APPLIED PHYSICS FOR MECHANICAL STREAM

A BRIEF NOTES

Contents

Part I

Module - 1 - Oscillations and Shock Waves

Chapter 1

Oscillations

1.1 Free Oscillations

1.1.1 Simple Harmonic Motion (SHM)

Oscillations: Oscillation is a repeating motion that occurs when a periodic force acts on the system. Oscillations are periodic motions.

Free Oscillations If the oscillations occur without the action of an external periodic force then such oscillations are called ***free oscillations***.

Simple Harmonic Motion The motion of an object is said to be simple harmonic motion if the restoring force (or acceleration) is directly proportional to the displacement and acts in the direction opposite to that of motion. Motion of the bob of an oscillating pendulum, spring mass system are the best examples of SHM.
In general the equation for the displacement in SHM is given by

$$y = Asn(\omega_0 t + \phi) \tag{1.1}$$

Here ϕ is the initial phase and A is the amplitude of SHM.

1.1.2 Mechanical Simple Harmonic Oscillator and Expression for SHM (Equation for Free Oscillations)

Consider a mass attached to spring of negligible mass which is suspended from a rigid support as shown in the figure. The oscillations in the mass are due to the restoring force developed in the spring.

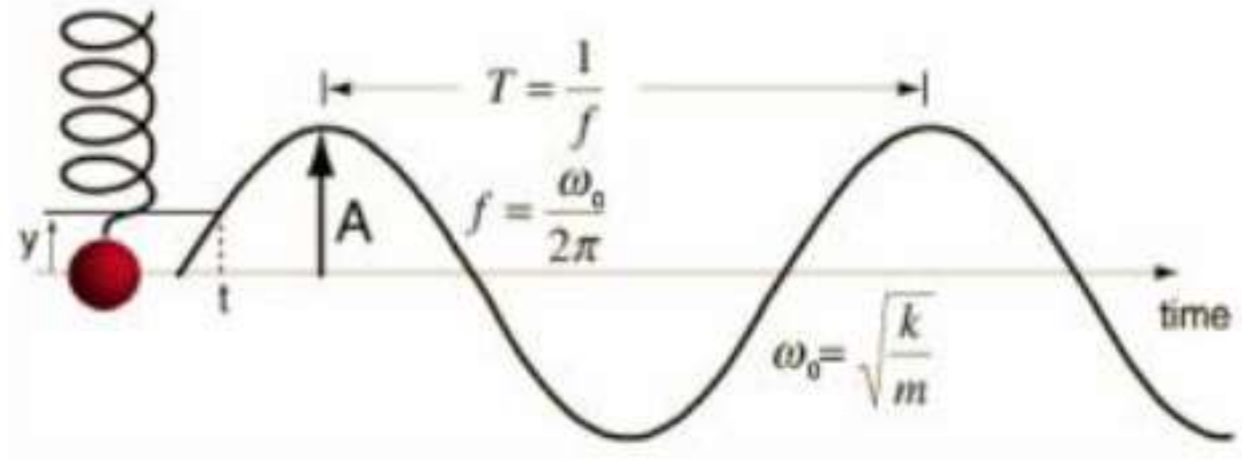


Figure 1.1: Oscillating Spring-Mass system

According to Hooke's Law the restoring force is directly proportional to displacement.

$$f = -ky \tag{1.2}$$

Here k is called spring constant or stiffness factor or force constant. Note : y is used since the displacement is along vertical direction.

$$ma = -ky$$

Applying equation of kinematics of motion the above equation could be written as

$$m\frac{d^2y}{dt^2} = -ky$$

$$\frac{d^2y}{dt^2} = \frac{-k}{m}y$$

$$\frac{d^2y}{dt^2} + \frac{k}{m}y = 0$$

$$\frac{d^2y}{dt^2} + \omega_0^2 y = 0 \tag{1.3}$$

Equation 1.3 is the second order homogeneous differential equation for SHM. Here $\omega_0 = \sqrt{\frac{k}{m}}$ is the angular frequency of the oscillations. The solution of equation 1.3 is given by

$$y = Asn(\omega_0 t + \phi) \tag{1.4}$$

Equation 1.4 also represents the equation of motion for free oscillations. A is the amplitude of the SHM and ϕ is the initial phase.

1.1.3 Characteristics of SHM

- Too and Fro motion
- Periodic Motion
- Acceleration or Force is proportional to the displacement.
- Acceleration is in the opposite direction of displacement.
- Restoring force is essential for SHM.

Amplitude (A): The Maximum displacement of the mass from equilibrium position. It can take values $+A$ and $-A$.

Phase Angle and Initial Phase : The value $(\omega + \phi)$ represents the state of the system and is called phase angle. The angle ϕ is called initial phase.

Angular velocity or Frequency (ω_0): It is the rate of change of angular displacement and is given by $\omega_0 = \sqrt{\frac{k}{m}}$

Frequency (f): Frequency of oscillations is defined as the no. of oscillations per second and is given by $f = \frac{\omega_0}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$.

Time Period (T) : It is the time taken to complete one oscillation and is given by $T = \frac{1}{f}$.

Velocity (v): The rate of change of displacement of the oscillating particle is its velocity. Hence we get

$$v = \frac{dy}{dt} = A\ \ _0Cos(\omega_0 t + \phi) \tag{1.5}$$

The velocity varies from $+A\ \ _0$ to $-A\ \ _0$.

Acceleration (a): The rate of change of velocity of oscillating particle is its acceleration.

$$a = \frac{dv}{dt} = -A\ \ _0^2Sn(\omega\ _0t + \phi) = -\omega_0^2 y \tag{1.6}$$

Acceleration varies from $+A\omega_0^2$ to $-A\ \ _0^2$.

1.2 Springs

1.2.1 Force constant and its Physical Significance

"It is the amount of force exerted when a spring is elongated/compressed by unit length." It determines the stiffness of the string. The SI unit of spring constant unit is Nm^{-1}). The physical significance of k is, if k large then higher force is required for unit extension and if k is small relatively lower force is required for unit extension in the spring.

$$f = kx \tag{1.7}$$

f is the restoring force, x is the displacement and k is spring constant or stiffness factor.

1.2.2 Natural Frequency

When a body exhibits free oscillations the frequency with which the oscillations occur is called *Natural Frequency*.

1.2.3 Springs in Series and Parallel

Consider two springs of negligible masses and with force constants k_1 and k_2. Let us calculate the effective spring constant when the two springs are connected in series to a mass and when connected in parallel, as shown in the figure 1.2.

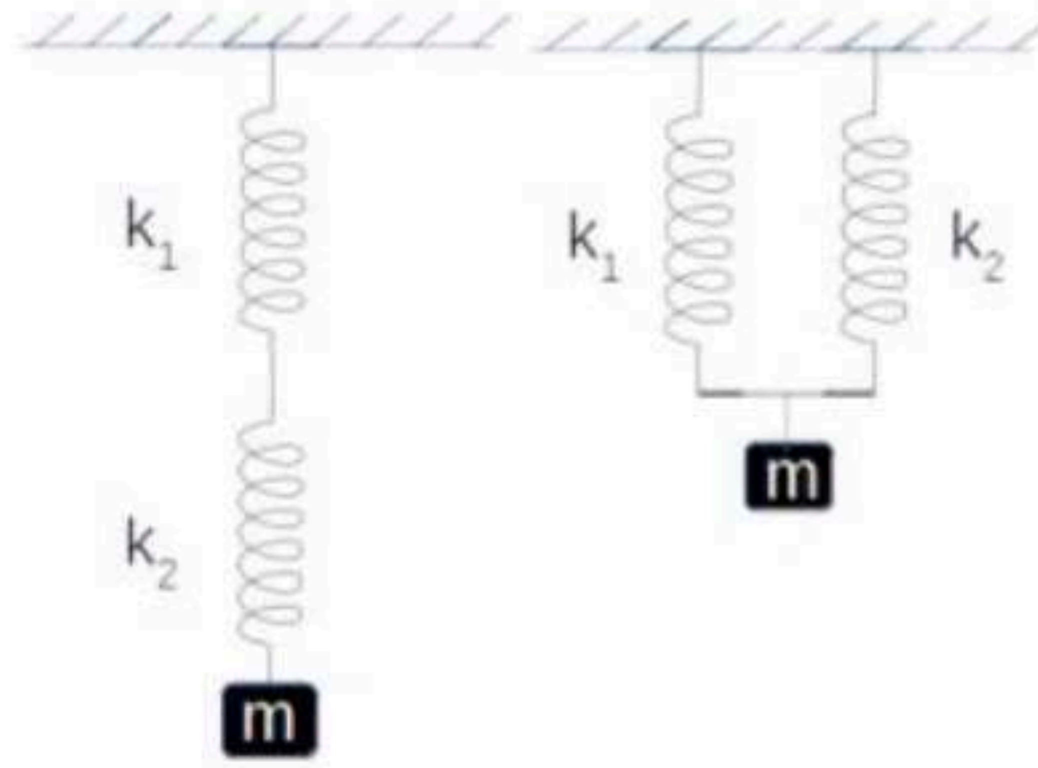


Figure 1.2: Springs in series and parallel

Springs in Series

When springs are in connected in series to a mass the net extension y is given by

$$y = y_1 + y_2 \tag{1.8}$$

Here y is the total extension, y_1 is the extension in the spring with force constant k_1 and y_2 is the extension in the spring with force constant k_2. If k_s is the effective force constant of the combination and if F is the force applied on the combination then we get

$$\frac{F}{k_s} = \frac{F}{k_1} + \frac{F}{k_2}$$

Thus

$$\frac{1}{k_s} = \frac{1}{k_1} + \frac{1}{k_2} \tag{1.9}$$

Thus from equation 1.9 effective force constant of the series combination is given by

$$k_s = \frac{k_1 k_2}{k_1 + k_2} \tag{1.10}$$

Springs in Parallel

When springs are in connected in parallel to a mass the net force F on the system of springs is given by

$$F = F_1 + F_2 \tag{1.11}$$

Here F_1 is the force on the spring with force constant k_1 and F_2 is the force on the spring with force constant k_2. If

k_p is the effective force constant of the combination and if y is the total extension in the combination then we get

$$k_p y = k_1 y + k_2 y$$

Thus the effective force constant of the parallel combination is given by

$$k_p = k_1 + k_2 \quad (1.12)$$

1.2.4 Applications of Springs

Compression Springs

Introduction : A compression spring is an elastic coil, made of spring steel - its spring characteristic is that it absorbs force or provides resistance.

