Electrical Properties of Materials and Applications

Explain the resistivity and mobility of electrons:

Consider a conductor carrying electric current with area of cross of section A perpendicular to the current. The resistance R is directly proportional to its area of cross section.

$$R \propto \frac{l}{A}$$

$$R = \rho \frac{l}{A}$$

The constant ρ is called resistivity and it is a measure of the opposition offered by the material to flow of current. Its unit is Ω-m.

Mobility of conduction electrons μ: It is defined as the drift velocity acquired by the conduction electrons per unit electric field strength, which is given by

$$\mu = \frac{Drift\ velocity}{Electric\ field}$$

$$\mu = \frac{\sigma}{ne}$$

Explain the concept of phonon. Explain the effect of temperature and impurity on electrical resistivity of metals (Matheissen's rule)

A phonon is a particle like entity which carries the energy of elastic field. It is quantum of lattice vibrations in which a lattice of atoms or molecules vibrates.

In a solid material the thermal energy make atoms to vibrate because elastic property of the material. These vibrations are called lattice vibrations which are quantized and quantum of the energy is called phonon. The phonon plays an important role in the thermal, electrical, acoustic properties, and also essential in the phenomenon of superconductivity.

Matheissen's rule:

A metal consists of lattice ions and impurities and the free electrons are moving inside the metal. During the motion, electrons undergo scattering by lattice ions and impurity atoms so that resistivity arises due to two regions.

1. Resistivity due to lattice vibrations: Scattering of electrons with the vibrating lattice ions. The resistivity of the metal is given by

$$\rho_{ph} = \frac{m}{ne^2\tau_{ph}}$$

Here resistivity is due to temperature dependent. (ph means phonons).

2. Scattering of electrons by the presence of impurities in the metal. At low temperature, the resistivity is due to the concentration of impurities in the metals. The resistivity of the metal due to the scattering of electrons by impurities is given by

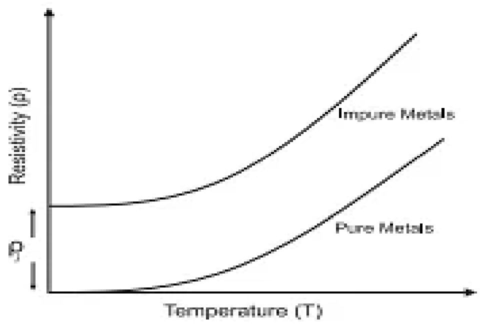
$$\rho_i = \frac{m}{ne^2\tau_i}$$

The total resistivity of the material is given by

$$\rho = \rho_{ph} + \rho_i$$

$$\rho = \frac{m}{ne^2\tau_{ph}} + \frac{m}{ne^2\tau_i}$$

The above equation is called **Matheissen's rule,** which states that the total resistivity of a metal is the sum of the resistivity due to phonon scattering which is temperature dependent and the resistivity due to the presence of impurities which is temperature independent. Below figure depicts the variation of resistivity with temperature and impurities.



Enumerate the failures of classical free electron theory and assumptions of quantum free electron theory.

i) Specific heats for metals
ii) Temperature dependence of electrical conductivity.
iii) Dependence of electrical conductivity on electron concentration.

i) Specific heats for metals:

According to classical free electron theory, the specific heat of metals is given by $C_V = \frac{3}{2}R = 12.5\ \text{Jmol}^{-1}\text{K}^{-1}$.

The experimental value of specific heat is $C_V = 10^{-4}RT$. This is very small and also temperature dependent.

Hence, classical free electron theory could not explain specific heat.

ii) Temperature dependence of electrical conductivity:

According to classical free electron theory of metals, the electrical conductivity is inversely proportional to square root of temperature, i.e

$$\sigma = \frac{ne^2\tau}{m} \Rightarrow \sigma \propto \frac{1}{\sqrt{T}}$$

But, experiments reveal that electrical conductivity is inversely proportional to temperature $\sigma \propto \frac{1}{T}$.

Hence, classical free electron theory fails to explain Temperature dependence of electrical conductivity.

iii) Dependence of electrical conductivity on electron concentration.

As per classical free electron theory, $\sigma = \frac{ne^2\tau}{m}$ that is $\sigma \propto n$

But experiments reveal that electrical conductivity of some metals as follows.

Metal	Electrical conductivity	Electron concentration
Copper	5.88 X10^7	8.45 X10^{28}
Aluminium	3.65 X10^7	18.06 X10^{28}

This indicates that $\sigma \propto n$ does not hold good. Hence, classical free electron theory fails to explain dependence of electrical conductivity on electron concentration.

Assumptions of quantum free electron theory.

