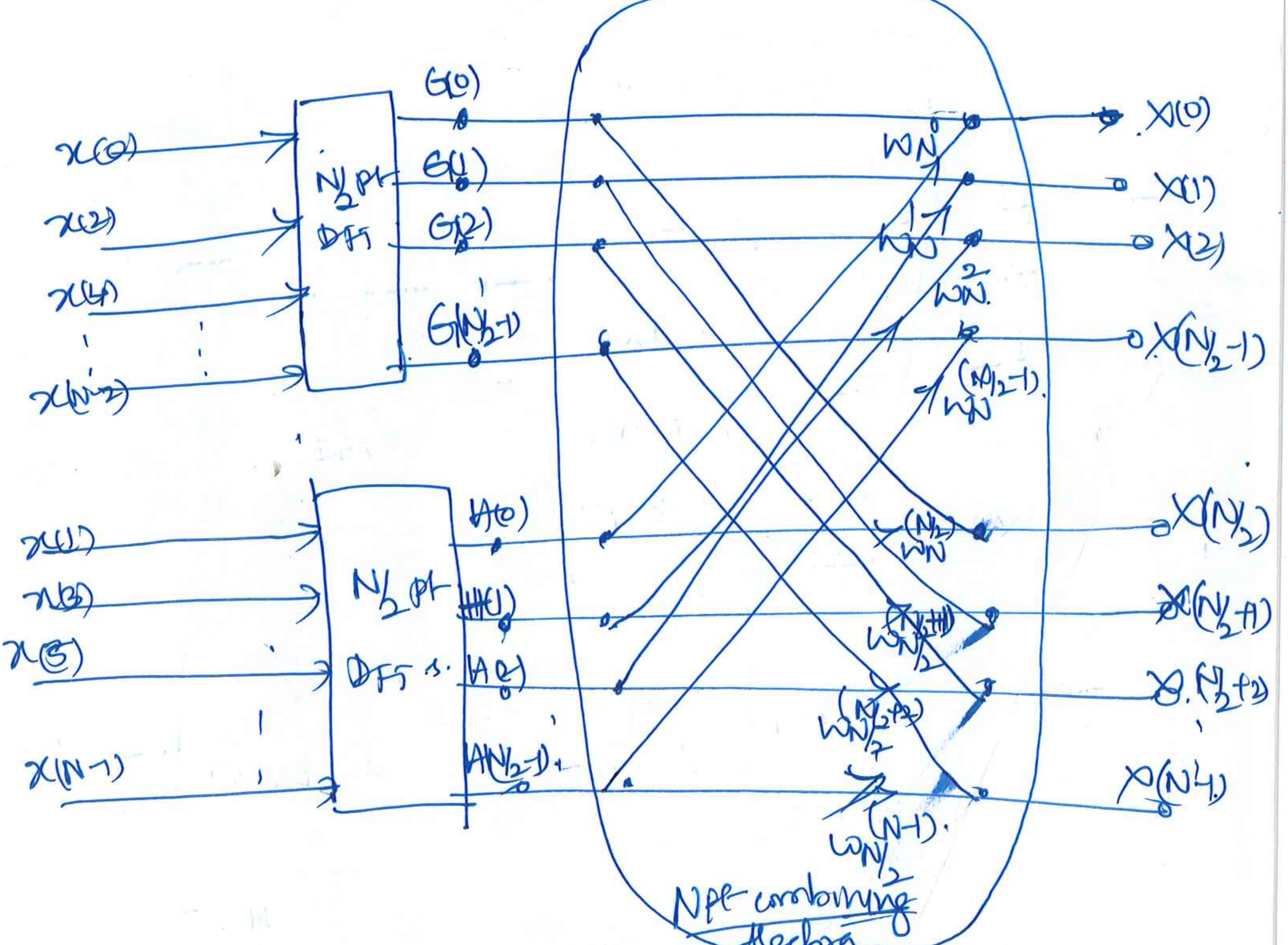
where $G(k)$ & $H(k)$ are $N/2$ pt DFT of even indexed samples & odd indexed samples;

$\therefore$ the eqns we required to get N pt DFT of $x(n)$ are

$$X(k) = G(k) + W_N^k H(k) \quad ; \quad 0 \le k \le N/2 - 1$$

$$X(k) = G(k + N/2) + W_N^k H(k + N/2) \quad N/2 \le k \le N-1$$

$\therefore$ Flow diagram for the first stage decomposition is



I st stage decomposition

Total no of CM's required to get N pt DFT's

$$\eta_1 = (N/2)^2 + (N/2)^2 + N = 2(N/2)^2 + N$$

$$\boxed{\eta_1 = \frac{N^2}{2} + N.}$$

Ex. $N = 8$ pt DFT $\therefore \eta_1 = \frac{64}{2} + 8 = 32 + 8 = 40$ CM.

64 → 40 CM

* Ist term indicates the CMs required for even numbered samples.
* IInd term indicates the CMs required for odd numbered samples.
* IIIrd terms represents the total no of CMs. required for the N-pt combining algebra..
* Each N/2 pt sequences are further decimated into 2 sequences of length N/4, then we get

$$G(K) = \sum_{r=0}^{N/2-1} g(r)\, W_{N/2}^{kr}. \quad \text{(general eqn)}.$$

∴ we can write this as 2 N/4 pt-DFT like below.

llly $\therefore$

$$G(K) = \sum_{l=0}^{N/4-1} g(2l)\, W_{N/2}^{2kl} + \sum_{l=0}^{N/4-1} g(2l+1)\, W_{N/2}^{k(2l+1)}.$$

$$G(K) = \sum_{l=0}^{N/4-1} g(2l)\, W_{N/4}^{kl} + W_{N/2}^{K} \sum_{l=0}^{N/4-1} g(2l+1)\, W_{N/4}^{kl}.$$

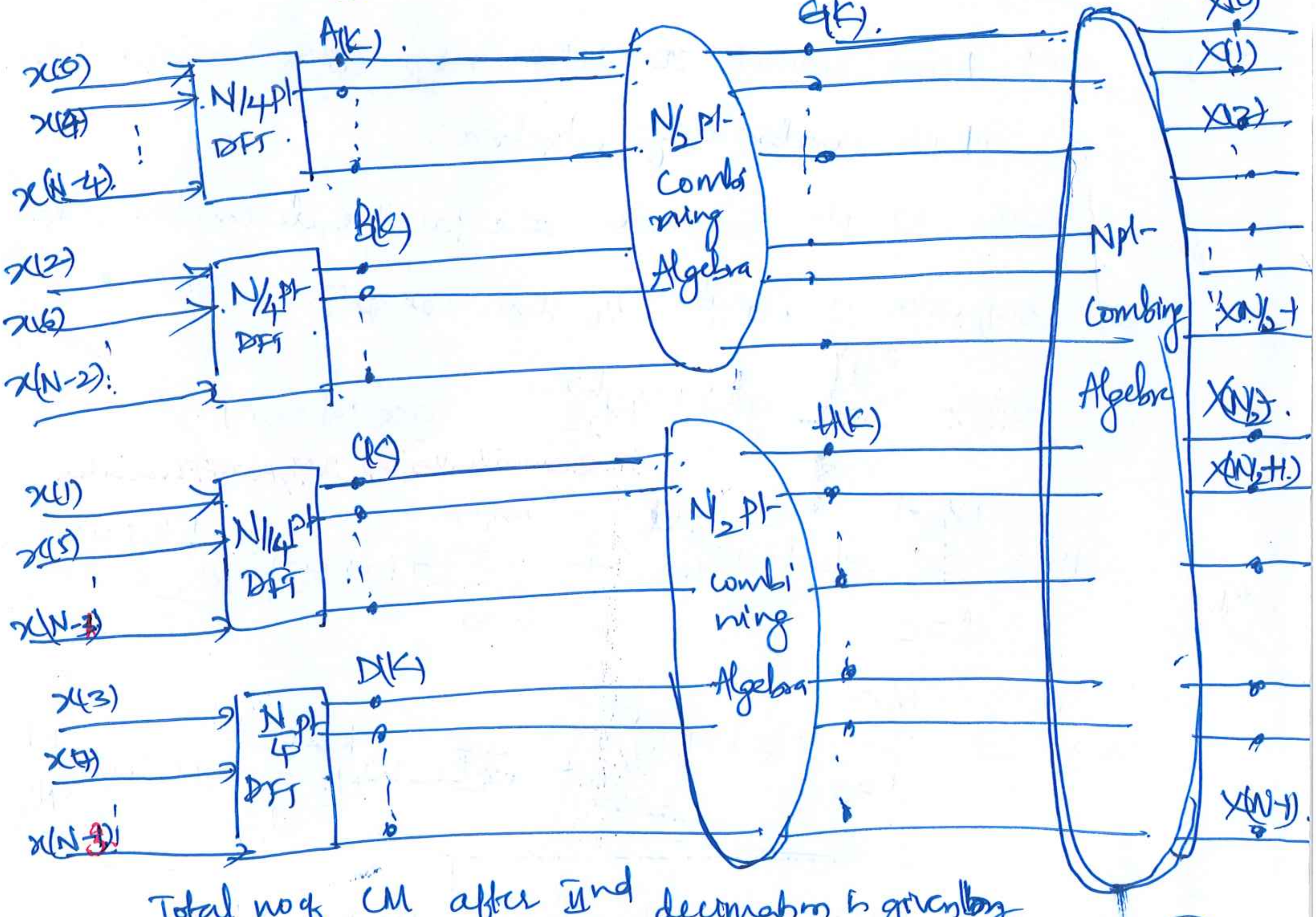
$$\boxed{G(K) = A(K) + W_{N/2}^{K}\, B(K)} \quad 0 \le K \le N/4 - 1$$

* where $A(K)$ & $B(K)$ are periodic with period equal to $N/4$, then we can write $N/2$ pt-DFT eqn for $G(K)$ & $H(K)$ as.

$$G(K) = \begin{cases} A(K) + W_{N/2}^{K} B(K) & 0 \le K \le N/4 - 1 \\ A(K+N/4) + W_{N/2}^{K} B(K+N/4) & N/4 \le K \le N/2 - 1 \end{cases}$$

11ly

$$H(K) = \begin{cases} C(K) + W_{N/2}^{K} D(K) & 0 \le K \le N/4 - 1 \\ C(K+N/4) + W_{N/2}^{K} D(K+N/4) & N/4 \le K \le N/2 - 1 \end{cases}$$



Total no of CM after IInd decimation is given by.

$$\eta_2 = 4\left(\frac{N}{4}\right)^2 + 2\left(\frac{N}{2}\right) + N$$

$$\eta_2 = \frac{N^2}{4} + N + N = \frac{N^2}{4} + 2N.$$

$64 \rightarrow 40 \rightarrow 32\ CM$:

For Ex. $N = 8$ pt DFT . then

$$\eta_2 = \frac{8^2}{4} + 2(8) = 16 + 16 = 32\ CM$$

Generally $N = 8$ pt $\quad N^2 \rightarrow \eta_1 \rightarrow \eta_2$.

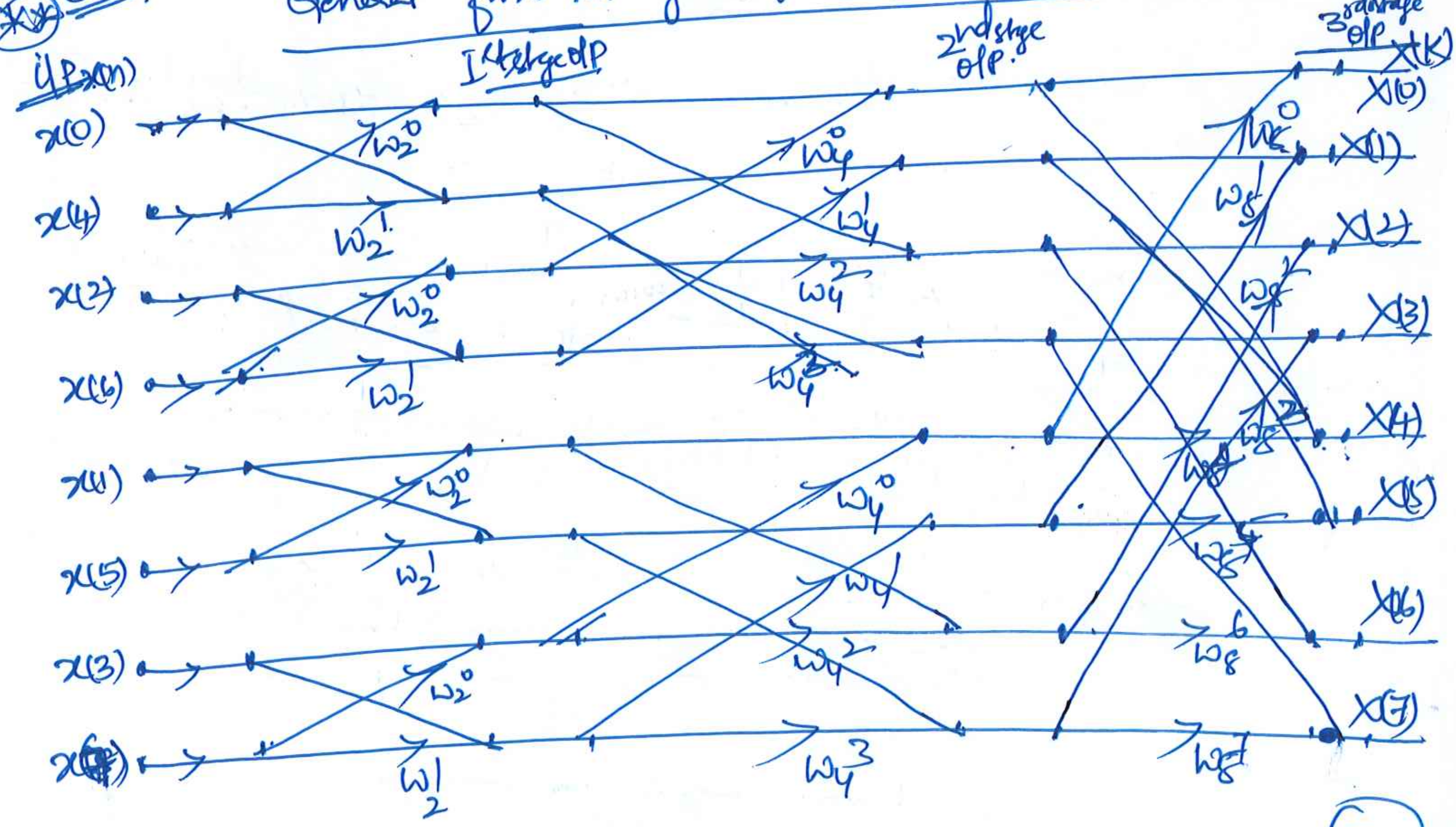
$N^2.\ 64 \rightarrow 40 \rightarrow 32 \rightarrow ?$

* Ist term gives no of CM required for Computation of $4\ N/4$ pt DFT

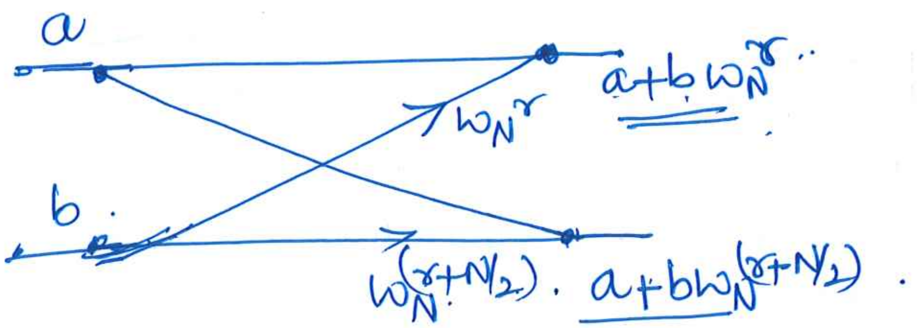
* 2nd & IIIrd terms represents the no of CM required for combining algebra for Ist & 2nd decimation.

CM

General flow diagram for 8-pt DIT-FFT algorithm



Basic butterfly dia in DIT-FFT.



* Further reduction of computation is done by using Cooley-Tukey algorithm.