Compression Spring

Structure and Working :

- Compression springs are very effective at building up energy. The compression spring has gaps between its coils in an unloaded state.
- The distance between the coils is reduced when the spring is loaded and compressed.

Uses : A compression spring can be used as a pure energy accumulator, shock absorber, suspensions, vibration, damper or force generator.

Life : To get the longest possible service life from a compression spring, it is important not to overload it.

Leaf Springs

Structure and Working :

- The Leaf Spring is made of arc-shaped slender pieces of steel that are stacked in smaller sizes and bolted together.
- Its construction creates a reinforced bow-like item. It is then attached to the rear axle and the chassis.

The overall purpose of a leaf spring is to provide support for a vehicle. It also creates a smoother ride, absorbing any bumps or potholes in the road.

Leaf Spring

Uses:

- The leaf springs are used control the height at which the vehicle rides and keep the tires aligned on the road.
- Leaf springs are most useful for train, trucks and vans hauling heavy loads.

1.3 Damped Oscillations

Definition If the amplitude of oscillations decreases with respect to time then they are called damped oscillations. The decay in amplitude of the oscillations is due to the involvement of resistive forces like friction resulting in energy loss. The following are the examples for damped oscillations.

1.3.1 Examples of Damped Oscillations

Spring Mass System With Mass Immersed in a Viscous Liquid

Consider a spring mass system in which the oscillating mass is immersed in a viscous fluid. During the oscillations the viscous force acting on the mass reduces the amplitude progressively.

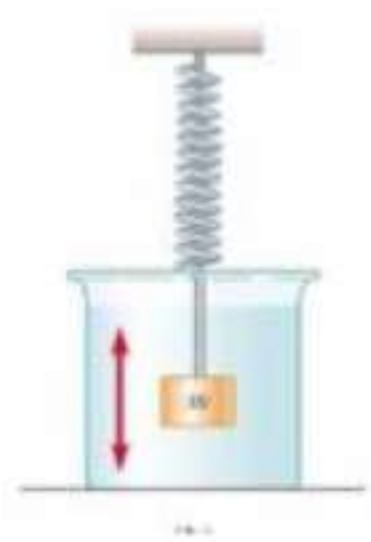


Figure 1.3: Spring Mass - Damped system

1.3.2 Expression for the Decay of the Amplitude in Damped Oscillations

Let us consider a simple harmonic oscillator system damped by viscous damping forces. The damping force is proportional to the velocity of the system. Thus the damping force is given by

$$f_d = -b\frac{dy}{dt} \quad (1.13)$$

here b is a constant that depends on the medium and the shape of the body called Damping Constant. Thus the equation for the damped simple harmonic oscillator could be obtained by adding the damping force term to the Hooke's law.

$$m\frac{d^2y}{dt^2} = -ky - b\frac{dy}{dt}$$

$$m\frac{d^2y}{dt^2} + b\frac{dy}{dt} + ky = 0$$

The above equation could be written as

$$\frac{d^2y}{dt^2} + \frac{b}{m}\frac{dy}{dt} + \frac{k}{m}y = 0 \quad (1.14)$$

$$\frac{d^2y}{dt^2} + \gamma\frac{dy}{dt} + \omega_0^2 y = 0 \quad (1.15)$$

Here $\gamma = \frac{b}{m}$ called damping ratio and $\omega_0 = \sqrt{\frac{k}{m}}$ is the natural frequency of the system. Thus the general solution for equation 1.14 could be written as

$$y(t) = Me^{\left(\frac{-b}{2m}+\frac{1}{2m}\sqrt{b^2-4mk}\right)t} + Ne^{\left(\frac{-b}{2m}-\frac{1}{2m}\sqrt{b^2-4mk}\right)t} \quad (1.16)$$

For small damping, the above equation could be reduced to

$$y(t) = Ae^{\frac{-b}{2m}t}cos(\omega + \phi) \quad (1.17)$$

here the frequency of damped oscillations $\omega = \sqrt{\omega_0^2 - \left(\frac{b}{2m}\right)^2}$.
The amplitude of damped oscillations is given by

$$Ae^{\frac{-b}{2m}t} \quad (1.18)$$

The amplitude of damped oscillations decreases with increase in time. The frequency of the damped oscillations ω is less than the natural frequency ω_0.

The damping is classified into three types as shown below

Under damping : Oscillations are said to be under damped or weakly damped if the retarding force is weaker than the restoring force. The amplitude of oscillations decreases with respect to time. The condition for damped oscillations is $b^2 < 4mk$.

Over Damping : Oscillations are said to be over damped or heavy damped when the system attains equilibrium state quite slowly without making oscillations. The condition for over damping is given by $b^2 > 4mk$

Critical Damping : When the system approaches equilibrium state quite quickly without making any oscillations is called Critical Damping. The condition for critical damping is given by $b^2 = 4mk$.

The above conditions are as shown in the below graph.

Figure 1.4: Under, Over, Critical damping

1.3.3 Quality factor and its Significance

Definition

The energy loss rate of a weakly damped oscillator is conveniently characterized in terms of a parameter Q called Quality factor. The quality factor is defined as 2π times the ratio of energy stored in the oscillator to the energy lost per time period. Q is also given by

$$Q = \frac{1}{\gamma}\sqrt{\frac{k}{m}} \quad (1.19)$$

This is because $\tau = \frac{1}{\gamma}$ and $\omega = \sqrt{\frac{k}{m}}$.

Significance

If the oscillator is weakly damped then the energy loss per period is relatively small and the Quality factor is much large than unity. Quality factor could be considered as the number of oscillations that the oscillator typically completes before its amplitude decays to a negligible value.

1.3.4 Engineering Applications of Damped Oscillations

Automatic Door Closures

Principle : The Heavily Damped system returns to the equilibrium position very slowly, without any oscillation. Heavy damping occurs when the resistive forces exceed those of critical damping.

Structure and Working :

- One end of the hydraulic damper is attached to the door, and the other end to the door frame.
- When door is opened, the hydraulic door closer pulls the door and politely closes it rather than slamming the door.

- This happens because the closer has a sealed tube which contains a strong spring as a damper.
- It also includes a fluid-filled chamber which releases the pressure to close the door in a slow manner rather than banging it.

Automobile Suspension System

principle The Automobile Suspension System works on the principle of Critical Damping. The damper in the suspension returns to the equilibrium quickly.

Structure and Working

- The automobile suspension consists of a compression spring of suitable damping connected between drive shaft and chassis.
- Spring must be flexible so that if can absorb road shocks.
- But if it is too flexible it may rebound excessively and repeatedly resulting in a a rough ride.
- For effective cushioning even a cylinder-piston damper is also fixed in vehicles.

1.4 Forced Oscillations

1.4.1 Definition and Examples

Forced Oscillations : The oscillations occur that under the action of an external periodic force are called forced oscillations. During forced oscillations the system oscillates with the frequency of the external periodic force.

Examples : Sonometer wire exited by an electromagnet or tuning fork and Resonance air column are the examples of forced oscillations.

1.4.2 Expression for Amplitude and Phase in Forced Oscillations

The forces acting on the system during forced oscillations

1. Restoring force acting in the direction opposite to the displacement.
2. Damping force due to the viscous medium.
3. External periodic force acting on the system.