1. All metals consist of large number of free electrons which are responsible for electrical conduction.

2. The energy values of the free electrons are discrete.

3. In metal there exists large number of closely spaced energy levels for free electrons which form a band.

3. The distribution of electrons in the various allowed energy levels is according to Pauli’s exclusion principle. According to that maximum of two electrons can occupy in a given energy level. This suggests the availability of two states for free electrons in the energy level corresponds to spin up and spin down states.

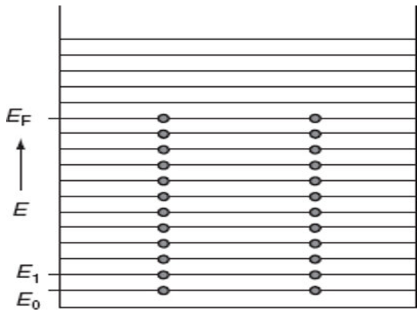
4. The mutual interaction between repulsion between electrons and the attraction between the electrons and lattice ions are ignored.

5. The electrons travel in a constant potential inside the metal but stay confined within its boundaries.

Explain Fermi energy and Fermi factor. Discuss the variation of Fermi factor with temperature and energy.

The highest filled energy level in a metal at absolute zero by free electrons is called Fermi level and corresponding energy is called Fermi Energy. It is denoted by E_F.

We know that, metal containing N number of atoms and there will be N number of allowed energy levels in each band. These energy levels are closely spaced. The energy levels in bands fill up as per Pauli Exclusion Principle. The free electrons in a metal start filling up the available energy levels from the lowest energy level of the valence band. Thus, when no electric field applied, at T=0K all the energy levels below Fermi level is completely filled and above the Fermi level are empty.



In Fermi Dirac statistics, the probability of occupation of an energy state (E) under thermal equilibrium is evaluated through a function called Fermi factor which is denoted as f (E) and it is given by

$$\mathbf{f(E) = \frac{1}{1 + e^{\left(\frac{E-E_F}{kT}\right)}}}$$

Here, E is the energy state being occupied by electron at temperature T, and E_F is the Fermi energy, k is the Boltzmann constant, $k = 1.38 \times 10^{-23}$ J/K.

The variation of Fermi factor with energy at absolute zero and higher temperature can be

explained as follows.

Case I: Probability of occupation of levels with energy $E < E_F$ and at T=0K,
Fermi factor is given by the relation

$$\mathbf{f(E) = \frac{1}{1 + e^{\left(\frac{E-E_F}{kT}\right)}}}$$

In this case E- E_F is negative and T=0K

$$\mathbf{f(E) = \frac{1}{1 + e^{(-\infty)}} = \frac{1}{1+0}}$$

$$\mathbf{f(E) = 1}$$

Therefore, f(E)=1 means the energy levels below Fermi level are occupied at T=0K.

Case II: Probability of occupation of levels with energy E >E_F and at T=0K.

In this case E- E_F is positive and T=0K

$$\mathbf{f(E) = \frac{1}{1+e^{\left(\frac{E-E_F}{kT}\right)}} = \frac{1}{1+e^{(\infty)}}}$$

$$\mathbf{f(E) = \frac{1}{1+\infty} = 0}$$

$$\mathbf{f(E) = 0}$$

Therefore, f(E)=0 means the energy levels above Fermi energy are empty at T=0K

Case III: Probability of occupation of levels with energy E = E_F and at T > 0K

In this case E- E_F =0, substituting the value in Fermi factor

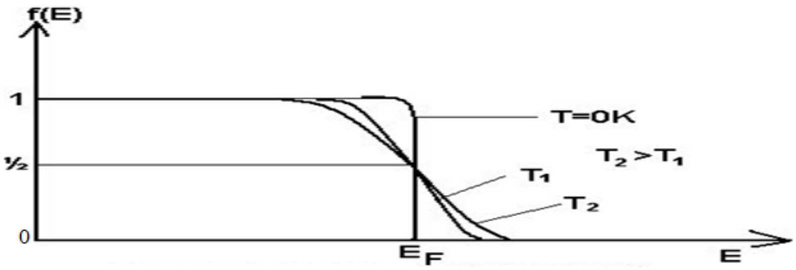
$$\mathbf{f(E) = \frac{1}{1+e^{\left(\frac{E-E_F}{kT}\right)}} = \frac{1}{1+e^{\left(\frac{0}{kT}\right)}}}$$

$$\mathbf{(E) == \frac{1}{1+1} = 0.5}$$

$$\mathbf{f(E) = 0.5}$$

Therefore for temperature above 0K the probability of occupation of energy level is ½

Figure shows the variation of Fermi factor with temperature



Define density of states and explain Fermi-Dirac statistics.

