Electromagnetic Waves

18EC55

Syllabus

Module 1

Coulomb's Law, Electric Field Intensity and Flux density: Experimental law of Coulomb, Electric field intensity, Field due to continuous volume charge distribution, Field of a line charge, Field due to Sheet of charge, Electric flux density, Numerical Problems. **(Text: Chapter 2.1 to 2.5, 3.1)**

Module 2

Gauss's law and Divergence: Gauss 'law, Application of Gauss' law to point charge, line charge, Surface charge and volume charge, Point (differential) form of Gauss law, Divergence. Maxwell's First equation (Electrostatics), Vector Operator ▼ and divergence theorem, Numerical Problems (**Text: Chapter 3.2 to 3.7).**

Energy, Potential and Conductors: Energy expended or work done in moving a point charge in an electric field, The line integral, Definition of potential difference and potential, The potential field of point charge, Potential gradient, Numerical Problems **(Text: Chapter 4.1 to 4.4 and 4.6).** Current and Current density, Continuity of current. **(Text: Chapter 5.1, 5.2)**

Module 3

Poisson’s and Laplace’s Equations: Derivation of Poisson‘s and Laplace‘s Equations, Uniqueness theorem, Examples of the solution of Laplace‘s equation, Numerical problems on Laplace equation **(Text: Chapter 7.1 to 7.3)**

Steady Magnetic Field: Biot-Savart Law, Ampere‘s circuital law, Curl, Stokes‘ theorem, Magnetic flux and magnetic flux density, Basic concepts Scalar and Vector Magnetic Potentials, Numerical problems. (**Text: Chapter 8.1 to 8.6)**

Module 4

Magnetic Forces: Force on a moving charge, differential current elements, Force between differential current elements, Numerical problems **(Text: Chapter 9.1 to 9.3).**

Magnetic Materials: Magnetization and permeability, Magnetic boundary conditions, The magnetic circuit, Potential energy and forces on magnetic materials, Inductance and mutual reactance, Numerical problems **(Text: Chapter 9.6 to 9.7).**

Faraday’ law of Electromagnetic Induction –Integral form and Point form, Numerical problems **(Text: Chapter 10.1)**

Module 5

Maxwell's equations Continuity equation, Inconsistency of Ampere's law with continuity equation, displacement current, Conduction current, Derivation of Maxwell's equations in point form, and integral form, Maxwell's equations for different media, Numerical problems **(Text: Chapter 10.2 to 10.4)**

Uniform Plane Wave: Plane wave, Uniform plane wave, Derivation of plane wave equations from Maxwell's equations, Solution of wave equation for perfect dielectric, Relation between E and H, Wave propagation in free space, Solution of wave equation for sinusoidal excitation, wave propagation in any conducting media (**γ, α, β, η**) and good conductors, Skin effect or Depth of penetration, Poynting's theorem and wave power, Numerical problems. **(Text: Chapter 12.1 to 12.4)**

Textbooks

- W.H. Hayt and J.A. Buck, ―Engineering Electromagnetics‖, 8th Edition, Tata McGraw-Hill, 2014, ISBN-978-93-392-0327-6.

Reference Books

1. Elements of Electromagnetics – Matthew N.O., Sadiku, Oxford university press, 4th Edn.
2. 2. Electromagnetic Waves and Radiating systems – E. C. Jordan and K.G. Balman, PHI, 2ndEdn.
3. 3. Electromagnetics- Joseph Edminister, Schaum Outline Series, McGraw Hill. N. Narayana Rao, —Fundamentals of Electromagnetics for Engineering‖, Pearson.

Course Outcomes

- Evaluate problems on electrostatic force, electric field due to point, linear, volume charges by applying conventional methods and charge in a volume..
- Apply Gauss law to evaluate Electric fields due to different charge distributions and Volume Charge distribution by using Divergence Theorem and determine potential and energy of a point charge
- Determine capacitance of a parallel plate capacitor, coaxial cylindrical capacitor with different charge distributions using Laplace equation and Apply Biot-Savart's and Ampere's laws for evaluating Magnetic field for different current configurations
- Calculate magnetic force, potential energy and Magnetization with respect to magnetic materials and voltage induced in electric circuits..
- Apply Maxwell's equations for time varying fields, EM waves in free space and conductors and Evaluate power associated with EM waves using Poynting theorem.

Module 1

- Revision of Vector Calculus –**(Text 1: Chapter 1)**

Coulomb's Law, Electric Field Intensity and Flux density: Experimental law of Coulomb, Electric field intensity, Field due to continuous volume charge distribution, Field of a line charge, Field due to Sheet of charge, Electric flux density, Numerical Problems. **(Text: Chapter 2.1 to 2.5, 3.1)**

Revision of Vector Calculus

What is Electromagnetics

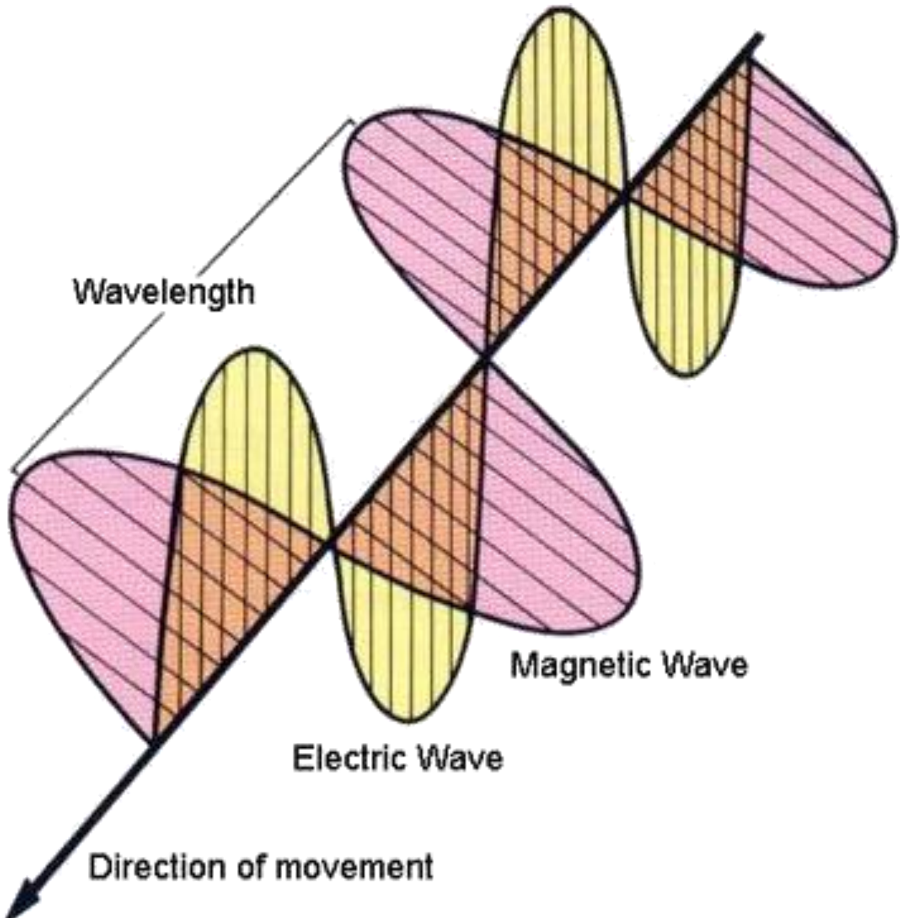
Electric field

Produced by the presence of electrically charged particles, and gives rise to the electric force.

Magnetic field

Produced by the motion of electric charges, or electric current, and gives rise to the magnetic force associated with magnets.

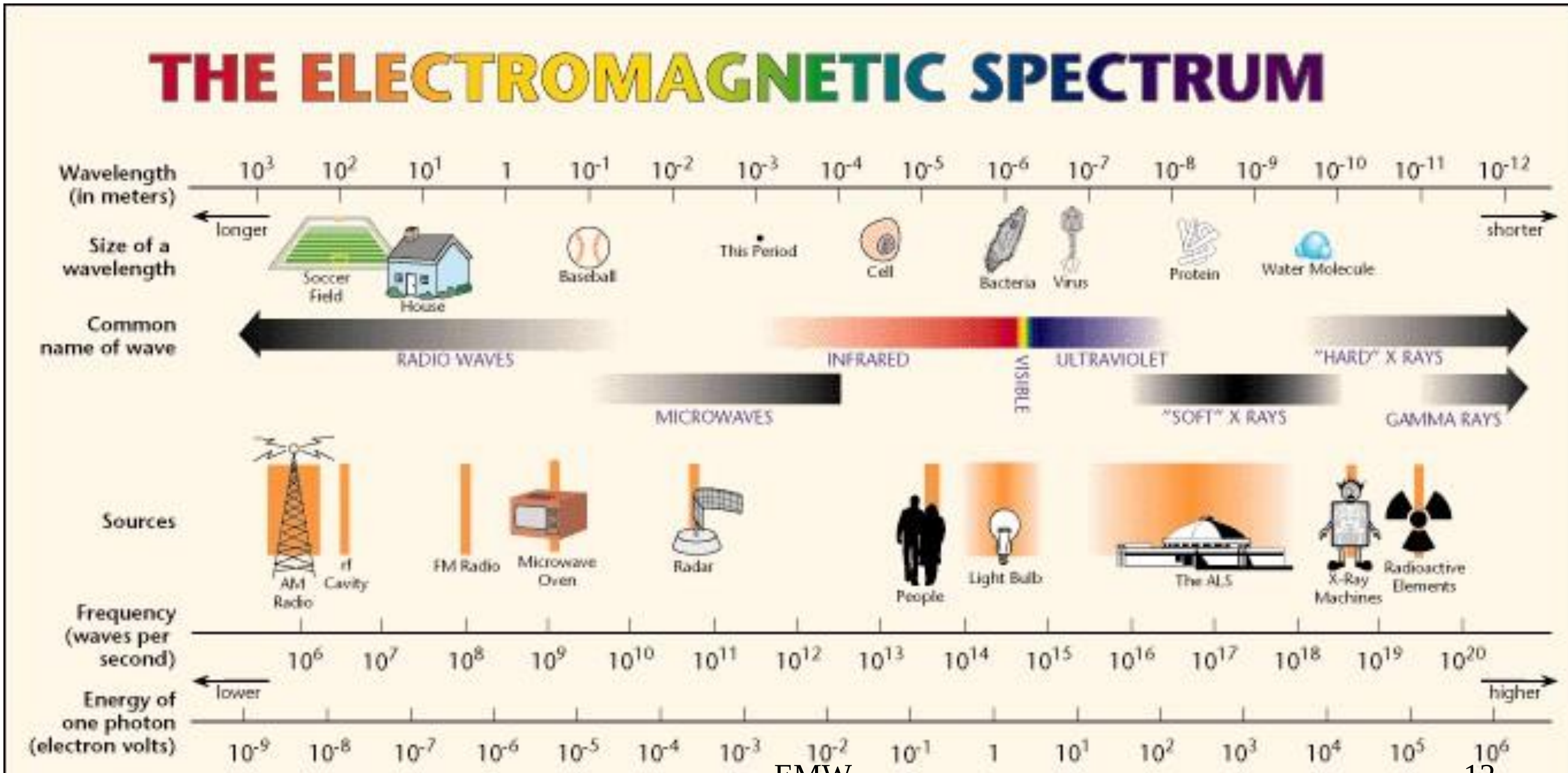
What is Electromagnetics?



- An electromagnetic field is generated when charged particles, such as electrons, are *accelerated*.
- All electrically charged particles are surrounded by electric fields.
- Charged particles in motion produce magnetic fields.
- When the velocity of a charged particle changes, an electromagnetic field is produced.

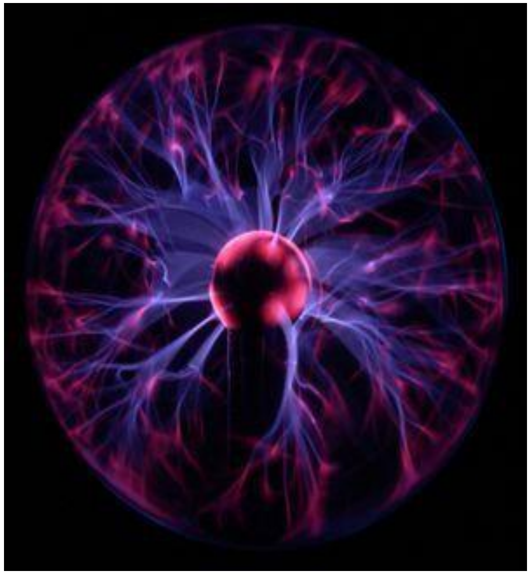
Why do we learn Engineering Electromagnetics

- ✓ Electric and magnetic field exist nearly everywhere.