Thus the equation oscillations in differential form could be written as

$$m\frac{d^2y}{dt^2} = -ky - b\frac{dy}{dt} + F_0cos(\omega)$$

$$m\frac{d^2y}{dt^2} + b\frac{dy}{dt} + ky = F_0cos(\omega)$$

The above equation could be written as

$$\frac{d^2y}{dt^2} + \frac{b}{m}\frac{dy}{dt} + \frac{k}{m}y = \frac{F_0}{m}cos(\omega) \tag{1.20}$$

$$\frac{d^2y}{dt^2} + \gamma\frac{dy}{dt} + \omega_0^2 y = \frac{F_0}{m}cos(\omega) \tag{1.21}$$

Here $\gamma = \frac{b}{m}$ called damping coefficient, $\omega_0 = \sqrt{\frac{k}{m}}$ is the natural frequency,$\frac{F_0}{m}cos(\omega)$ is the applied periodic force and ω is the frequency of the applied periodic force.The above equation could be represented in the complex exponential form as

$$\frac{d^2z}{dt^2} + \gamma\frac{dz}{dt} + \omega_0^2 z = \frac{F_0}{m}e^{i\omega t} \tag{1.22}$$

let us assume the following solution fo

r the above equation

$$z = A\ ^{i(\omega t - \phi)} \tag{1.23}$$

Substituting equation 1.23 in the equation 1.22 and simplifying we can obtain the relation for the amplitude and phase as follows.

$$A = \frac{\frac{F_0}{m}}{\sqrt{(\omega_0^2 - \omega^2)^2 + (\gamma\omega)^2}} \tag{1.24}$$

$$\tan\phi = \frac{\gamma\omega}{\omega_0^2 - \omega^2} \tag{1.25}$$

.

1.4.3 Different conditions of forced oscillations

1. If $\omega << \omega_0$: If the frequency of the applied force is very less than the natural frequency Then the equation 1.24 becomes
$$A = \frac{F_0}{m\omega_0^2} \tag{1.26}$$
Hence the system oscillates with frequency ω and its amplitude depends on $\frac{F_0}{m}$ and independent of ω. Equation 1.25 becomes
$$tan(\phi) = \frac{\gamma\omega}{\omega_0^2} \approx 0 \tag{1.27}$$
2. If $\omega = \omega_0$ the equation 1.24 reduces to
$$A = \frac{F_0}{m\gamma\omega_0} = \frac{F_0}{b\omega_0} \tag{1.28}$$
Here the amplitude of the oscillations is maximum since there is no square for ω in the denominator when compared to the previous case. This condition is called resonance. The equation for phase difference could be obtained from equation 1.25 by substituting $\omega = \omega_0$. Since the denominator is zero $tan\phi = \infty$ and hence the phase angle between displacement and the applied periodic force is $\frac{\pi}{2}$.

3. If $\omega >> \omega_0$ This case is significant only when the damping forces are very small (for small γ). The equation 1.24 reduces to

$$A = \frac{\frac{F_0}{m}}{\sqrt{(\gamma\omega)^2 + \omega^4}}$$

$$A = \frac{\frac{F_0}{m}}{\omega^2} = \frac{F_0}{m\omega^2} \quad (1.29)$$

the equation 1.25 becomes

$$tan\phi = -\frac{\gamma}{\omega} \quad (1.30)$$

for small γ the above equation is zero. Thus $tan(\phi) = 0$ and the phase difference between the displacement and the applied periodic force is $-\pi$.

The graph representing the variation of amplitude as a function of driver frequency and damping is as shown in the below figure 1.5.

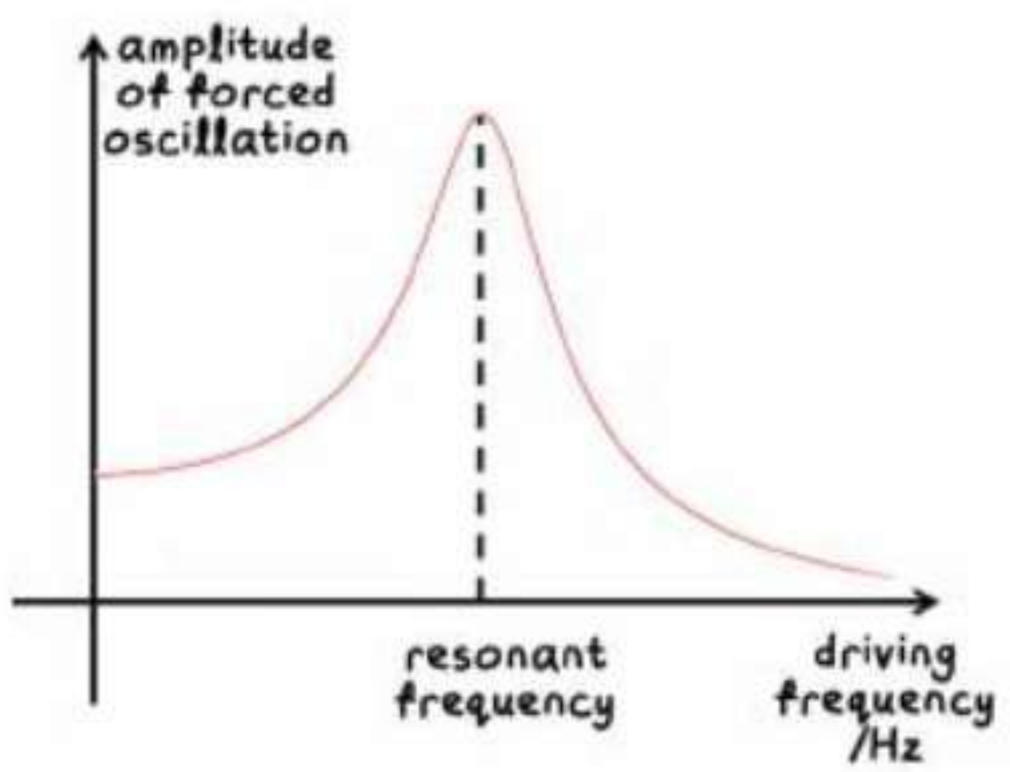


Figure 1.5: Variation of amplitude with driver force frequency

The graph representing the variation of phase with respect to dirver frequency is as shown in the below figure 1.6.

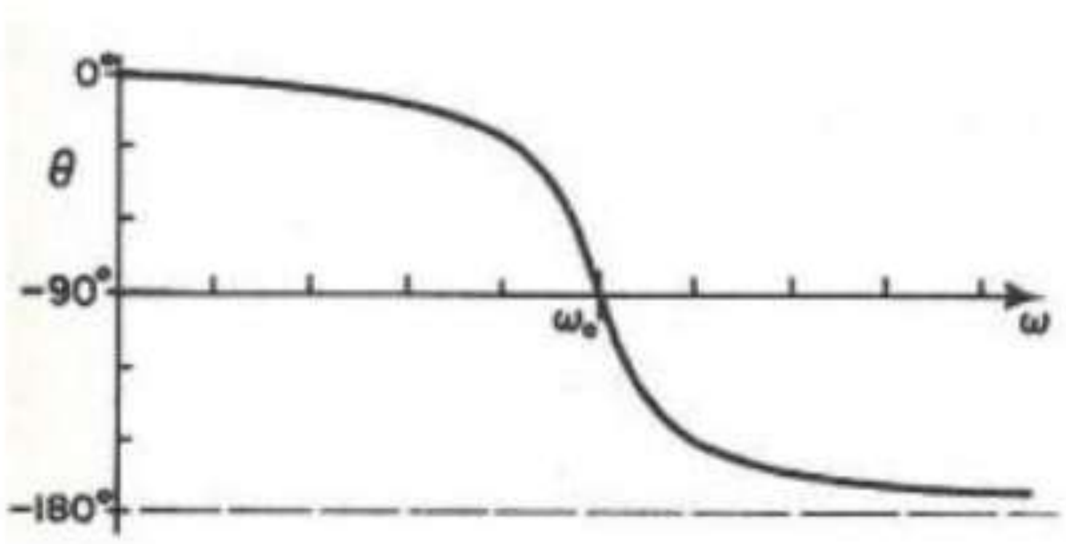


Figure 1.6: Variation of phase with driver force frequecy

1.4.4 Resonance

Consider a system under forced oscillations in which the frequecy of the applied periodic force is varied (tuning). During the course of tuning, if the frequency of the applied periodic force ω matches with the natural frequency ω_0 of oscillations of the system then the amplitude of the oscillations will be maximum and the condition is called resonance.

$$A = \frac{F_0}{m\gamma\omega_0} = \frac{F_0}{b\omega_0} \quad (1.31)$$

As shown in the figure 1.5 the amplitude of oscillations is maximum at resonant frequency.

Examples of Resonance

The following are the examples of resonance in different oscillating systems under forced oscillations.

1. In sonometer when the natural frequency of the stretched string is equal to the frequency of the tuning fork the amplitude of oscillation is maximum.
2. Helmoltz resonator
3. Resonance in LCR circuits, an example for electrical resonance.
4. The absorption of energy by electrons in atoms.
5. Resonance air column.

1.4.5 Sharpness of Resonance

Definition and Significance

During the tuning of oscillating system the rate at which the amplitude varies near resonance depends on damping.

The sharpness of resonance is the rate of change of amplitude with respect to a small change in frequency of the applied external periodic force, at resonance. Mathematically

$$Shapnessofresonane = \frac{\Delta A}{\Delta\omega}$$

Here ΔA is the change in amplitude with respect to $\Delta\omega$ the change in frequency at resonance. The following expression could be derived for sharpness of resonance.

$$Shapnessofresonane = \frac{F_0}{m\gamma\omega_0\omega} = \frac{F_0}{b\omega_0\omega} \quad (1.32)$$

Effect of damping on the sharpness of resonance

The graph representing the variation of amplitude of forced oscillations with respect to damping is as shown in the below figure 1.7.