Density of states is the number of energy states available per unit volume of the material in the unit energy range in the valence band of the material. Mathematically, the density of states in the energy range E and E+dE per unit volume of the material is given by

$$g(E)dE = \left(\frac{8\sqrt{2}\ \ \pi m^{\left(\frac{3}{2}\right)}}{h^3}\right)\sqrt{E}\ \ dE$$

It is denoted by g(E)dE

From the above relations, π, m, and h are constants, therefore, $g(E)dE$ is proportional to $\sqrt{E}$ and its variation of $g(E)dE$ as a function of E.

Fermi-Dirac distribution function :

Fermi–Dirac statistics describes a distribution of particles over energy states in a systems consisting of identical particles with half-integer spin that obey "Pauli exclusion principle" under thermal equilibrium and such particles are called fermions. Since, electrons with spin half satisfy these conditions and they obey Fermi-Dirac statistics.

The allowed energy levels in each band are closely spaced. The occupation of these energy levels by free electrons in the valence band of a metal is according to Pauli Exclusion Principle.

F-D statistics permits the evaluation of the probability of occupation of an energy level in certain energy range under thermal equilibrium. This is done through function called Fermi factor f (E) which is given by

$$\mathbf{f(E) = \frac{1}{1 + e^{\left(\frac{E - E_F}{kT}\right)}}}$$

Where, E is the energy state being occupied by electron at temperature T, and E_F is the Fermi energy, k is the Boltzmann constant, $k = 1.38X10^{-23}$J/K.

Numerical Problems:

1. Calculate the probability of an electron occupying an energy level 0.02eV above the Fermi level at 200K and 400k.

   Solution: Given, energy level E occupied by the electron above E_F, $E= E_F+ 0.02eV$
   At T= 200K
   $E= E_F+ 0.02\ X1.609X10^{-19}J$

$$\text{Fermi factor, } f(E) = \frac{1}{1+e^{\left(\frac{E-E_F}{kT}\right)}}$$

$$f(E) = \frac{1}{1 + e^{\left(\frac{E_F + 0.02\ X1.609X10^{-19} - E_F}{1.38X10^{-23} * 200}\right)}} = 0.24$$

$$f(E) = 0.24$$

At T= 200K

Fermi factor, $f(E) = \frac{1}{1+e^{\left(\frac{E-E_F}{kT}\right)}}$

$$f(E) = \frac{1}{1 + e^{\left(\frac{E_F + 0.02\ X1.609X10^{-19} - E_F}{1.38X10^{-23} * 400}\right)}}$$

$$f(E) = 0.36$$

2. Calculate the probability of an electron occupying an energy level 0.02eV below the Fermi level at 200K.

Solution: Given, energy level E occupied by the electron below E_F, E= E_F - 0.02eV

E= E_F - 0.02 X1.609X10^{-19}J. at T= 200K

Fermi factor, $f(E) = \frac{1}{1+e^{\left(\frac{E-E_F}{kT}\right)}} = \frac{1}{1+e^{\left(\frac{E_F - 0.02\ X1.609X10^{-19} - E_F}{1.38X10^{-23} * 200}\right)}} = 0.76$

Discuss the effect of temperature and impurity on electrical resistivity of conductors and hence explain for superconductors.

Or

(Explain temperature dependence of resistivity of metal and superconductor).

A metal consists of lattice ions and impurity atoms that are held by together free electrons. The free electrons are moving inside the metal. During the motion, electrons undergo scattering by lattice ions and impurity atoms so that resistivity arises due to two regions;

3. Resistivity due to scattering of electrons by lattice vibrations (phonons) which is increasing with temperature. Therefore it is temperature dependent $\rho(T)$
4. Resistivity due to scattering of electrons by the presence of impurities. This scattering is temperature independent and resistivity at T=0K is called residual resistivity ρ_0

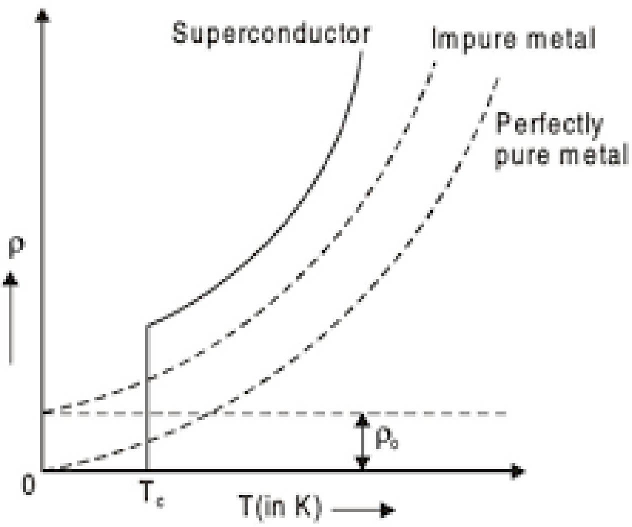
Therefore, the total resistivity of a metal is the sum of the resistivity due to phonon scattering which is temperature dependent and the resistivity due to the presence of impurities which is temperature independent.

