

## **Electrical Properties of Materials and Applications**

### **Syllabus:**

Classical and Quantum Free Electron Theory of Metals:

Classical Free Electron Theory of Metals:

Electrical Conductivity and Resistivity ,Mobility of conduction electrons,Concept of Phonon Mathiesen's Rule

Failures of classical free electron theory of metals ; Electronic specific heat of, Dependence of  $\sigma$  on temperature, Dependence of  $\sigma$  on the number density  $n$ .

Quantum free electron theory of metals

Assumptions, Fermi energy , Density of States (DoS) ,Fermi-Dirac distribution and Fermi factor Dependence of Fermi factor on energy and temperature

Superconductivity

Introduction to Superconductivity ,Meissner's Effect,Critical Field and its Temperature Dependence . Types of Superconductors BCS Theory of Superconductivity, High Temperature Superconductivity Quantum Tunneling, AC and DC Josephson Junctions, DC Josephson Effects , AC Josephson Effect ,DC and RF SQUIDS; DC SQUID , RF (AC)

Applications of Superconductivity in Quantum Computing ,Charge Qubit , Flux Qubit ,Phase Qubit Model Questions Numerical Problems .

## Classical Free Electron Theory of Metals

### Electrical Conductivity in metals

Consider a conductor carrying electric current with area of cross of section A perpendicular to the current.

Electric current through a conductor is  $\mathbf{I} = n e A \mathbf{V}_d$

Where, n is the number of free electrons per unit volume or electron concentration which can be evaluated by the relation as

$$n = \frac{(\text{no of free electron per atom}) * \text{Avogadro's number} * \text{density}}{\text{atomic weight}}$$

$$n = \frac{(\text{no of free electron per atom}) * 6.02 * 10^{26} * D}{W}$$

The charge on electron,  $e = 1.609 \times 10^{-19} \text{ C}$ , mass of an electron,  $m = 9.1 \times 10^{-31} \text{ kg}$

### Drift velocity $\mathbf{V}_d$ :

It is the average velocity acquired by the electrons in a direction opposite to the direction of the applied electric field, which is given by

$$v_d = \frac{eE\tau}{m}$$

Where E is the strength of the applied electric field and  $\tau$  is the relaxation time

**Current density  $\mathbf{J}$ :** It is defined as the ratio of current to the cross section A of a conductor, Hence  $J = \frac{I}{A}$  A/m<sup>2</sup>

It is observed that current density is proportional to the applied electric field

$$J \propto E$$

$$J = \sigma E$$

The constant of proportionality  $\sigma$  is called electrical conductivity of the conductor.

**Electrical resistivity**  $\rho = \frac{1}{\sigma}$ :

It is the reciprocal of the electrical conductivity of the material and it is the property of the material by virtue of which it opposes the flow of current through it.

### **Mobility of conduction electrons $\mu$ :**

It is defined as the drift velocity acquired by the conduction electrons per unit electric field strength, which is given by

$$\mu = \frac{v_d}{E} = \frac{\left(\frac{eE\tau}{m}\right)}{E} = \frac{e\tau}{m}$$

On the basis of classical free electron theory, the electrical conductivity can be expressed as

$$\sigma = \frac{ne^2\tau}{m}$$

From the above relations,

$$\mu = \frac{ne}{\sigma} \quad \text{or} \quad \mu = \rho ne \quad \text{m}^2/\text{v.s}$$

### **Concept of Phonon:**

A phonon is a particle like entity which carries the energy of elastic field in a particular mode is called phonon. It is quantum of lattice vibrations in which a lattice of atoms or molecules vibrates. The name phonon comes from the Greek word which translates to sound or voice, because long-wavelength phonons give rise to sound.

A solid material consists of periodic array of atoms in 3-D is called lattice. According classical free electrons theory, the thermal energy make lattice to vibrate because elastic property of the material. This generates mechanical waves that carry heat and sound through the material. A packet of these waves can travel throughout the material with a definite energy and momentum. The energy of these lattice vibrations is quantized and quantum of the energy is called phonon.

The phonon plays an important role in the thermal, electrical, acoustic properties, and also essential in the phenomenon of superconductivity.

### **Matheissen's rule:**

A metal consists of lattice ions and impurity atoms that are held by together free electrons. The free electrons are moving inside the metal. During the motion, electrons undergo scattering by lattice ions and impurity atoms so that resistivity arises due to two regions;

1. Scattering of electrons with the vibrating lattice ions. The resistivity of the metal due to electron lattice ions scattering is given by

$$\rho_{ph} = \frac{m}{ne^2\tau_{ph}}$$

2. Scattering of electrons by the presence of impurities present in the metal. The resistivity also occurs from the lattice dislocations and grain boundaries. The resistivity of the metal due to such scattering is given by

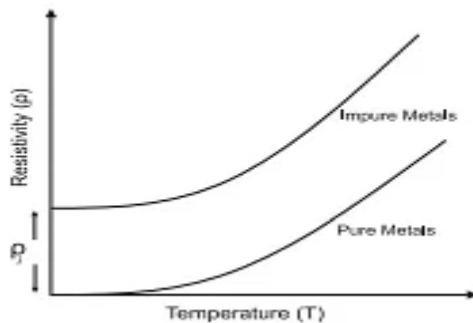
$$\rho_i = \frac{m}{ne^2\tau_i}$$

The total resistivity of the material is given by

$$\rho = \rho_{ph} + \rho_i$$

$$\rho = \frac{m}{ne^2\tau_{ph}} + \frac{m}{ne^2\tau_i}$$

The above equation is called **Matheissen's rule**, which states that the total resistivity of a metal is the sum of the resistivity due to phonon scattering which is temperature dependent and the resistivity due to the presence of impurities which is temperature independent. Below figure depicts the variation of resistivity with temperature and impurities.