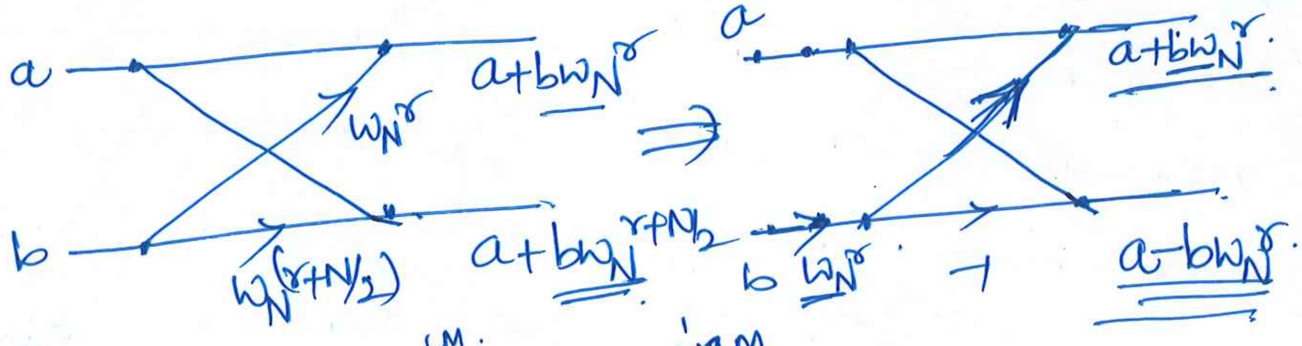
* By using Cooley-Tukey Algorithm, we can reduce the CM per butterfly by 1. [ie 2 to 1.]

ie $W_N^{r+N/2} = \left(e^{-j\frac{2\pi}{N}}\right)^{(r+N/2)} = e^{-j\frac{2\pi}{N}r} \cdot \underbrace{e^{-\frac{j2\pi}{N}\cdot\frac{N}{2}}}_{(-1)}$

$\therefore W_N^{r+N/2} = -W_N^r.$

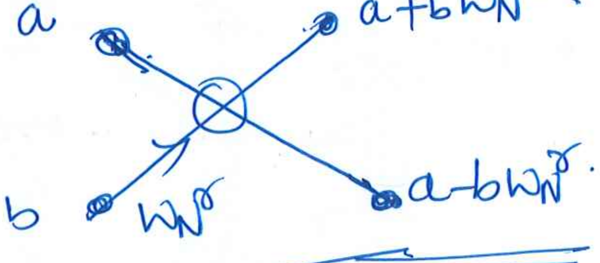
$\therefore$ we can write:

Final reduced method



CM
2M ⟶ 1CM

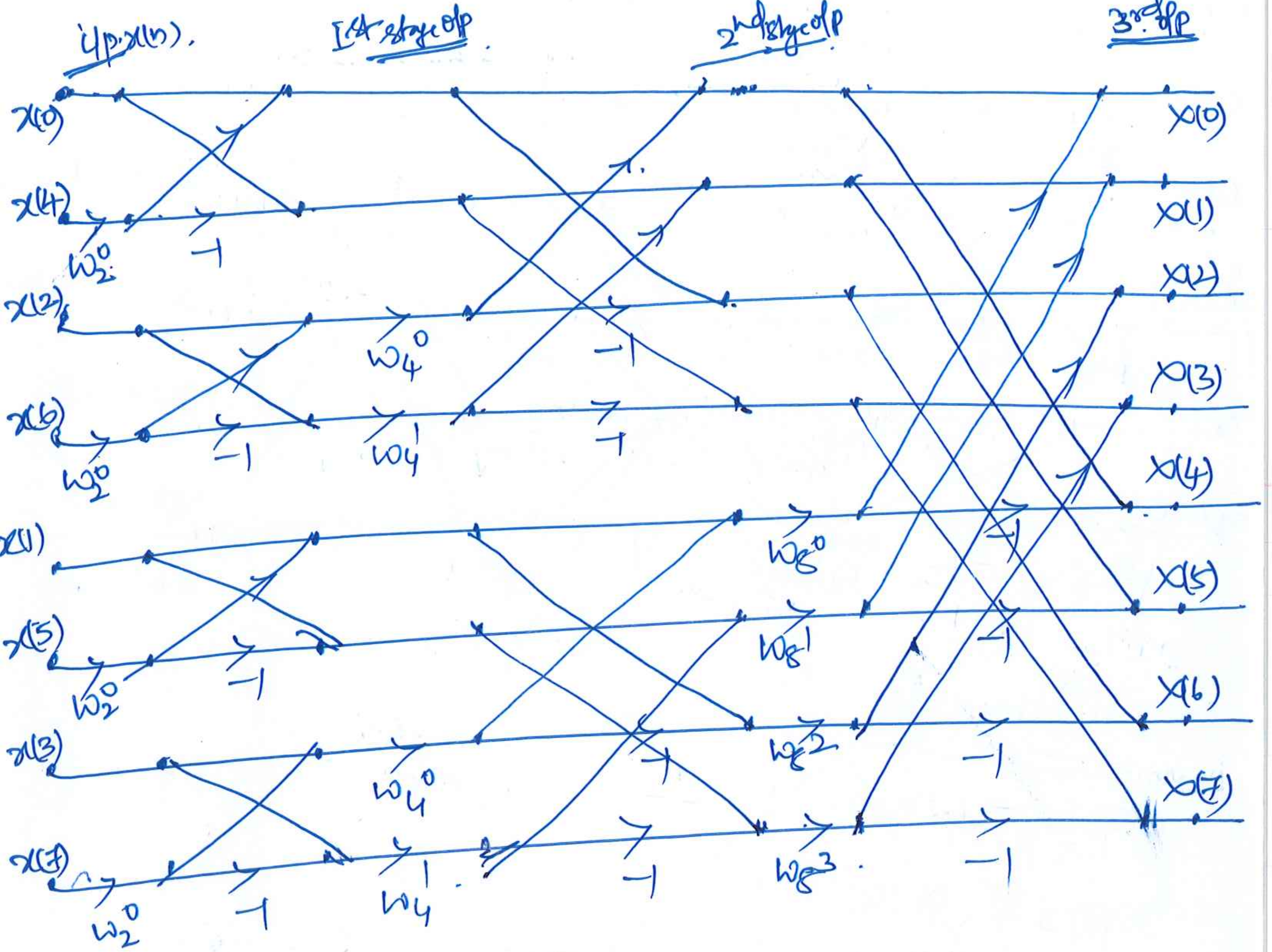
or ⇒



* Reduced flowgraph (efficient method) of 8 pt- DIT-FFT algorithm

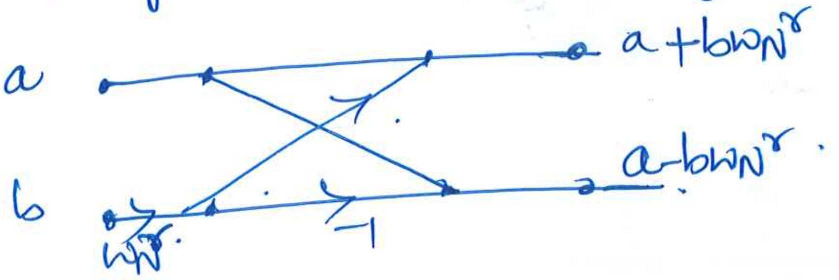
OR

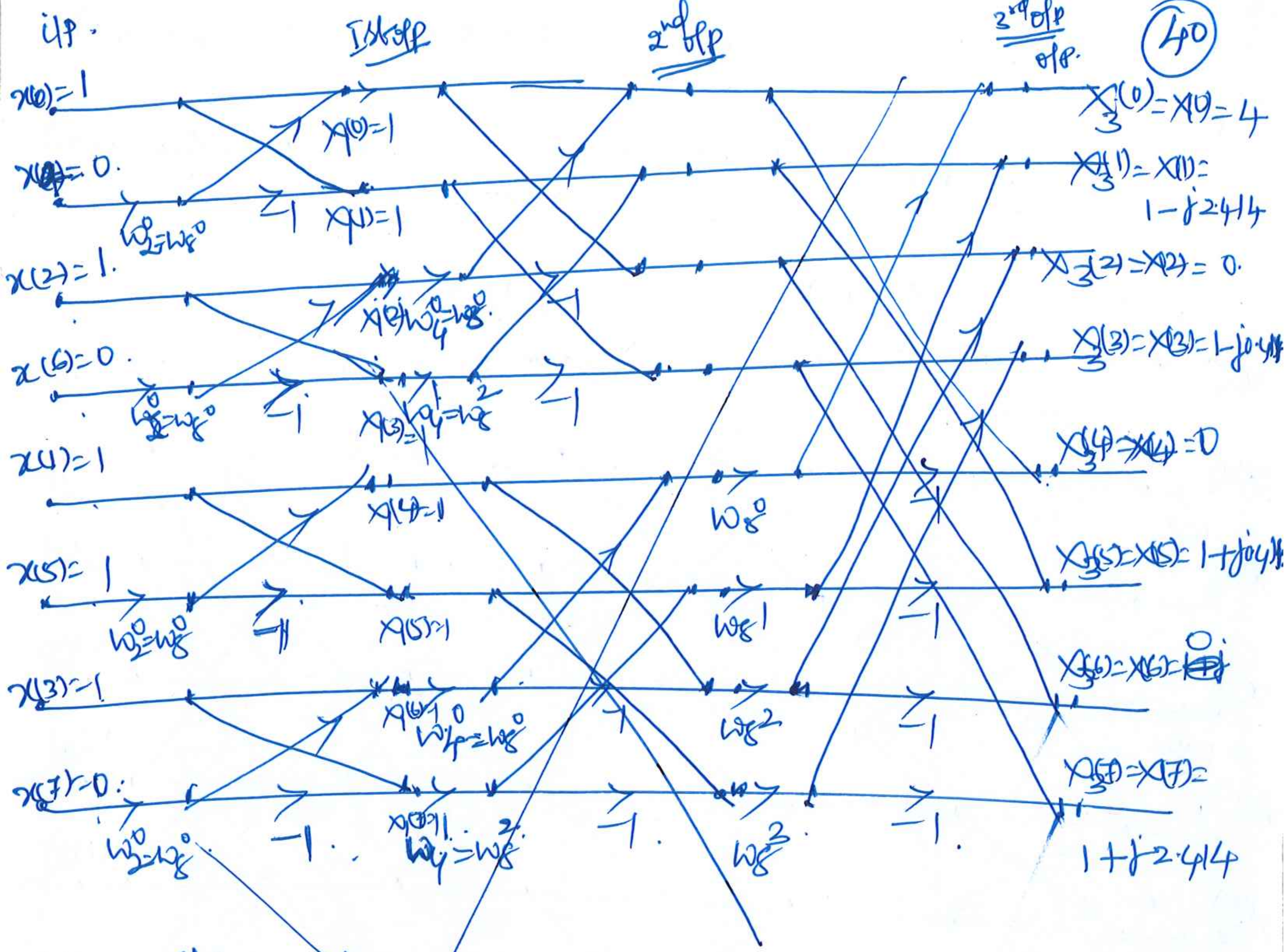
Reduced flowgraph for 8pt-DFT using cooley-Tuokey algorithm



Problems.

1) Find the 8pt- DFT of a sequence $x(n) = \{1, 1, 1, 1, 0, 0, 0, 0\}$ using DIT-FFT radix-2 algorithm. Use the butterfly diagram of cooley-Tuokey algorithm; show intermediate o/p also.





Ist stage o/p

$X_1(0) = 1 + 0 \times W_8^0 = 1$

$X_1(1) = 1 - 0 \times W_8^0 = 1$

3rd stage o/p.

$X(0) = 2 + 2 \times W_8^0 = 2+2 = 4$

$X(1) = (1-j) + (1+j) W_8^1 = 1 - j2.414$

$X(2) = 0 + 0 W_8^2 = 0.$

$X(3) = (1+j) + (1+j) \cdot (-1/\sqrt{2} - j1/\sqrt{2}) = \underline{1 - j0.414}$

$X(4) = 2 - 2 W_8^0 = 0.$

$X(5) = (1-j) - (1-j)(1/\sqrt{2} - j1/\sqrt{2}) = 1 + j0.414.$

$X(6) = 0 - 0 W_8^2 = 0.$

$X(7) = (1+j) - (1-j)(-1/\sqrt{2} - j \cdot 1/\sqrt{2})$

$X(7) = 1 + j2.414.$

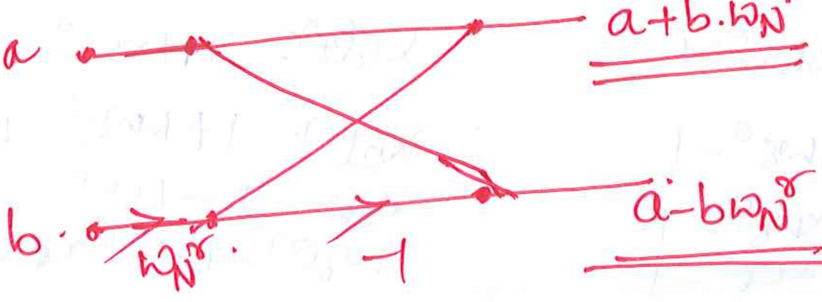
$$\boxed{\therefore X(k) = \{4, 1-j2.414, 0, 1-j0.414, 0, 1+j0.414, 0, 1+j2.414\}.}$$

(2) Repeat the problem for $x(n) = \{1, 0, 1, 0, 1, 0, 1, 0\}$

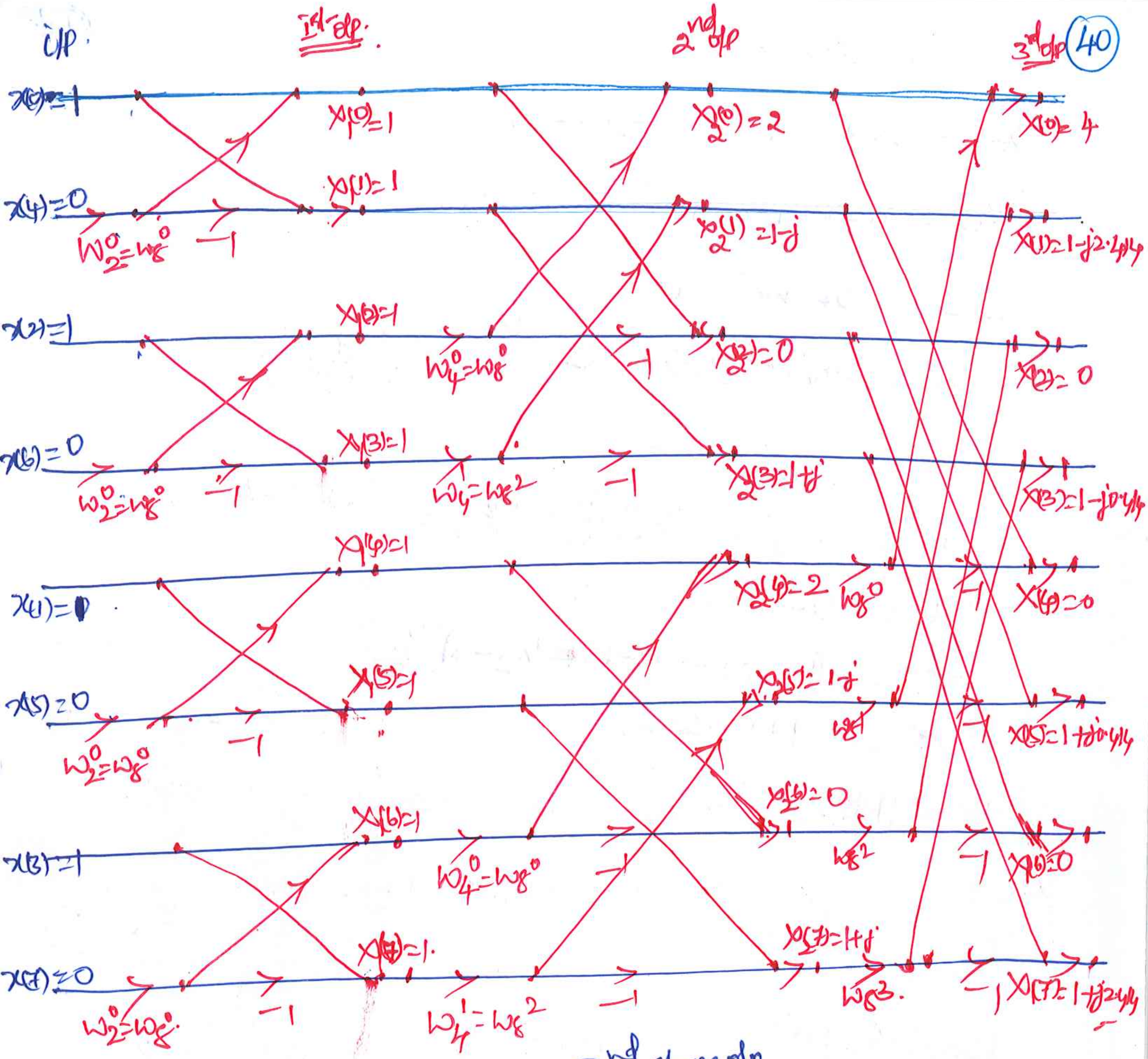
(or).

Find 8 pt DFT of the sequence $x(n) = \{1, 0, 1, 0, 1, 0, 1, 0\}$ using DIT-FFT radix 2 ~~basic~~ Algorithm. Use Butterfly dia given. Show all intermediate o/p.