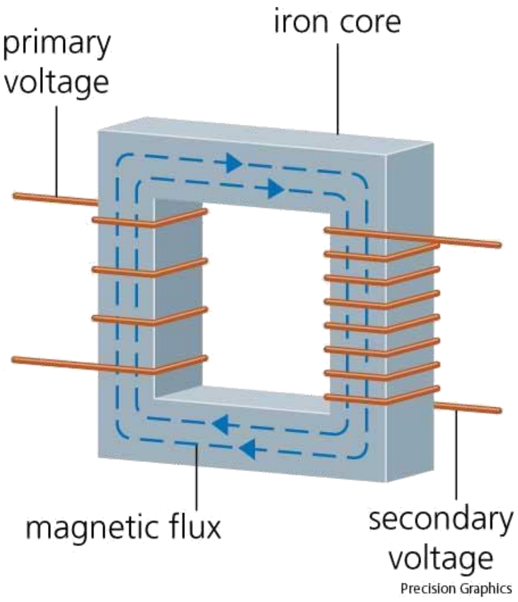
Applications

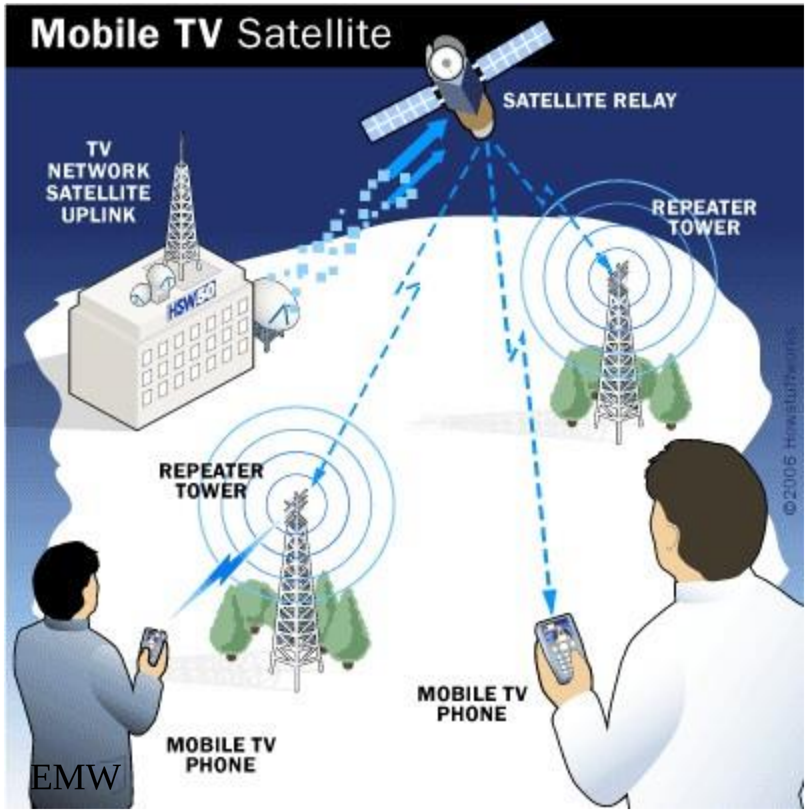
✓ Electromagnetic principles find application in various disciplines such as microwaves, x-rays, antennas, electric machines, plasmas, etc.



Applications

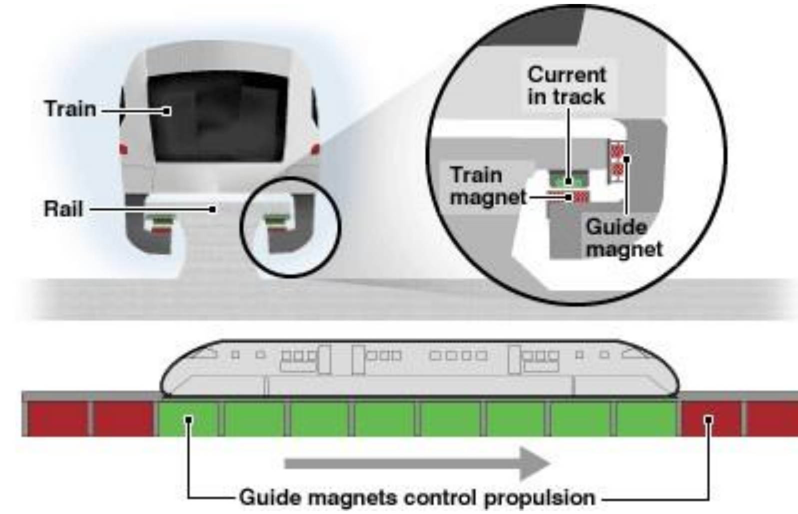
- ✓ Electromagnetic fields are used in induction heaters for melting, forging, annealing, surface hardening, and soldering operation.
- ✓ Electromagnetic devices include transformers, radio, television, mobile phones, radars, lasers, etc.



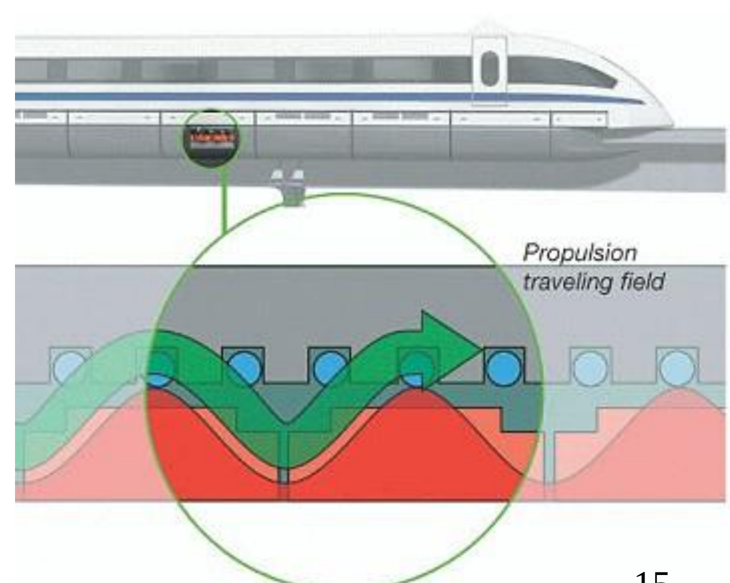


Applications

Transrapid Train



- **A magnetic traveling field moves the vehicle without contact.**
- **The speed can be continuously regulated by varying the frequency of the alternating current.**



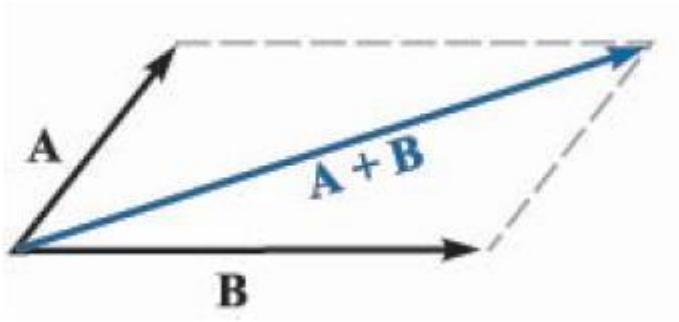
Scalars and Vectors

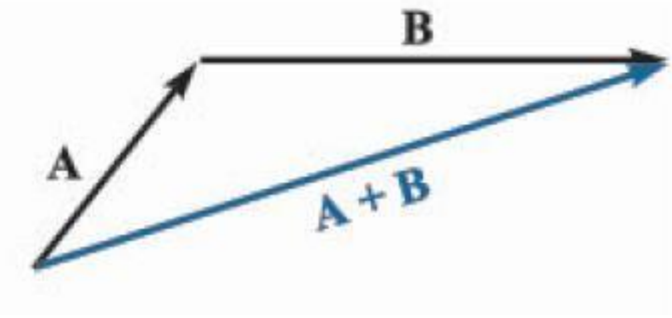
- ✓ Scalar refers to a quantity whose value may be represented by a single (positive or negative) real number.
- ✓ Some examples include distance, temperature, mass, density, pressure, volume, and time.

- ✓ A vector quantity has both a magnitude and a direction in space. We especially concerned with two- and three-dimensional spaces only.
- ✓ Displacement, velocity, acceleration, and force are examples of vectors.

  - Scalar notation: *A* or A (*italic* or plain)
  - Vector notation: **A** or $\vec{A}$ (**bold** or plain with arrow)

Vector Algebra





$$\mathbf{A}+\mathbf{B}=\mathbf{B}+\mathbf{A}$$

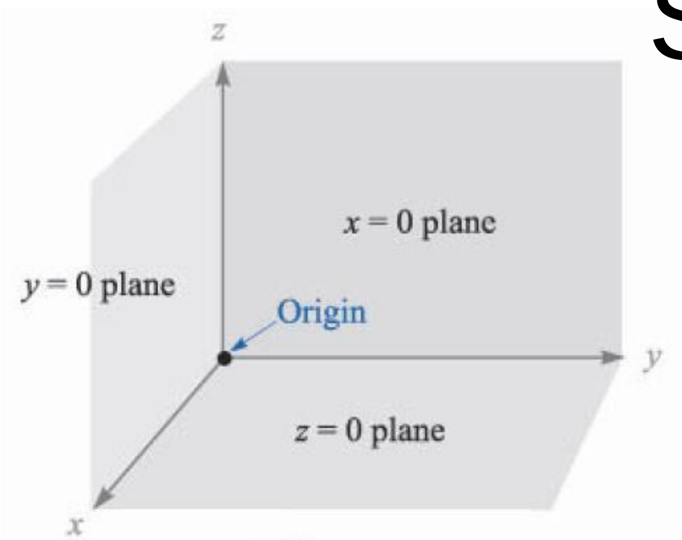
$$\mathbf{A}+(\mathbf{B}+\mathbf{C})=(\mathbf{A}+\mathbf{B})+\mathbf{C}$$