### FAILURES OF CLASSICAL FREE ELECTRON THEORY:

#### 1. Specific heats for metals:

According to classical free electron theory, the specific heat of metals is given by

$$C_V = \frac{3}{2} R = 12.5 \text{ Jmol}^{-1}\text{K}^{-1}$$

The experimental value of specific heat is  $C_V = 10^{-4} RT$ . This is very small and also temperature dependent. Hence, classical free electron theory could not explain specific heat.

#### 2. Temperature dependence of electrical conductivity:

According to classical free electron theory of metals, the electrical conductivity is inversely proportional to square root of temperature, i.e

$$\sigma = \frac{ne^2\tau}{m} \Rightarrow \sigma \propto \frac{1}{\sqrt{T}}$$

But, experiments reveal that electrical conductivity is inversely proportional to temperature

$$\sigma \propto \frac{1}{T}$$

Hence, classical free electron theory fails to explain Temperature dependence of electrical conductivity

### 3. Dependence of electrical conductivity on electron concentration:

As per classical free electron theory,  $\sigma = \frac{ne^2\tau}{m}$  that is  $\sigma \propto n$

But experiments reveal that electrical conductivity of some metals as follows. The electron concentration 'n' of zinc and Aluminium are  $13.10 \times 10^{28}/m^3$  and  $18.06 \times 10^{28}/m^3$  respectively. But these metals comparatively less conducting than Cu and Ag which have values of 'n' is  $8.45 \times 10^{28}/m^3$  and  $5.85 \times 10^{28}/m^3$  respectively. This indicates that  $\sigma \propto n$  does not hold good. Hence, classical free electron theory fails to explain dependence of electrical conductivity on electron concentration.

### Quantum Free Electron Theory

The failure of classical free electron theory led to the development of quantum free electron theory and was proposed by Arnold Sommerfeld in the year 1928. The quantum free electron theory is based on the following assumptions.

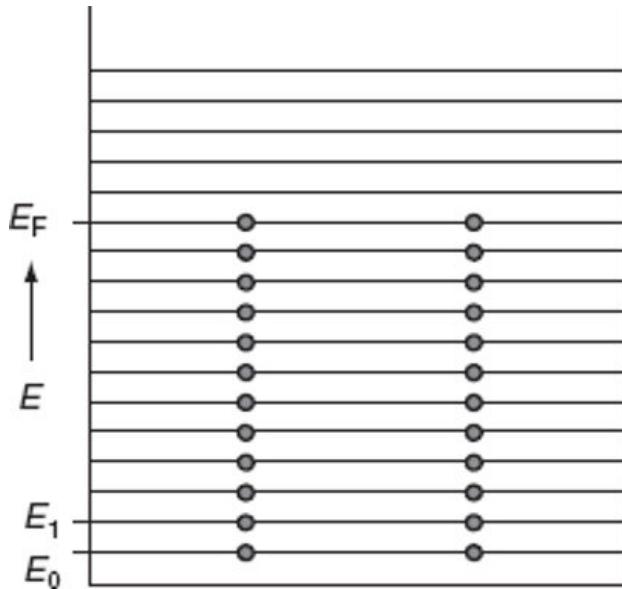
#### **Assumptions**

1. All metals consist of large number of free electrons which are responsible for electrical conduction.
2. The energy values of the free electrons are discrete.
3. In metal there exists large number of closely spaced energy levels for free electrons which form a band.
3. The distribution of electrons in the various allowed energy levels is according to Pauli's exclusion principle. According to that maximum of two electrons can occupy in a given energy level. This suggests the availability of two states for free electrons in the energy level corresponds to spin up and spin down states.
4. The mutual interaction between repulsion between electrons and the attraction between the electrons and lattice ions are ignored.
5. The electrons travel in a constant potential inside the metal but stay confined within its boundaries.

**Fermi Energy:** It is denoted by  $E_F$ .

We know that, metal containing N number of atoms and there will be N number of allowed energy levels in each band. These energy levels are closely spaced. The energy levels in bands fill up as per Pauli Exclusion Principle. The free electrons in a metal start filling up the

available energy levels from the lowest energy level of the valence band. **The highest filled energy level in a metal at absolute zero by free electrons is called Fermi level and corresponding energy is called Fermi Energy.** Thus, at  $T=0K$  and no electric field applied all the energy levels below Fermi energy is completely filled and above the Fermi energy are empty.



### Density of States: It is denoted by $g(E)dE$ .

The free electrons are quantum particles and the distribution of electrons amongst various energy levels is as per Pauli Exclusion Principle. According to the band theory, the energy bands are formed in solids and in energy bands the spacing between two successive energy levels decreases with increase in energy.