Answer $X(k) = \{4, 0, 0, 0, 4, 0, 0, 0\}$



40



1st stage o/p

$X_1(0) = 1 + 0 \times W_8^0 = \underline{1}$

$X_1(1) = 1 - 0 \times W_8^0 = 1$

$X_1(2) = 1 + 0 \times W_8^0 = 1$

$X_1(3) = 1 - 0 \times W_8^0 = 1$

$X_1(4) = 1 + 0 \times W_8^0 = 1$

$X_1(5) = 1 - 0 \times W_8^0 = 1$

$X_1(6) = 1 + 0 \times W_8^0 = 1$

$X_1(7) = 1 - 0 \times W_8^0 = 1$

2nd stage o/p

$X_2(0) = 1 + W_8^0 \times 1 = 2$

$X_2(1) = 1 + W_8^2 \cdot 1 = \underline{1 - j}$

$X_2(2) = 1 - W_8^0(1) = \underline{0}$

$X_2(3) = 1 - W_8^2(1) = 1 + j$

$X_2(4) = 1 + 1 \times W_8^0 = 1 + 1(1) = \underline{2}$

$X_2(5) = 1 + 1 \times W_8^2 = 1 - j$

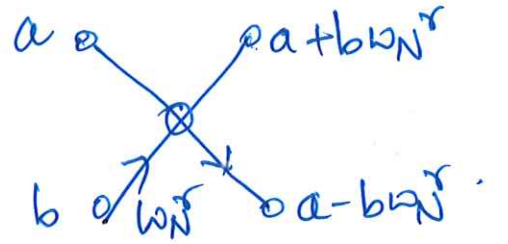
$X_2(6) = 1 - 1\, W_8^0 = 0$

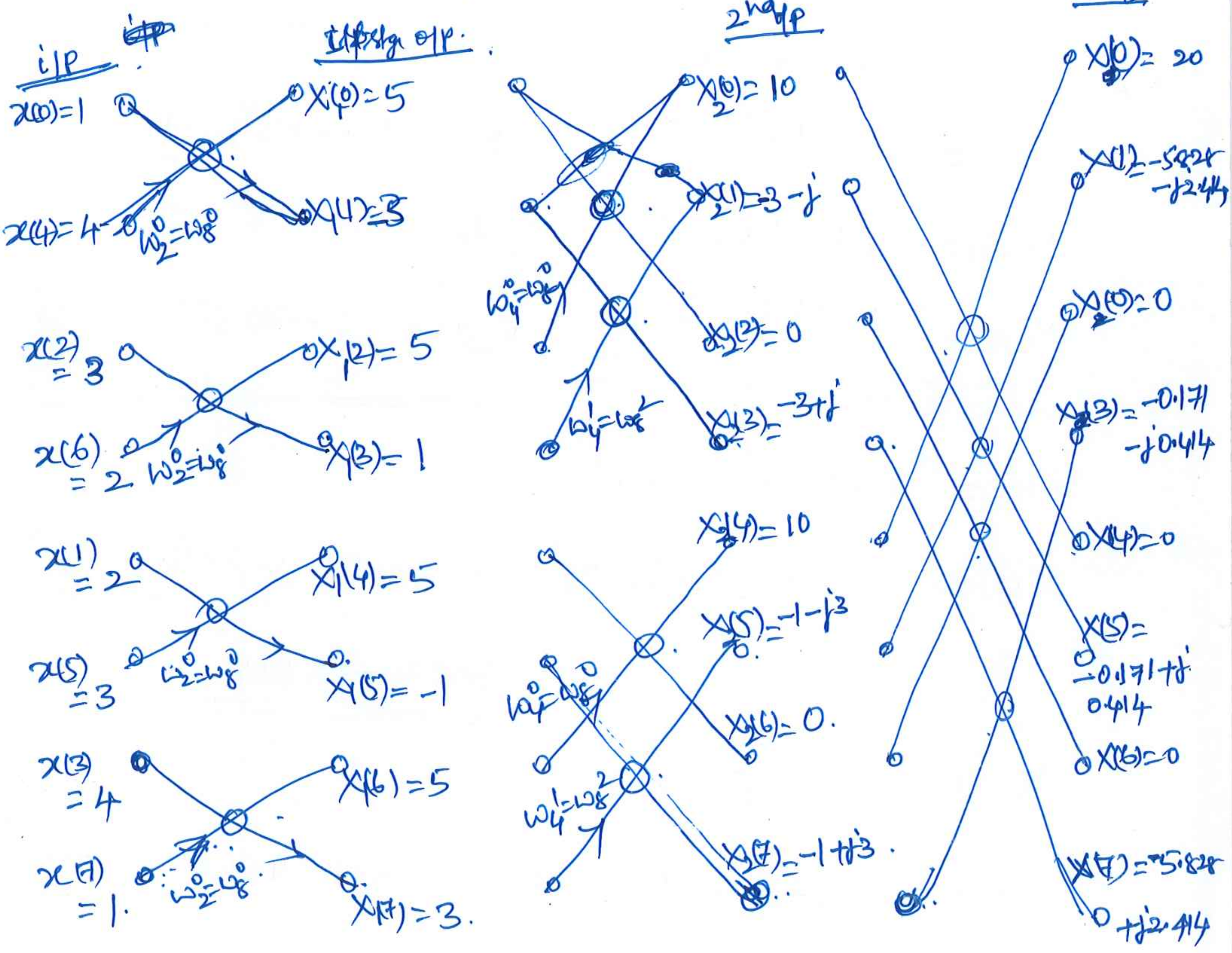
$W_8(7) = 1 - 1 \times W_8^2 = \underline{1 + j}$

3). Find 8 pt DFT of a sequence.

$x(n) = \{1, 2, 3, 4, 4, 3, 2, 1\}$

using DIT-FFT algorithm. use the basic butterfly dia shown below:





1^{st} stage o/p

$X_1(0) = 1 + 4W_8^0 = 5$

$X_1(1) = 1 - 4W_8^0$
$1 - 4 = -3$

$X_1(2) = 3 + 2W_8^0 = 5$

$X_1(3) = 3 - 2W_8^0 = 1.$

$X_1(4) = 2 + 3W_8^0 = 5$

$X_1(5) = 2 - 3W_8^0 = -1$

$X_1(6) = 4 + 1W_8^0 = 5$

$X_1(7) = 4 - 1W_8^0 = 3.$

2^{nd} stage o/p

$X_2(0) = 5 + 5W_8^0 = 10$

$X_2(1) = -3 + 1W_8^2 = -3 - j$

$X_2(2) = 5 - 5W_8^0 = 0$

$X_2(3) = -3 - 1(-j) = -3 + j$

$X_2(4) = 5 + 5W_8^0 = 10$

$X_2(5) = -1 + 3W_8^2 = -1 - j$

$X_2(6) = 5 - 5W_8^0 = 0$

$X_2(7) = -1 - 3(W_8^2) = -1 + 3j$

$X(0) = 10 + 10W_8^0 = 20$

$X(1) = -3 - j + (-j - j3)W_8^1 = -3 - j + (-j - j3)\left(\frac{1}{\sqrt{2}} - j\frac{1}{\sqrt{2}}\right)$

$= -5.828 - j2.414$

$X(2) = 0 + 0\,W_8^2 = 0$

$X(3) = -3 + j - (1 + j3)W_8^3 = -3 + j - (1 + j3)\left(-\frac{1}{\sqrt{2}} - j\frac{1}{\sqrt{2}}\right)$

$= -0.171 - j0.414$

$X(4) = 10 - 10W_8^0 = 0$

$X(5) = (-3 - j)\left[-(-1 - j3)\right]W_8 = -0.171 + j0.414$

$X(6) = 0 - 0 \cdot W_8^2 = 0$

$X(7) = (-3+j)[-(j+j3)](-1/\sqrt{2} - j1/\sqrt{2}) = \underline{-5.828 + j2.414}$

$X(K) = X^*(8-K)$ is satisfied, Hence it's symmetric for

H(W) (4) Repeat for $\{4, 3, 2, 1, 1, 2, 3, 4\}$

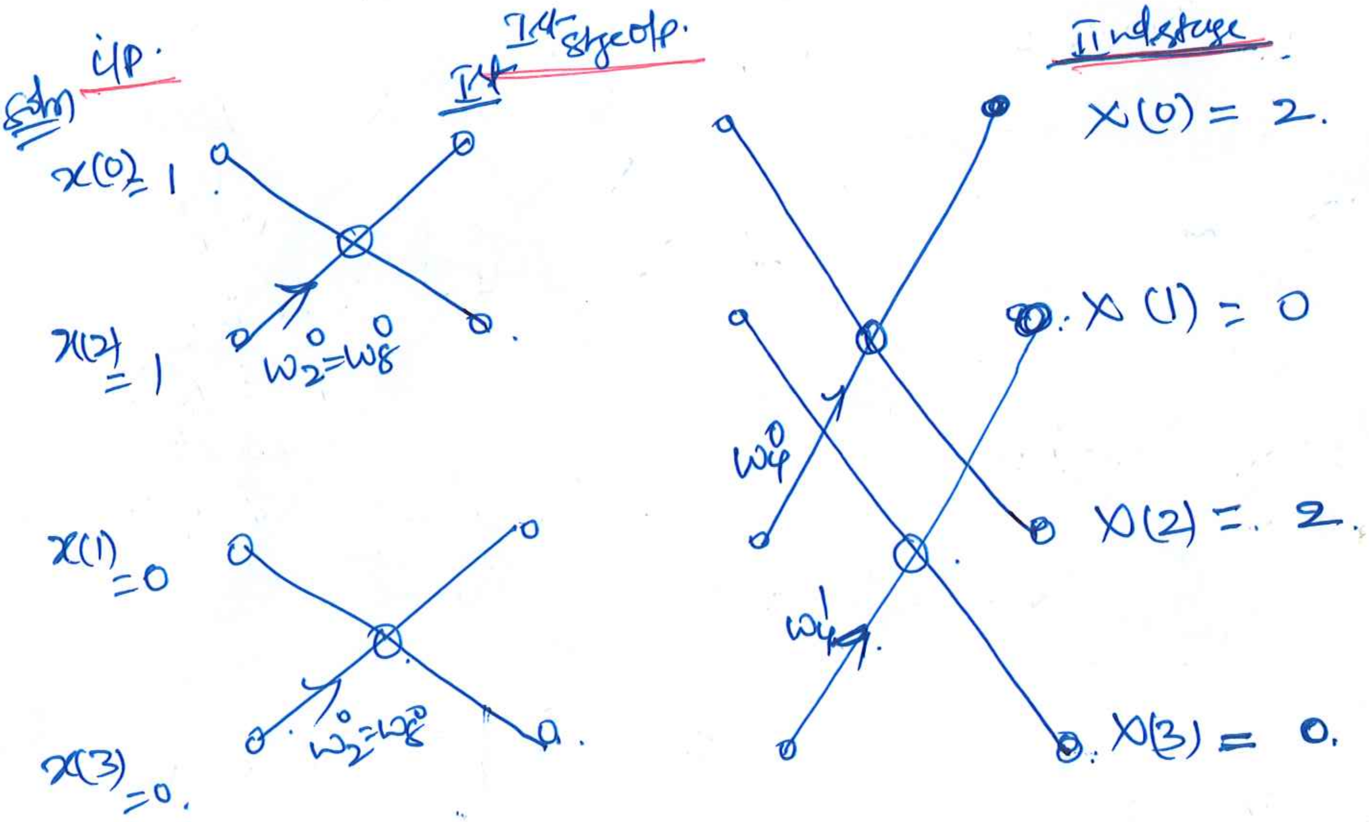
XX 8M

(5) compute 4 pt DFT of the sequence.

$x(n) = \{1, 0, 1, 0\}$. using DIT-FFT

algorithm. Find also IFFT of the sequence. [IDFT]

Soln



$X(K) = \{2, 0, 2, 0\}$

* computing $x(n)$ from DIT-FFT algorithm

$x(n)$ = IFFT method

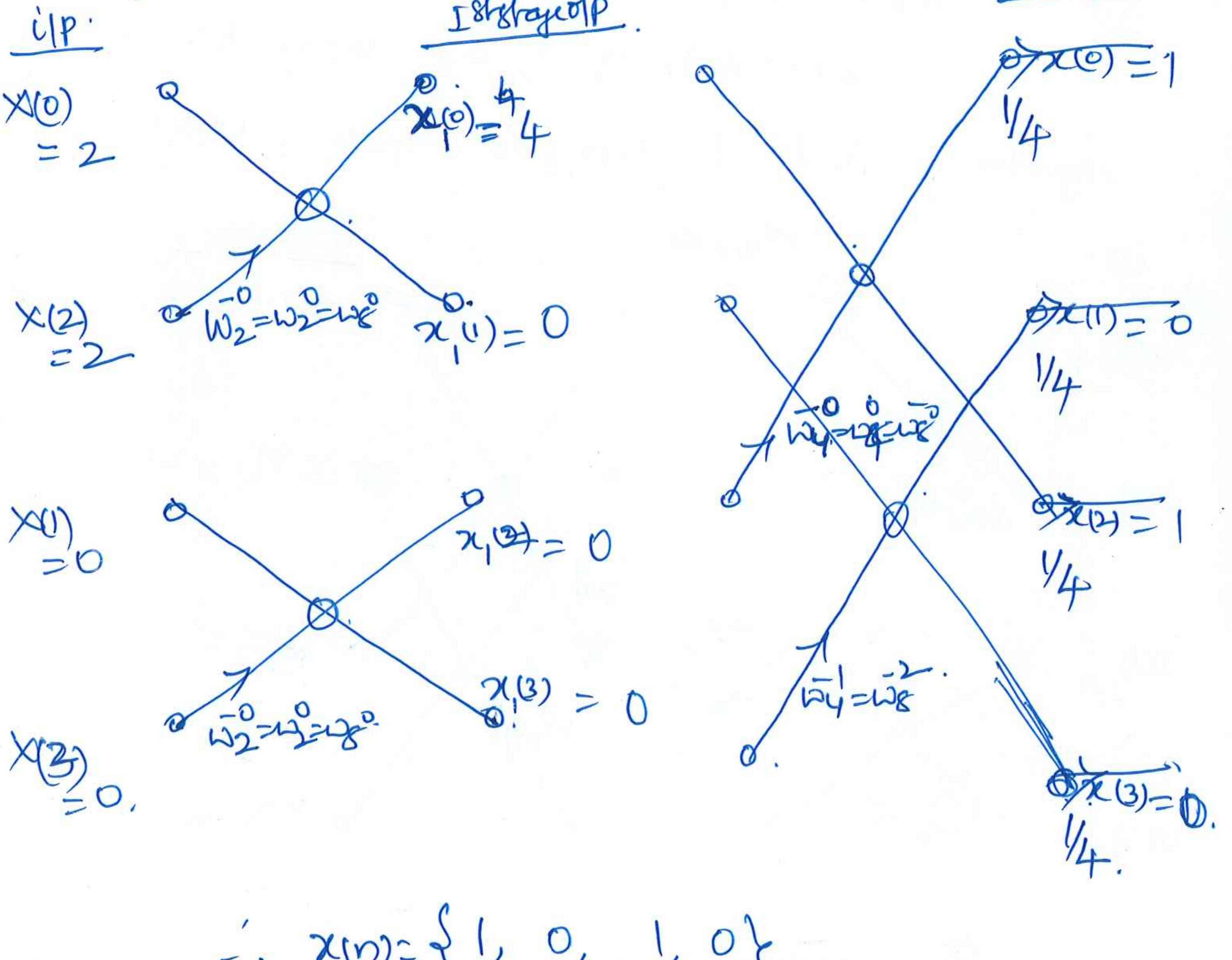
2 steps

1) Write o/p as i/p & i/p as o/p.

but i/p in bit reversed form & o/p as normal.

2) divide o/p of butterfly by $1/N$ to get final o/p.

3) write W_N^{+} as W_N^{-ve}.



$\therefore x(n) = \{1, 0, 1, 0\}$

HW.

6) Repeat for $x(n) = \{0, 1, 0, 1\}$

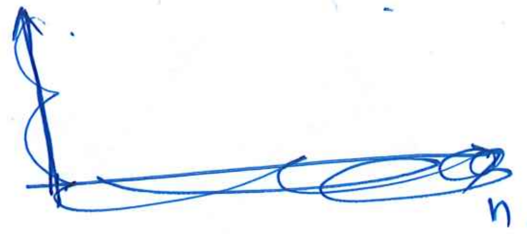
(7). For the sequences $x_1(n)$ & $x_2(n)$ shown, compute the circular convolution.

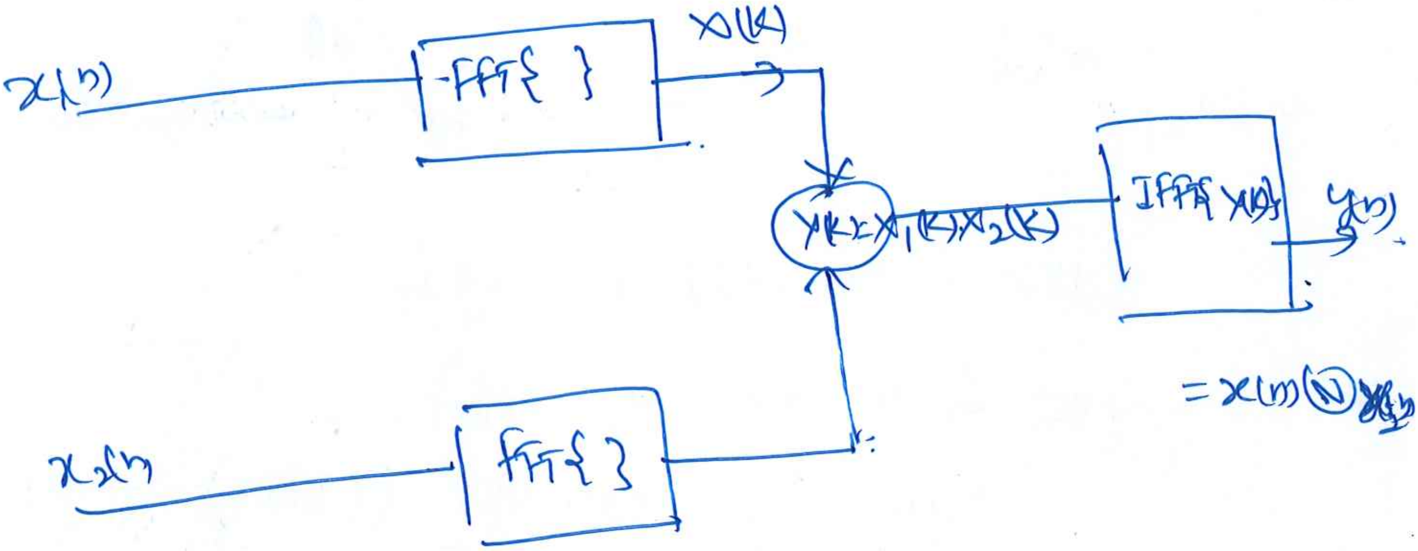
$x_1(n) \circledN x_2$ for $N=4$, use DIT-FFT Algorithm.

$\boxed{\text{FFT \& IFFT}}$. $x_1(n) = \{2, 1, 1, 2\}$.