$$\mathbf{A}-\mathbf{B}=\mathbf{A}+(-\mathbf{B})$$

$$\frac{\mathbf{A}}{n}=\frac{1}{n}\mathbf{A}$$

$$\mathbf{A}-\mathbf{B}=0\rightarrow\mathbf{A}=\mathbf{B}$$

Rectangular Coordinate System

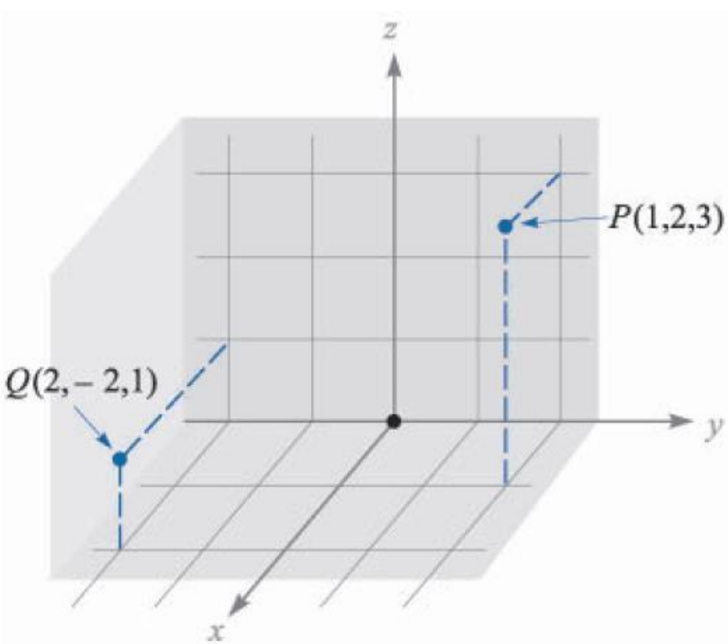


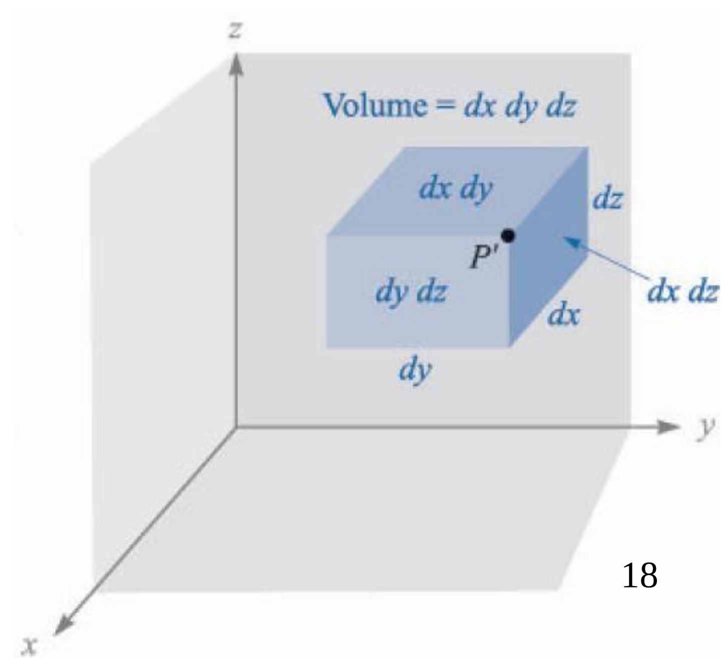
- **Differential surface units:**

$$dx \cdot dy$$
$$dy \cdot dz$$
$$dx \cdot dz$$

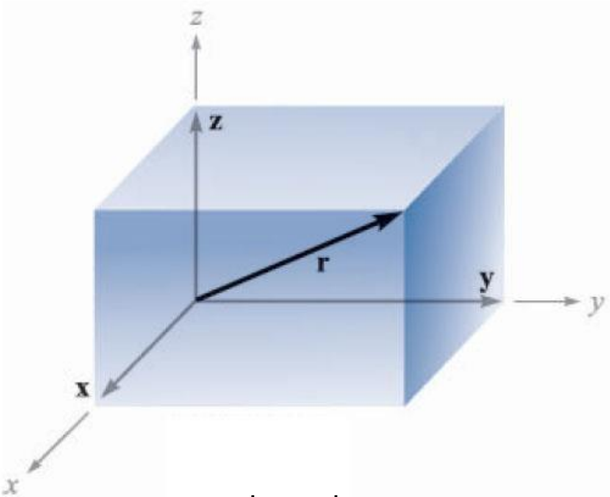
- **Differential volume unit :**

$$dx \cdot dy \cdot dz$$





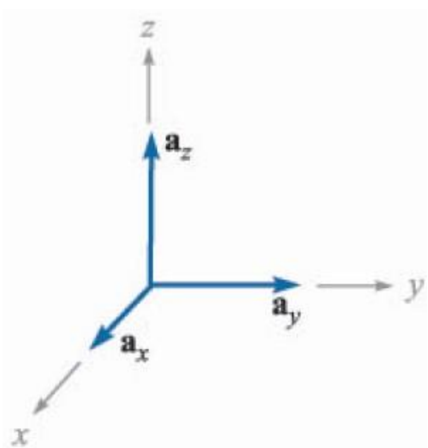
Vector Components and Unit Vectors

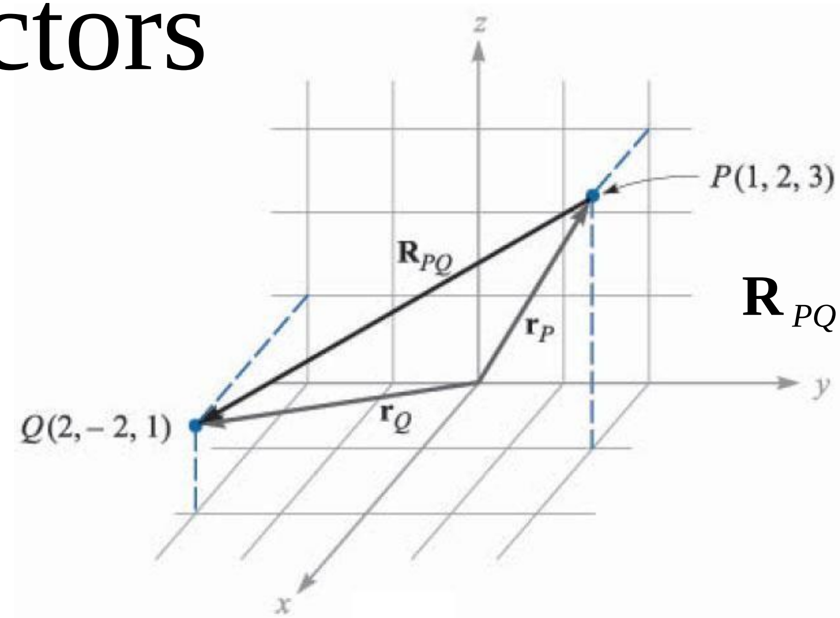


$$\mathbf{r} = \mathbf{x} + \mathbf{y} + \mathbf{z}$$

$$\mathbf{r} = x\mathbf{a}_x + y\mathbf{a}_y + z\mathbf{a}_z$$

$\mathbf{a}_x, \mathbf{a}_y, \mathbf{a}_z$: unit vectors





$\mathbf{R}_{PQ}$?

$$\mathbf{R}_{PQ} = \mathbf{r}_Q - \mathbf{r}_P$$

$$= (2\mathbf{a}_x - 2\mathbf{a}_y + \mathbf{a}_z) - (1\mathbf{a}_x + 2\mathbf{a}_y + 3\mathbf{a}_z)$$

$$= \mathbf{a}_x - 4\mathbf{a}_y - 2\mathbf{a}_z$$

Vector Components and Unit Vectors

- For any vector **B**, $\mathbf{B} = B_x\mathbf{a}_x + B_y\mathbf{a}_y + B_z\mathbf{a}_z$:

$$|\mathbf{B}| = \sqrt{B_x^2 + B_y^2 + B_z^2} = B$$

Magnitude of **B**

$$\mathbf{a}_B = \frac{\mathbf{B}}{\sqrt{B_x^2 + B_y^2 + B_z^2}} = \frac{\mathbf{B}}{|\mathbf{B}|}$$

Unit vector in the direction of **B**

- Example

Given points *M*(–1,2,1) and *N*(3,–3,0), find $\mathbf{R}_{MN}$ and $\mathbf{a}_{MN}$.

$$\mathbf{R}_{MN} = (3\mathbf{a}_x - 3\mathbf{a}_y + 0\mathbf{a}_z) - (-1\mathbf{a}_x + 2\mathbf{a}_y + 1\mathbf{a}_z) = \underline{\underline{4\mathbf{a}_x - 5\mathbf{a}_y - \mathbf{a}_z}}$$

$$\mathbf{a}_{MN} = \frac{\mathbf{R}_{MN}}{|\mathbf{R}_{MN}|} = \frac{4\mathbf{a}_x - 5\mathbf{a}_y - 1\mathbf{a}_z}{\sqrt{4^2 + (-5)^2 + (-1)^2}} = \underline{\underline{0.617\mathbf{a}_x - 0.772\mathbf{a}_y - 0.154\mathbf{a}_z}}$$

The Dot Product

- ✓ Given two vectors **A** and **B**, the *dot product*, or *scalar product*, is defines as the product of the magnitude of **A**, the magnitude of **B**, and the cosine of the smaller angle between them:

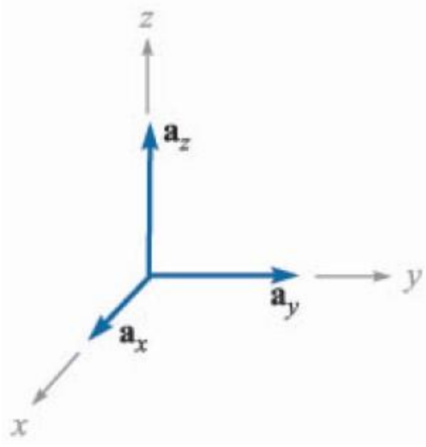
$$\mathbf{A} \cdot \mathbf{B} = |\mathbf{A}||\mathbf{B}| \cos \theta_{AB}$$

- ✓ The dot product is a scalar, and it obeys the commutative law:

$$\mathbf{A} \cdot \mathbf{B} = \mathbf{B} \cdot \mathbf{A}$$

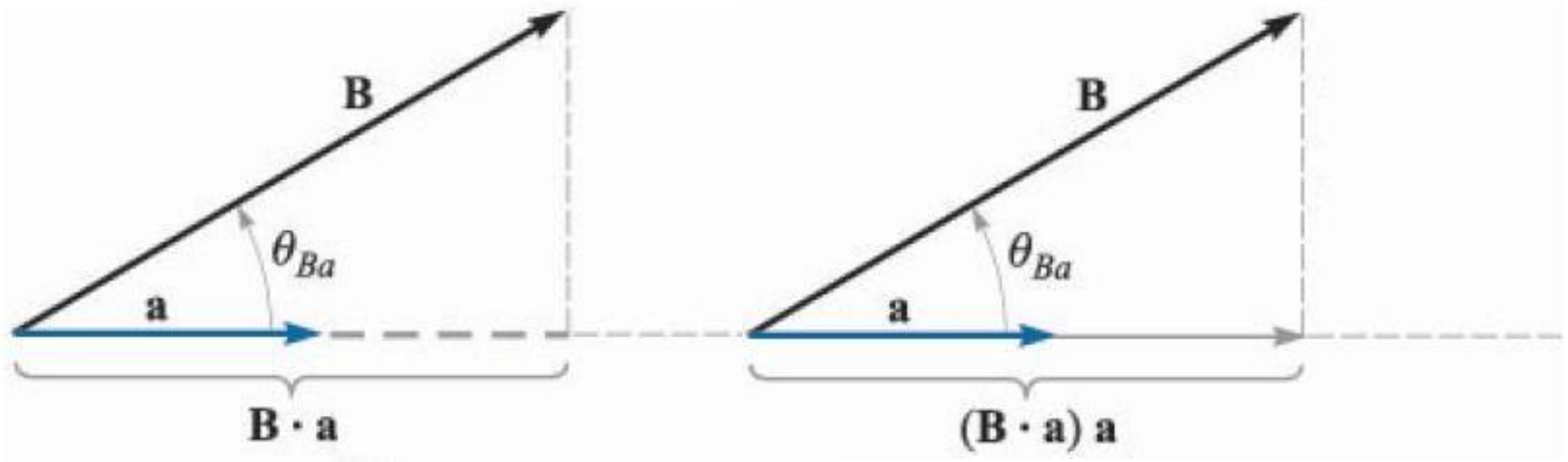
- ✓ For any vector $\mathbf{A} = A_x \mathbf{a}_x + A_y \mathbf{a}_y + A_z \mathbf{a}_z$ and $\mathbf{B} = B_x \mathbf{a}_x + B_y \mathbf{a}_y + B_z \mathbf{a}_z$,

$$\mathbf{A} \cdot \mathbf{B} = A_x B_x + A_y B_y + A_z B_z$$



The Dot Product

✓ One of the most important applications of the dot product is that of finding the component of a vector in a given direction.



- The scalar component of **B** in the direction of the unit vector **a** is **B**·**a**
- The vector component of **B** in the direction of the unit vector **a** is (**B**·**a**)**a**

$$\mathbf{B}\cdot\mathbf{a} = |\mathbf{B}||\mathbf{a}|\cos\theta_{Ba} = |\mathbf{B}|\cos\theta_{Ba}$$

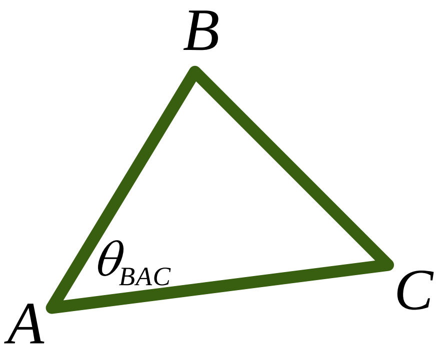
The Dot Product

✓ <u>Example</u>

The three vertices of a triangle are located at $A(6,–1,2)$, $B(–2,3,–4)$, and $C(–3,1,5)$. Find: (*a*) $\mathbf{R}_{AB}$; (*b*) $\mathbf{R}_{AC}$; (*c*) the angle θ_{BAC} at vertex *A*; (*d*) the vector projection of $\mathbf{R}_{AB}$ on $\mathbf{R}_{AC}$.

$$\mathbf{R}_{AB} = (-2\mathbf{a}_x + 3\mathbf{a}_y - 4\mathbf{a}_z) - (6\mathbf{a}_x - \mathbf{a}_y + 2\mathbf{a}_z) = \underline{\underline{-8\mathbf{a}_x + 4\mathbf{a}_y - 6\mathbf{a}_z}}$$

$$\mathbf{R}_{AC} = (-3\mathbf{a}_x + 1\mathbf{a}_y + 5\mathbf{a}_z) - (6\mathbf{a}_x - \mathbf{a}_y + 2\mathbf{a}_z) = \underline{\underline{-9\mathbf{a}_x + 2\mathbf{a}_y + 3\mathbf{a}_z}}$$



$$\mathbf{R}_{AB} \cdot \mathbf{R}_{AC} = |\mathbf{R}_{AB}||\mathbf{R}_{AC}|\cos\theta_{BAC}$$