**The define density of states defined as “the number of energy states available per unit volume of the material in the unit energy range in the valence band of the material. Mathematically, the density of states in the energy range  $E$  and  $E+dE$  per unit volume of the material is given by**

$$g(E)dE = \left( \frac{8\sqrt{2} \pi m^{(3/2)}}{h^3} \right) \sqrt{E} dE$$

From the above relations,  $\pi$ ,  $m$ , and  $h$  are constants, therefore,  **$g(E)dE$  is proportional to  $\sqrt{E}$**  and its variation of  $g(E)dE$  as a function of  $E$  is shown in figure as

### Fermi-Dirac distribution function and Fermi factor.

The occupation of energy levels by free electrons in the valence of a metal is according to Pauli Exclusion Principle. This distribution is not random but is statistical in nature which is called Fermi-Dirac statistics. It is named after Enrico Fermi and Paul Dirac.

Fermi-Dirac statistics describes a distribution of particles over energy states in a systems consisting of identical particles with half-integer spin that obey "Pauli exclusion principle" under thermal equilibrium and such particles are called fermions. Since, electrons with spin half satisfy these conditions and they obey Fermi-Dirac statistics.

In F-D statistics, the probability of occupation of an energy level (E) at temperature (T) under thermal equilibrium is evaluated using a function called finding Fermi factor or probability factor, which is denoted as  $f(E)$ .

$$f(E) = \frac{1}{1 + e^{\left(\frac{E-E_F}{kT}\right)}}$$

**Where**, E is the energy state being occupied by electron at temperature T, and  $E_F$  is the Fermi energy, k is the Boltzmann constant,  $k = 1.38 \times 10^{-23} \text{ J/K}$ .

### **Variation of Fermi Factor with Temperature and Energy:**

As Fermi factor is a function of energy and temperature. This dependence can be explained for energy levels below and above Fermi level at absolute zero and higher temperature.

**Case I:** Probability of occupation of levels with energy  $E < E_F$  and at  $T=0\text{K}$ ,

Fermi factor is given by the relation

$$f(E) = \frac{1}{1 + e^{\left(\frac{E-E_F}{kT}\right)}}$$

In this case  $E - E_F$  is negative and  $T=0\text{K}$

$$f(E) = \frac{1}{1 + e^{(-\infty)}} = \frac{1}{1 + 0}$$

$$f(E) = 1$$

Therefore,  $f(E)=1$  at  $T=0\text{K}$  means all the energy level the energy levels below Fermi level are completely occupied.

**Case II:** Probability of occupation of levels with energy  $E > E_F$  and at  $T=0\text{K}$ .

In this case  $E - E_F$  is positive and  $T=0\text{K}$

$$f(E) = \frac{1}{1 + e^{\left(\frac{E-E_F}{kT}\right)}} = \frac{1}{1 + e^{(\infty)}}$$

$$f(E) = \frac{1}{1 + \infty} = 0$$

$$f(E) = 0$$

Therefore,  $f(E)=0$  at  $T=0\text{K}$  means the energy levels above Fermi energy are empty.

In view of the above two cases, the variation of  $f(E)$  is a step function as shown in figure.

**Case III:** Probability of occupation of levels with energy  $E = E_F$  and at  $T > 0K$

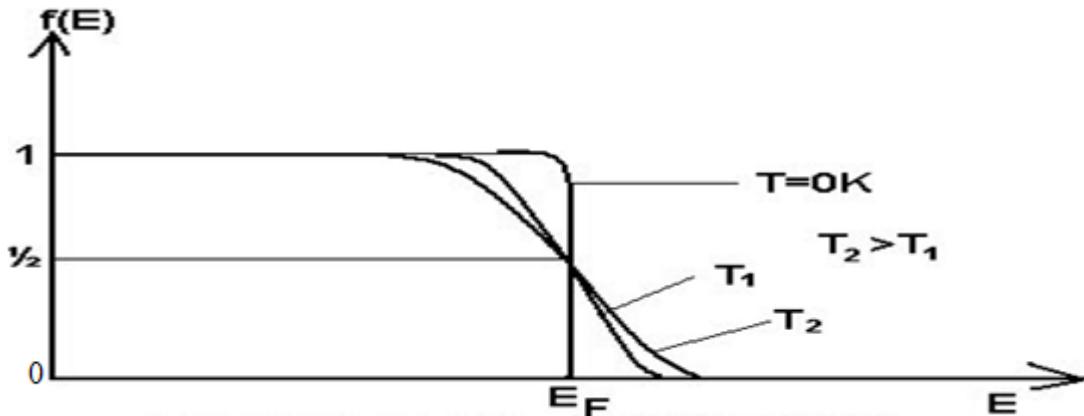
In this case  $E - E_F = 0$ , substituting the value in Fermi factor

$$f(E) = \frac{1}{1 + e^{\left(\frac{E-E_F}{kT}\right)}} = \frac{1}{1 + e^{\left(\frac{0}{kT}\right)}}$$

$$f(E) = \frac{1}{1 + 1} = 0.5$$

$$f(E) = 0.5$$

Therefore for all the temperature above 0K the probability of occupation of energy level is  $\frac{1}{2}$   
Figure shows the variation of Fermi factor with temperature