Therefore, According to Mattheissen's rule, the total resistivity of materials is

$$\rho = \rho_0 + \rho(T)$$

Here, $\rho(T)$ is the resistivity at higher temperature, and ρ_0 is the resistivity at T=0K is called residual resistivity.

Therefore, resistivity of materials depends on the temperature. This variation of the resistivity materials with temperature is as shown in figure below (shown by dotted lines)



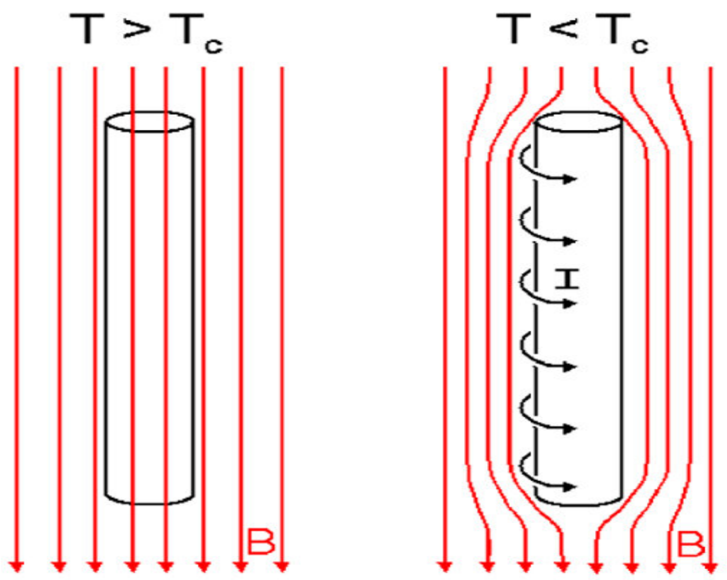
From the above figure, we can see that resistivity of certain materials decreases with decrease in temperature up to a particular temperature Tc (shown by solid line). The temperature at which the resistivity drops to zero is called **critical temperature Tc**. Critical temperature is different for different materials. T_c is also called as transition temperature because when T>Tc material is said to be in normal state which offers resistance to the flow of current. If T<Tc, material is said to be in superconducting state which offers zero resistance to the flow of current and is called super current.

Therefore, such material whose resistivity is zero below critical temperature is called **superconductors**. The phenomenon in which resistance of materials like certain metals, alloys and compounds drops to zero abruptly below certain temperature is called **superconductivity**.

Decribe Meissner's Effect. Explain the variation of critical field with temperature.

Meissner showed that when a superconducting material is magnetized (allows the magnetic flux though it) when it is placed in a magnetic field if its temperature is above Tc (T>Tc). Further, if the temperature is reduced below its critical temperature T<Tc then it expels all the flux lines out of the material and exhibits perfect diamagnetism. This is known as Meissner's effect.

Since superconductor exhibits perfect diamagnetism below the critical temperature Tc, then magnetic flux density inside the material is zero.



The magnetic flux density is given by

$$B = \mu_0(H + M)$$

$$H + M = \frac{B}{\mu_0}$$

Here B is Magnetic Flux Density, M is Magnetization and H is the applied magnetic field strength.

According to Meisner effect, when the material is in superconducting state, B = 0 at T<Tc

$$H + M = 0$$

$$H = -M$$

Therefore Magnetic susceptibility $\chi = \frac{M}{H}$ = -1. This is the indication for a perfect diamagnetic material.

Variation of critical field with temperature:

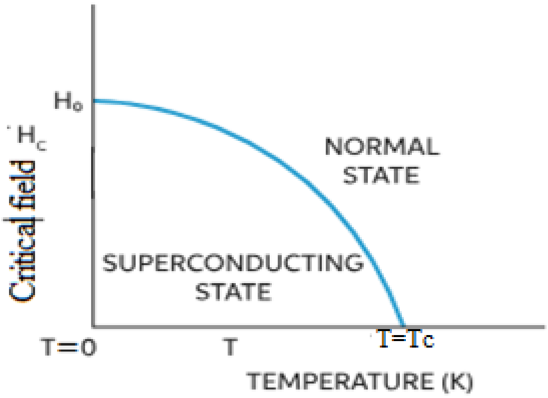
As we know that when superconductor is placed in a magnetic field it exhibits a perfect diamagnetism. But if the strength of the magnetic field is further increased, it is found that for a particular value of the magnetic field, material loses its superconducting property and becomes a normal conductor. Therefore, **The strength of the magnetic field at which the transition occurs from the Superconducting state to Normal state is called Critical Field or Critical Magnetic Field is denoted by Hc** .