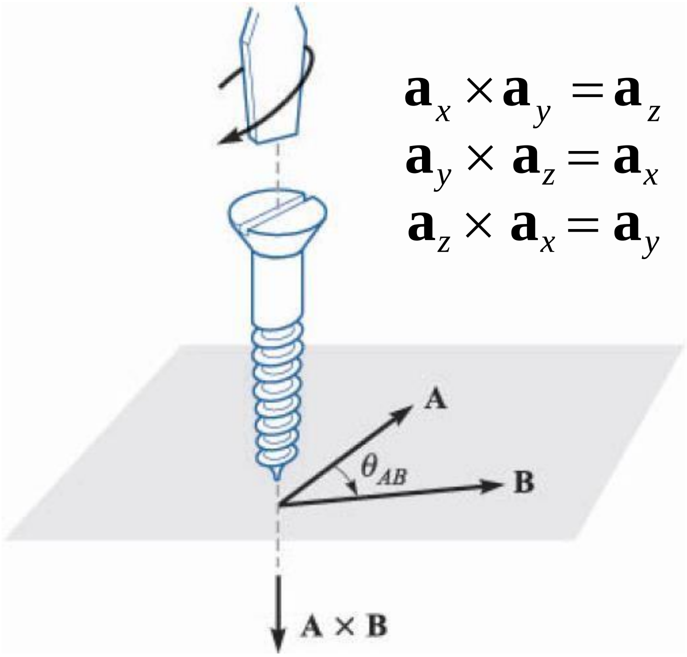
$$\Rightarrow \cos\theta_{BAC} = \frac{\mathbf{R}_{AB} \cdot \mathbf{R}_{AC}}{|\mathbf{R}_{AB}||\mathbf{R}_{AC}|} = \frac{(-8\mathbf{a}_x + 4\mathbf{a}_y - 6\mathbf{a}_z) \cdot (-9\mathbf{a}_x + 2\mathbf{a}_y + 3\mathbf{a}_z)}{\left|\sqrt{(-8)^2 + (4)^2 + (-6)^2}\right|\left|\sqrt{(-9)^2 + (2)^2 + (3)^2}\right|} = \frac{\square 62}{|\sqrt{116}||\sqrt{94}|} = 0.594$$

$$\Rightarrow \theta_{BAC} = \cos_{-1}(0.594) = \underline{\underline{53.56^\circ}}$$

The Cross Product

- Given two vectors **A** and **B**, the magnitude of the *cross product*, or *vector product*, written as **A**×**B**, is defines as the product of the magnitude of **A**, the magnitude of **B**, and the sine of the smaller angle between them.
- The direction of **A**×**B** is perpendicular to the plane containing **A** and **B** and is in the direction of advance of a right-handed screw as **A** is turned into **B**.

$$\mathbf{A}\times\mathbf{B} = \mathbf{a}_N \,|\mathbf{A}||\mathbf{B}|\sin\theta_{AB}$$



$$\mathbf{a}_x \times \mathbf{a}_y = \mathbf{a}_z$$
$$\mathbf{a}_y \times \mathbf{a}_z = \mathbf{a}_x$$
$$\mathbf{a}_z \times \mathbf{a}_x = \mathbf{a}_y$$

- The cross product is a vector, and it is not commutative:

$$(\mathbf{B}\times\mathbf{A}) = -(\mathbf{A}\times\mathbf{B})$$

The Cross Product

✓ <u>Example</u>

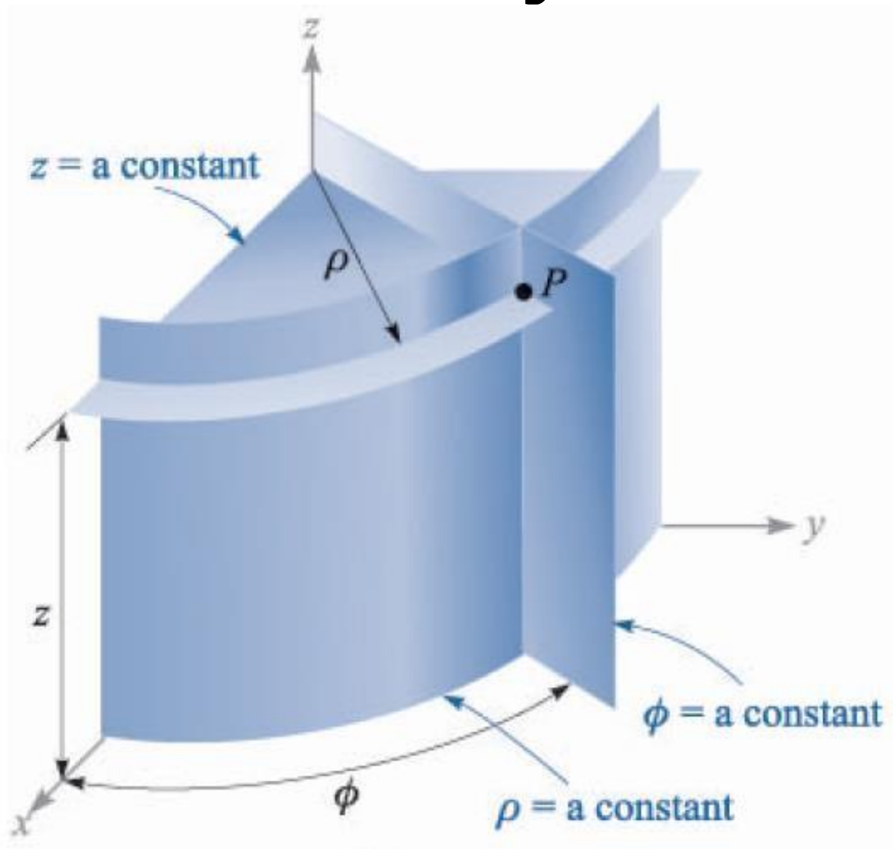
Given **A** = 2$\mathbf{a}_x$–3$\mathbf{a}_y$+$\mathbf{a}_z$ and **B** = –4$\mathbf{a}_x$–2$\mathbf{a}_y$+5$\mathbf{a}_z$, find **A**×**B**.

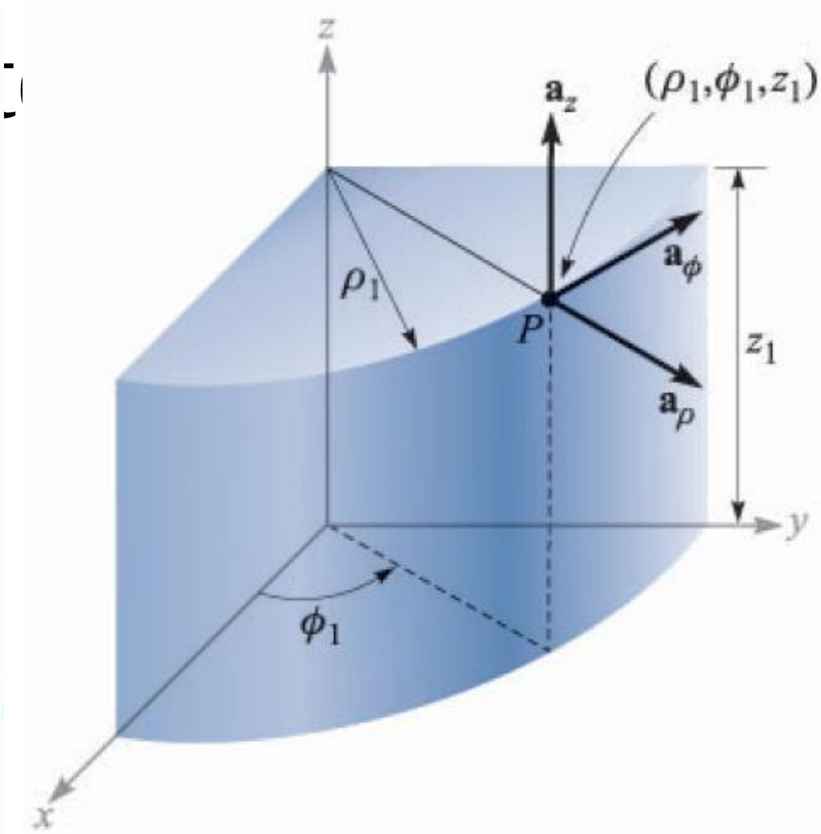
$$\mathbf{A}\times\mathbf{B} = (A_yB_z - A_zB_y)\mathbf{a}_x + (A_zB_x - A_xB_z)\mathbf{a}_y + (A_xB_y - A_yB_x)\mathbf{a}_z$$

$$= \big((-3)(5) - (1)(-2)\big)\mathbf{a}_x + \big((1)(-4) - (2)(5)\big)\mathbf{a}_y + \big((2)(-2) - (-3)(-4)\big)\mathbf{a}_z$$

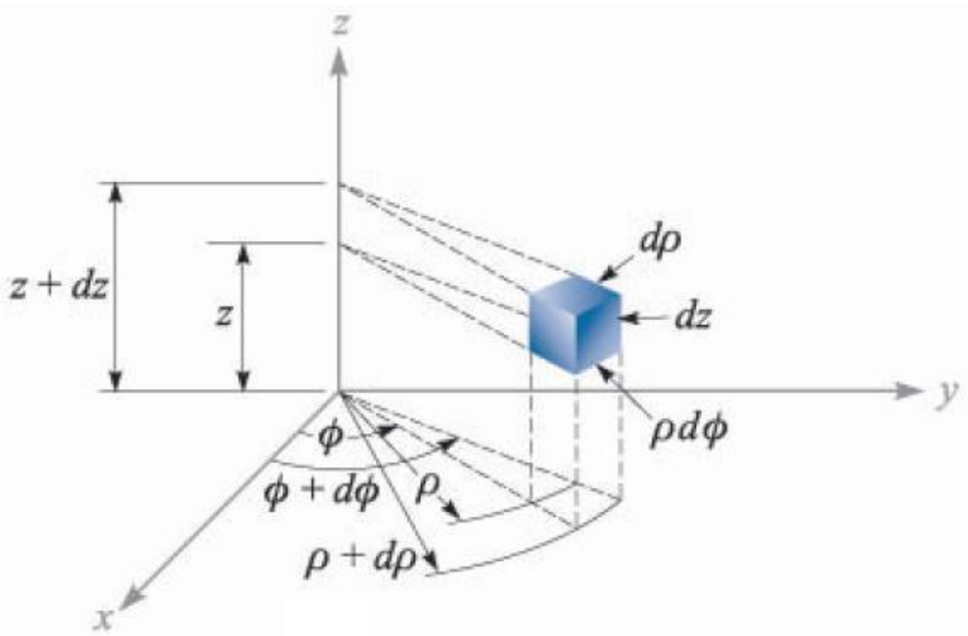
$$= \underline{\underline{-13\mathbf{a}_x - 14\mathbf{a}_y - 16\mathbf{a}_z}}$$

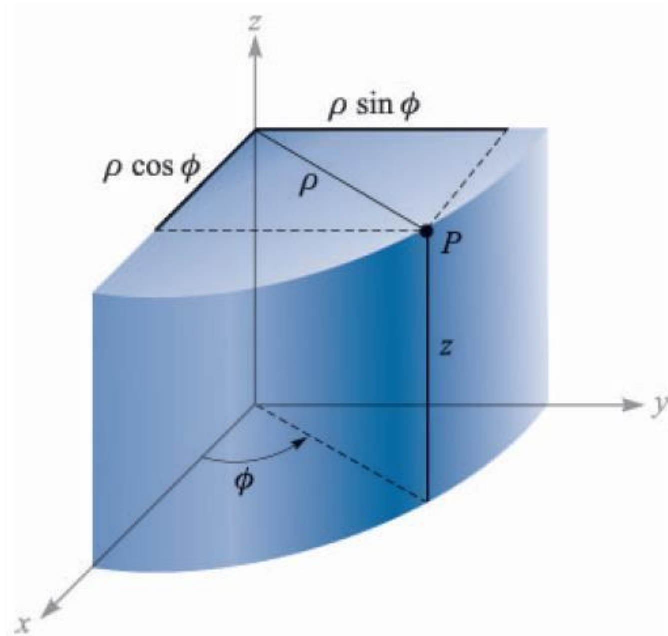
The Cylindrical Coordinate





The Cylindrical Coordinate System





- **Differential surface units:**

$$d\rho \cdot dz$$
$$\rho d\phi \cdot dz$$
$$d\rho \cdot \rho d\phi$$

- **Differential volume unit :**

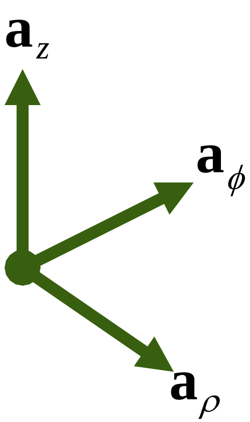
$$d\rho \cdot \rho d\phi \cdot dz$$

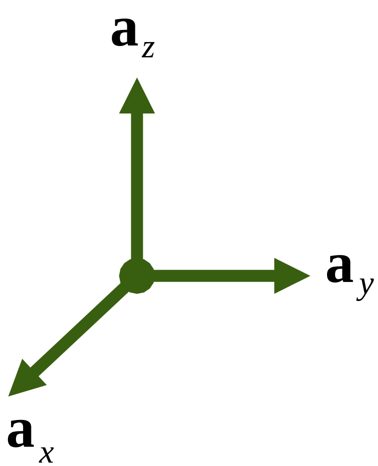
- **Relation between the rectangular and the cylindrical coordinate systems**