### 3.2 Superconductivity

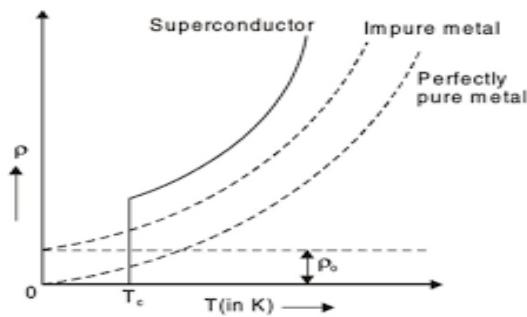
#### Introduction to Superconductivity Discovery:

Lord Kamerlingh Onnes discovered the phenomenon of superconductivity in the year 1911. When he was studying the temperature dependence of resistance of Mercury at very low temperature he found that resistance of Mercury decreases with temperature with the decrease in temperature up to a particular temperature  $T_c = 4.15K$ . Below this temperature the resistance of mercury abruptly drops to zero. Between 4.15K and 0K Mercury offered no resistance for the flow of electric current. This phenomenon is reversible and material becomes normal once again when temperature was increased above 4.15K. This phenomenon is called superconductivity and material which exhibits this property is named superconductor.

**Definition of superconductivity:** The Superconductivity is defined as “The phenomenon in which resistance of certain metals, alloys and compounds drops to zero abruptly, below certain temperature is called superconductivity

### **Temperature dependence of resistivity:**

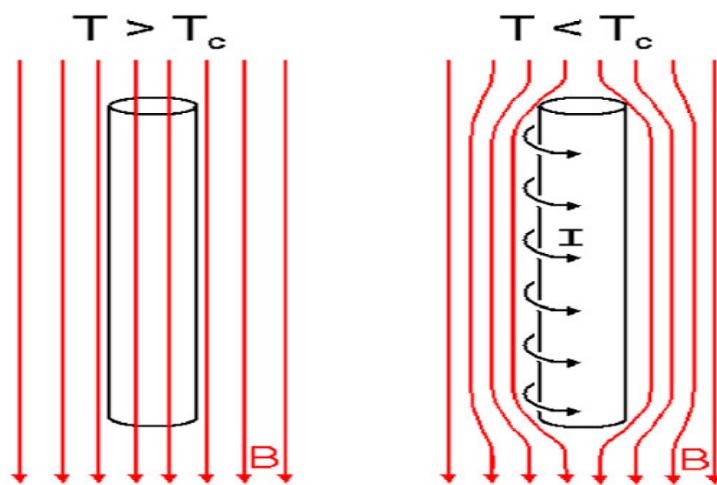
As we know that the resistivity of materials depends on the temperature. This variation of the resistivity of a superconductor, pure and impure metals with temperature is as shown below.



**The temperature, below which materials exhibit superconducting property is called critical temperature, denoted by  $T_c$ .** Critical temperature is different for different substances. The materials, which exhibit superconducting property, are called superconductors. Above critical temperature material is said to be in normal state and offers resistance for the flow of electric current. Below critical temperature material is said to be in superconducting state so that  $T_c$  is also called as transition temperature.

### **Meissner's Effect**

In 1933, Meissner showed that when a superconducting material is placed in a magnetic field it allows magnetic flux to pass through, if it's temperature is above  $T_c$ . If the temperature is reduced below the critical temperature  $T_c$  then it expels all the flux lines completely out of the material and exhibits perfect diamagnetism. This is known as Meissner's effect. Since superconductor exhibits perfect diamagnetism below the critical temperature  $T_c$ , then magnetic flux density inside the material is zero.



The expression for magnetic flux density is given by

$$B = \mu_0(H + M)$$

Here B is Magnetic Flux Density, M is Magnetization and H is the applied magnetic field strength.

According to Meissner effect, when the material is in superconducting state,  $B = 0$  at  $T < T_c$

We get,  $H = -M$  which signifies the negative magnetic moment associated with superconductors

Therefore Magnetic susceptibility  $\chi = \frac{M}{H} = -1$ . This is the indication for a perfect diamagnetic material.

### **Critical Field and its temperature dependence:**

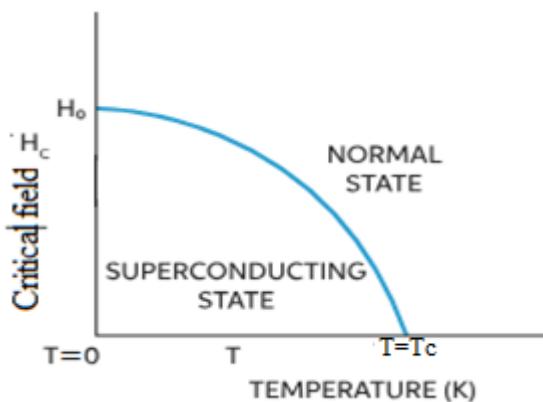
As we know that when superconductor is placed in a magnetic field it exhibits a perfect diamagnetism. But if the strength of the magnetic field is further increased, it is found that for a particular value of the magnetic field, material loses its superconducting property and becomes a normal conductor. **The value of the magnetic field at which the transition occurs from the Superconducting state to Normal Conducting state is called Critical Field or Critical Magnetic Field and is denoted by  $H_c$ .**

It is found that by reducing the temperature of the material further superconducting property of the material could be restored. Thus, critical field does not destroy the superconducting property of the material completely but only reduces the critical temperature of the material.