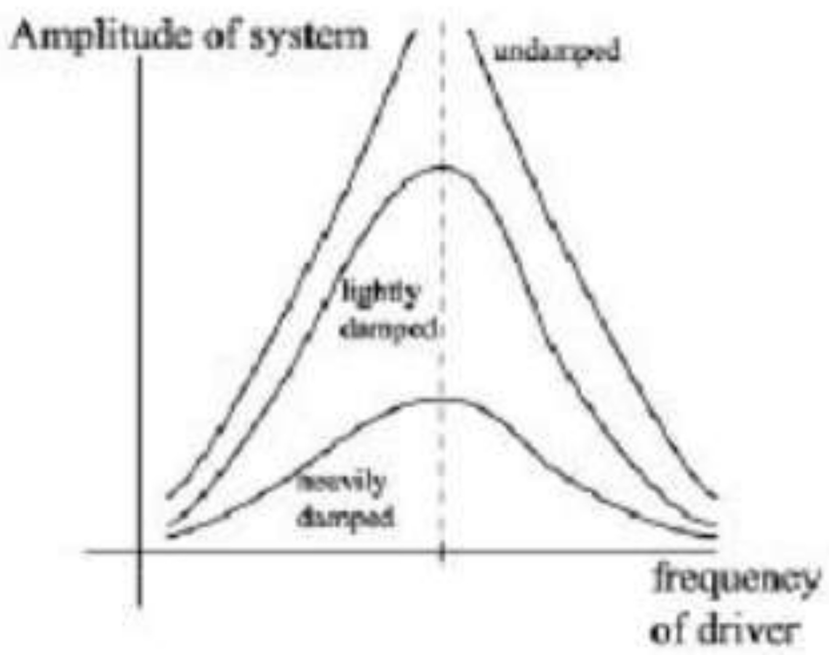


Figure 1.7: Variation of amplitude with damping

With reference to equation 1.32 the maximum amplitude at resonance is a function of damping. Higher the damping lower will be the amplitude at resonance. Thus the sharpness will be higher at lower damping and vise-versa is the significance.

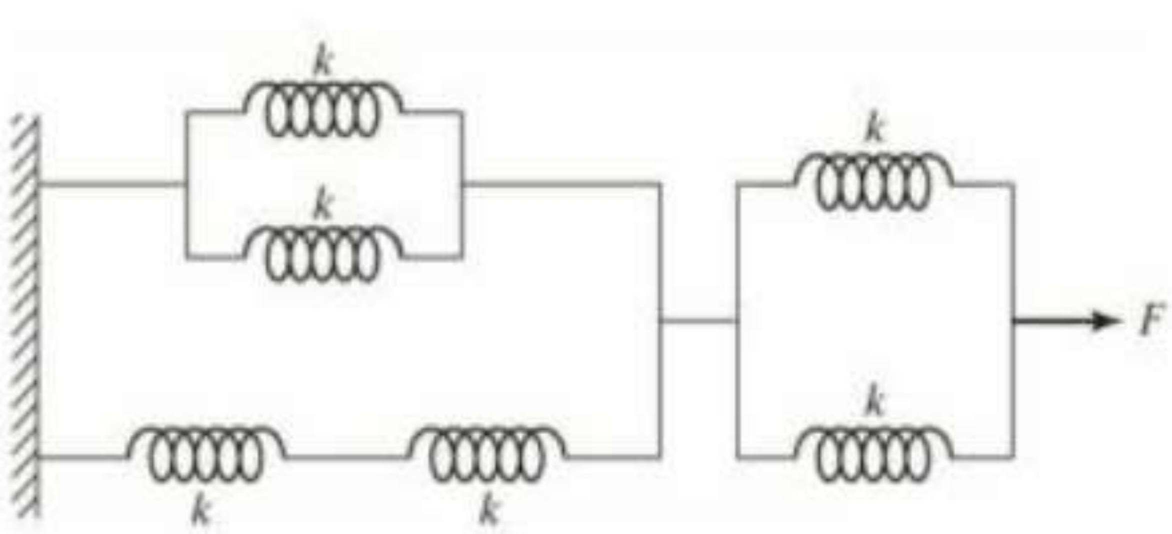
1.5 Model Questions

1. Define SHM and Explain its characteristics.
2. Set up Differential equation for the SHM and arrive at the solution.
3. Derive the equation of motion for Free Oscillations and arrive at the solution.
4. Define force constant and its physical significance.
5. Discuss the effective spring constant of sprigs in series and parallel.
6. Describe damped oscillations with examples.
7. Discuss the theory of damped oscillations and classify damped oscillations in to under damping, critical damping and over damping.
8. Discuss the theory of forced vibrations.
9. Explain the dependence of amplitude and frequency on the applied periodic force in forced oscillations.
10. Describe resonance and its physical significance.
11. Explain the sharpness of resonance.

1.6 Numerical Problems

1. Find the frequency of oscillations of a free particle executing SHM of amplitude 0.35 m if he maximum velocity it can attain is 220 m/s. Ans 100 Hz.
2. Find the displacement at the end of 3 seconds and also the amplitude of oscillations of a body executing simple harmonic motion in a straight line if its period is 10 seconds, and if its velocity is 1 m/s, at the time 2 seconds after crossing the equilibrium. Assume that there are no resistive forces. Ans:Amplitude 5.15 m, Displacement at 3 sec = 4.9 m.
3. Find the frequency of vibration of a sonometer wire which reaches a maximum velocity of 6.28 m/s, when its amplitude of vibration is 1 cm. (Assume free vibrations). Ans: frequency = 100 Hz.
4. Evaluate the resonance frequency of a spring of force constant 2467 N/m, carrying a mass of 100gm. frequency = 25 Hz.
5. Given the force constant as 9.8 N/m for a spring, estimate the number of oscillations it would complete in 1 minute if it is set for oscillations with a load of 89.37 gm. Assume there are no external forces acting on it. Ans : frequency = 100 Cycles / min.
6. A Mass of 10 kg is suspended from the free end of a spring. When set for oscillations, the system executes 100 oscillations in 5 minute. Calculate the force constant of the spring. Ans 2.27 N/m.
7. A mass of 100 kg is mounted on 4 springs each of which has spring constant $4x10^3$ N/m. The motor moves only in vertical direction. Find the natural frequency of the system.Ans : Frequency = 2 Hz.
8. A mass of 4.3 gm is attached to spring of negligible mass of force constant 17 N/m. this mass spring system is executing simple harmonic oscillations. Find out the frequency of the external force which excites resonance in the system. f=10 Hz.
9. An arrangement of identical springs is as shown in the figure 1.8. If the spring constant of each spring is $100Nm^{-1}$ calculate the effective spring constant of the combination. Also calculate the frequency of oscillation of the system when a mass of 1 kg is attached. f=1.677 Hz.

Figure 1.8:



10. A mass of 500 g is attached to a spring and the system is driven by an external periodic force of amplitude 15 N, and frequency of 0.796 Hz. The spring extends by a length of 88 mm under the given load. Calculate the amplitude of oscillations if the resistance co-efficient of the medium is 5.05 kg/s. Ignore the mass of the spring. Ans: Amplitude 0.3 m.

11. Calculate the resonance amplitude of the vibration of the system whose natural frequency is 1000 Hz when it oscillates in the resistive medium for which the value of damping per unit mass is 0.008 rad/s under the action of an external periodic force/unit mass of amplitude 5 N/kg, with tunable frequency. Ans: 0.1 m

12. A 20 gm oscillator with natural angular frequency 10 rad/sec is vibrating in damping medium. the damping force is proportional to the velocity of the vibrator. If the damping coefficient is 0.17, how does the oscillations decay. Ans: Its a case of underdamping.

Chapter 2

Shockwaves

2.1 Mach number

It is defined as the ratio of speed of the object (v_0) to the speed of the sound (v_s) in the surrounding medium. It is a dimensionless quantity. Mathematically Mach number M is given by

$$M = \frac{v_o}{v_s} \tag{2.1}$$

Mach 0.5 represents half the speed of sound and Mach 2 represents twice the speed of sound. The Mach number is named after the scientist Ernst Mach.

2.2 Classification of waves and Mach regimes

The classification of waves is as follows

2.2.1 Types of waves

Acoustics : It is the study which deals with the study of waves through gases, liquids and solids.

Infrasonics : It is the study of waves which have frequencies lower than the audible sound (<20kHz)

Ultrasonics : It is the study of waves which have frequencies higher than the audible sound (>20kHz).

Subsonic : Waves which travel with speeds less than the speed of sound in a medium are called subsonic waves ($M < 1$).

Transonic : The speed over lapping between subsonic and supersonic speeds is the transonic waves ($0.8 < M < 1.2$) .

Supersonic : Waves which travel with speeds greater than the speed of sound in a medium are called supersonic waves ($M > 1$).

2.2.2 Mach regimes

The following is a table representing the regimes of fluid flow of moving object in a fluid with regard to Mach number.

Mach No.	Regime
Subsonic	0.8
Transonic	0.8 - 1.2
Sonic	1.0
Supersonic	1.0 - 5.0
Hypersonic	5.0 - 10.0
High-Hypersonic	>10

2.3 Shock Waves and its applications

2.3.1 Shock Waves

When the speed of a moving object in a medium exceeds the speed of sound in the medium then the wave fronts lag behind the source forming a cone-shaped region with the source at a vertex. The edge of the cone forms a supersonic wave front with an unusually large amplitude called a "shock wave". A sonic boom is heard when shock waves reach an observer. Shock waves generated when a fighter jet attains the speed of sound is as shown in the figure 2.1.