$x = \rho \cdot \cos\phi$	$\rho = \sqrt{x^2 + y^2}$
$y = \rho \cdot \sin\phi$	$\phi = \tan^{-1}\dfrac{y}{x}$
$z = z$	$z = z$

The Cylindrical Coordinate System

$$\mathbf{A} = A_x\mathbf{a}_x + A_y\mathbf{a}_y + A_z\mathbf{a}_z \overset{?}{\Rightarrow} \mathbf{A} = A_\rho\mathbf{a}_\rho + A_\phi\mathbf{a}_\phi + A_z\mathbf{a}_z$$





$$\begin{aligned} A_\rho &= \mathbf{A}\cdot\mathbf{a}_\rho \\ &= (A_x\mathbf{a}_x + A_y\mathbf{a}_y + A_z\mathbf{a}_z)\cdot\mathbf{a}_\rho \\ &= A_x\mathbf{a}_x\cdot\mathbf{a}_\rho + A_y\mathbf{a}_y\cdot\mathbf{a}_\rho + A_z\mathbf{a}_z\cdot\mathbf{a}_\rho \\ &= A_x\cos\phi + A_y\sin\phi \end{aligned}$$

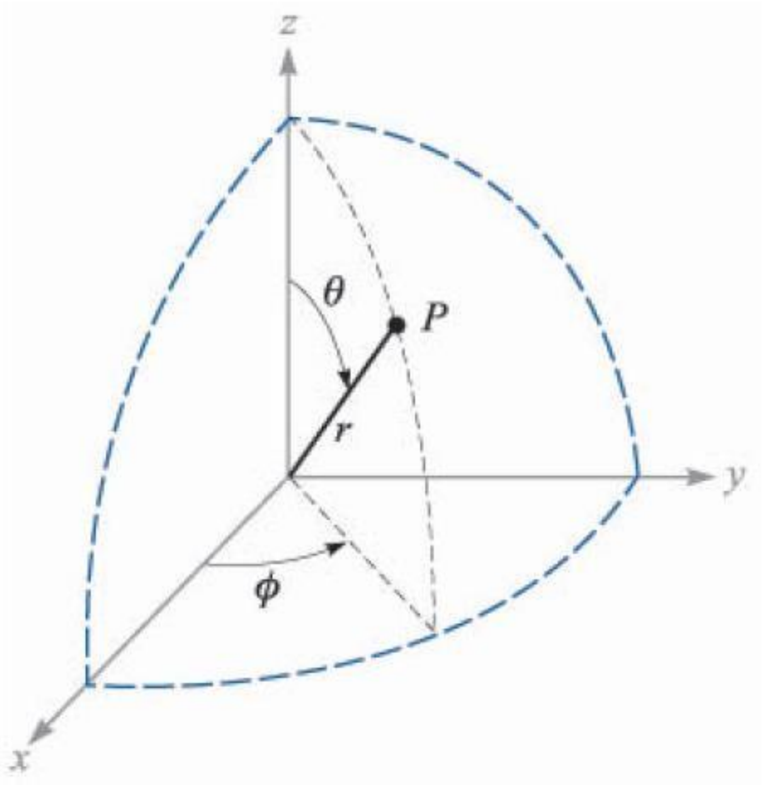
$$\begin{aligned} A_\phi &= \mathbf{A}\cdot\mathbf{a}_\phi \\ &= (A_x\mathbf{a}_x + A_y\mathbf{a}_y + A_z\mathbf{a}_z)\cdot\mathbf{a}_\phi \\ &= A_x\mathbf{a}_x\cdot\mathbf{a}_\phi + A_y\mathbf{a}_y\cdot\mathbf{a}_\phi + A_z\mathbf{a}_z\cdot\mathbf{a}_\phi \\ &= -A_x\sin\phi + A_y\cos\phi \end{aligned}$$

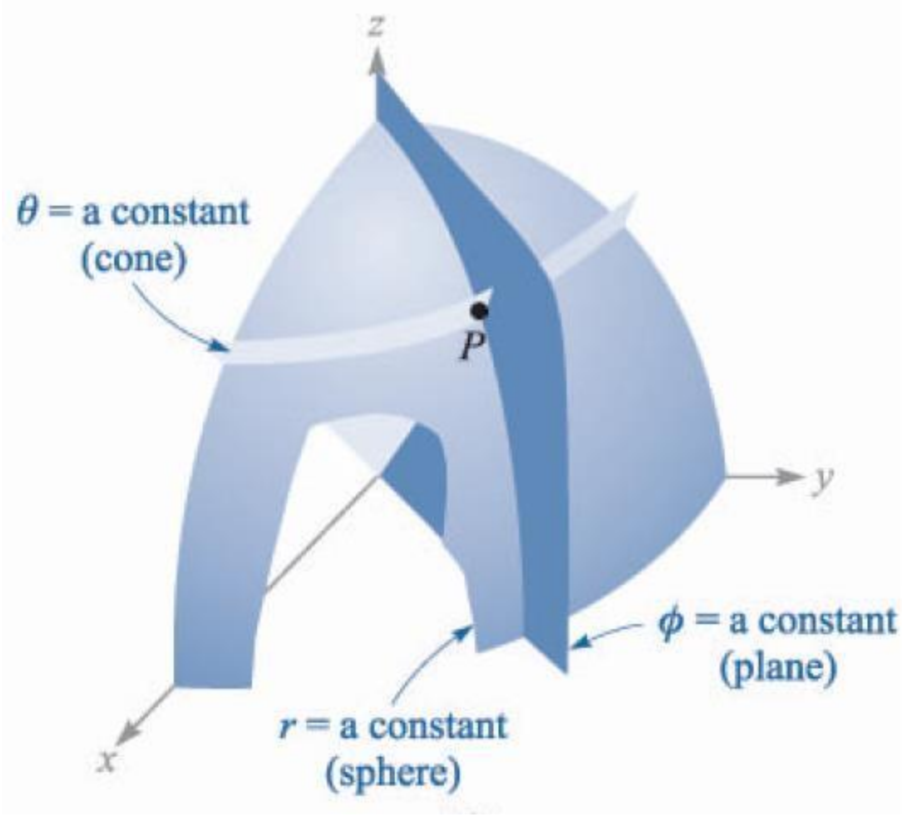
$$\begin{aligned} A_z &= \mathbf{A}\cdot\mathbf{a}_z \\ &= (A_x\mathbf{a}_x + A_y\mathbf{a}_y + A_z\mathbf{a}_z)\cdot\mathbf{a}_z \\ &= A_x\mathbf{a}_x\cdot\mathbf{a}_z + A_y\mathbf{a}_y\cdot\mathbf{a}_z + A_z\mathbf{a}_z\cdot\mathbf{a}_z \\ &= A_z \end{aligned}$$

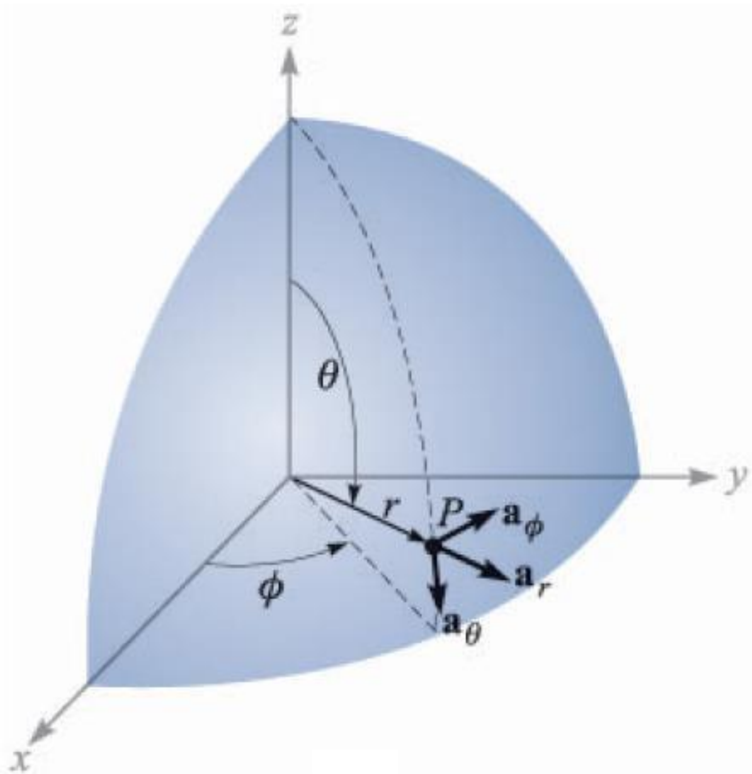
- **Dot products of unit vectors in cylindrical and rectangular coordinate systems**

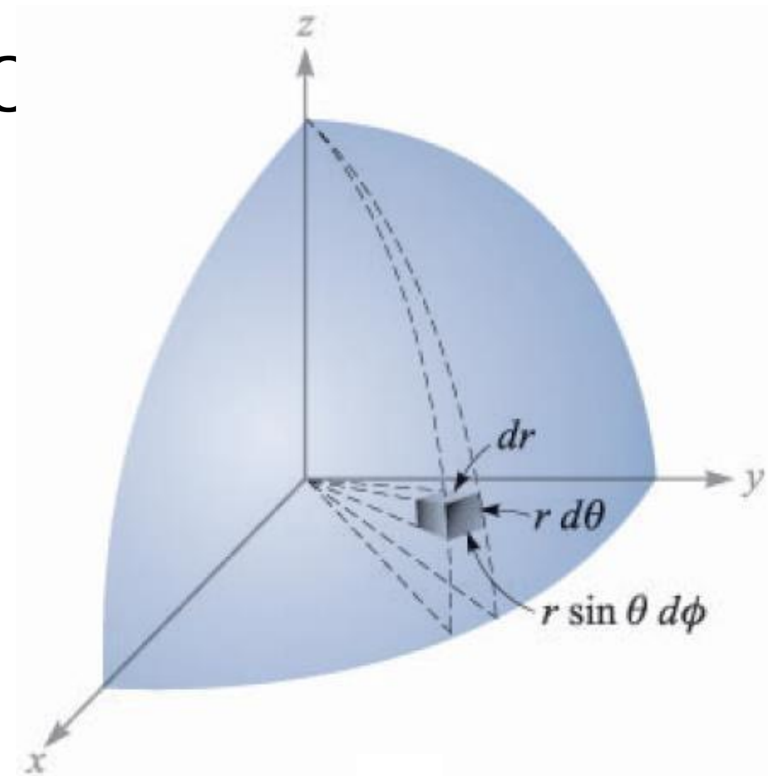
	$\mathbf{a}_\rho$	$\mathbf{a}_\phi$	$\mathbf{a}_z$
$\mathbf{a}_x\cdot$	$\cos\phi$	$-\sin\phi$	0
$\mathbf{a}_y\cdot$	$\sin\phi$	$\cos\phi$	0
$\mathbf{a}_z\cdot$	0	0	1

The Spherical Coordinate System









- **Differential surface units:**

  $dr \cdot rd\theta$

  $dr \cdot r\sin\theta d\phi$

  $rd\theta \cdot r\sin\theta d\phi$

- **Differential volume unit :**

  $dr \cdot rd\theta \cdot r\sin\theta d\phi$

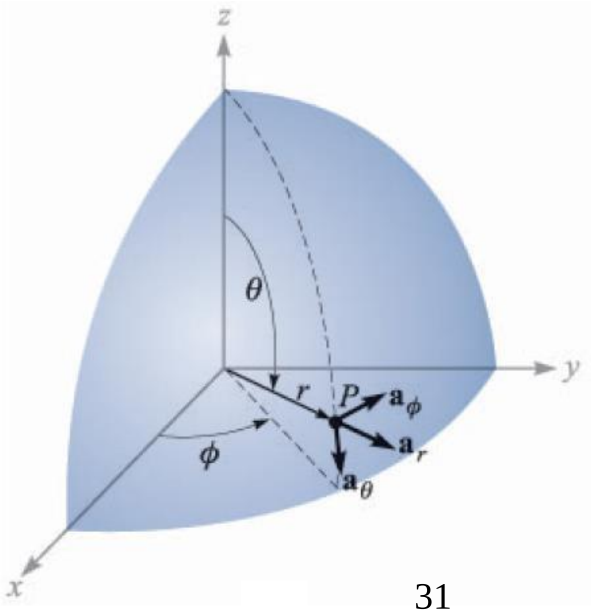
The Spherical Coordinate System

- **Relation between the rectangular and the spherical coordinate systems**

$x = r\sin\theta\cos\phi$	$r = \sqrt{x^2 + y^2 + z^2}, \quad r \geq 0$
$y = r\sin\theta\sin\phi$	$\theta = \cos^{-1}\dfrac{z}{\sqrt{x^2+y^2+z^2}}, \quad 0° \leq \theta \leq 180°$
$z = r\cos\theta$	$\phi = \tan^{-1}\dfrac{y}{x}$