The variation of Critical field with temperature below the critical temperature is given by

$$H_c = H_0 \left[ 1 - \frac{T^2}{T_c^2} \right]$$

Here  $H_c$  is the Critical field at any temperature  $T < T_c$  and  $H_0$  is the critical field at  $T=0K$ .



### **Types of Super Conductors:**

Superconductors are classified into two types;

1. Type I Superconductor or Soft Superconductor.
2. Type II Superconductor or Hard Superconductor.

#### **Type -1 superconductor (Soft superconductors)**

Type I superconductors exhibit complete Meissner's Effect. The graph of magnetic moment  $V/s$  magnetic field is as shown in the Figure 1. As the field strength increases the material becomes more diamagnetic until  $H$  becomes equal to  $H_c$ . Above  $H_c$  the material allows the magnetic flux to pass through and exhibits normal conductivity. The value of critical field  $H_c$  is very small for soft superconductors. Therefore soft superconductors cannot withstand high magnetic fields. Therefore they cannot be used for making superconducting magnets. Ex. Hg, Pb and Zn.

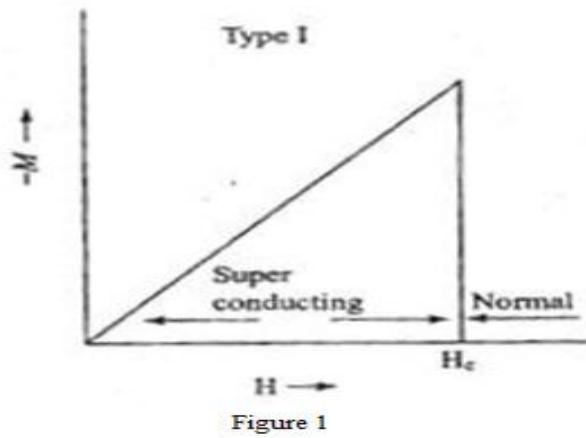


Figure 1

#### **Type -2 Superconductors (Hard Superconductors):**

They do not exhibit complete Meissner effect. The superconductors are characterized by two critical fields  $H_{c1}$  and  $H_{c2}$  namely lower critical field and upper critical field respectively. The graph of magnetic moment  $V/s$  magnetic field is as shown in the Figure 2. When  $H < H_{c1}$  material exhibits perfect diamagnetism. Beyond  $H_{c1}$  partial flux penetrates and the material is said to be Vortex State. Thus flux penetration occurs through small-channelized regions called filaments. As  $H > H_{c1}$  more and more flux fills the body and thereby decreasing the diamagnetic property of the material. At  $H_{c2}$  flux fills the body completely and material loses its diamagnetic property as well as superconducting property completely. The  $H_{c2}$  value is greater than  $H_c$  of soft superconductors. Therefore they are used for making powerful superconducting magnets. Examples: NbTi,  $Nb_3Sn$ .

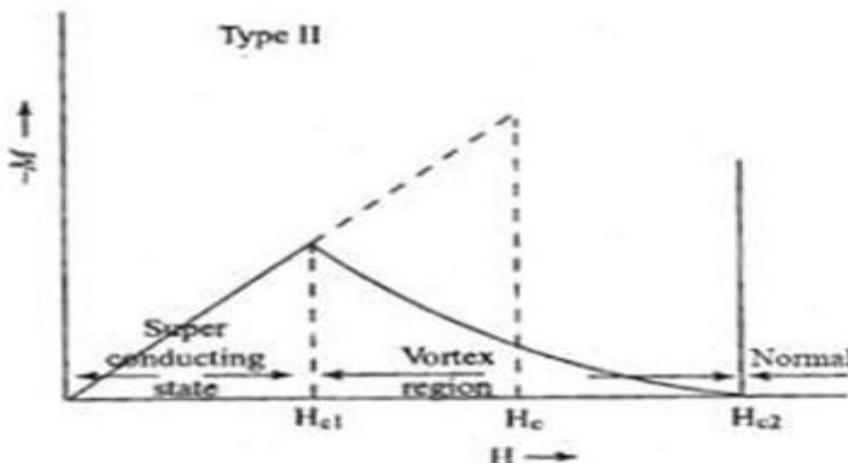


Figure 2

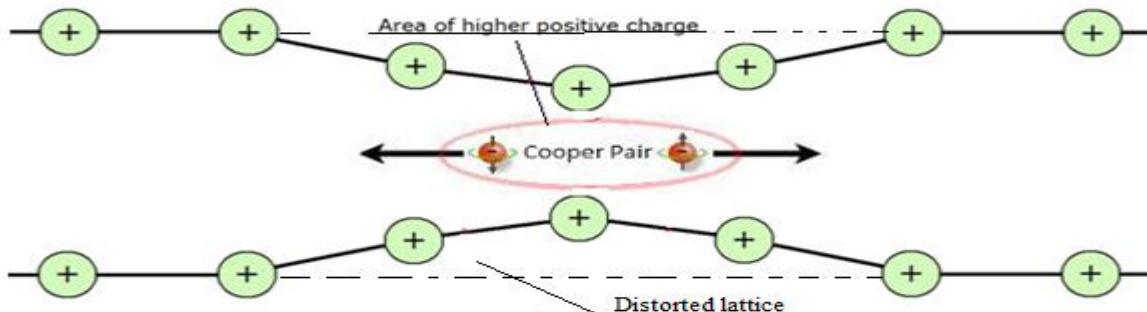
### BCS theory of superconductivity:

Bardeen, Cooper and Schrieffer explained the phenomenon of superconductivity in the year 1957. According to this theory, Superconductivity is due to the formation of cooper pairs. "Cooper pairs are the pair of electrons formed by the interaction between electrons with opposite spin and momentum in a phonon field.

Consider an electron approaching a positive ion core and suffers attractive coulomb interaction. Due to this attraction, ion core is set in motion and distorts that lattice. Let a second electron come in the way of distorted lattice and interaction between the two occurs which lowers the energy of the second electron. The two electrons therefore interact indirectly through the lattice distortion or the phonon field which lowers the energy of the electrons. This interaction is interpreted as electron - Lattice - electron interaction through phonon field.

It was shown by Cooper that, this attractive force becomes maximum if two electrons have opposite spins and momentum. The attractive force may exceed coulombs repulsive force between the two electrons below the critical temperature, which results in the formation of bound pair of electrons called cooper pairs. Each cooper pair causes the formation of many numbers of such pairs, causing the formation of cloud of cooper pairs.

Below the critical temperature the dense cloud of Cooper pairs forms a collective state and the motion all Cooper pairs is correlated resulting in zero resistance of the material.



### High Temperature superconductivity:

Superconducting materials which exhibit superconductivity at relatively higher temperatures are called high temperature superconductors. Thus high temperature superconductors possess higher value of critical temperature compared to conventional superconductors. Most of the high temperature superconductors are found to fall into the category of ceramics. All high temperature superconductors are oxides of copper and bear Perovskite crystal structure characterized by large number of copper-oxygen layers. It was found that addition of extra copper-oxygen layer pushes the critical temperature  $T_c$  to higher values. The super currents are strong in the copper-oxygen layer and weak in the direction perpendicular to the planes.

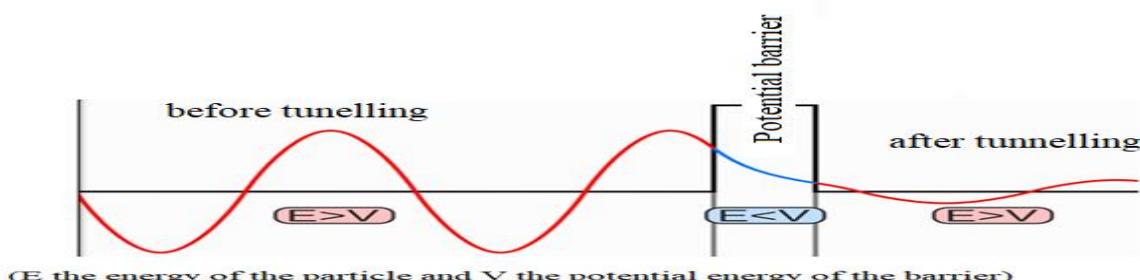
Ex: In 1986 George Bednorz and Alex Muller discovered a compound containing Lanthanum, Barium, Copper and Oxygen having  $T_c = 30K$  was developed. In 1987 scientists developed a compound which is an oxide of the form  $YBa_2Cu_3O_7$  which is referred to as 1-2-3 compound with  $T_c > 90K$  was discovered.

Following is the list of High Temperature Superconductors.

Material	$T_c$ [K]
Pb	7.2
Nb	9.2
Nb-Ti alloys	~ 9.6
$Nb_3Sn$	18.1
$Nb_3Ge$	23.2
$YBa_2Cu_3O_{7-x}$ (YBCO)	90
$Bi_2Sr_2Ca_1Cu_2O$ (Bi2212)	80
$Bi_2Sr_2Ca_2Cu_3O$ (Bi2223)	125

### Quantum Tunnelling:

In Physics, quantum tunnelling or barrier penetration is a quantum mechanical phenomenon in which a particle such as an electron or atom passes through a potential energy barrier. This concept is not possible, according to classical mechanics.



Tunneling is an outcome of wave nature of matter and is found in low mass particles like electrons, protons etc. Probability of transmission of a wave through a barrier decreases

exponentially with the barrier height. When the quantum wave reaches the barrier, its amplitude will decrease exponentially which corresponds to probability of finding the particle further into barrier. If the barrier is thin enough then the amplitude may be non-zero on the other side. This implies there is a finite probability that some of the particles will tunnel through a barrier.