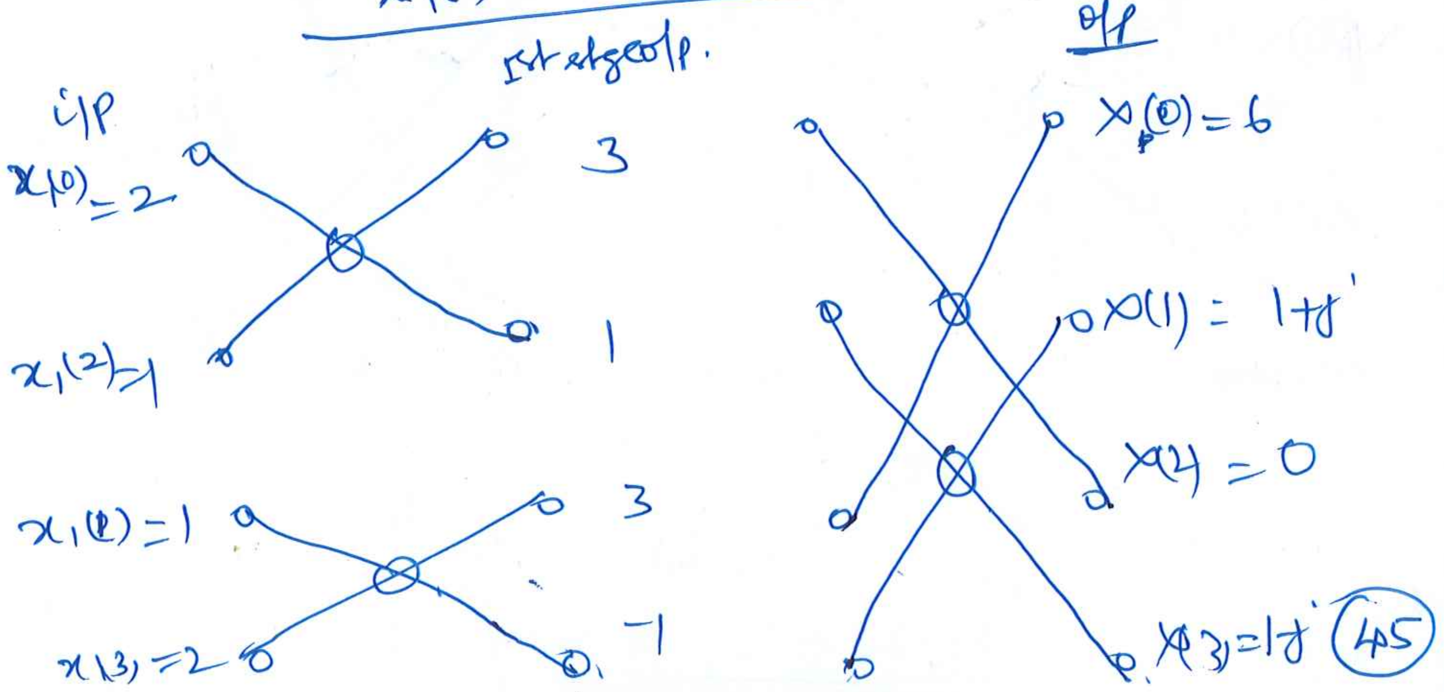
$x_2(n) = \{1, -1, -1, 1\}$

Soln



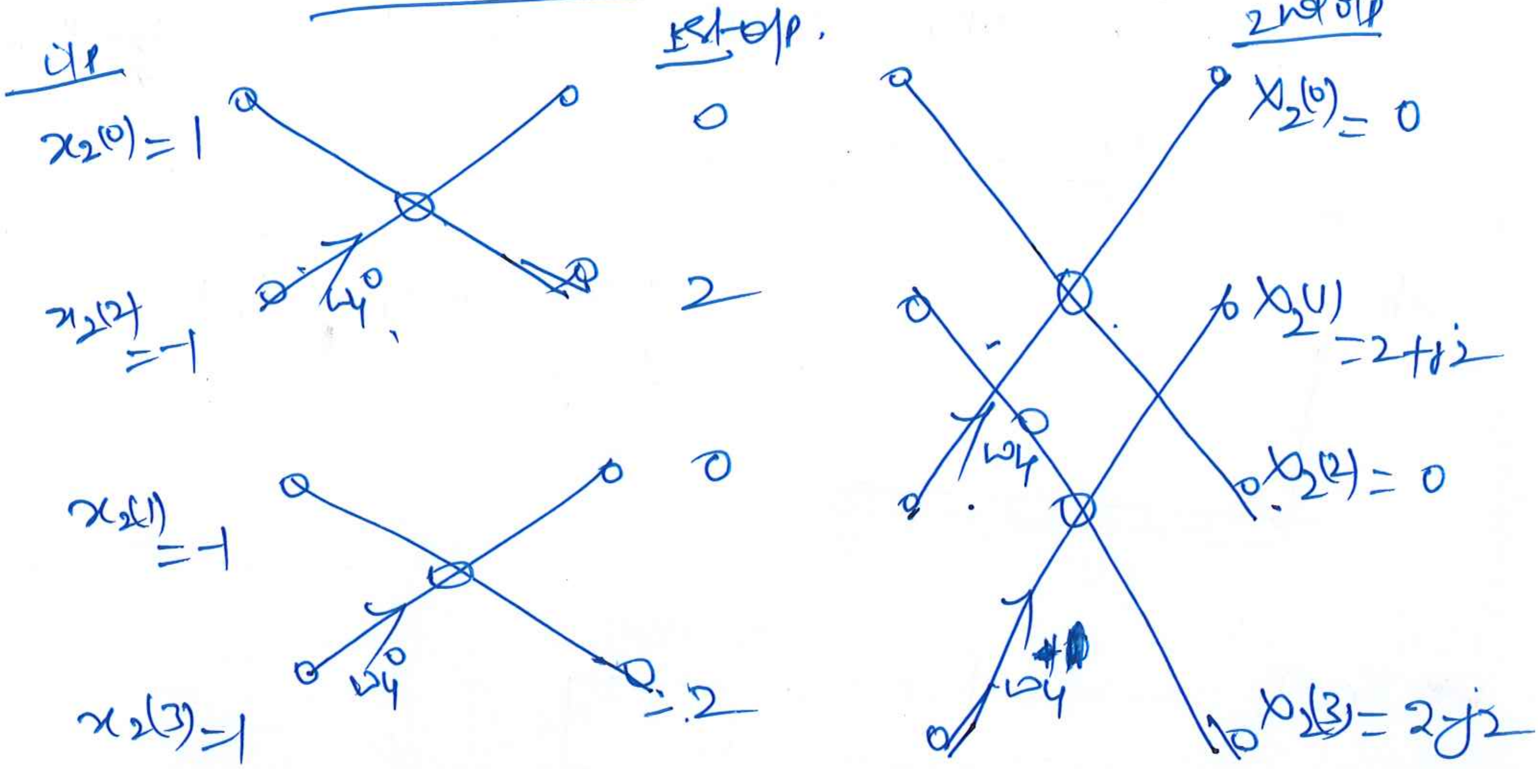


$X_1(k)$ using FFT



$x_2(n) = \{1, -1, -1, 1\}$

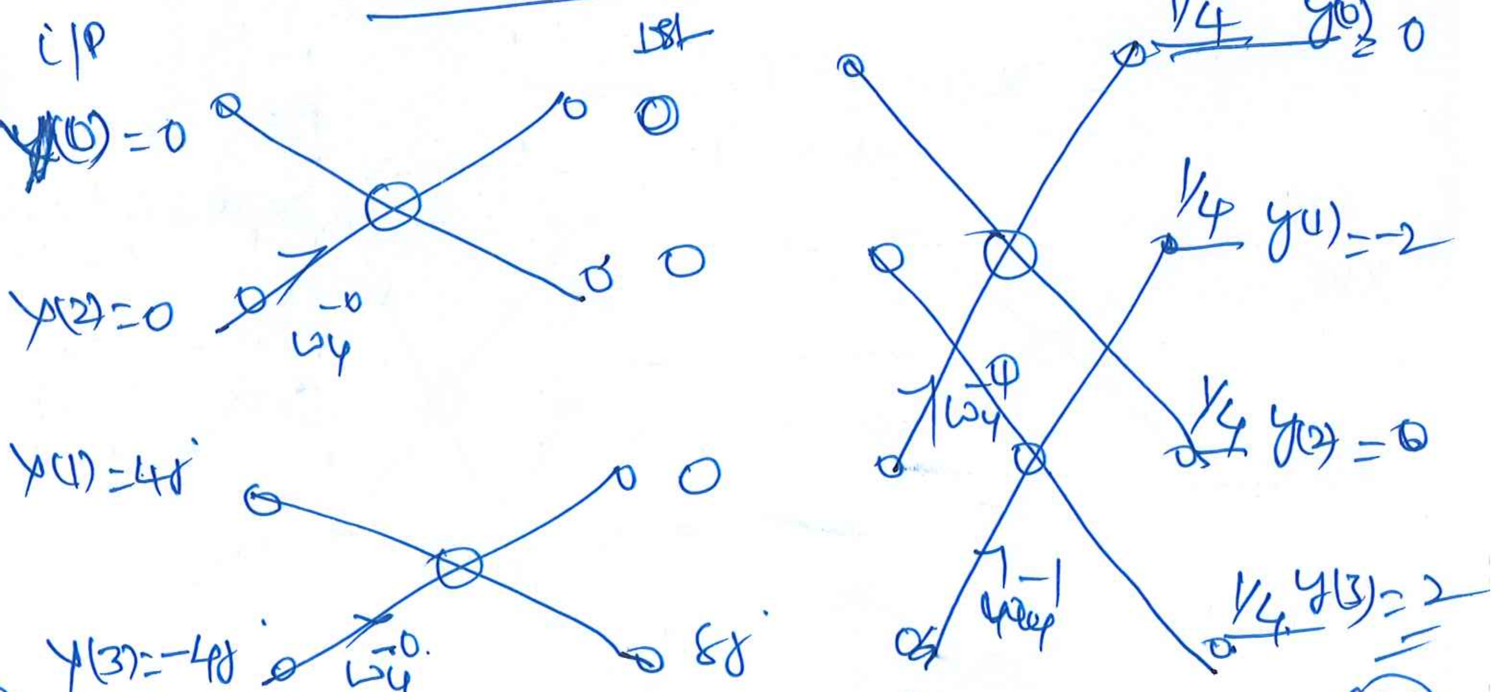
$X_2(K)$ using DIT FFT



$X_2(K) = \{0, 2+2j, 0, 2-j2\}$.

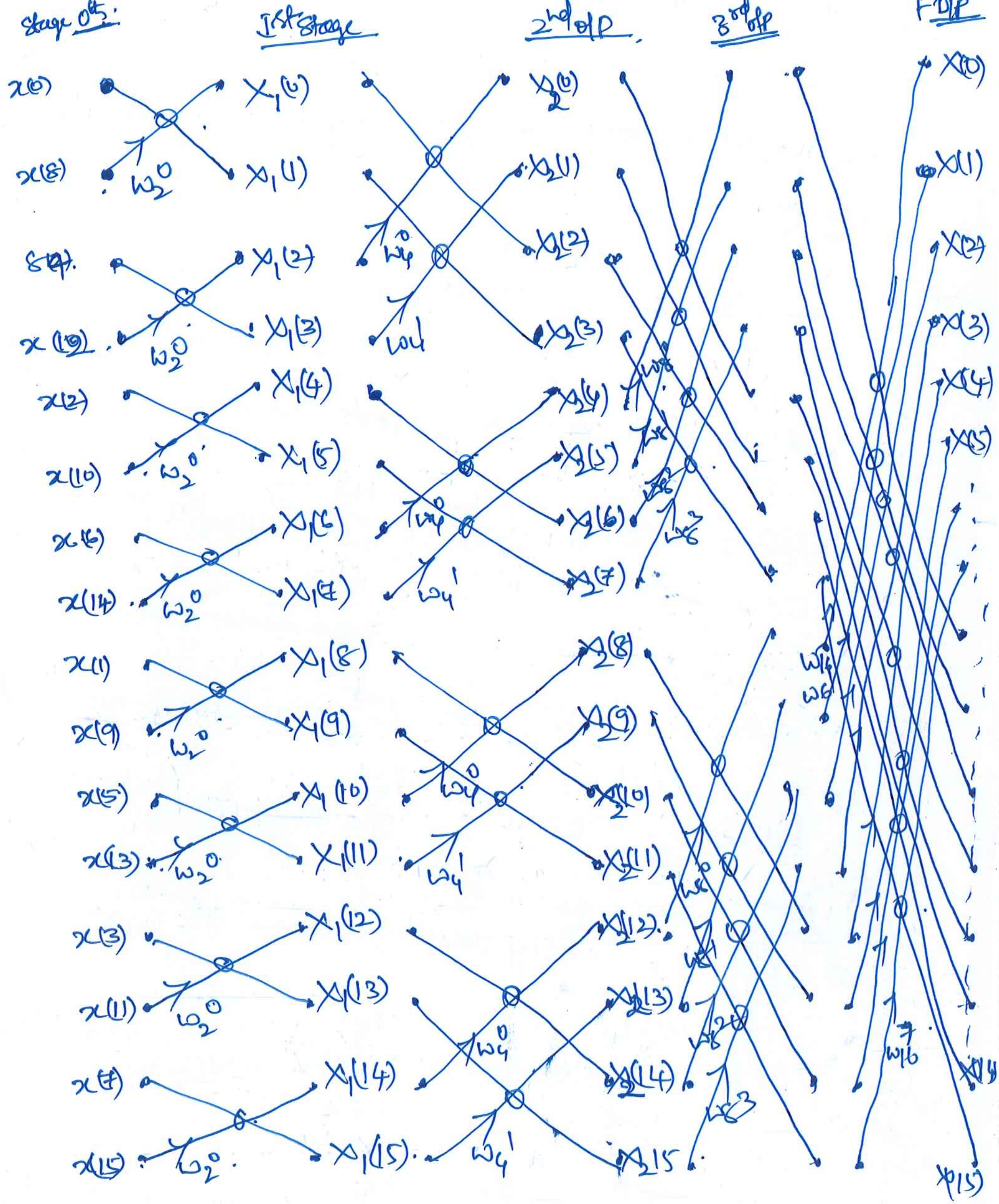
$Y(K) = \{0.\ 4j, 0, -4j\}$.

$y(n)$ using DIT-FFT ((IFFT)



(8) Repeat. $x_1(n) = \{1, 2, 2, 1\}$, $x_2(n) = \{-1, 1, 1, -1\}$

(**) (SM) Q) Draw the flow dia for a 16 pt Radix-2 DIT-FFT algorithm. Show all multipliers approximately.



Decimation in Frequency FFT.

* In This algorithm X(K) is divided into smaller & smaller subsequences like DITFFT. Here we evaluate odd numbered samples & even no of frequency samples. separately.

* We KT

$$X(K) = \sum_{n=0}^{N-1} x(n)\, W_N^{+kn}$$

$k = 0, 1, 2 \ldots N-1$

* let us break X(K) into 2 parts ie

$$X(K) = \sum_{n=0}^{N/2-1} x(n)\, W_N^{kn} + \sum_{n=N/2}^{N-1} x(n) W_N^{kn}$$

* sub $r = n - N/2$ into II^nd summation, we get

$$X(K) = \sum_{n=0}^{N/2-1} x(n) W_N^{kn} + \sum_{r=0}^{N/2-1} x(r + N/2)\, W_N^{k(r+N/2)}$$

when $n = N/2$, $r + N/2 = N/2 \Rightarrow r = 0$. $n = N-1$, $r + N/2 = N-1 \Rightarrow r = N/2 - 1$. $\therefore \sum_{r=0}^{N/2-1}$

since r is dummy variable, change into n, we get $r \to n$.

$$X(K) = \sum_{n=0}^{N/2-1} x(n) W_N^{kn} + \sum_{n=0}^{N/2-1} x(n + N/2)\, W_N^{k(n+N/2)}$$

$$X(K) = \sum_{n=0}^{N/2-1} x(n) W_N^{kn} + \sum_{n=0}^{N/2-1} x(n+N/2)\, W_N^{kn} \cdot W_N^{kN/2}$$

$$\therefore X(K) = \sum_{n=0}^{N/2-1} x(n) W_N^{kn} + W_N^{KN/2} \sum_{n=0}^{N/2-1} x(n+N/2) W_N^{kn}.$$

$$\therefore \boxed{X(K) = \sum_{n=0}^{N/2-1} \left[x(n) + (-1)^k x(n+N/2)\right] W_N^{kn}} \rightarrow (1)$$

$$\left[\because W_N^{KN/2} = \left(e^{\frac{-j2\pi}{N}}\right)^{KN/2} = e^{-j\pi K} = \cos \pi K - j\cancel{\sin \pi K}^{\,0} = (-1)^K. \right]$$

$\therefore$ ~~X(K)~~ Decimation in frequency is carried out by taking odd & even samples of $X(K)$ separately i.e.