- **Dot products of unit vectors in spherical and rectangular coordinate systems**

	$\mathbf{a}_r$	$\mathbf{a}_\theta$	$\mathbf{a}_\phi$
$\mathbf{a}_x \cdot$	$\sin\theta\cos\phi$	$\cos\theta\cos\phi$	$-\sin\phi$
$\mathbf{a}_y \cdot$	$\sin\theta\sin\phi$	$\cos\theta\sin\phi$	$\cos\phi$
$\mathbf{a}_z \cdot$	$\cos\theta$	$-\sin\theta$	0

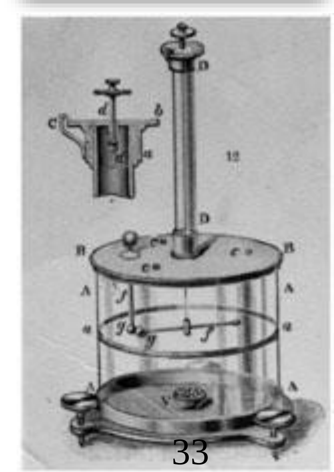


COULOMB'S LAW AND ELECTRIC FIELD INTENSITY

The Experimental Law of Coulomb

- In 1600, Dr. Gilbert, a physician from England, published the first major classification of electric and non-electric materials.
- He stated that glass, sulfur, amber, and some other materials "not only draw to themselves straw, and chaff, but all metals, wood, leaves, stone, earths, even water and oil."

- In the following century, a French Army Engineer, Colonel Charles Coulomb, performed an elaborate series of experiments using devices invented by himself.
- Coulomb could determine quantitatively the force exerted between two objects, each having a static charge of electricity.
- He wrote seven important treatises on electric and magnetism, developed a theory of attraction and repulsion between bodies of the opposite and the same electrical charge.



The Experimental Law of Coulomb

- ✓ Coulomb stated that the **force** between two very small objects separated in vacuum or free space by a distance which is large compared to their size is **proportional** to the **charge** on each and **inversely proportional** to the **square of the distance** between them.

$$F = k\,\frac{Q_1 Q_2}{R^2}$$

- ✓ In SI Units, the quantities of charge *Q* are measured in coulombs (C), the separation *R* in meters (m), and the force *F* should be newtons (N).
- ✓ This will be achieved if the constant of proportionality *k* is written as:

$$k = \frac{1}{4\pi\varepsilon_0}$$

The Experimental Law of Coulomb

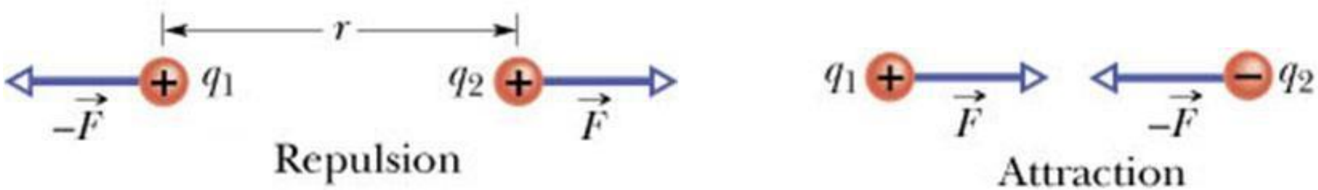
- The *permittivity of free space ε* is measured in farads per meter (F/m), and has the magnitude of:

$$\varepsilon_0 = 8.854 \times 10^{-12} \doteq \frac{1}{36\pi} 10^{-9} \text{ F/m}$$

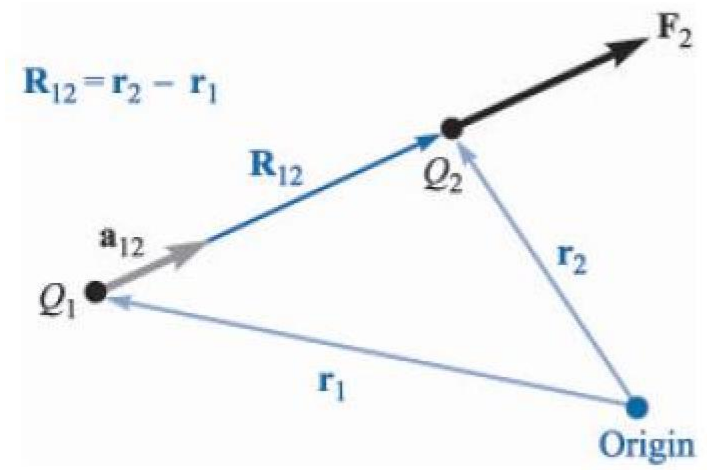
- The Coulomb's law is now:

$$F = \frac{1}{4\pi\varepsilon_0} \frac{Q_1 Q_2}{R^2}$$

- The force *F* acts along the line joining the two charges. It is repulsive if the charges are alike in sign and attractive if the are of opposite sign.



The Experimental Law of Coulomb



$$\mathbf{a}_{12} = \frac{\mathbf{R}_{12}}{|\mathbf{R}_{12}|} = \frac{\mathbf{R}_{12}}{R_{12}} = \frac{\mathbf{r}_2 - \mathbf{r}_1}{|\mathbf{r}_2 - \mathbf{r}_1|}$$

- In vector form, Coulomb's law is written as

$$\mathbf{F}_2 = \frac{1}{4\pi\varepsilon_0}\frac{Q_1 Q_2}{R_{12}^2}\mathbf{a}_{12}$$

- $\mathbf{F}_2$ is the force on Q_2, for the case where Q_1 and Q_2 have the same sign, while $\mathbf{a}_{12}$ is the unit vector in the direction of R_{12}, the line segment from Q_1 to Q_2.

Electric Field Intensity

- Let us consider one charge, say Q_1, fixed in position in space.
- Now, imagine that we can introduce a second charge, Q_t, as a "unit test charge", that we can move around.
- We know that there exists everywhere a force on this second charge ► This second charge is displaying the existence of a force field.

- The force on it is given by Coulomb's law as:

$$\mathbf{F}_t = \frac{1}{4\pi\varepsilon_0}\frac{Q_1 Q_t}{R_{1t}^2}\mathbf{a}_{1t}$$

- Writing this force as a "force per unit charge" gives:

$$\frac{\mathbf{F}_t}{Q_t} = \frac{1}{4\pi\varepsilon_0}\frac{Q_1}{R_{1t}^2}\mathbf{a}_{1t}$$

Vector Field,
Electric Field Intensity

Electric Field Intensity

- We define the electric field intensity as the vector of force on a unit positive test charge.
- Electric field intensity, E, is measured by the unit newtons per coulomb (N/C) or volts per meter (V/m).

$$\mathbf{E} = \frac{\mathbf{F}_t}{Q_t} = \frac{1}{4\pi\varepsilon_0}\frac{Q_1}{R_{1t}^2}\mathbf{a}_{1t}$$

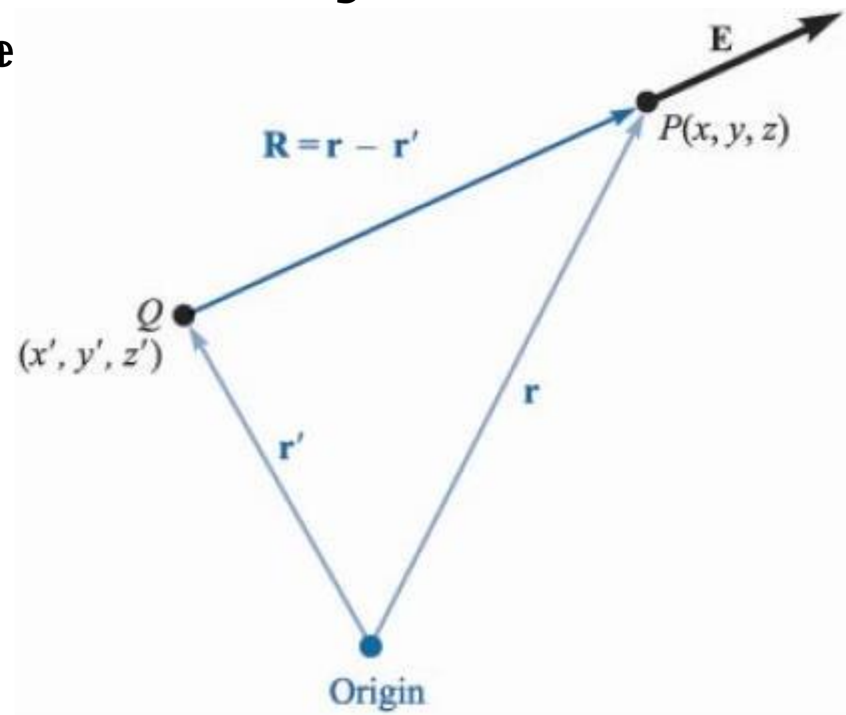
- The field of a single point charge can be written as

$$\mathbf{E} = \frac{1}{4\pi\varepsilon_0}\frac{Q}{R^2}\mathbf{a}_R$$

- a_R is a unit vector in the direction from the point at which the point charge Q is located, to the point at which E is desired/measured.

Electric Field Intensity

✓ For a charge which is not at the origin of the coordinate, the electric field intensity is:



$$\mathbf{E}(\mathbf{r}) = \frac{1}{4\pi\varepsilon_0}\frac{Q}{|\mathbf{r}-\mathbf{r}'|^2}\frac{\mathbf{r}-\mathbf{r}'}{|\mathbf{r}-\mathbf{r}'|}$$

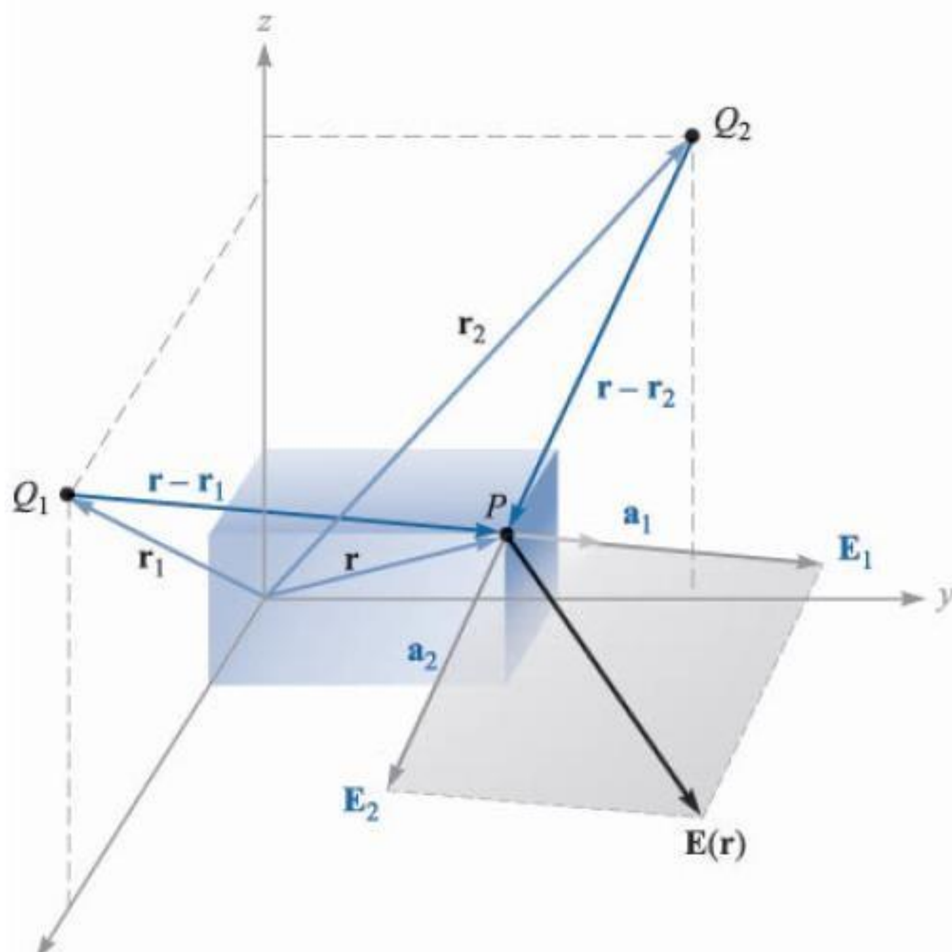
$$= \frac{1}{4\pi\varepsilon_0}\frac{Q(\mathbf{r}-\mathbf{r}')}{|\mathbf{r}-\mathbf{r}'|^3}$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{Q\left[(x-x')\mathbf{a}_x+(y-y')\mathbf{a}_y+(z-z')\mathbf{a}_z\right]}{\left[(x-x')^2+(y-y')^2+(z-z')^2\right]^{3/2}}$$