It is found that by reducing the temperature of the material below its critical temperature the superconducting property of the material could be restored. Thus, critical field does not destroy the superconducting property of the material completely but only reduces the critical temperature of the material.

The variation of Critical field with temperature below the critical temperature is given by

$$\mathbf{H_c = H_0\left[1 - \frac{T^2}{T_c^2}\right]}$$

Here $\mathbf{H_c}$ is the Critical field at any temperature T <Tc and $\mathbf{H_0}$ is the critical field at T=0K.



Distinguish between Type –I and Type –II superconductors.

Types of Super Conductors:

Superconductors are classified into two types;

1. Type I Superconductor or Soft Superconductor.

2. Type II Superconductor or Hard Superconductor.

Type -1 superconductor (Soft superconductors)

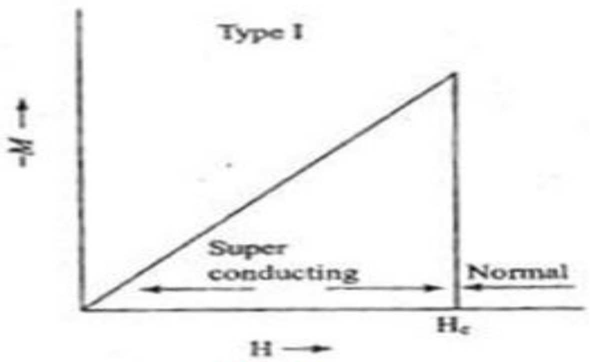


Figure 1

Type I superconductors exhibit complete Meissener's Effect. The graph of magnetic moment V/s magnetic field is as shown in the Figure 1. As the field strength increases the material becomes more diamagnetic until H becomes equal to Hc. Above Hc the material allows the magnetic flux to pass through and exhibits normal conductivity. The value of critical field Hc is very small for soft superconductors.

Therefore soft superconductors cannot withstand high magnetic fields. Therefore they cannot be used for making superconducting magnets. Ex. Hg, Pb and Zn.

Type -2 Superconductors (Hard Superconductors):

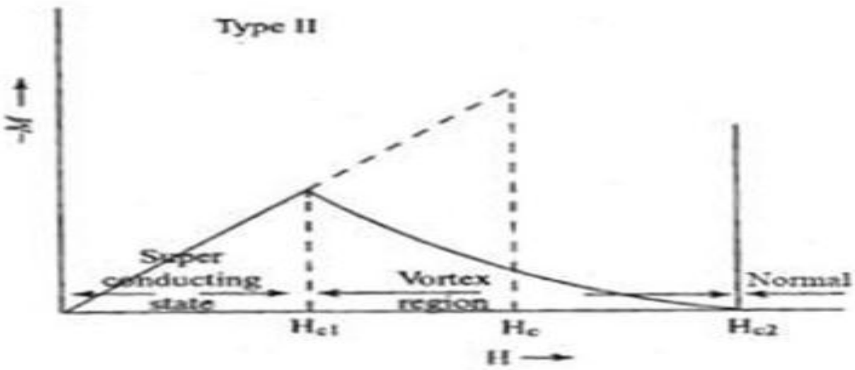


Figure 2

They do not exhibit complete Meisner effect. The superconductors are characterized by two critical fields H_{c1} and H_{c2} namely lower critical field and upper critical filed respectively. The graph of magnetic moment V/s magnetic field is as shown in the Figure 2. When $H<H_{c1}$ material exhibits perfect diamagnetism. Beyond H_{c1} partial flux penetrates and the material is said to be vortex State. As $H> H_{c1}$ more and more flux fills the body and thereby decreasing the diamagnetic property of the material. At H_{c2} flux fills the body completely and material losses its diamagnetic property as well as superconducting property completely.