\*Tunnelling applications include quantum computing, flash memory and the scanning tunnelling microscope (STM). Tunnelling limits the minimum size of devices used in microelectronics because electrons tunnel readily through insulating layers and transistors that are thinner than about 1 nm\*

### **Josephson Junctions (Qualitative):**

In 1962, Brian Josephson predicted that cooper pairs could tunnel through thin insulating layer separating two superconductors. The superconductor –insulator-superconductor layer constitutes Josephson **Junctions**.

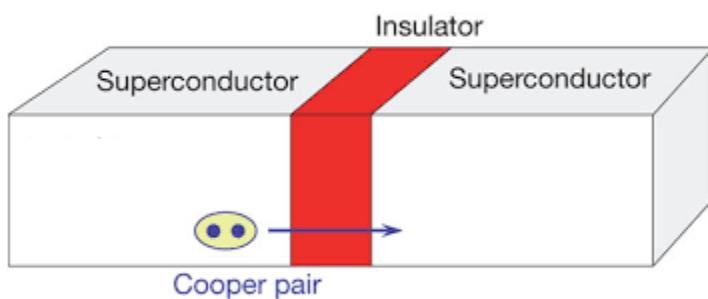
This phenomenon of flow of current between two pieces of superconductor separated by a normal material is called as Josephson effect and the current is called Josephson current. The current flows through the junction even in the absence of external DC voltage. Hence the Josephson current is present in the absence of supply voltage. If the external DC voltage is applied, current oscillates rapidly with a frequency of several GHz, leading to the development of AC voltage.

### **DC Josephson Effects:**

Consider a Josephson junction consisting of two superconducting films separated by a thin oxide layer of 1nm to 2nm thick. Let it be connected in a circuit as shown in figure. The cooper pairs in superconductors can be represented by a wavefunction. The cooper pairs tunnel through the oxide layer. The effect of the insulating layer introduces a phase difference between wavefunctions of cooper pairs. Due to this phase difference, super current flow through the junction, even in the absence of external source (zero voltage). This is known as DC Josephson effect. The super current through the junction is

$$I_s = I_c \sin\phi_0$$

$I_c$  = critical current at zero voltage, which depends on the thickness of the junction layer and the temperature,  $\phi_0$  = Phase difference between the wave functions of cooper pairs



### **AC Josephson Effects:**

When dc voltage is applied across the Josephson junction, it introduces an additional phase change on cooper pairs during tunnelling and generates an alternating current. This effect is called as AC Josephson effect. Thus the current is given by

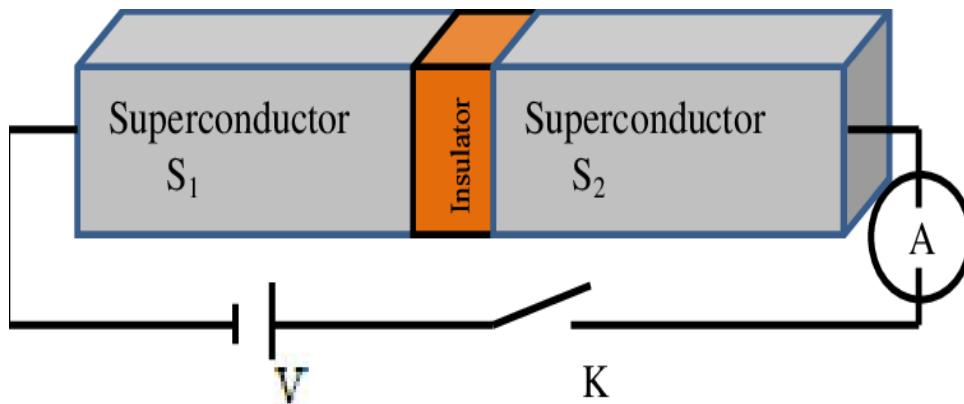
$$I_s = I_c \sin (\Phi_0 + \Delta\Phi)$$

$\Delta\Phi$  = Phase difference,  $I_c$  is the critical current

The frequency of the generated AC is

$$f = \frac{2eV}{h}$$

For example, if a voltage of about  $V=1\mu V$  is applied, and AC frequency of about 484 MHz can be obtained.



Because of the DC voltage applied across the barrier, the energy difference of cooper pairs on both sides is of the order of 2eV.

### **SQUIDS**

SQUIDS is the Superconducting Quantum Interference Device used to measure extremely weak magnetic field of the order of  $10^{-13}T$ . Hence it is a sensitive magnetometer made of a superconducting ring. Heart of the SQUID is a super conducting ring containing one or more Josephson junctions. Two types of SQUIDS are available namely DC SQUID and RF SQUID. It works on the principle of Josephson effect.

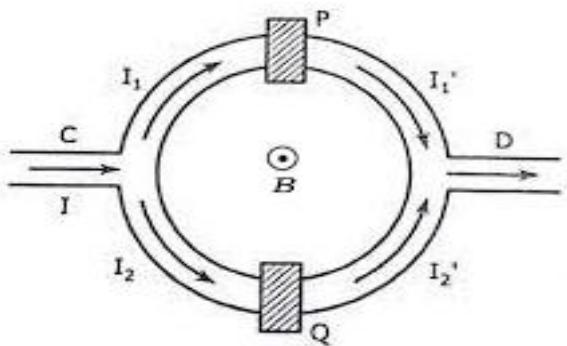
#### **1. DC SQUID:**

It has two Josephson junctions connected in parallel and works on the interference of current from two junctions. It works on the principle of DC Josephson effect which is the phenomenon of flow of super current through the junction even in the absence of external voltage.