Figure 2.1: Shock Waves

2.3.2 Mach Angle

Mach angle is half of the vertex angle of a Mach cone whose sine is the ratio of the speed of sound to the speed of a moving body.

$$\sin\theta = \frac{v_s}{v_o} = \frac{1}{M} \tag{2.2}$$

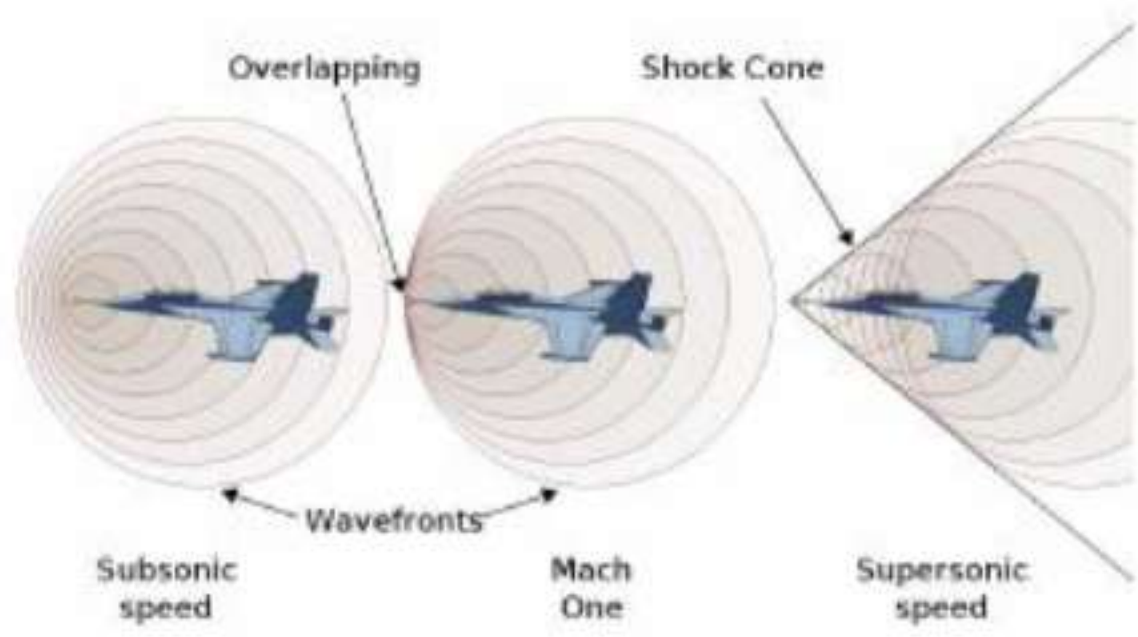


Figure 2.2: Shock Waves schematic

The Mach angle depends on the speed of the object. For transonic speeds Mach angle is 90°and for supersonic speeds Mach angle will be < 90°.

2.3.3 Properties of Shock waves

1. Shock waves travel in a medium with mach (M > 1).
2. Shock waves obey the laws of fluid dynamics.
3. When shock waves pass through a medium the entropy of the system increases.
4. When shock waves pass through a medium the changes are adiabatic.
5. General wave properties cannot be associated with shock waves.

2.3.4 Applications of Shock waves

The following are the applications of the shock waves.

1. In Aerospace the shock waves are used to study the behavior of gases and liquids at high speeds.
2. They are also used in the design of supersonic and hypersonic vehicles
3. They are used in the study of re-entry dynamics and the behavior of spacecraft.
4. Shock waves are used in chemical kinetics to determine and rate of thermal decomposition of NO_2
5. Shock waves are used tn the study of explosions the blast wave signature is a very vital parameter
6. Mach reflection of a shock wave is used to remove micron size dust particles from the surface of silicon wafers.
7. Shock waves are used in medical therapy in orthopedics and for breaking kidney stones.
8. Shock waves are used in pencil industries to impregnate preservatives into wood slats.
9. Shock waves are used in sandal oil extraction.

2.3.5 Shock Tubes

A device used to produce shock waves of required strength in the laboratory is called shock tube. The following are some of the types of shock tubes.

1. Compression Driven
2. Blast Driven
3. Piston Driven
4. Reddy Tube

2.3.6 Reddy Shock Tube

Dr. K.P.J. Reddy and associates developed a miniaturized simple hand-operated piston-driven shock tube named 'Reddy Shock Tube'. The Reddy shock tube is as shown in the figure 2.3.

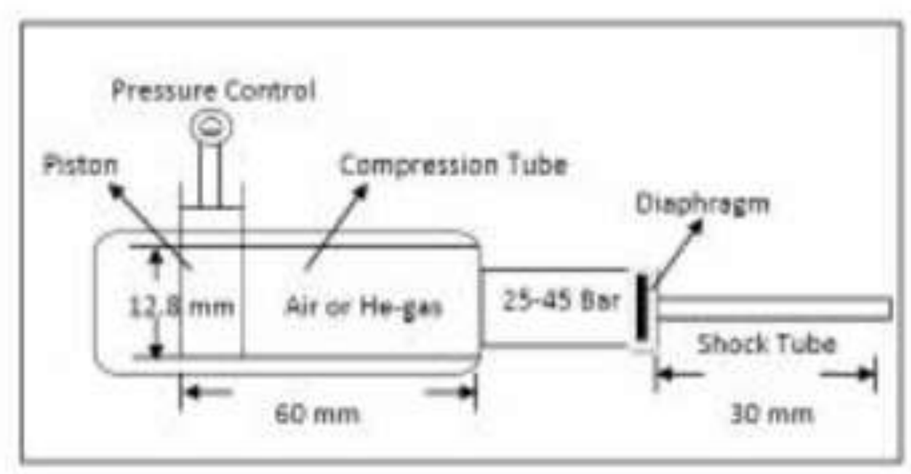


Figure 2.3: Reddy Shock Tube

Construction : Reddy tube operates on the principle of conventional piston driven shock tube is made of simple medical syringe with the following dimensions

- Compression chamber is of length 60mm and dia 12.8mm.
- A plastic piston with plunger with outer dia 12.8mm fitted closely inside the compression chamber.
- O-rings on the piston head prevent the leakage of leaking of gas.
- The driven tube is a 30 mm long SS tube of 1 mm internal diameter and wall thickness of 0.8 mm.
- The compression tube and the shock tube (hypodermic needle) are separated by a 50 μm thick plastic diaphragm.
- The elliptic free end of the hypodermic needle (shock tube) is made circular by grinding and is open to atmospheric air.

Working :

- Diaphragm and the needle are detached from the compression tube.
- the piston is moved outwards to fill the compression chamber with ambient air at atmospheric pressure.
- The diaphragm and the needle are attached to the compression tube.
- The piston is pushed into the compression chamber so that the air inside is compressed and pressure builds.
- As the piston moves the pressure increases and finally the diaphragm ruptures resulting in the very high speed flow of the compressed air.
- Schlieren images of the flow at the exit of the Reddy tube confirm the generation of shock waves.
- Thus shock waves of mach 1.5 to 2.0 are generated at the tip of the needle.

Model Questions

1. Classify the waves in to different types.
2. Define Mach number. Give the classification of waves based on Mach-number.
3. What are Shock Waves. Mention the properties of Shock wave.
4. Define control volume and hence explain the conservation laws as applied to the propagation of shock-waves in a fluid.
5. What is a shock tube? Explain in detail the construction and working of Reddy shock tube and its advantages over other shock tubes.
6. Explain briefly the applications of shock-waves.

Numerical Problem

1. The distance between two pressure sensors in a shock tube is 100mm. Shock waves travel between the two sensors in 100μs. Find the mach number of the shock wave if he velocity of sound under the condition is 330m/s.
2. Calculate the mach angle of the given the speed of source 660m/s and the velocity of the sound in the medium is 330m/s. Ans: 30°

Part II

Module - 2 - Elastic Properties of Materials

Chapter 3

Elastic Properties of Materials

3.1 Introduction

The forces that deform or tend to deform a body are called **deforming forces**. The body which undergoes change in dimensions is called a deformed or strained body. This chapter is intended to study the elastic properties of the materials.

3.2 Elasticity and Plasticity

3.2.1 Elasticity and Plasticity

Elasticity : The property of the body by the virtue of which it tends to regain its shape and size after the deforming forces are withdrawn is called Elasticity and the body is said to be elastic.

Plasticity : The property of the body by the virtue of which it cannot regain its shape and size after the deforming forces are withdrawn is called Plasticity and the body is said to be Plastic.

3.3 Stress and Strain

3.3.1 Stress

The restoring force per unit area is called Stress. It has the unit of pressure (Pa). mathematically

$$Stress = \frac{Restoring\ Foce}{Aea} = \frac{F}{A}\ Nm^{-2} \qquad (3.1)$$

3.3.2 Strain

The fractional change in the dimension of the body is called strain. Mathematically

$$Stran = \frac{Change\ in\ dimensonof\ the\ body}{Original\ dimenson\ of\ the\ body} \qquad (3.2)$$

3.3.3 Classification of Stress and Strain

- Tensile stress and Linear strain.
- Bulk stress and Volume strain.
- Shearing stress and Shearing strain.