1) For even terms, Assume $K = 2r$ & $r = 0, 1, 2 \ldots \frac{N}{2}-1$.

we get

$$X(2r) = \sum_{n=0}^{N/2-1} \left[x(n) + (-1)^{2r} x(n+N/2)\right] W_N^{n(2r)}.$$

$$\text{ie } X(2r) = \sum_{n=0}^{N/2-1} \left[x(n) + x(n+N/2)\right] W_{N/2}^{nr} \rightarrow (2)$$

Illy For odd sequences ie odd values of K

$$X(2r+1) = \sum_{n=0}^{N/2-1} \left[x(n) + (-1)^{2r+1} x(n+N/2)\right] W_N^{n(2r+1)}$$

$$X(2r+1) = \sum_{n=0}^{N/2-1} \left[x(n) - x(n+N/2)\right] W_N^{n} \cdot W_{N/2}^{nr} \rightarrow (3).$$

$$X(2r+1) = \sum_{n=0}^{N/2-1} \left[x(n) - x(n+N/2)\right] W_N^{k}$$

∴ Even and odd fn are defined as.

$$e(n) = x(n) + x(n+N/2)$$

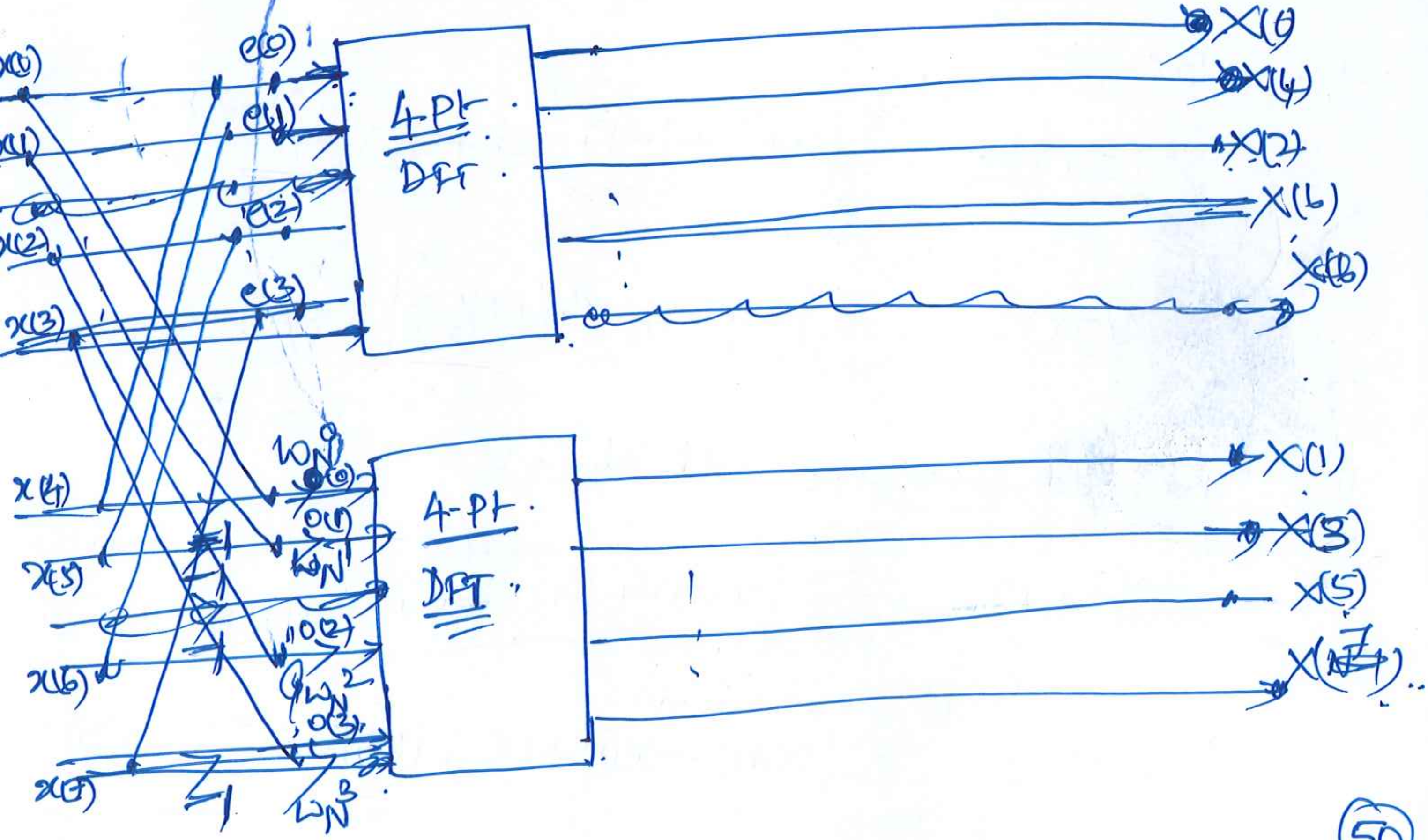
$$o(n) = \left[x(n) - x(n+N/2)\right] W_N^{n}$$

then eqn ② & ③ becomes

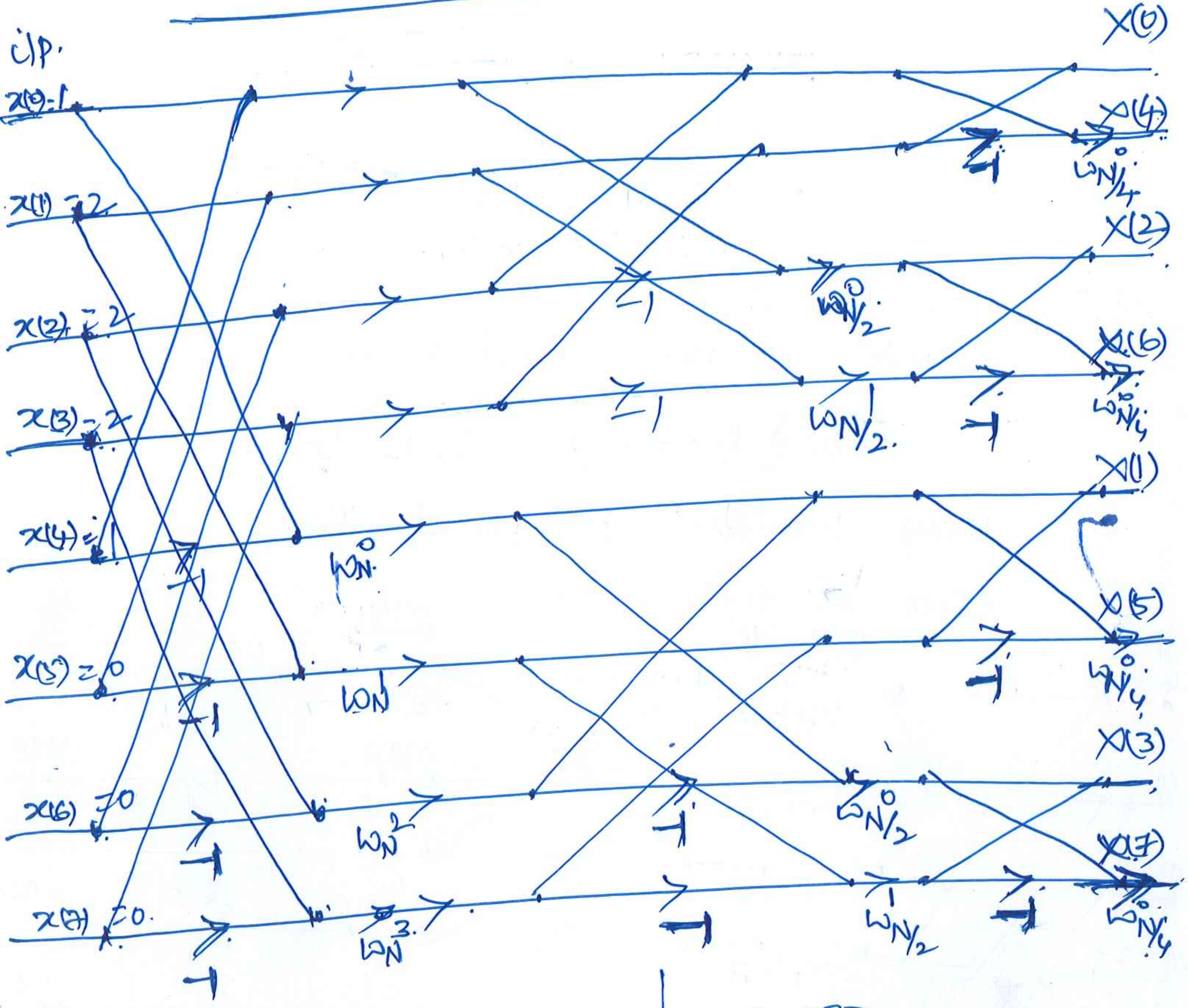
$$X(2r) = \sum_{n=0}^{N/2-1} e(n)\, W_{N/2}^{rn}$$

$$X(2r+1) = \sum_{n=0}^{N/2-1} o(n)\, W_{N/2}^{rn} \quad \left[r=0,1 \ldots N/2-1\right]$$

※ Flow dia for 8-pt DFT using DIF-FFT.

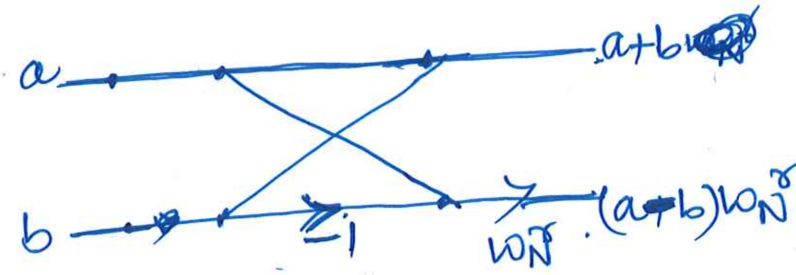


complete DIF-FFT; N=8.



✱. DIT-FFT.	DIF-FFT
1) Flow diagram is of exactly reverse form.	
2). o/p is normal order	i/p is normal order
3) i/p is bit reversed order	o/p is bit reversed.
5). Twiddle factor is multiplied at the beginning of BD	multiplied at the end of BD.
4) Frequency samples are evaluated one after the other	odd set of samples & even set of samples are evaluated seperatly.

Basic butterfly dia of DIF FFT.



Problems

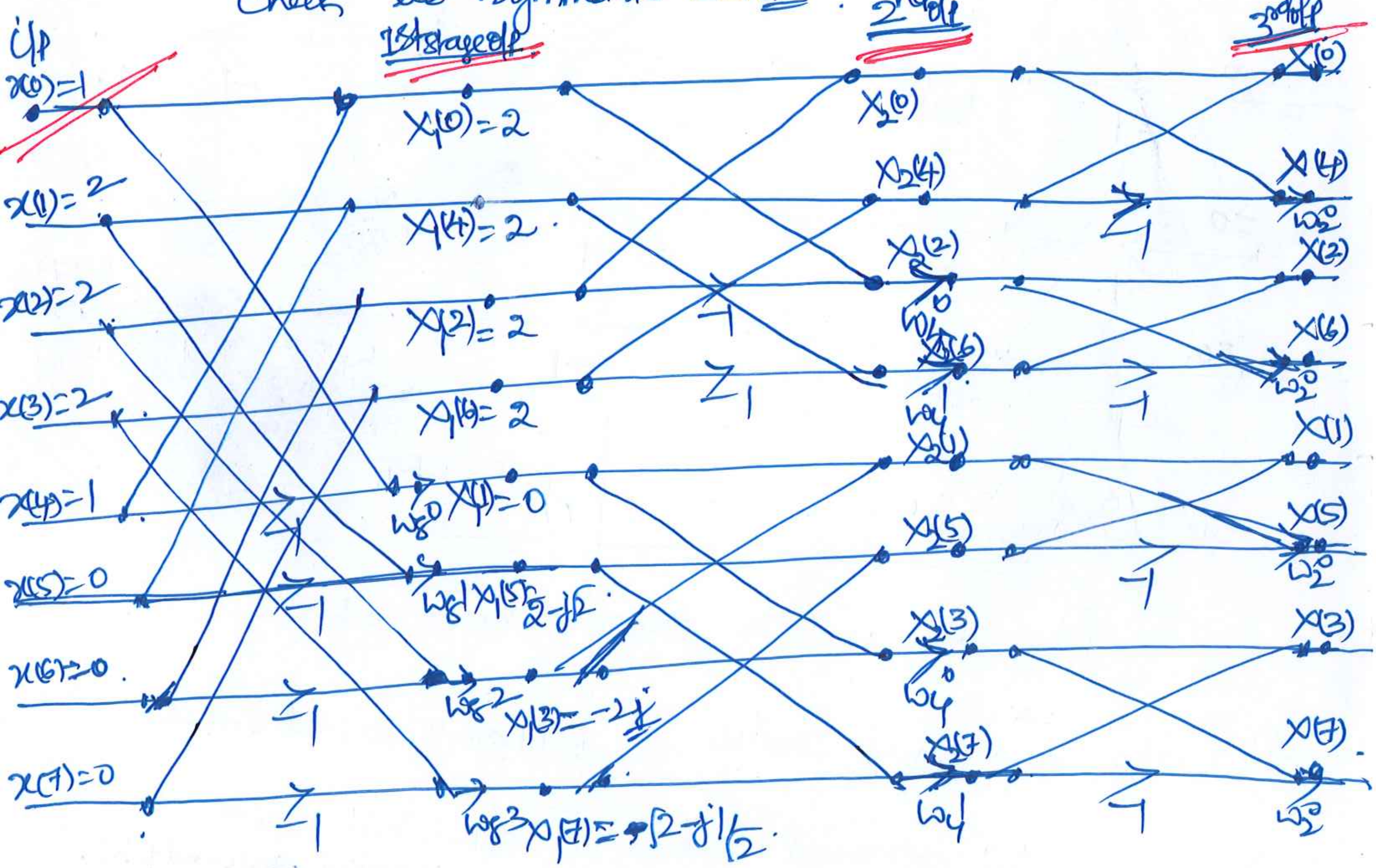
8M 10M

1) compute 8 pt DFT of a real sequence

$x(n) = \{1, 2, 2, 2, 1, 0, 0, 0\}$

Using Decimation in Freq FFT algorithm.

Check symmetric condn.



O/p of Ist stage

$x_1(0) = 1+1 = 2 = e(0)$

$x_1(4) = (2+0) = 2 = e(1)$

$x_1(2) = 2+0 = 2 = e_2$

$x_1(6) = 2+0 = 2 = e(3)$

$x_1(1) = (1-1) w_8^0 = 0.$

$x_1(5) = (2-0) w_8^1 = (2-0)(1/\sqrt{2} - j 1/\sqrt{2}) = \sqrt{2} - j\sqrt{2}.$

$x_1(3) = (2-0) w_8^2 = (2-0)(-j) = -2j.$

$x_1(7) = (2-0) w_8^3$

$x_1(7) = (2-0)(-1/\sqrt{2} - j 1/\sqrt{2}) = -\sqrt{2} - j\sqrt{2}.$

2nd stage o/p.

$X_2(0) = 2+2 = 4$

$X_2(4) = 2+2 = 4$

$X_2(2) = (2-2) w_4^0 = 0$

$X_2(6) = (2-2) w_4^1 = 0$

$X_2(1) = 0 + (-j2) = -j2.$

$X_2(5) = \sqrt{2}(1-j) + \sqrt{2}(1+j) = -2j\sqrt{2}.$

$X_2(3) = (0+j2) w_4^0 = j2$

$X_2(7) = [\sqrt{2}(1-j) - \sqrt{2}(1+j)] w_8^2$

$X_2(7) = 2\sqrt{2}(-j).$

$X(0) = 4+4 = 8$

$X(4) = (4-4)\, W_8^0 = 0.$

$X(2) = (0+0) = 0.$

$X(6) = (0-0)\, W_8^0 = 0.$

$X(1) = (-j2 - j2\sqrt{2}) = -j2\,(1+\sqrt{2})$

$X(5) = (-j2 + j2\sqrt{2})\, W_8^0 = -j2\,(1-\sqrt{2})$

$X(3) = (j2 - j2\sqrt{2}) = j2\,(1-\sqrt{2})$

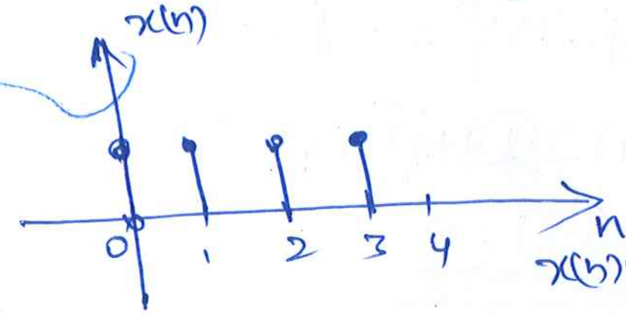
$X(7) = (j2 + j2\sqrt{2})\, W_8^0 = j2\,(1+\sqrt{2}).$

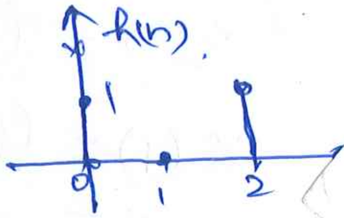
$$\boxed{X(k) = \{\, 8,\ -j2(1+\sqrt{2}),\ 0,\ j2(1-\sqrt{2}),\ 0,\ -j2(1-\sqrt{2}),\ 0,\ j2(1+\sqrt{2}) \,\}.}$$

(2) Repeat the problem for $x(n) = \{1, 1, 1, 1, 0, 0, 0, 0\}$.

Answers: $X(k) = \{\, 4,\ 1-j2.414,\ 0,\ 1-j0.414,\ 0,\ 1+j0.414,\ 0,\ 1+j2.414 \,\}.$

(3) Find the 4 pt CC of $x(n)$ & $h(n)$ shown using Radix-2 DIF-FFT algorithm.