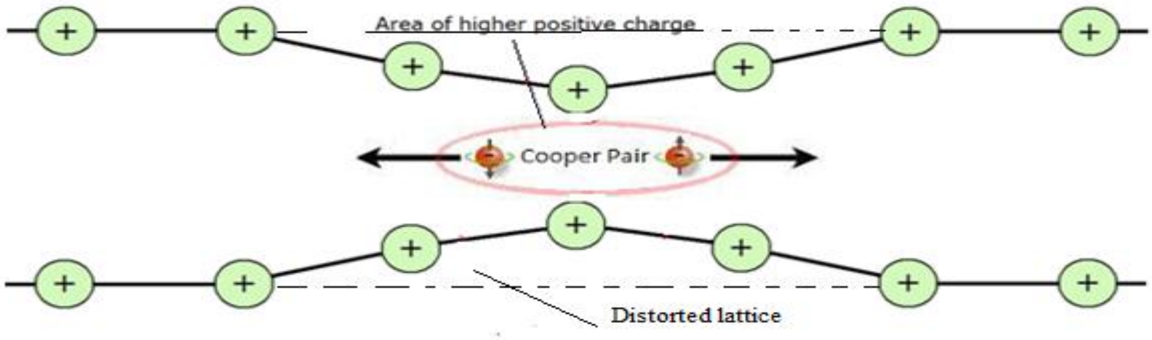
The H_{c2} value is greater than Hc of soft superconductors. Therefore they are used for making powerful superconducting magnets. Examples: NbTi, Nb_3Sn.

BCS theory of superconductivity:

Bardeen, Cooper and Schrieffer explained the phenomenon of superconductivity in the year 1957. According to this theory, Superconductivity is due to the formation of cooper pairs. "Cooper pairs are the pair of electrons formed by the interaction between electrons with opposite spin and momentum in a phonon field.

Consider an electron approaching a positive ion core and suffers attractive coulomb interaction. Due to this attraction, lattice gets distorted. Let another electron come in the way of distorted lattice and interaction between the two occurs. The two electrons therefore interact through the lattice distortion (phonon field) which lowers the energy of the electrons. This interaction is called as electron - lattice - electron interaction. It was shown by Cooper that, this attractive force becomes maximum if two electrons have opposite spins and momentum. The attractive force may exceed coulombs repulsive force between the two electrons below the critical temperature, which results in the formation of bound pair of electrons called cooper pairs. Each cooper pair causes the formation of many numbers of such pairs, causing the formation of cloud of cooper pairs.

Below the critical temperature the dense cloud of Cooper pairs forms a collective state and the motion all Cooper pairs is correlated resulting in zero resistance of the material.



High Temperature superconductivity:

Superconducting materials which exhibit superconductivity at relatively higher temperatures are called high temperature superconductors. Thus high temperature superconductor's posses higher value of critical temperature compared to conventional superconductors. Most of the high temperature superconductors are found to fall into the category of ceramics. All high temperature superconductors are oxides of copper and bear Perovskite crystal structure characterized by large number of copper-oxygen layers. It was found that addition of extra copper-oxygen layer pushes the critical temperature Tc to higher values. The super currents are strong in the copper-oxygen layer and weak in the direction perpendicular to the planes.

Ex: In 1986 George Bednorz and Alex Muller discovered a compound containing Lanthanum, Barium, Copper and Oxygen having Tc =30K was developed. In 1987 scientists developed a compound which is an oxide of the form $YBa_2Cu_3O_7$ which is referred to as 1-2-3 compound with Tc > 90K was discovered.

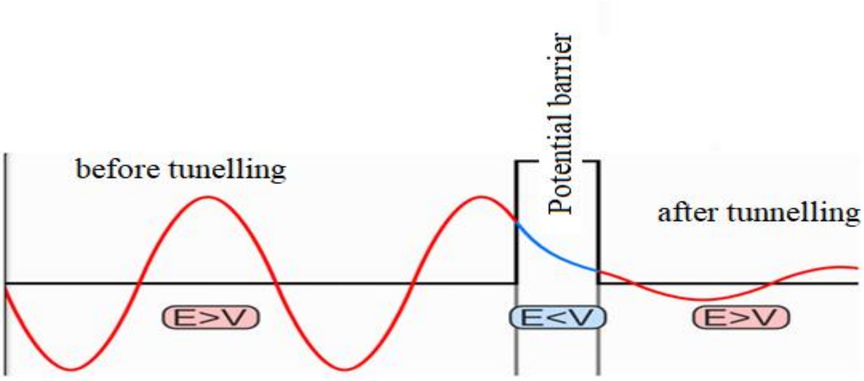
Following is the list of High Temperature Superconductors.