3.4 Stress vs Strain Curve and Hooke's Law

3.4.1 Hooke's Law

For an elastic material the stress is proportional to the strain within the elastic limit. This proportionality is called Hooke's Law. Mathematically

$$Stress \propto Stran$$

$$Stress = Elastic\ Modulus \times Stran$$

Thus the elastic modulus is defined as

$$Elasic\ Modulus = \frac{Stress}{Stran} \qquad (3.3)$$

The constant of proportionality is called Elastic Modulus. The unit of Elastic Modulus is same as stress $Nm^{-2} o Pa$. Strain has no unit.

3.4.2 Stress vs Strain curve

Consider a bar or wire of uniform cross section fixed at one end and loaded at the other. The stress-strain relationship could be understood by making a plot of stress vs strain and is called stress-strain diagram. It is as shown in the figure.

1. The region "OA" is the elastic region and Hooke's Law is valid. The point "A" is the "Proportionality Limit".
2. Between the points "A" and "B" the material exhibits elastic behavior. But Hooke's Law is not valid. The point "B" is the elastic limit.
3. Further for any point 'C' the material possess permanent strain and plastic behavior is observed. While unloading the dotted curve is traced.

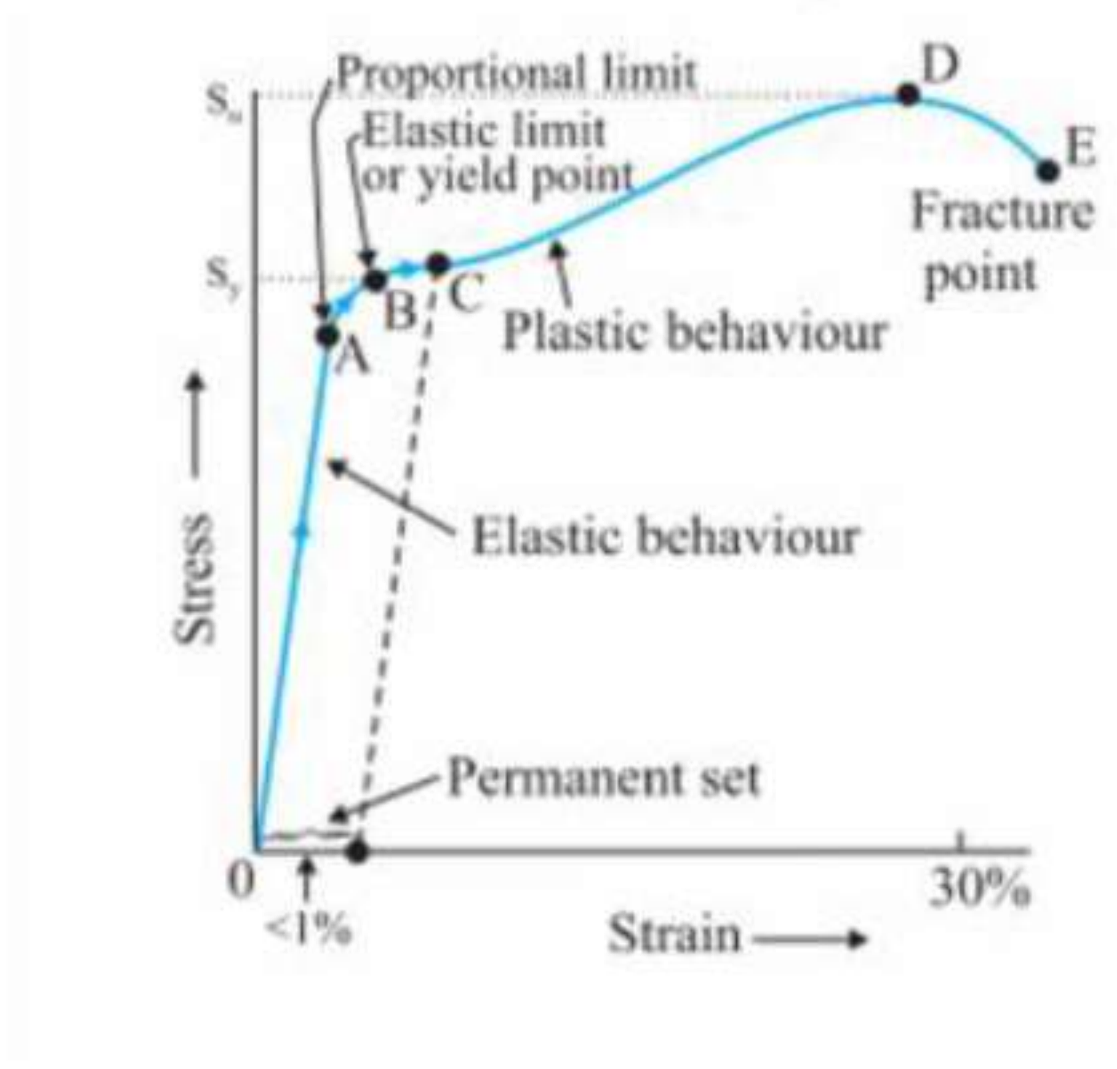


Figure 3.1: Tensile Stress and Linear Strain

4. The point "D" corresponds to the maximum stress that the material can withstand and is called "Point of ultimate strength".

5. At the point "E" material breaks and the body no more exists in single piece. The point "E" is called "Fracture Point". The stress corresponding to the point "E" is called "Breaking Stress".

3.5 Strain Hardening and Softening

3.5.1 Strain Hardening

The strain hardening is the process of making a metal harder by plastic deformation. Consider a material stressed and deformed beyond elastic limit and then unloaded gradually. The curve traced in the graph is "BC" with residual strain "OC".

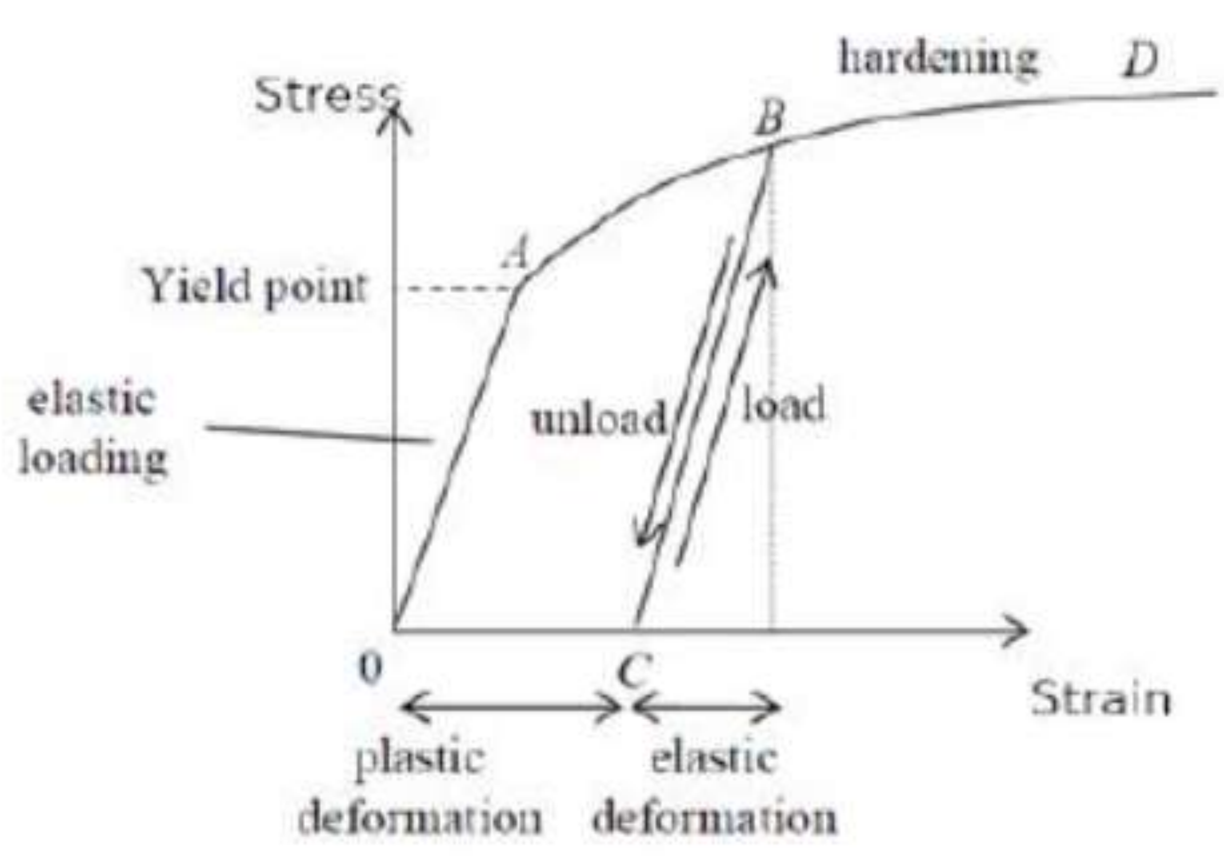


Figure 3.2: Strain Hardening

if the loading is repeated again in steps and the graph is plotted with C as origin curve CB is observed. Thus the material exhibits proportionality to a higher value of stress. Hence the material can withstand higher stresses and is known as strain hardening.

Example : copper-zinc alloy, brass, used for ammunition cartridges and the aluminum-magnesium alloys in beverage cans.