Electric Field Intensity

✓ The electric field intensity due to two point charges, say Q_1 at r_1 and Q_2 at r_2, is the sum of the electric field intensity on Q_t caused by Q_1 and Q_2 acting alone (Superposition Principle).

$$\mathbf{E}(\mathbf{r}) = \frac{1}{4\pi\varepsilon_0}\frac{Q}{|\mathbf{r}-\mathbf{r}_1|^2}\mathbf{a}_1 + \frac{1}{4\pi\varepsilon_0}\frac{Q}{|\mathbf{r}-\mathbf{r}_2|^2}\mathbf{a}_2$$



Field Due to a Continuous Volume Charge Distribution

- We denote the volume charge density by ρ_v, having the units of coulombs per cubic meter (C/m^3).
- The small amount of charge ΔQ in a small volume Δv is

$$\Delta Q = \rho_v \Delta v$$

- We may define ρ_v mathematically by using a limit on the above equation:

$$\rho_v = \lim_{\Delta v \to 0} \frac{\Delta Q}{\Delta v}$$

- The total charge within some finite volume is obtained by integrating throughout that volume:

$$Q = \int_{\text{vol}} \rho_v \, dv$$

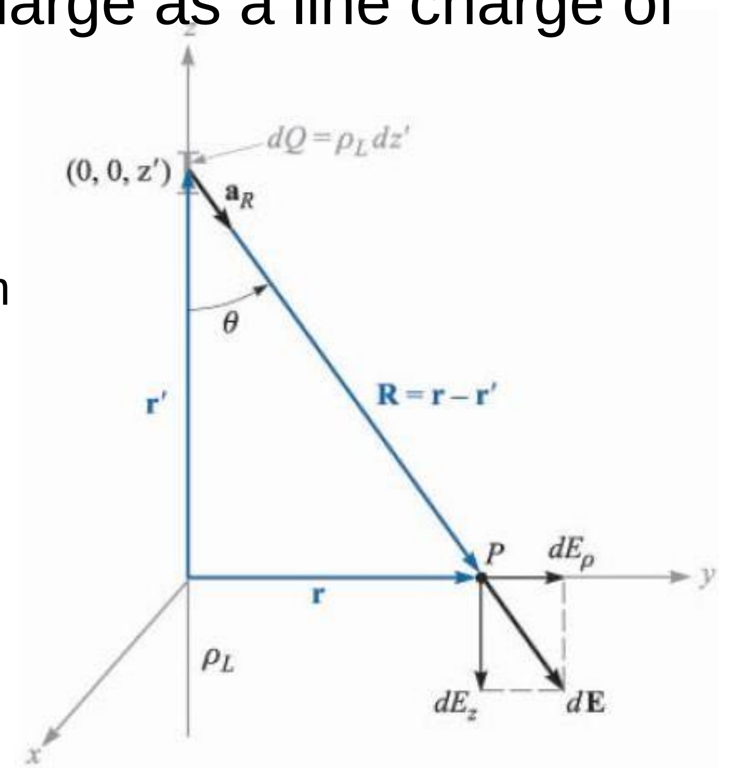
Field Due to a Continuous Volume Charge Distribution

✓ The contributions of all the volume charge in a given region, as the volume element Δv approaches zero, is an integral in the form of:

$$\mathbf{E}(\mathbf{r}) = \int_{\text{vol}} \frac{1}{4\pi\varepsilon_0} \frac{\rho_v(\mathbf{r}')\,dv'}{|\mathbf{r}-\mathbf{r}'|^2} \frac{\mathbf{r}-\mathbf{r}'}{|\mathbf{r}-\mathbf{r}'|}$$

Field of a Line Charge

- Now we consider a filamentlike distribution of volume charge density. It is convenient to treat the charge as a line charge of density ρ_L C/m.



- Let us assume a straight-line charge extending along the *z* axis in a cylindrical coordinate system from –∞ to +∞.
- We desire the electric field intensity E at any point resulting from a uniform line charge density ρ_L.

$$d\mathbf{E} = d\mathrm{E}_\rho\,\mathbf{a}_\rho + dE_z\mathbf{a}_z$$

Field of a Line Charge

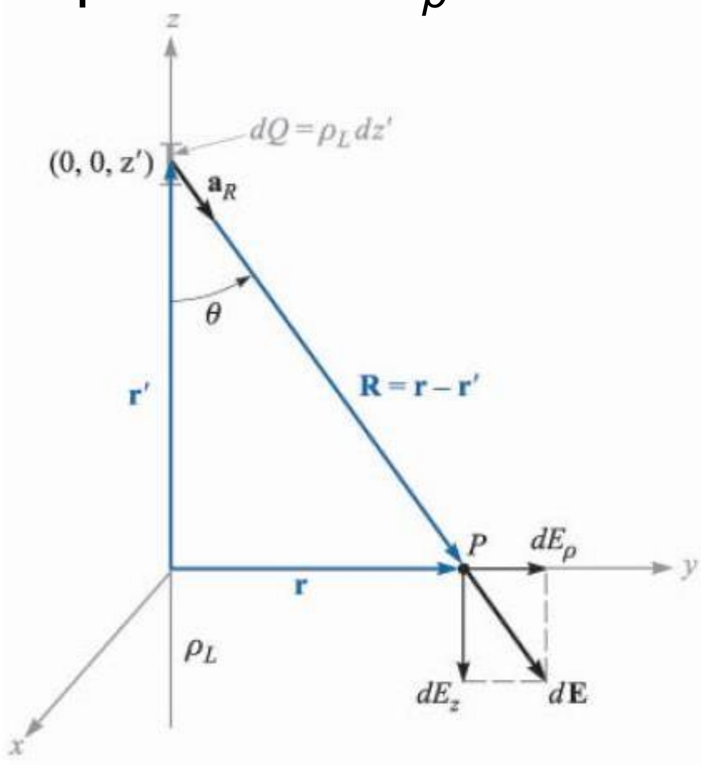
✓ The incremental field dE only has the components in $\mathbf{a}_\rho$ and $\mathbf{a}_z$ direction, and no $\mathbf{a}_\Phi$ direction.

- **Why?**

✓ The component dE_z is the result of symmetrical contributions of line segments above and below the observation point *P*.

✓ Since the length is infinity, they are canceling each other ► $dEz = 0$.

✓ The component dE_ρ exists, and from the Coulomb's law we know that dE_ρ will be inversely proportional to the distance to the line charge, ρ.



$$d\mathbf{E} = d\mathrm{E}_\rho\, \mathbf{a}_\rho + dE_z \mathbf{a}_z$$

Field of a Line Charge

- ✓ Take $P(0,y,0)$,

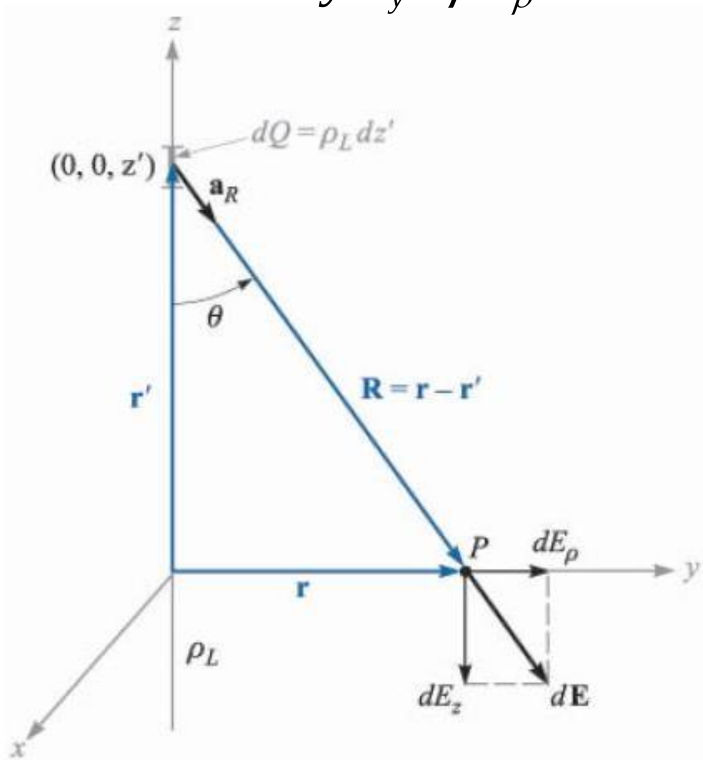
$$\mathbf{r}' = z'\mathbf{a}_z$$

$$\mathbf{r} = y\mathbf{a}_y = \rho\mathbf{a}_\rho$$

$$d\mathbf{E} = \frac{1}{4\pi\varepsilon_0}\frac{dQ(\mathbf{r}-\mathbf{r}')}{|\mathbf{r}-\mathbf{r}'|^3}$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{\rho_L dz'(\rho\mathbf{a}_\rho - z'\mathbf{a}_z)}{(\rho^2+z'^2)^{3/2}}$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{\rho_L \rho\mathbf{a}_\rho dz'}{(\rho^2+z'^2)^{3/2}}$$



$$d\mathbf{E} = dE_\rho\mathbf{a}_\rho + dE_z\mathbf{a}_z$$

$$E_\rho = \int_{-\infty}^{+\infty}\frac{1}{4\pi\varepsilon_0}\frac{\rho_L \rho dz'}{(\rho^2+z'^2)^{3/2}}$$

$$= \frac{\rho_L\rho}{4\pi\varepsilon_0}\left[\frac{z'}{\rho^2(\rho^2+z'^2)^{1/2}}\right]_{-\infty}^{+\infty}$$

$$E_\rho = \frac{\rho_L}{2\pi\varepsilon_0\rho}$$

$$\int\frac{du}{(a^2+u^2)^{3/2}} = \frac{u}{a^2\sqrt{a^2+u^2}} + C$$

Field of a Line Charge

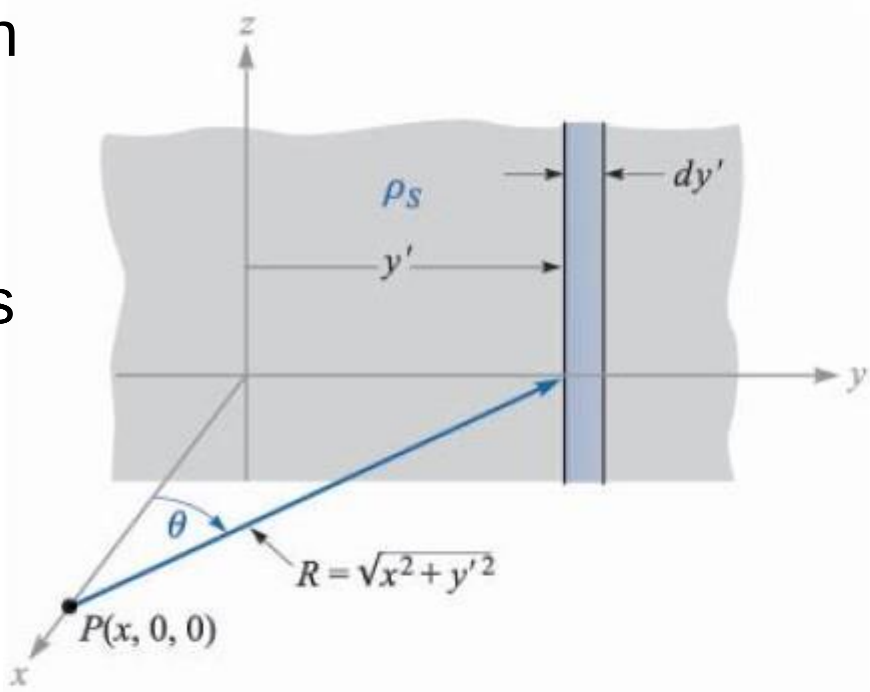
- Now let us analyze the answer itself:

$$\mathbf{E} = \frac{\rho_L}{2\pi\varepsilon_0 \rho}\mathbf{a}_\rho$$

- The field falls off inversely with the distance to the charged line, as compared with the point charge, where the field decreased with the square of the distance.