$x(n) = \{1, 1, 1, 1\}$ $h(n) = \{1, \ldots$

$\therefore \sum_{k=0}^{N-1} \left[e^{j2\pi \frac{(m-n-l)}{N}} \right]^k = \begin{cases} N. & \text{when } m-n-l \text{ is multiple of } N. \\ 0. & \text{otherwise}. \end{cases}$ 56.

$$\therefore x_3(m) = \frac{1}{\cancel{N}} \sum_{n=0}^{N-1} x_1(n) \sum_{l=0}^{N-1} x_2(l) [\cancel{N}]$$

$$\therefore x_3(m) = \sum_{n=0}^{N-1} x_1(n) \sum_{l=0}^{N-1} x_2(l) \rightarrow (2).$$

* let $m-n-l = pN$ where p is an integer can be +ve or -ve.

$m-n-l = -pN$.

$l = m-n+pN$.

Sub this into eq (2). we get

$$x_3(m) = \sum_{n=0}^{N-1} x_1(n)\, x_2(m-n+pN)$$

[summation of 'l' is dropped as no 'l' terms in the above eqn.]

* since $x_2(m-n+pN)$ is a periodic sequence with period 'N' we can write this as.

i.e $x_2(m-n+pN) = x_2((m-n))_N$ [$x_2(m)$ is circularly shifted by 'N' samples]

Hence the proof.

$$\boxed{x_3(m) = \sum_{n=0}^{N-1} x_1(n)\, x_2((m-n))_N} \equiv \text{RHS} \quad x_1(n) \,(N)\, x_2(n) \equiv \text{LHS}$$

56

Case 1

when $m-n-l = N, 2N, 3N \ldots$ multiple of 'N'

then we get $a = e^{\frac{j2\pi(m-n-l)}{N}} = 1.$

$$e^{\frac{j2\pi[1N, 2N, 3N \ldots nN]}{N}} = e^{j2\pi n} = 1 \quad \text{where } n = 0,1,2,3 \ldots n$$

$$\therefore \sum_{k=0}^{N-1} a^k = \sum_{k=0}^{N-1} \left[e^{\frac{j2\pi(m-n-l)}{N}}\right]^k$$

$$= \sum_{k=0}^{N-1} (1)^k = N.$$

Case - 2

i.e $a \neq 1$, when $(m-n-l)$ is not multiple of 'N'

$$\text{i.e} \quad \sum_{k=0}^{N-1} a^k = \frac{1 - a^N}{1 - a} = \frac{1 - \left[e^{\frac{j2\pi(m-n-l)}{N}}\right]^N}{\left[1 - e^{\frac{j2\pi(m-n-l)}{N}}\right]}$$

$$\sum_{k=0}^{N-1} a^k = \frac{1 - e^{j2\pi(m-n-l)}}{1 - e^{\frac{j2\pi(m-n-l)}{N}}} = \frac{1 - 1}{1 - e^{\frac{j2\pi(m-n-l)}{N}}}$$

$$\therefore \sum_{k=0}^{N-1} a^k = \frac{0}{1 - e^{\frac{j2\pi(m-n-l)}{N}}} = 0$$

Problems

** 6M.

1) Find cc of 2 sequences..

$x_1(n) = \{2, 1, 2, 1\}$

$x_2(n) = \{1, 2, 3, 4\}$.

Soln

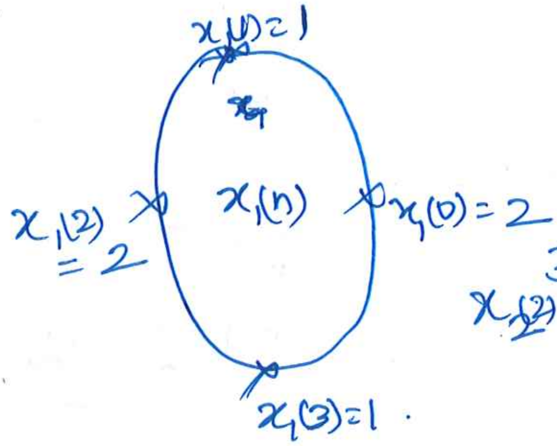
$x_3(m) = x_1(n) \, (N) \, x_2(n) = x_1(n) \, (4) \, x_2(n)$

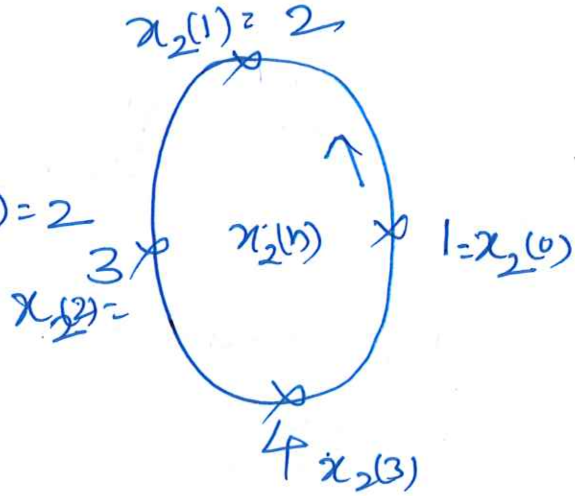
$$x_3(m) = \sum_{n=0}^{3} x_1(n) \, x_2((m-n))_4 .$$

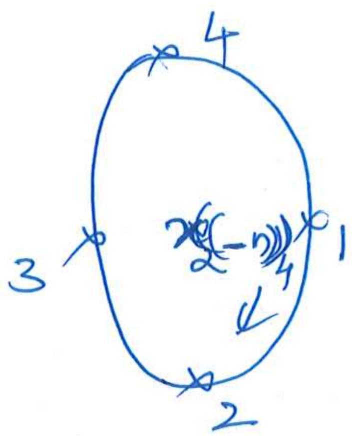
$m = 0, 1, 2 \, \& \, 3$.

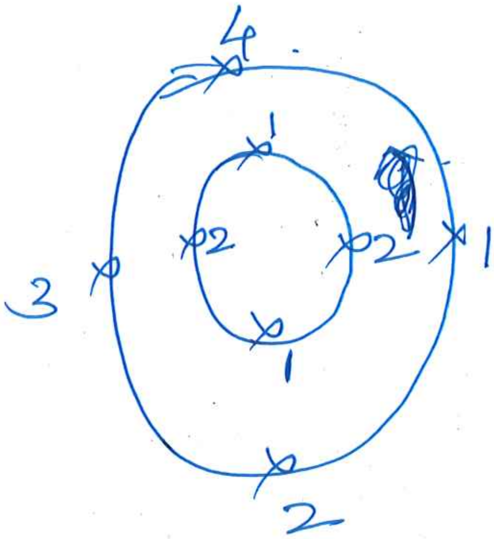
$m = 0$.

$$x_3(0) = \sum_{n=0}^{3} x_1(n) \, x_2((-n))_4$$









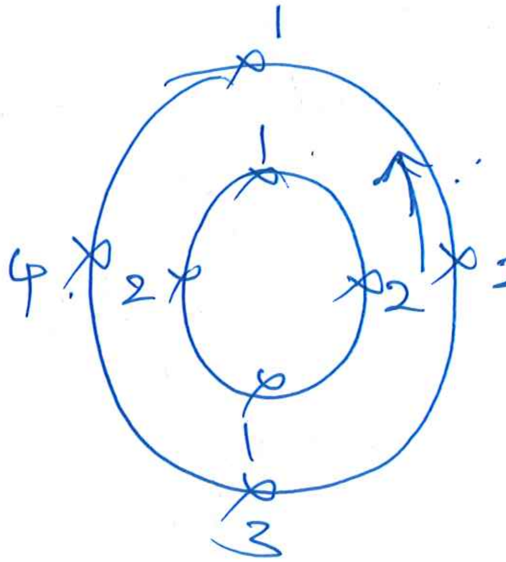
$= 2\times 1 + 1\times 4 + 3\times 2 + 2\times 1$

$= 14.$

$\therefore \; x_3(m) = \{14, 16, 14, 16\}$
$\underset{m=0}{\uparrow}$

$m=1$

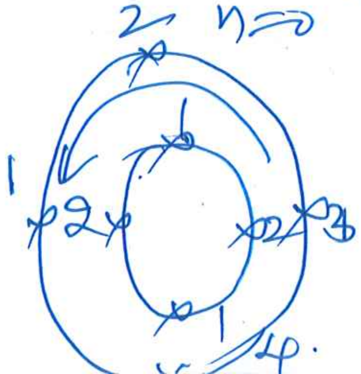
$x_3(1) = \sum_{n=0}^{3} x_1(n)\, x_2((1-n))_4.$



$= 2\times 2 + 1\times 1 + 2\times 4 + 1\times 3 = \underline{16}$

$\therefore \; m=2$

$x_3(2) = \sum_{n=0}^{3} x_1(n)\, x_2((2-m))_4$

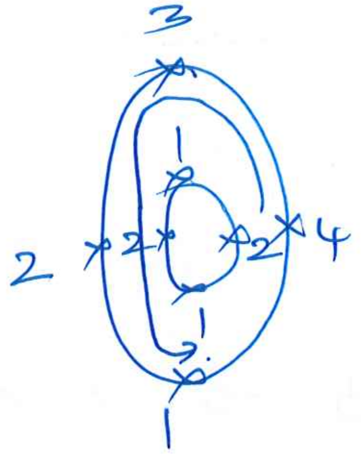


$= 3\times 2 + 2\times 1 + 2\times 1 + 1\times 4$

$= 14.$

$\underline{m=3}$

$$x_3(3) = \sum_{n=0}^{3} x_1(n)\, x_2((3-m))_4$$



$$= 2\times4 + 1\times3 + 2\times2 + 1\times1$$

$$= 8 + 3 + 4 + 1 = \underline{16}$$

$$\boxed{\therefore\; x_3(m) = \{14, 16, 14, 16\}}$$

(R) 2). compute CC for the following sequences.

$x_1(n) = \{1, 2, 1, 2\}$

$x_2(n) = \{4, 3, 2, 1\}$.

3) compute CC of the following sequences & compare the results with LC.

$x(n) = \{1, 1, 1, 1, -1, -1, -1, -1\}$

$h(n) = \{0, 1, 2, 3, 4, 3, 2, 1\}$

Soln   LC by Multiplication method → Method ①.

$x(n) =$ 1 1 1 1 −1 −1 −1 −1.

$h(n) =$ 0 1 2 3 4 3 2 1 *

							1	1	1	1	−1	−1	−1	−1
						2	2	2	2	−2	−2	−2	−2	x
					3	3	3	+3	−3	−3	−3	−3	x	x
				4	4	4	4	−4	−4	−4	−4	x	x	x
			3	3	3	3	−3	−3	−3	−3	x	x	x	x
		2	2	2	2	−2	−2	−2	−2	x	x	x	x	x
	1	1	1	1	−1	−1	−1	−1	x	x	x	x	x	x
0	0	0	0	0	0	0	0	x	x	x	x	x	x	x

{0, 1, 3, 6, 10, 11, 9, 4, −4, −9, −11, −10, −6, −3, −1.}

$x(n) * h(n) = \underline{N_1 + N_2 - 1} = 8 + 8 - 1 = 15$ length sequences

Cross check by Tabular column method

h(n) \ x(n)	1	1	1	1	−1	−1	−1	−1
0	0	0	0	0	0	0	0	0
1	1	1	1	1	−1	−1	−1	−1
2	2	2	2	2	−2	−2	−2	−2
3	3	3	3	3	−3	−3	−3	−3
4	4	4	4	4	−4	−4	−4	−4
3	3	3	3	3	−3	−3	−3	−3
2	2	2	2	2	−2	−2	−2	−2
1	1	1	1	1	−1	−1	−1	−1

$x(n) * h(n) = \{ 0, 1, 3, 6, 10, 11, 9,$

$h(n)$ \ $x(n)$	1	1	1	1	−1	−1	−1	−1
0	0	0	0	0	0	0	0	0
1	1	1	1	1	−1	−1	−1	−1
2	2	2	2	2	−2	−2	−2	−2
3	3	3	3	3	−3	−3	−3	−3
4	4	4	4	4	−4	−4	−4	−4
3	3	3	3	3	−3	−3	−3	−3
2	2	2	2	2	−2	−2	−2	−2
1	1	1	1	1	−1	−1	−1	−1

$x(n)*h(n) = \{0, 1, 3, 6, 10, 11, 9, 4, -4, -9, -11, -10, -6, -3, -1\}$

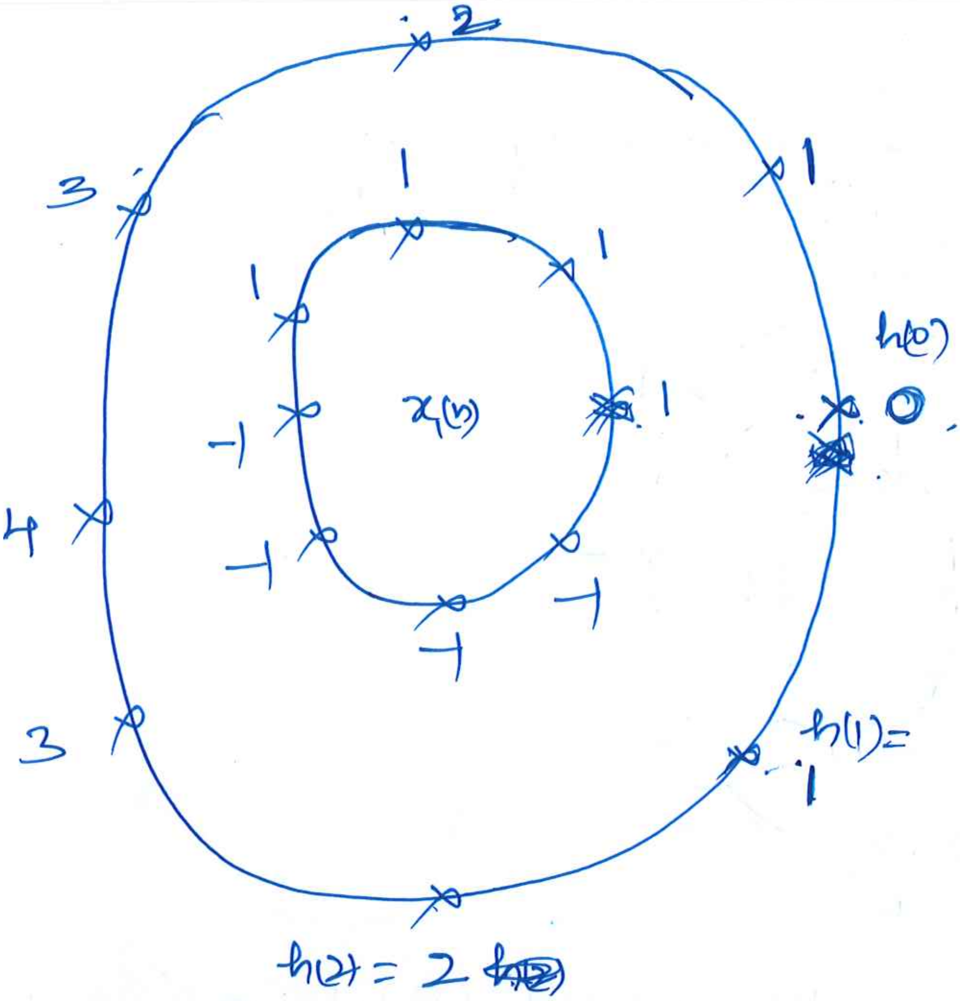
$y(n) = 8+8-1 = \underline{\underline{15}}$