3.5.2 Strain Softening

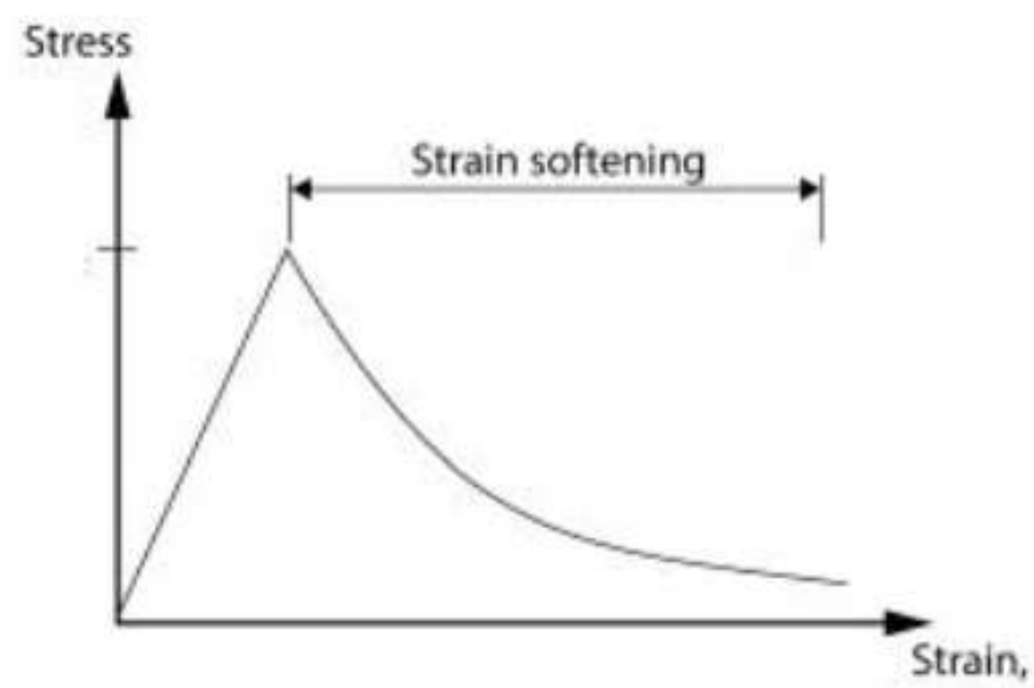


Figure 3.3: Strain Softening

In case of certain materials, when they are stressed beyond elastic region the stress-strain curve takes negative slope. Since the material can withstand only reduced stresses in this region it is called strain softening.

Example : Typically observed at a continuum level in damaged quasi brittle materials, including fibre reinforced composites and concrete.

3.6 Elastic Moduli

3.6.1 Young's Modulus(Y, q or E)

It is defined as the ratio of tensile (Longitudinal) stress to longitudinal (Linear) strain.

$$Y = \frac{Lonitudinal\ Sress}{Linear\ Srain}$$

$$Y = \frac{\frac{F}{A}}{\frac{x}{L}} = \frac{FL}{A}\ pascal\ o\ Nm^{-2} \tag{3.4}$$

Here F is the force acting on area A, x is the change in length and L is the original length.

3.6.2 Bulk Modulus (K)

It is defied as the ratio of compressive stress to volume strain.

$$k = \frac{Compessve\ Sress}{Volume\ Sran}$$

$$k = \frac{P}{\frac{\Delta V}{V}} = \frac{P}{\Delta V}\ pascal\ o\ Nm^{-2} \quad (3.5)$$

Here P is the uniform pressure applied, ΔV is the change in volume, V is the initial volume.

3.6.3 Rigidity Modulus (n or η)

It is defied as the ratio of Shearing stress to Shearing strain

$$n = \frac{Logtudinal\ Stress}{Linea\ Stran}$$

$$n = \frac{\frac{F}{A}}{\theta} = \frac{\frac{F}{A}}{\frac{x}{L}} = \frac{FL}{A}\ pascal\ o\ Nm^{-2} \quad (3.6)$$

Here F is the tangential force acting on area A, θ shearing strain or shearing angle, x is the change is the displacement along F and L is the original length.

3.7 Strain Coefficients

3.7.1 Longitudinal Strain Coefficient (α)

It is defined as the longitudinal strain produced per unit stress. The longitudinal strain is given by $\frac{x}{L}$. If T is the applied stress then the longitudinal strain coefficient is given by

$$\alpha = \frac{\frac{x}{T}}{T} = \frac{x}{L} \quad (3.7)$$

$$\Rightarrow Longtudina\ Stran = x = \alpha LT \quad (3.8)$$

3.7.2 Lateral Strain Co-efficient (β)

If a material of cylindrical shape is subjected to stretching the length increases and the diameter decreases. Hence lateral strain is observed and is defined as the ratio of change in diameter to original diameter.

$$Laeral\ Strain = \frac{d}{D}$$

For an applied stress of T

$$Laeral\ Strain\ Co-efficen = \beta = \frac{d}{TD} \quad (3.9)$$

Thus the later strain in terms of β is given by

$$d = \beta D \quad (3.10)$$

3.8 Poisson's Ratio (σ)

It is defined as the ratio of Lateral Strain to Longitudinal strain within the elastic limit.

$$\sigma = \frac{\frac{d}{D}}{\frac{x}{L}} = \frac{\frac{d}{TD}}{\frac{x}{TL}} = \frac{\beta}{\alpha} \quad (3.11)$$

3.8.1 Expressions for Y, k and n in-terms of α and β

The relations are as follows.

$$Y = \frac{1}{\alpha} \quad (3.12)$$

$$n = \frac{1}{2(\alpha+\beta)} \quad (3.13)$$

$$K = \frac{1}{3(\alpha-2\beta)} \quad (3.14)$$

3.9 Relation between Elastic Moduli

3.9.1 Relation between Y, n and σ

Consider an elastic material in the shape of cube which is glued to a rigid surface as shown in the figure.

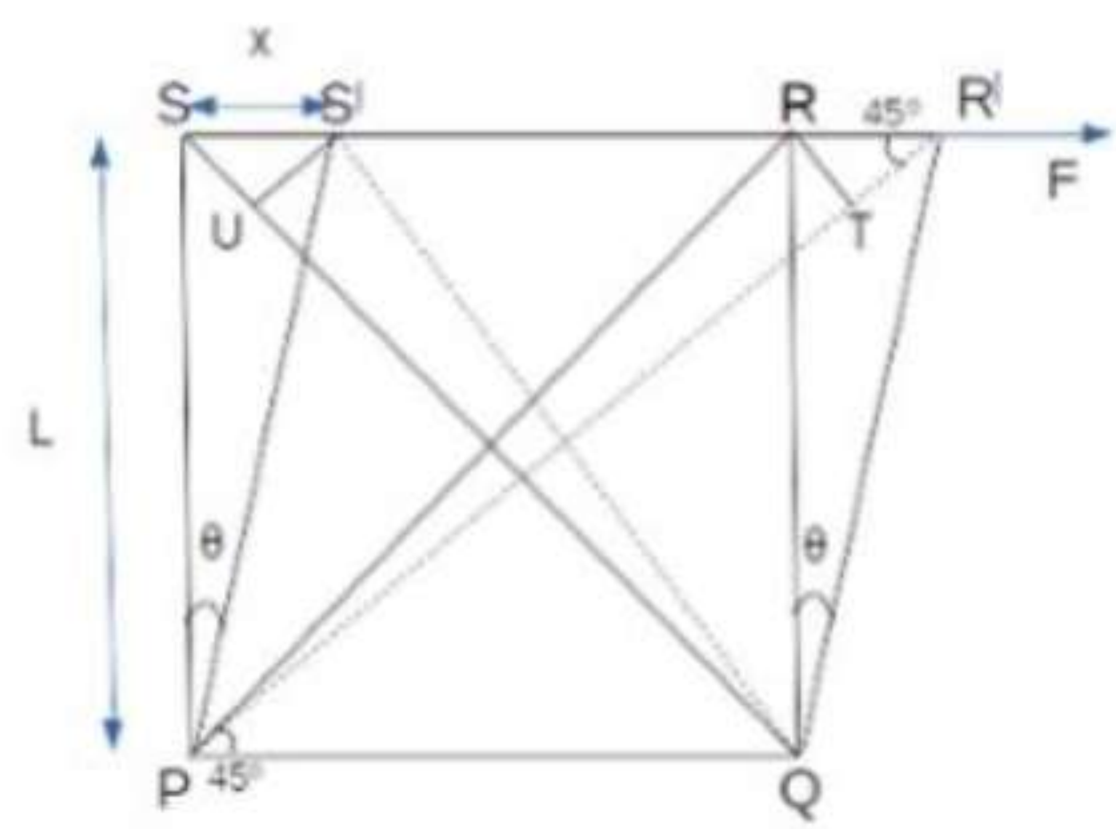


Figure 3.4: Shearing

Let α and β the longitudinal and lateral stress coefficients. The top surface shears through a distance x due to the applied tangential force F on area A. Hence PR increases to PR'. The shearing strain could be considered as due to both longitudinal strain along PR and Lateral strain along QS.