#### **Construction and Working of DC SQUID:**

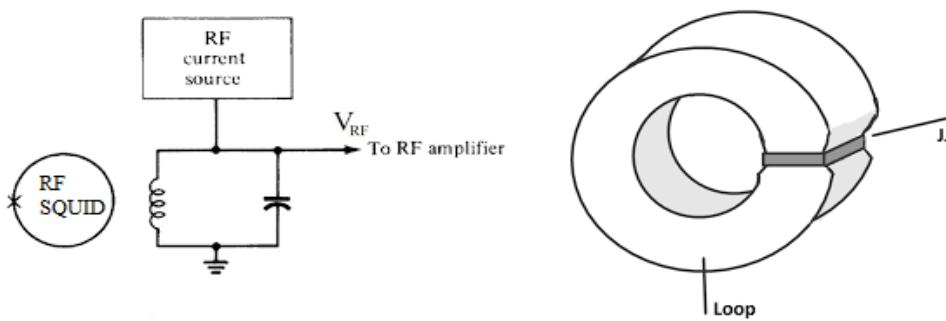
The cross sectional view of the arrangement is shown in figure. P and Q are two Josephson Junctions arranged in parallel. When current I flow through the point C, it divides into  $I_1$  and  $I_2$ . Hence the wave functions due to these super currents (cooper pairs) experience a phase shift at 1 and 2.

In the absence of applied magnetic field, the phase difference between the wave functions is zero. If the magnetic field is applied perpendicular to the current loop, then phase difference between the wave functions will not be zero. This can be identified by the sum of the currents  $I_1'$  and  $I_2'$ . The magnitude of phase difference is proportional to applied magnetic field. Hence, Even if there is a weak magnetic field in the region will be detected.



## 2. RF SQUID:

It works on the principle of AC Josephson effect - When dc voltage is applied across the Josephson junction, it leads to the development of oscillating current. It has single Josephson Junction as shown in figure. Magnetic field is applied perpendicular to the plane of the current loop. The flux is coupled into a loop containing a single Josephson junction through an input coil and an RF source. Hence when the RF current changes, there is corresponding change in the flux linked with the coil. This variation is very sensitive and is measured. It is also used in the detection of low magnetic field. It is less sensitive compared to DC SQUID.

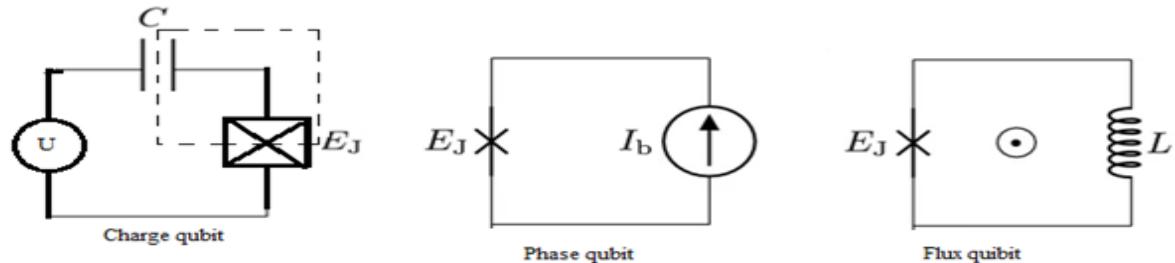


### Applications of superconductivity in Quantum Computing:

There are three basic designs for Josephson-junction qubits are in quantum computing are as follows; Charge qubits, Flux qubits and Phase qubits. The charge qubit is a box for charge,

controlled by an external voltage  $U$ , The phase qubit is a Josephson junction biased by a current  $I_b$ . The flux qubit is a loop, controlled by an external magnetic flux  $\Phi_{ext}$ .

Three basic Josephson-junction qubit circuits as shown in figure below.  $E_J$  Josephson energy,  $C$  the capacitor and  $L$  is the inductance.



### 1. Charge qubits:

In quantum computing, a charge qubit is also known as Cooper-Pair Box (CPB). It was one of the first superconducting qubits developed. It is a qubit whose basis states are charge states. The state represents the presence or absence of excess Cooper pairs in the island (dotted line in the figure). In superconducting quantum computing, a charge qubit is formed by a tiny superconducting island coupled by Josephson junction to a superconducting reservoir.

### 2. Phase qubits:

A phase qubit is a current-biased Josephson junction, operated in the zero voltage state with a non-zero current bias. This employs a single Josephson and the two levels are identified by quantum oscillations of the phase difference between the electrodes of the junction. DC SQUID is a type of phase qubit.

### 3. Flux qubits:

The flux qubit is another simple Josephson-junction qubit. It is also known as a persistent-current qubit. Flux qubits are micrometer sized loops of superconducting metal that is interrupted by a number of Josephson-junctions. These devices function as qubit. The Josephson-junctions are designed so that persistent current will flow continuously when external magnetic is applied.