C C Method

$$y(m) = x(n) \; \text{Ⓝ} \; h(n) = \sum_{n=0}^{7} x(n)\, h((m-n))_8$$

$m=0$

$$y(0) = \sum_{n=0}^{7} x(n)\, h((-n))_8$$



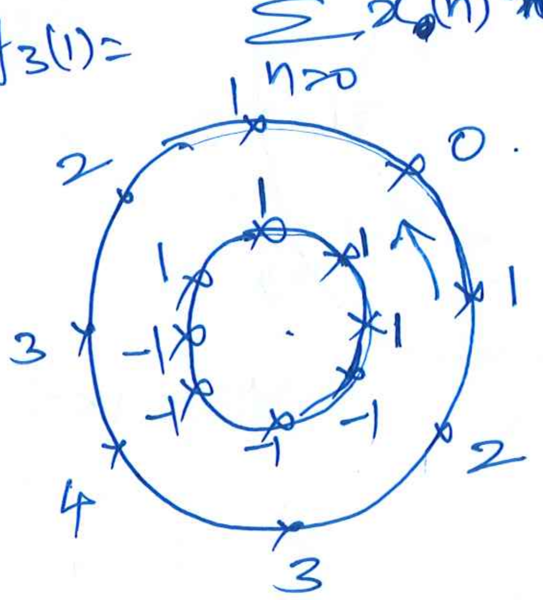
$$y(0) = 1\times0 + \cancel{1\times1} + \cancel{1\times2} + \cancel{1\times3} + (-4) \cancel{-3}$$

$$\cancel{-2}\ \cancel{-1} = \underline{\underline{-4}}.$$

$$y_3(m) = \{-4, -8, -8, -4, 4, 8, 8, 4\}$$

$m=1$

$$y_3(1) = \sum_{n=0}^{7} x_1(n)\, h_2((1-m))_8.$$

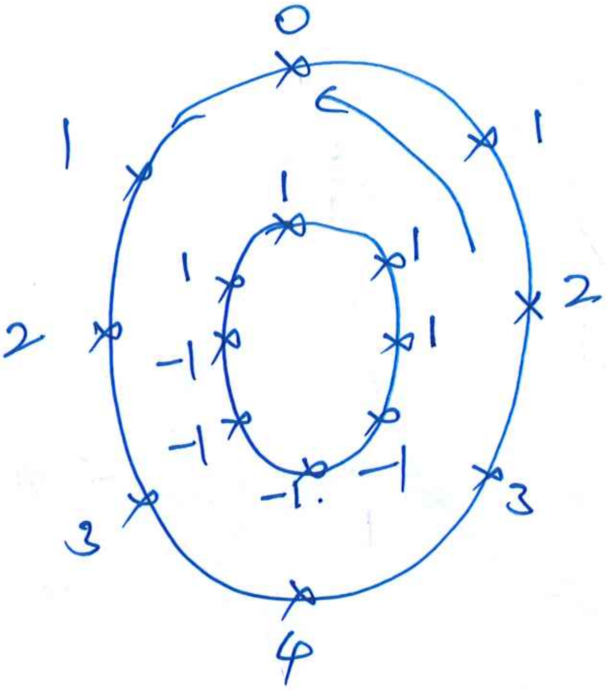


$$= 1\times1 + \times1\times0 + 1\times1 + \cancel{1\times2}$$

$$+3 - 4, -3, \cancel{-2} = \underline{\underline{-8}}$$

$m=2$

$$y(2) = \sum_{n=0}^{7} x_1(n)\, h((2-m))_8 .$$

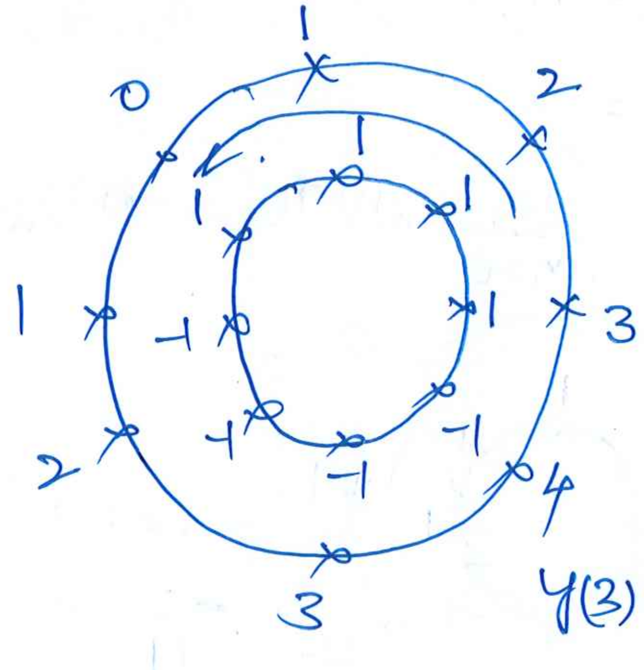


$\therefore y(2) = 1\times 2 + \cancel{\times 1\times 1} + \cancel{0} + \cancel{1\times 1} + \cancel{2} - 3 - 4 - \cancel{3}.$

$y(2) = -8$

$m=3$

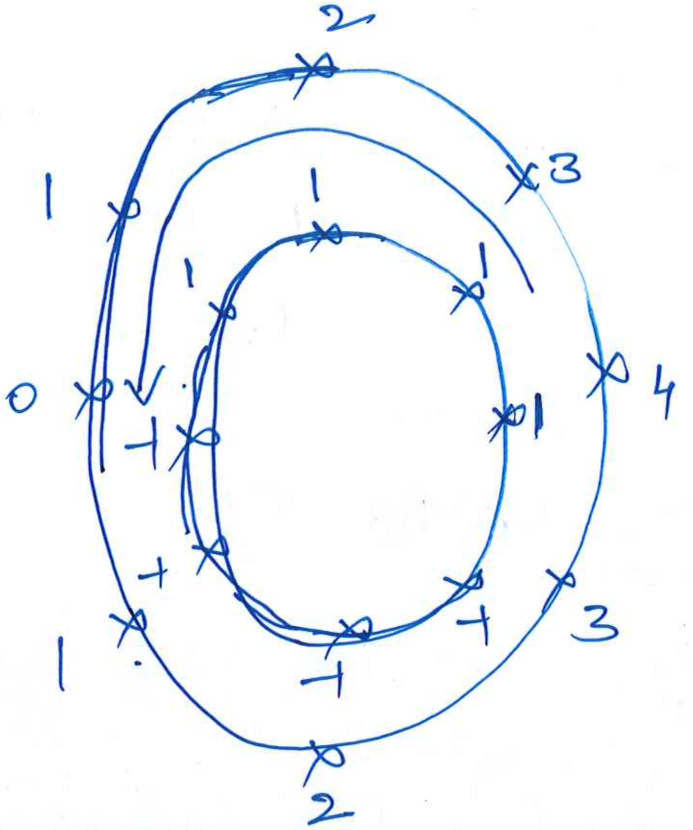
$$y(3) = \sum_{n=0}^{7} x(n)\, h((3-m))_8$$



$y(3) = \cancel{1\times 3} + 2 + \cancel{1} + 0 - 1$
$- 2 - 3 - \cancel{4}$
$= -4$

$m = 4.$

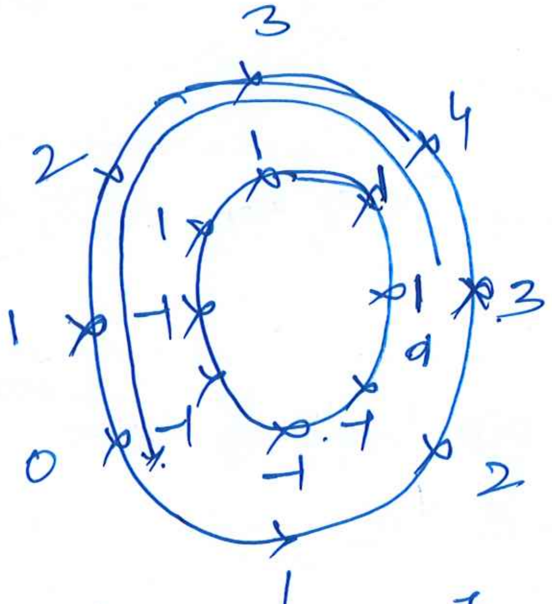
$$y(4) = \sum_{n=0}^{7} x(n)\, h((4-n))_8 .$$



$$= 4 + 3 + 2 + 1 + 0 - 1 - 2 - 3 = \underline{4}$$

$m = 5$

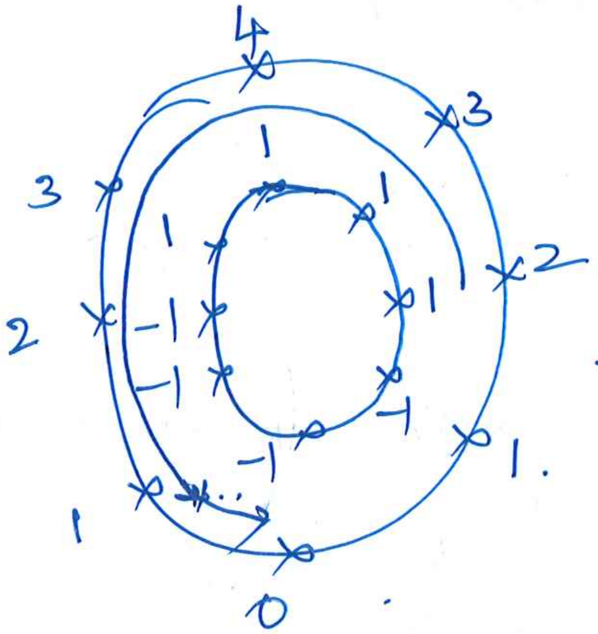
$$y(5) = \sum_{n=0}^{7} x(n)\, h((5-n))_8 .$$



$$= 3 + 4 + 3 + 2 - 1 - 0 - 1 - 2 = \underline{\underline{8}}$$

$m = 6$

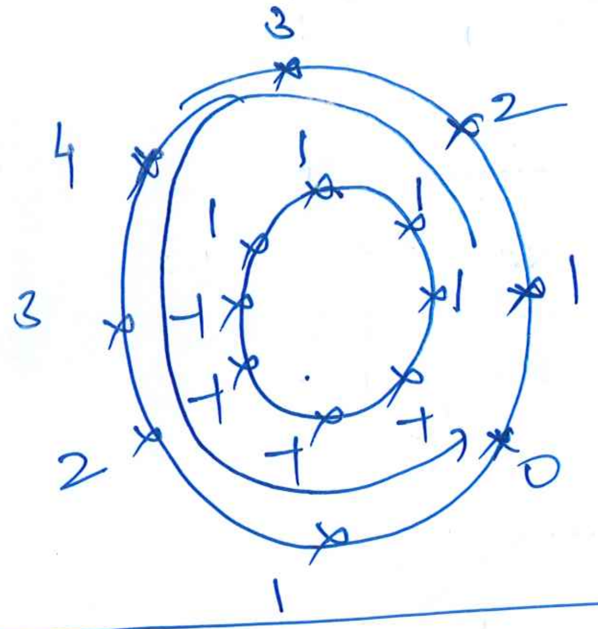
$$y(6) = \sum_{n=0}^{7} x(n)\, h((6-n))_8$$



$= 2 + 3 + 4 + 3 - 2 - 1 - 0 - 1 = \underline{8}$

$m = 7$

$$y(7) = \sum_{n=0}^{7} x(n)\, h((7-n))_8$$



$= 1 + 2 + 3 + 4 - 3 - 2 - 1 - 0 = 0$

$\underline{\underline{4}}$

$$\boxed{y(n) = \{-4, -8, -8, -4, 4, 8, 8, 4\}}$$

(*) for case of CC the o/p length is same as that of i/p of either x(n) or h(n), whereas in LC the length of o/p sequence is equal to length of x & length of h -1. ie $\underline{\underline{N_1 + N_2 - 1 = \text{length of LC}}}$.

3) LC denoted by → (*)

* . The o/p sequence is entirely different in both the cases. i) zero padding required in CC & not needed in LC.

H.W. R. i) compute CC for $x(n)=\{1, 0.5, 1, 0.5, 1, 0.5, 1, 0.5\}$

$h(n)=\{0, 1, 2, 3\}, 0, 0, 0.$

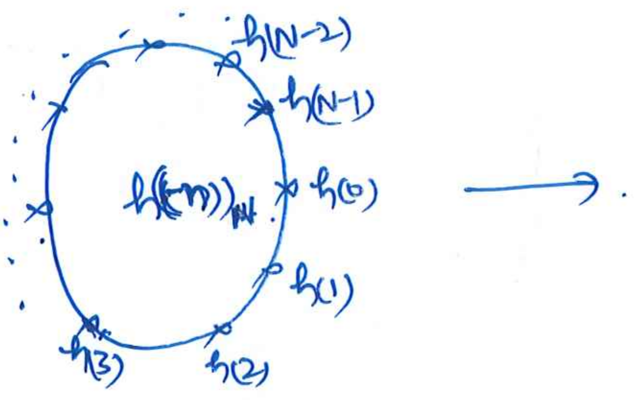
soln LC.
$x(n) * h(n) = \{0, 1, 2.5, 5, 4, 5, 4, 5, 4, 4.1.5\}$

$x(n) \, (N) \, h(n) = \{4.5, 4.5, 4.5, 4.5 \ldots\} \quad N=8$

$\boxed{\text{Technique 2}}$ circular convolution by Matrix method

$y(n) = h(n) \, (N) \, x(n).$

$$\begin{bmatrix} y(0) \\ y(1) \\ y(2) \\ \vdots \\ y(N-2) \\ y(N-1) \end{bmatrix} = \begin{bmatrix} h(0) & h(N-1) & h(N-2) & \cdots & h(2) & h(1) \\ h(1) & h(0) & h(N-1) & h(N-2) \cdots & h(3) & h(2) \\ h(2) & h(1) & h(0) & \cdots & h(4) & h(3) \\ h(3) & \cdots & & & \cdots & h(4) \\ \vdots & & & & & \\ h(N-1) & h(N-2) & \cdots & & \cdots & h(0) \end{bmatrix} \begin{bmatrix} x(0) \\ x(1) \\ x(2) \\ \vdots \\ x(N-1) \end{bmatrix}$$



given $h(-n) = \{\underline{0}, 1, 2, 3, 0, 0, \underline{0}, \underline{0}\}$.

$x(n) = \{1, 0.5, 1, 0.5, 1, 0.5, 1, 0.5, 1, 0.5\}$.

$$\begin{bmatrix} y(0) \\ y(1) \\ y(2) \\ y(3) \\ y(4) \\ y(5) \\ y(6) \\ y(7) \end{bmatrix} = \begin{array}{cccccccc} h(0) & h(N-1) & h(N-2) & & & & & \\ \end{array} \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 3 & 2 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 & 3 & 2 \\ 2 & 1 & 0 & 0 & 0 & 0 & 0 & 3 \\ 3 & 2 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 3 & 2 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 3 & 2 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 3 & 2 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 3 & 2 & 1 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 0.5 \\ 1 \\ 0.5 \\ 1 \\ 0.5 \\ 1 \\ 0.5 \end{bmatrix}$$

$$\begin{bmatrix} y(0) \\ y(1) \\ y(2) \\ y(3) \\ y(4) \\ y(5) \\ y(6) \\ y(7) \end{bmatrix} = \begin{bmatrix} 0+0+0+0+0+1.5+2+0.5 \\ \\ \\ \\ \\ \\ \\ \\ \end{bmatrix} = \begin{bmatrix} 4 \\ 5 \\ 4 \\ 5 \\ 4 \\ 5 \\ 4 \\ 5 \end{bmatrix}$$

R ** 1) Find CC by matrix method for the following sequences.

$x(n) = \{0, 1, 2, 3, 4, 3, 2, 1\}$

$h(n) = \{1, 2, 1, 2, 1, 2, 1, 2\}$.

R 2) Find CC for the following by matrix method

$x(n) = \{1, 1, 1, 1, -1, -1, -1, -1\}$

$h(n) = \{0, 1, 2, 3, 4, 3, 2, 1\}$.

** **Method-3.** circular convolution by DFT & IDFT method

$x(n) = \{2, 1, 2, 1\}$

$h(n) = \{1, 2, 3, 4\}$

Soln Wkt CC of $x_1(n) \, (N) \, x_2(n) = X_1(K) \cdot X_2(K) = X_3(K)$

$x_1(n) = \{2, 1, 2, 1\}$ Then for $X_3(K)$ using IDFT find out $x_3(n) = \frac{1}{4}[W_4]^* X_4$.

$X_4 = [W_4] x_4$

$$\begin{bmatrix} X(0) \\ X(1) \\ X(2) \\ X(3) \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix} \begin{bmatrix} 2 \\ 1 \\ 2 \\ 1 \end{bmatrix} = \begin{bmatrix} 6 \\ 0 \\ 2 \\ 0 \end{bmatrix}$$

lly. $X_2(k) = ?$

$$\begin{bmatrix} X_2(0) \\ X_2(1) \\ X_2(2) \\ X_2(3) \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix} = \begin{bmatrix} 10 \\ -2+2j \\ -2 \\ -2-2j \end{bmatrix}$$

$\therefore X_3(k) = X_1(k) \cdot X_2(k) \rightarrow$ By DFT.

$$\begin{bmatrix} X_3(0) \\ X_3(1) \\ X_3(2) \\ X_3(3) \end{bmatrix} = \begin{bmatrix} 60 \\ 0 \\ -4 \\ 0 \end{bmatrix}$$

By IDFT:

$x_3(n)$ from IDFT of $X_3(k)$.

$$x_3(n) = \frac{1}{4} \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix} \begin{bmatrix} 60 \\ 0 \\ -4 \\ 0 \end{bmatrix} = \frac{1}{4} \begin{bmatrix} 56 \\ 64 \\ 56 \\ 64 \end{bmatrix} = \begin{bmatrix} 14 \\ 16 \\ 14 \\ 16 \end{bmatrix}$$

$\therefore x_3(n) = \{14, 16, 14, 16\}$

Note: the method out of 3 is very efficient as DFT & IDFT can be computed very fast & efficient by using FFT algorithm.

H/W) 2) compute CC by ~~...~~ using DFT & IDFT method

$$x_1(n) = \{2, 3, 1, 1\}$$

$$x_2(n) = \{1, 3, 5, 3\}.$$

by DFT $\longrightarrow$ $\{7, 1-j2, -1, 1+j2\}$

$X_2(k) = \{12, -4, 0, -4\}$

$X_3(k) = \{84, -4+j8, 0, -4-j8\}$

By IDFT $\longrightarrow$ $x_3(n) = \{19, 17, 23, 25\}$

---

3) let $x(n)$ be the sequence

$$x(n) = \delta(n) + 2\delta(n-2) + \delta(n-3).$$

a) Find 4 pt DFT of $x(n)$

b) If $y(n)$ is the 4 pt circular convolution of $x(n)$ with itself.

Find $y(n)$ & 4pt DFT of $Y(K)$.

Soln $x(n) = \delta(n) + 2\delta(n-2) + \delta(n-3).$

$x(n) = \{1, 0, 2, 1\}.$

$$X_4 = [W_4]\, x_4$$

$$\begin{bmatrix} X(0) \\ X(1) \\ X(2) \\ X(3) \end{bmatrix} = \begin{bmatrix} 4 \\ -1+j \\ 2 \\ -1-j \end{bmatrix}$$

<u>given</u>. $y(n) = x(n) \; (N) \; x(n) \xleftrightarrow[N]{DFT} X(K) \cdot X(K) = Y(K)$

$$= \begin{bmatrix} 4 \\ -1+j \\ 2 \\ -1-j \end{bmatrix} \times \begin{bmatrix} 4 \\ -1+j \\ 2 \\ -1-j \end{bmatrix} = \begin{bmatrix} 16 \\ -j2 \\ 4 \\ j2 \end{bmatrix}$$

IDFT of Y(K) given.

$$y(n) = \frac{1}{4} [W_4] Y(K).$$

$$= 1/4 [W_4] \begin{bmatrix} 16 \\ -j2 \\ 4 \\ j2 \end{bmatrix}$$

$$\boxed{y(n) = [5, 4, 5, 2]}$$

HW (5) Let $x(n)$ be the sequence.