Field of a Sheet of Charge

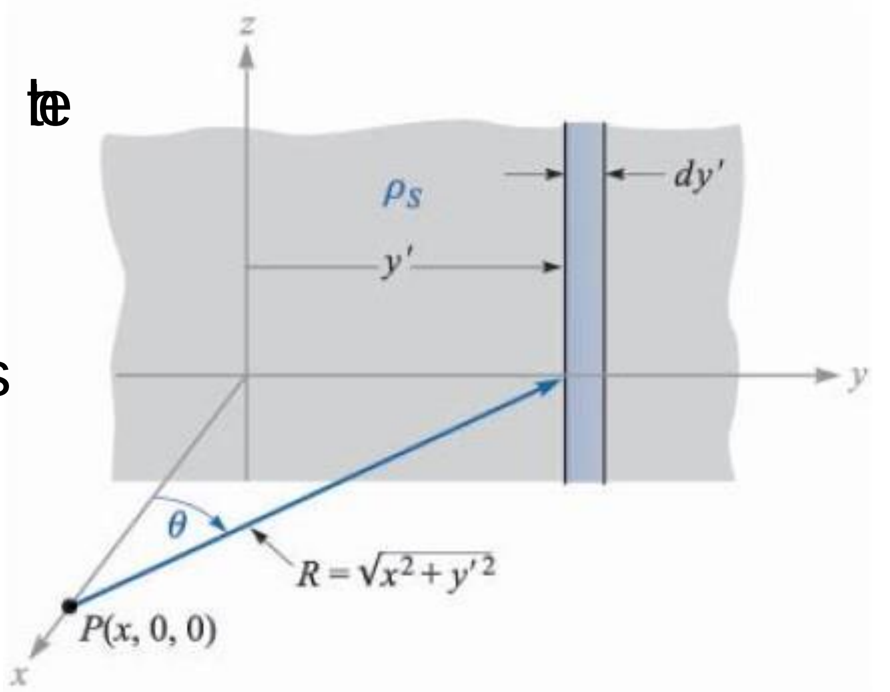
- Another basic charge configuration is the infinite sheet of charge having a uniform density of ρ_S C/m^2.
- The charge-distribution family is now complete: point (Q), line (ρ_L), surface (ρ_S), and volume (ρ_v).



- Let us examine a sheet of charge above, which is placed in the yz plane.
- The plane can be seen to be assembled from an infinite number of line charge, extending along the z axis, from $-\infty$ to $+\infty$.

Field of a Sheet of Charge

- For a differential width strip dy', the line charge density is given by $\rho_L = \rho_S dy'$.
- The component dE_z at P is zero, because the differential segments above and below the y axis will cancel each other.
- The component dE_y at P is also zero, because the differential segments to the right and to the left of z axis will cancel each other.
- Only dE_x is present, and this component is a function of x alone.

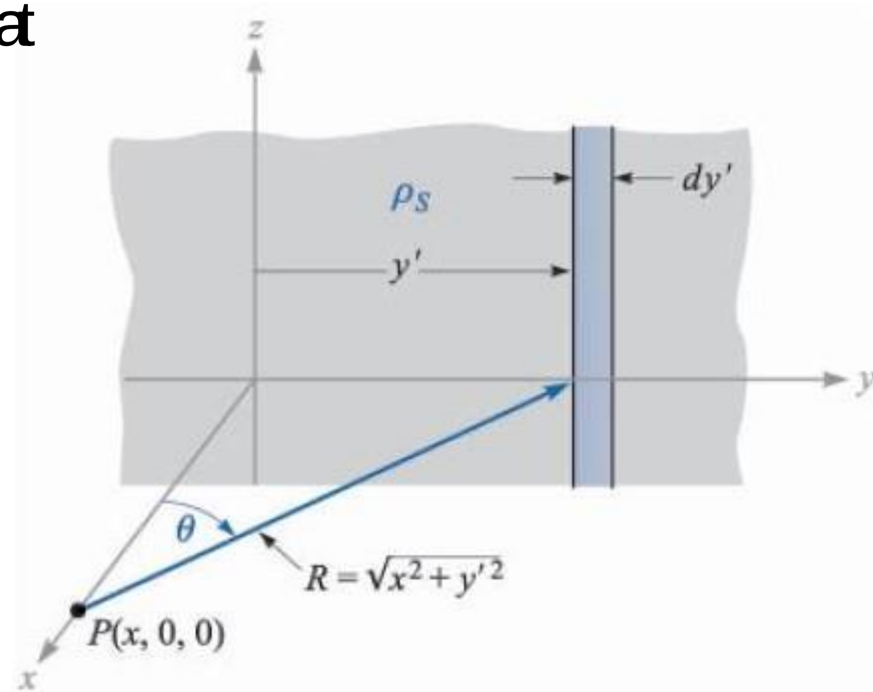


Field of a Sheet of Charge

- ✓ The contribution of a strip to E_x at P is given by:

$$dE_x = \frac{\rho_s dy'}{2\pi\varepsilon_0\sqrt{x^2 + y'^2}}\cos\theta$$

$$= \frac{\rho_s}{2\pi\varepsilon_0}\frac{x dy'}{x^2 + y'^2}$$



- ✓ Adding the effects of all the strips,

$$E_x = \frac{\rho_s}{2\pi\varepsilon_0}\int_{-\infty}^{+\infty}\frac{x dy'}{x^2 + y'^2}$$

$$= \frac{\rho_s}{2\pi\varepsilon_0}\left[\tan^{-1}\frac{y'}{x}\right]_{-\infty}^{+\infty}$$

$$= \frac{\rho_s}{2\varepsilon_0}$$

$$\int \frac{du}{a^2 + u^2} = \frac{1}{a}\tan^{-1}\frac{u}{a} + C$$

Field of a Sheet of Charge

- **Fact**: The electric field is always directed away from the positive charge, into the negative charge.
- We now introduce a unit vector $\mathbf{a}_N$, which is normal to the sheet and directed away from it.

$$\mathbf{E} = \frac{\rho_s}{2\varepsilon_0}\mathbf{a}_N$$

- The field of a sheet of charge is constant in magnitude and direction. It is not a function of distance.

Electric Flux Density

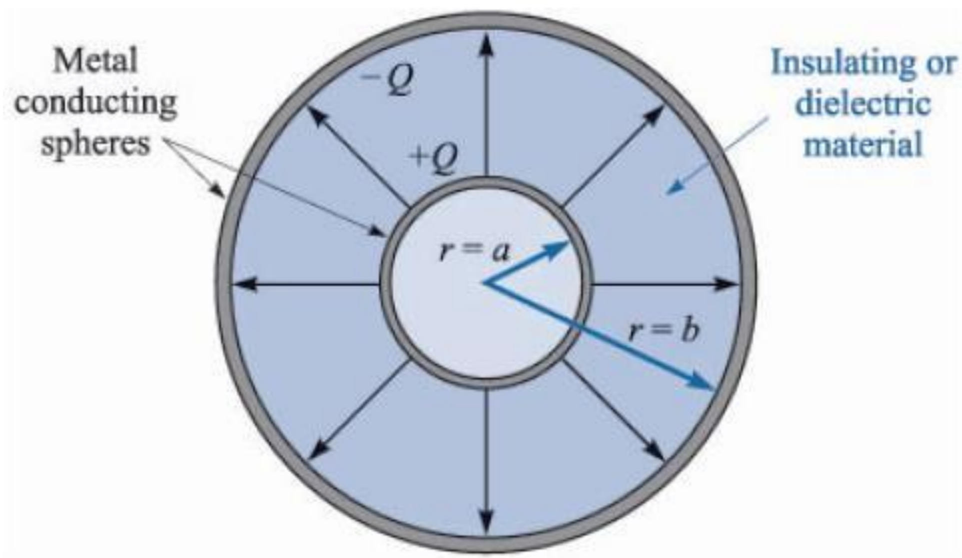
ELECTRIC FLUX DENSITY

Electric Flux Density

- ✓ About 1837, the Director of the Royal Society in London, Michael Faraday, was interested in static electric fields and the effect of various insulating materials on these fields.
- ✓ This is the lead to his famous invention, the electric motor.
- ✓ He found that if he moved a magnet through a loop of wire, an electric current flowed in the wire. The current also flowed if the loop was moved over a stationary magnet.
- ► Changing magnetic field produces an electric field.

Electric Flux Density

- In his experiments, Faraday had a pair of concentric metallic spheres constructed, the outer one consisting of two hemispheres that could be firmly clamed together.
- He also prepared shells of insulating material (or *dielectric* material), which would occupy the entire volume between the concentric spheres.



Electric Flux Density

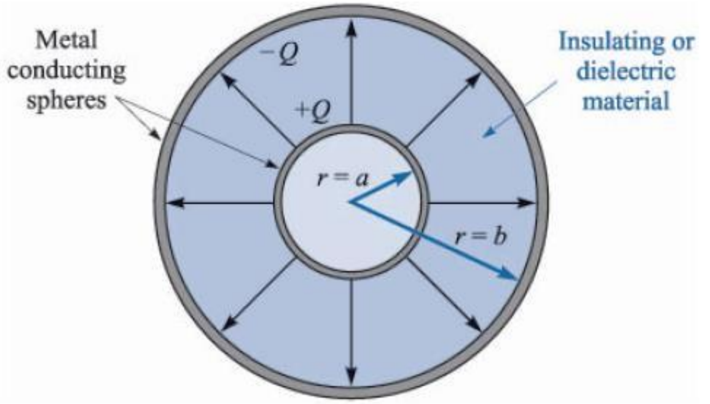
- ✓Faraday found out, that there was a sort of “charge displacement” from the inner sphere to the outer sphere, which was independent of the medium.
- ✓We refer to this flow as **displacement**, **displacement flux**, or simply **electric flux**.

$$\psi = Q$$

- ✓ Where ψ is the electric flux, measured in coulombs, and Q is the total charge on the inner sphere, also in coulombs.

Electric Flux Density

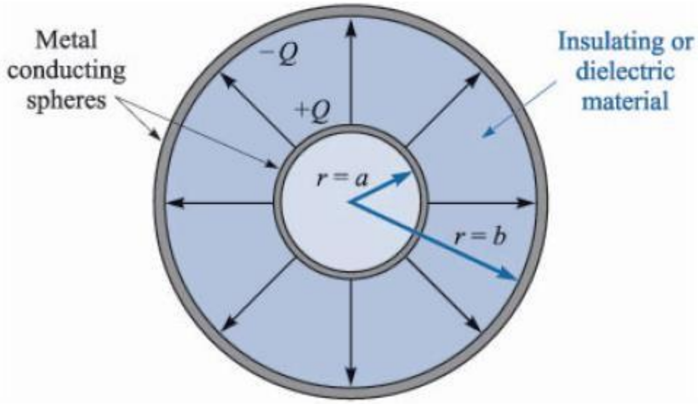
- At the surface of the inner sphere, ψ coulombs of electric flux are produced by the given charge Q coulombs, and distributed uniformly over a surface having an area of $4\pi a^2$ m^2.
- The density of the flux at this surface is $\psi/4\pi a^2$ or $Q/4\pi a^2$ C/m^2.



- The new quantity, *electric flux density*, is measured in C/m^2 and denoted with **D**.
- The direction of **D** at a point is the direction of the flux lines at that point.
- The magnitude of **D** is given by the number of flux lines crossing a surface normal to the lines divided by the surface area.

Electric Flux Density

- ✓ Referring again to the concentric spheres, the electric flux density is in the radial direction :



$$\mathbf{D}\big|_{r=a} = \frac{Q}{4\pi a^2}\mathbf{a}_r \qquad \text{(inner sphere)}$$

$$\mathbf{D}\big|_{r=b} = \frac{Q}{4\pi b^2}\mathbf{a}_r \qquad \text{(outer sphere)}$$

- ✓ At a distance *r*, where $a \leq r \leq b$, $\mathbf{D} = \frac{Q}{4\pi r^2}\mathbf{a}_r$

- ✓ If we make the inner sphere smaller and smaller, it becomes a point charge while still retaining a charge of *Q*. The electrix flux density at a point *r* meters away is still given by:

$$\mathbf{D} = \frac{Q}{4\pi r^2}\mathbf{a}_r$$

Electric Flux Density

- ✓ Comparing with the radial electric field intensity of a point charge in free space is:

$$\mathbf{E} = \frac{Q}{4\pi\varepsilon_0 r^2}\mathbf{a}_r$$

- ✓ Therefore, in free space, the following relation applies:

$$\mathbf{D} = \varepsilon_0 \mathbf{E}$$

- ✓ For a general volume charge distribution in free space:

$$\mathbf{E} = \int_{\text{vol}} \frac{\rho_v dv}{4\pi\varepsilon_0 R^2}\mathbf{a}_R$$

$$\mathbf{D} = \int_{\text{vol}} \frac{\rho_v dv}{4\pi R^2}\mathbf{a}_R$$