Material	T_c [K]
Pb	7.2
Nb	9.2
Nb-Ti alloys	~ 9.6
Nb_3Sn	18.1
Nb_3Ge	23.2
$Y_1Ba_2Cu_3O_{7-x}$ (YBCO)	90
$BI_2Sr_2Ca_1Cu_2O$ (Bi2212)	80
$BI_2Sr_2Ca_2Cu_3O$ (Bi2223)	125

<u>Quantum Tunnelling</u>:

In Physics, quantum tunnelling or barrier penetration is a quantum mechanical phenomenon in which a particle such as an electron or atom passes through a potential energy barrier

Probability of transmission of a matter wave through a barrier decreases exponentially with the barrier height. If the barrier is thin enough then there is a finite probability that particles will tunnel through a barrier



(E the energy of the particle and V the potential energy of the barrier)

<u>Josephson Junctions (Qualitative):</u>

In 1962, Brain Josephson predicted that cooper pairs could tunnel through thin insulating layer separating two superconductors. The superconductor –insulator-superconductor layer constitutes Josephson **<u>Junctions.</u>**

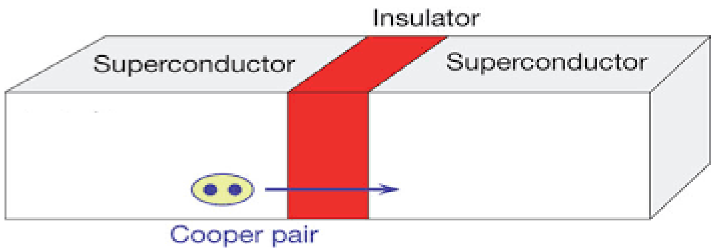
The phenomenon of flow of current between two superconductor separated by a thin insulating material is called as Josephson effect and current is called Josephson current. The two Josephson effects as follows ;DC Josephson Effects and AC Josephson Effects

<u>DC Josephson Effects</u> :

Consider a Josephson junction consisting of two superconductors separated by a thin insulating layer of 1nm to 2nm thick. The cooper pairs tunnel through the insulating layer which causes direct current flows through it. The insulating layer introduces a phase difference so that the Josephson current flow through the junction in the absence of external source (zero voltage). This is known as DC Josephson effect. The Josephson (super) current through the junction is

$$\mathbf{I_S = I_c \sin\phi_0}$$

$\mathbf{I_c}$ = critical current at zero voltage, which depends on the thickness of the junction layer and the temperature, $\boldsymbol{\phi_0}$= Phase difference between the wave functions of cooper pairs relative to the superconductors.



AC Josephson Effects:

When dc voltage is applied across the Josephson junction, it introduces an additional phase change on cooper pairs during tunnelling and generates an alternating current. This effect is called as AC Josephson effect. Thus the current is given by

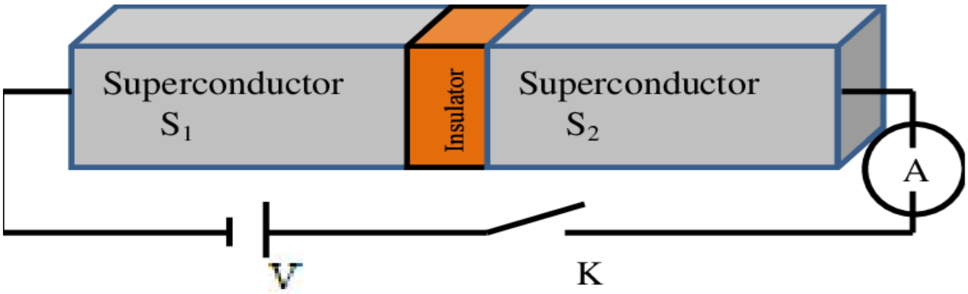
$$\mathbf{I_S = I_c \sin\,(\phi_0 + \Delta\phi)}$$

$\boldsymbol{\Delta\phi}$ = Phase difference, $\mathbf{I_c}$ is the critical current

The frequency of the generated AC is using (2eV=hf)

$$\mathbf{f = \frac{2eV}{h}}$$

For example, if a voltage of about V=1μV is applied, and AC frequency of about 484 MHz can be obtained.



Because of the DC voltage applied across the barrier, the energy difference of cooper pairs on both sides is of the order of 2eV.

SQUIDs

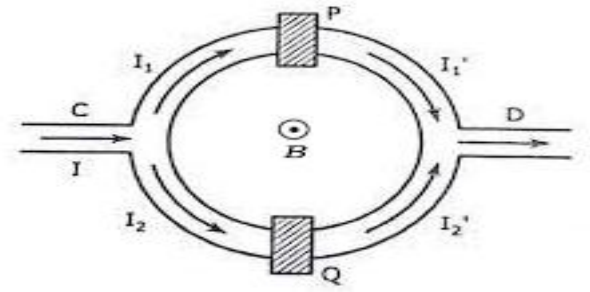
SQUIDs is the Superconducting Quantum Interference Device used to measure extremely weak magnetic field of the order of 10^{-13}T. Hence it is a sensitive magnetometer made of a superconducting ring. SQUID is a super conducting ring containing one or more Josephson junctions. Two types of SQUIDS are available namely DC SQUID and RF SQUID. It works on the principle of Josephson effect.