$x(n) = 2\delta(n) + \delta(n-1) + \delta(n-3)$. Find the sequence

$y(n) = x(n) \; (5) \; x(n)$ i.e 5 pt-CC of $x(n)$ with itself.

<u>soln</u>. $x(n) = \{2, 1, 0, 1\}$

$x(n) = \{2, 1, 0, 1, 0\}$ [∵ 5 pt- sequence]

$x(n) = \{2, 1, 0, 1, 0\}$.

$$y(n)=\begin{bmatrix}2 & 0 & 0 & 0 & 0\\ 1 & 2 & 0 & 1 & 0\\ 0 & 1 & 2 & 0 & 1\\ 1 & 0 & 1 & 2 & 0\\ 0 & 1 & 0 & 1 & 2\end{bmatrix}\begin{bmatrix}2\\1\\0\\1\\0\end{bmatrix}=\begin{bmatrix}4\\5\\1\\4\\2\end{bmatrix}$$

$y(n)=\{4, 5, 1, 4, 2\}$..

6) Determine C.C using DFT & IDFT method for the following sequences.

(HW).

$x(n)=\{1, 2, -1, -1\}$

$h(n)=\{-3, -2, -1, 0\}$ & verify the same using matrix method.

soln $X(K)=\{ \ldots \}$

$H(K)=\{ \ldots \}$

$Y(K)=\{ \quad \} \Rightarrow \quad y(n)=\{0, -7, 2, 3\}$.

By matrix method Verification.

$x(n)=\{1, 2, -1, -1\}$

$h(n)=\{-3, -2, -1, 0\}$

$$y(n)=\begin{bmatrix}-3 & 0 & -1 & -2\\ -2 & -3 & 0 & -1\\ -1 & -2 & -3 & 0\\ 0 & -1 & -2 & -3\end{bmatrix}\begin{bmatrix}1\\2\\-1\\-1\end{bmatrix}=\begin{bmatrix}0\\-7\\2\\3\end{bmatrix}$$

Additional DFT properties.

1) circular correlation properties.

i.e. circular cross correlation property – Extended convoln property

2) circular Auto correlation property

3) parseval's Theorem.

circular cross correlation property.

Statement: It states that if

$$x(n) \xleftrightarrow[N]{DFT} X(K) \text{ \& }$$

$$y(n) \xleftrightarrow[N]{DFT} Y(K) \text{ then}$$

$$r_{xy}(l) \xleftrightarrow[N]{DFT} R_{xy}(k) = X(K) \cdot Y^*(K)$$

Where $r_{xy}(l)$ is called circular cross correlation in time domain & is given by.

$$r_{xy}(l) = \sum_{n=0}^{N-1} x(n)\, y^*((n-l))_N .$$

[∵ Extended convolution Expression i.e $y(m) = \sum_{n=0}^{N-1} x(n)\, y((m-n))_N$]

i.e. multiplication of DFT of one sequence & conjugate DFT of another sequence is eqt to crosscorrelation.

of 2 sequences in time domain.

proof: Circular ~~Auto~~ cross correlation is given by.

$$* \quad r_{xy}(l) = \sum_{n=0}^{N-1} x(n)\, y^*((n-l))_N \xrightarrow{(std)} ①$$

Sub $y^*((n-l))_N = y^*((-[l-n]))_N$ in the above eqn

$$r_{xy}(l) = \sum_{n=0}^{N-1} x(n)\, y^*((-[l-n]))_N \longrightarrow ②$$

* compare this equation with normal convolution, we get.

$$r_{xy}(l) = x(l) \; (N) \; y^*(-l)$$

* By using circular convolution property we

$$DFT\{r_{xy}(l)\} = DFT\{x(l)\} \cdot DFT\{y^*(-l)\}$$

[∵ Multiplication of 2 DFT RHS = $x(l) \; (N) \; y^*(-l)$ Lets]

$$\therefore R_{xy}(K) = X(K) \cdot \underbrace{DFT\{y^*(-l)\}}^{Y^*(K)} \longrightarrow ③$$

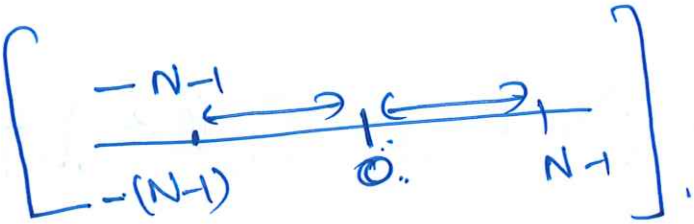
$$\therefore DFT\{y^*(-l)\} = \sum_{l=0}^{N-1} y^*(-l)\, e^{\frac{-j2\pi Kl}{N}}$$

* Put $n = -l$ in the above limit.

[when $l=0$, $n=0$; when $l=N-1$, $n=-(N-1)$; $\sum_{n=0}^{-(N-1)}$]

$$\therefore \; DFT\{y^*(-l)\} = \sum_{n=0}^{-(N-1)} y^*(n)\, e^{\frac{-j2\pi k(-n)}{N}}$$ (76)

$$DFT\{y^*(-l)\} = \sum_{n=0}^{-(N-1)} y^*(n)\, e^{\frac{j2\pi kn}{N}}$$



$$DFT\{y^*(-l)\} = \sum_{n=0}^{N-1} y^*(n)\, e^{\frac{j2\pi kn}{N}}$$

$\left[\because \; 0 \text{ to } N-1 \cong -(N-1) \text{ to } 0 \right]$

$$DFT\{y^*(-l)\} = \sum_{n=0}^{N-1} \left[y(n)\, e^{\frac{-j2\pi kn}{N}} \right]^*$$

$$DFT\{y^*(-l)\} = \underline{Y^*(K)}$$

$$\therefore \; \boxed{R_{xy}(K) = X(K) \cdot Y^*(K)}$$ Hence the proof.

HW. <u>Circular Auto correlation property.</u>

<u>Statement</u>: It states that $x(n) \xleftrightarrow[N]{DFT} X(K)$ then PT

$$r_{xx}(l) \xleftrightarrow[N]{DFT} \underline{R_{xx}(K) = X(K) \cdot X^*(K)}$$

where $r_{xx}(l)$ is the circular Autocorrelation eq given by (76)

of 2 sequences in time domain.

proof: Circular ~~Auto~~ cross correlation is given by

$$* \quad r_{xy}(l) = \sum_{n=0}^{N-1} x(n)\, y^*((n-l))_N \xrightarrow{(std)} ①$$

Sub $y^*((n-l))_N = y^*((-[l-n]))_N$ in the above eqn

$$r_{xy}(l) = \sum_{n=0}^{N-1} x(n)\, y^*((-[l-n]))_N \longrightarrow ②$$

* compare this equation with normal convolution, we get.

$$r_{xy}(l) = x(l) \, Ⓝ \, y^*(-l)$$

* By using circular convolution property we

$$DFT\{r_{xy}(l)\} = DFT\{x(l)\} \cdot DFT\{y^*(-l)\}$$

[$\because$ Multiplication of 2 DFT RHS = $x(l)$ Ⓝ $y^*(-l)$]

$$\therefore R_{xy}(K) = X(K) \cdot \underbrace{DFT\{y^*(-l)\}}_{Y^*(k)} \longrightarrow ③$$

$$\therefore DFT\{y^*(-l)\} = \sum_{l=0}^{N-1} y^*(-l)\, e^{\frac{-j2\pi kl}{N}}$$

* Put $n = -l$ in the above limit.

[when $l = 0$, $n = 0$; when $l = N-1$, $n = -(N-1)$; $\sum_{n=0}^{-(N-1)}$]

$$\therefore \ DFT\{y^*(-l)\} = \sum_{n=0}^{-(N-1)} y^*(n)\, e^{\frac{-j2\pi k(-n)}{N}}$$

$$DFT\{y^*(-l)\} = \sum_{n=0}^{-(N-1)} y^*(n)\, e^{\frac{j2\pi kn}{N}}$$

$$DFT\{y^*(-l)\} = \sum_{n=0}^{N-1} y^*(n)\, e^{\frac{j2\pi kn}{N}} \qquad \left[\because 0 \text{ to } N-1 \cong -(N-1) \text{ to } 0\right]$$

$$DFT\{y^*(-l)\} = \left[\sum_{n=0}^{N-1} y(n)\, e^{\frac{-j2\pi kn}{N}}\right]^*$$

$$DFT\{y^*(-l)\} = Y^*(K)$$

$$\therefore \boxed{R_{xy}(K) = X(K)\,.\,Y^*(K)}$$ Hence the proof.

HW. **Circular Auto correlation property.**

Statement: It states that $x(n) \xleftrightarrow[N]{DFT} X(K)$ then PT

$$r_{xx}(l) \xleftrightarrow[N]{DFT} R_{xx}(K) = X(K).X^*(K)$$

where $r_{xx}(l)$ is the circular Autocorrelation of sequence

of 2 sequences in time domain.

proof. Circular ~~Auto~~ cross correlation is given by.

$$* \quad r_{xy}(l) = \sum_{n=0}^{N-1} x(n)\, y^*((n-l))_N \xrightarrow{(std)} ①$$

Sub $y^*((n-l))_N = y^*((-[l-n]))_N$ in the above eqn.

$$r_{xy}(l) = \sum_{n=0}^{N-1} x(n)\, y^*((-[l-n]))_N \longrightarrow ②$$

* compare this equation with normal convolution, we get.

$$r_{xy}(l) = x(l) \, ⓝ \, y^*(-l)$$

* By using circular convolution property we

$$DFT\{r_{xy}(l)\} = DFT\{x(l)\} . DFT\{y^*(-l)\}$$

[∴ Multiplication of 2 DFT RHS = LHS $x(l) \, ⓝ \, y^*(-l)$]

$$\therefore R_{xy}(K) = X(K) . \underbrace{DFT\{y^*(-l)\}}_{} \longrightarrow ③$$

($Y^*(K)$)

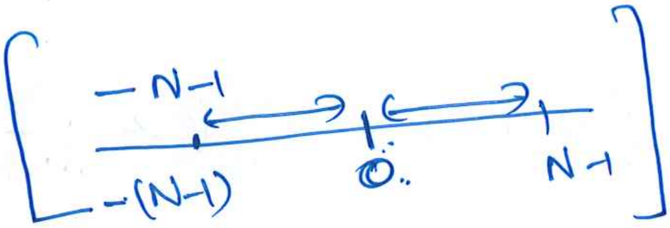
$$\therefore DFT\{y^*(-l)\} = \sum_{l=0}^{N-1} y^*(-l)\, e^{\frac{-j2\pi Kl}{N}}.$$

* Put $n = -l$ in the above limit.

[when $l=0$, $n=0$; when $l=N-1$, $n = -(N-1)$ ∴ $\sum_{n=0}^{-(N-1)}$]

$$\therefore \; DFT\{y^*(-l)\} = \sum_{n=0}^{-(N-1)} y^*(n)\, e^{\frac{-j2\pi k(-n)}{N}}$$

$$DFT\{y^*(-l)\} = \sum_{n=0}^{-(N-1)} y^*(n)\, e^{\frac{j2\pi kn}{N}}$$



$$DFT\{y^*(-l)\} = \sum_{n=0}^{N-1} y^*(n)\, e^{\frac{j2\pi kn}{N}}$$

$\left[\because 0 \text{ to } N-1 \cong -(N-1) \text{ to } 0\right]$

$$DFT\{y^*(-l)\} = \sum_{n=0}^{N-1} \left[y(n)\, e^{\frac{-j2\pi kn}{N}} \right]^*$$

$$DFT\{y^*(-l)\} = \underline{Y^*(K)}$$

$$\therefore \boxed{R_{xy}(k) = X(K) \cdot Y^*(K)}$$

Hence the proof.

HW. **Circular Auto correlation property.**

Statement: It states that $x(n) \xleftrightarrow[N]{DFT} X(K)$ then PT

$$\gamma_{xx}(l) \xleftrightarrow[N]{DFT} R_{xx}(K) = \underline{X(K) \cdot X^*(K)}$$

where $\gamma_{xx}(l)$ is the circular Autocorrelation eq given by 76

$$r_{xx}(l) = \sum_{n=0}^{N-1} x(n)\, x^{*}((n-l))_N .$$

* ie multiplication of DFT of one sequence & the conjugate DFT of the same sequence is eqt to circular ~~convolution~~ Auto correlation of these 2 sequence in time domain.

Proof → Same as above. sub. wherever $y(n) = x(n)$ in above.

Parseval.s theorem.

Statement. For a complex valued sequences $x(n)$ & $y(n)$

if $x(n) \xleftrightarrow[N]{DFT} X(k)$ &

$y(n) \xleftrightarrow[N]{DFT} Y(k)$. then PT

$$\sum_{n=0}^{N-1} x(n) y^{*}(n) = \frac{1}{N} \sum_{k=0}^{N-1} X(k) Y^{*}(k) \rightarrow (1)$$

When $y(n) = x(n)$ then

$$\sum_{n=0}^{N-1} |x(n)|^2 = \frac{1}{N} \sum_{k=0}^{N-1} |X(k)|^2 \rightarrow (2)$$

The above 2 equations gives relations for parseval's theorem. ie it gives the relations for energy of finite duration sequences in terms of its freq components.

Proof. circular cross correlation is given by

* $$r_{xy}(l) = \sum_{n=0}^{N-1} x(n)\, y^*((n-l))_N \longrightarrow \underline{std}.$$

For $l=0$ in the above eqn.

$$r_{xy}(0) = \sum_{n=0}^{N-1} x(n)\, y^*(n) \longrightarrow (1)$$

* From the defn of circular cross correlation property

$$DFT\{r_{xy}(l)\} = X(K)\cdot Y^*(K)$$

Take IDFT on both sides we get

$$r_{xy}(l) = \underline{\underline{IDFT\{X(K)\cdot Y^*(K)\}}}$$

By IDFT ~~we can write~~.

$$\therefore r_{xy}(l) = \frac{1}{N}\sum_{K=0}^{N-1}[X(K)\,Y^*(K)]\,(e^{\frac{j2\pi kl}{N}})$$

$$\left[\therefore IDFT\{X(K)\} = \frac{1}{N}\sum_{K=0}^{N-1} X(K)\, e^{\frac{j2\pi kl}{N}}\right]$$

sub $l=0$ in the above eqn

$$\therefore r_{xy}(0) = \frac{1}{N}\sum_{K=0}^{N-1}[X(K)\,Y^*(K)]\,(1) \longrightarrow (2)$$

Equating ① & ②, we get

$$\boxed{\sum_{n=0}^{N-1} x(n)\, y^*(n) = \frac{1}{N}\sum_{k=0}^{N-1} X(k)\cdot Y^*(k)} \longrightarrow ③$$

Hence the parseval's theorem is prooved.

Why Repeat parseval's theorem for circular autocorrelation:

* . std when $y(n) = x(n)$ then PT.

$$\sum_{n=0}^{N-1} (x(n))^2 = \frac{1}{N}\sum_{k=0}^{N-1} |X(k)|^2$$

proof

circular auto correlation is given by.

$$\gamma_{xx}(l) = \sum_{n=0}^{N-1} x(n)\, x^*((n-l))_N \rightarrow \text{std}$$

* Sub $l=0$. in the above eqn

$$\gamma_{xy}(0) = \sum_{n=0}^{N-1} x(n)\, x^*(n) \longrightarrow ①$$

* From the defn of circular autocorrelation prop.

$$DFT\{\gamma_{xx}(l)\} = X(K)\cdot X^*(K)$$

Take IDFT on bs.

$$\gamma_{xx}(l) = IDFT\{X(K)\cdot X^*(K)\}$$

By IDFT

$$\gamma_{xx}(l) = \frac{1}{N}\sum_{k=0}^{N-1} [X(k)\, X^*(k)]\, e^{\frac{j2\pi kl}{N}}$$

$$\left[\because \; IDFT\{X(k)\} = \frac{1}{N}\sum_{k=0}^{N-1} X(k)\, e^{\frac{j2\pi kl}{N}}\right]$$

sub $l=0$ in the above eqn, we get

$$r_{xy}(0) = \frac{1}{N}\sum_{k=0}^{N-1}\left[X(k)\cdot X^*(k)\right] \;(1) \longrightarrow (2)$$

$\therefore$ equating (1) & (2) we get

$$\sum_{n=0}^{N-1} x(n)\cdot x^*(n) = \frac{1}{N}\sum_{k=0}^{N-1}|X(k)|^2$$

$\because x(n) = x^*(n)$

i.e
$$\boxed{\sum_{n=0}^{N-1}|x(n)|^2 = \frac{1}{N}\sum_{k=0}^{N-1}|X(k)|^2} \longrightarrow (3)$$

$\therefore$ Hence parseval.s theorem for circular Autocorrelation proved.

Problems

1) For the sequence given $x(n) = \{1, 2, 0, 3, -2, 4, 7, 5\}$. compute the following.

a) $X(0)$ (b) $X(4)$ (c) $\sum_{k=0}^{7} X(k)$ (d). $\sum_{k=0}^{7}|X(k)|^2$

Soln.

$x(n) = \{1, 2, 0, 3, -2, 4, 7, 5\}$

a) wkt $X(k) = \sum_{n=0}^{7} x(n)\, W_8^{nk}$ $k=0,1,2,\ldots 7$.

$k=0$. $X(0) = \sum_{n=0}^{7} x(n)(W_8^0) = \sum_{n=0}^{7} x(n)$.