DC SQUID: It works on the principle of DC Josephson effect which is the phenomenon of flow of super current through the junction even in the absence of external voltage.

Construction and Working of DC SQUID:

It consists of two Josephson junctions connected in parallel which forms loop. The cross sectional view of the arrangement is shown in figure.

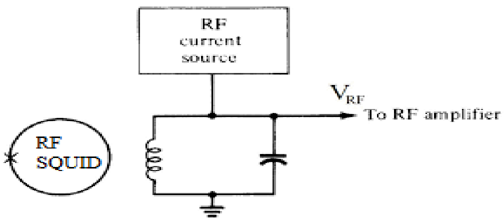
P and Q are two Josephson Junctions arranged in parallel. When current I flow through the point C, it divides into I_1 and I_2.

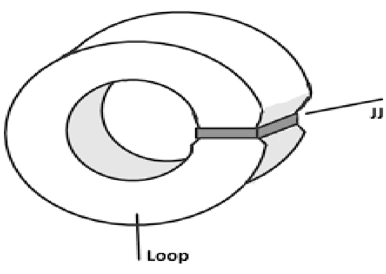


In the absence of applied magnetic field, the phase difference between the wave functions is zero.

If the magnetic field is applied perpendicular to the current loop, the wave functions due to these super currents (cooper pairs) experience a phase shift at 1 and 2. This can be shown as current I_1' and I_2' in the figure. The magnitude of phase difference is proportional to applied magnetic field. Hence, even if there is a weak magnetic field in the region will be detected.

RF SQUID:



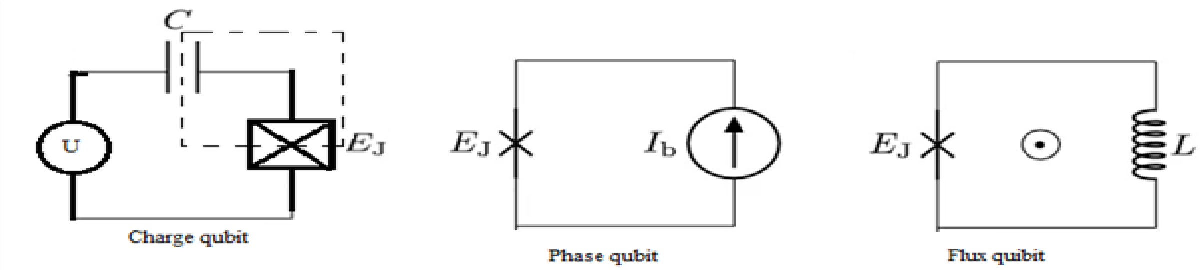


It works on the principle of AC Josephson effect. It has single Josephson junction as shown in figure. Magnetic field is applied perpendicular to the plane of the current loop. The flux is coupled into a loop containing a single Josephson junction through input coil and an RF source. Hence when RF current changes, there is corresponding change in the flux linked with the coil. This variation can be measured and it can be used in the detection of low magnetic field. It is less sensitive compared to DC Squid.

Applications of superconductivity in Quantum Computing:

There are three basic designs for Josephson-junction qubits are in quantum computing are as follows; Charge qubits, Flux qubits and Phase qubits. The charge qubit is a box for charge, controlled by an external voltage U, The phase qubit is a Josephson junction biased by a current I_b. The flux qubit is a loop, controlled by an external magnetic flux Φext.

Three basic Josephson-junction qubit circuits as shown in figure below. E_J Josephson energy, C the capacitor and L is the inductance.



1. **Charge qubits:**.
   In quantum computing, a charge qubit is also known as Cooper-Pair Box (CPB). It was one of the first superconducting qubits developed. It is a qubit whose basis states are charge states. The state represents the presence or absence of excess Cooper pairs in the island (dotted line in the figure). In superconducting quantum computing, a charge qubit is formed by a tiny superconducting island coupled by Josephson junction to a superconducting reservoir.
2. **Phase qubits:**
   A phase qubit is a current-biased Josephson junction, operated in the zero voltage state with a non-zero current bias. This employs a single Josephson and the two levels are identified by quantum oscillations of the phase difference between the electrodes of the junction. DC SQUID is a type of phase qubit.
3. **Flux qubits:**
   The flux qubit is another simple Josephson-junction qubit. It is also known as a persistent-current qubit. Flux qubits are micrometer sized loops of superconducting metal that is interrupted by a number of Josephson-junctions. These devices function as qubit. The Josephson-junctions are designed so that persistent current will flow continuously when external magnetic is applied.