$$TR' = T(PR)\alpha + T(QS)\beta$$

$$TR' = T(PR)(\alpha+\beta)$$

Because $PR = QS$ and also using Pythagoras theorem we get $PR = L\sqrt{2}$

$$TR' = TL\sqrt{2}(\alpha + \beta) \tag{3.15}$$

Consider the angle of shear ϕ to be very small and hence $\angle PR = \angle PR\ 'Q = 45°$. Thus from Right angled $\Delta^{le} RR'Q$, we have

$$Cos(45^0) = \frac{TR'}{RR'}$$

$$TR' = \frac{RR'}{\sqrt{2}} = \frac{x}{\sqrt{2}} \tag{3.16}$$

Substituting the above in equation 3.15 we get

$$\frac{x}{\sqrt{2}} = TL\sqrt{2}(\alpha + \beta)$$

$$\frac{TL}{x} = \frac{1}{2(\alpha + \beta)}$$

$$\frac{T}{\frac{x}{L}} = \frac{1}{2\alpha\left(1 + \frac{\beta}{\alpha}\right)} \tag{3.17}$$

We know that $\frac{1}{\alpha} = Y, \frac{x}{L} = \theta, \frac{\beta}{\alpha} = \sigma and \frac{T}{\theta} = n$. Hence equation 3.17 becomes

$$n = \frac{Y}{2\left(1 + \frac{\beta}{\alpha}\right)}$$

Hence the relation between $Y, n\ and\ \sigma$ is given by

$$Y = 2\ n\ (1 + \sigma) \tag{3.18}$$

3.9.2 Relation between Y, K and σ

The relation between Y, K and σ is as follows.

$$Y = 3K(1 - 2\sigma) \tag{3.19}$$

3.9.3 Relation between Y, K and n

$$Y = \frac{9Kn}{3K + n}$$

The relation between Y, K and n is as follows. Here Y is Young's Modulus, K is Bulk Modulus and n is Rigidity Modulus.

3.9.4 Limiting vaues of σ

Consider the relations

$$Y = 2n(1 + \sigma) \tag{3.20}$$

$$Y = 3K(1 - 2\sigma) \tag{3.21}$$

equating equations 3.20 and 3.21 we get

$$2n(1 + \sigma) = 3K(1 - 2\sigma) \tag{3.22}$$

- If $\sigma > 0.5$ then LHS will be "+Ve" and RHS will be "-Ve".
- If $\sigma < -1$ then LHS will be "-Ve" and RHS will be "+Ve".
- Since both sides represent Young's Modulus they must result in the same positive value.
- Hence for both sides to be "+Ve" the σ can take values in the range $-1 < \sigma < 0.5$.
- Since σ cannot take negative values $0 < \sigma < 0.5$.
- Thus the value of σ rages from 0 to 0.5.

3.10 Bending of Beams

A beam is a bar or rod of uniform cross section whose length is much greater as compared to its other dimensions.These are used in buildings to support roofs and in bridges to support the load of vehicles passing over them.

3.10.1 Assumptions

1. The weight of the beam is negligible compared to the load applied.
2. There are no shearing forces.
3. The cross section and thus geometrical MI of the beam remains unaltered.
4. The curvature of the beam is small.

3.11 Neutral surface and Neutral axis

Whenever a beam is subjected to load it is bent longitudinally. The longitudinal filaments on the convex side of the beam are elongated and those on concave side are compressed. In between these filaments there is a filament which is neither elongated nor compressed and is called Neutral filament. The length of the neutral filaments remains unchanged even after loading the beam. ***Neutral surface*** *is that layer of a uniform beam which does not undergo any changes in its dimensions when the beam is subjected to bending within the elastic limit.* ***Neutral axis*** *is a longitudinal line of intersection of neutral surface and plane of bending.*

3.12 Types of Beams

Beams are classified into the following five types

1. A Simply supported beam is a bar resting upon supports at its ends and this is the most commonly used.

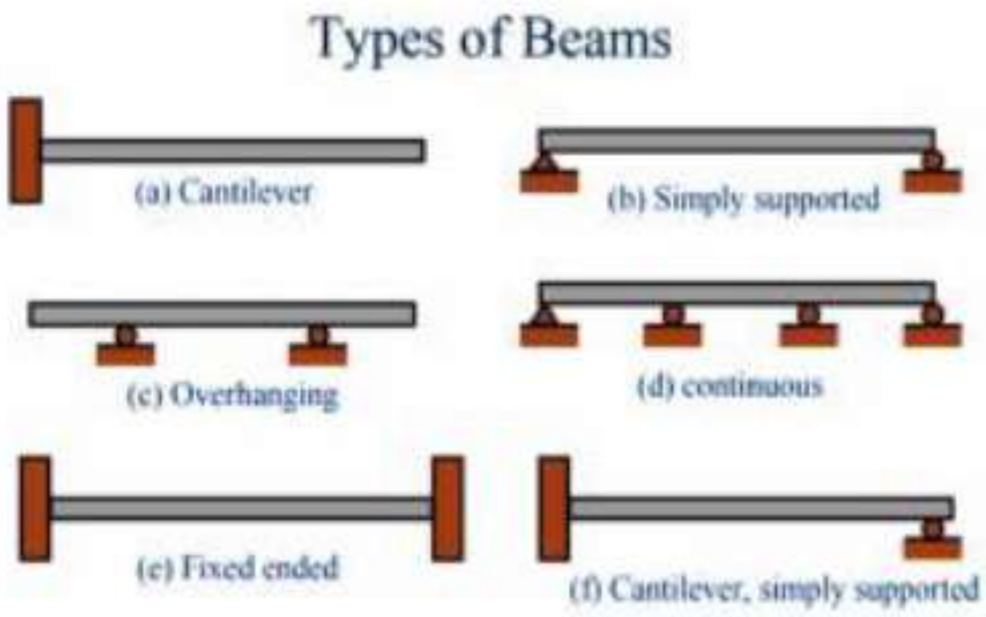


Figure 3.5: Types of Beams

2. A Continuous beam is a bar resting upon more than two supports
3. A Cantilever beam is a beam whose one end is fixed and other end is free.
4. A Fixed ended beam is beam whose both ends are fixed.
5. A Overhanging beam has its one or both ends stretching out past its supports.

3.13 Bending Moment and Expression

3.13.1 Bending Moment

Consider a cantilever beam of uniform cross-section which is bent by applying load. The moment of applied couple due to which the beam bends longitudinally is called **Bending Moment**.

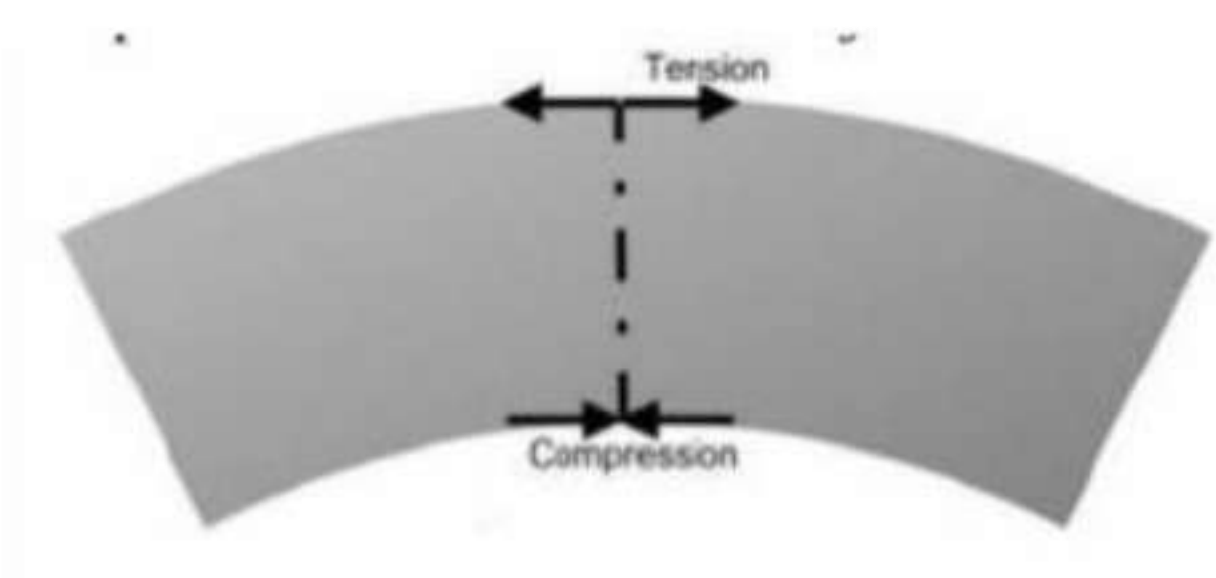


Figure 3.6: Bending of Beam

Definitions

The opposing moment developed in the material due to elasticity is called **Restoring Moment**.

3.13.2 Expression for Bending Moment

Introduction : Consider a beam of uniform cross-section. Consider three layers AB, CD and EF be three layers. Before applying the load $AB = CD = EF$. The beam bent by applying load and is as shown in the figure 3.7. On bending AB extends to $A^{'}B^{'}$ and EF contracts to $E^{'}F^{'}$. The layer CD is the neutral surface. The length CD forms an arc of a circle of radius R and subtends and angle θ at the center. The layer $A^{'}B^{'}$ is concentric to CD with radius $R + r$.

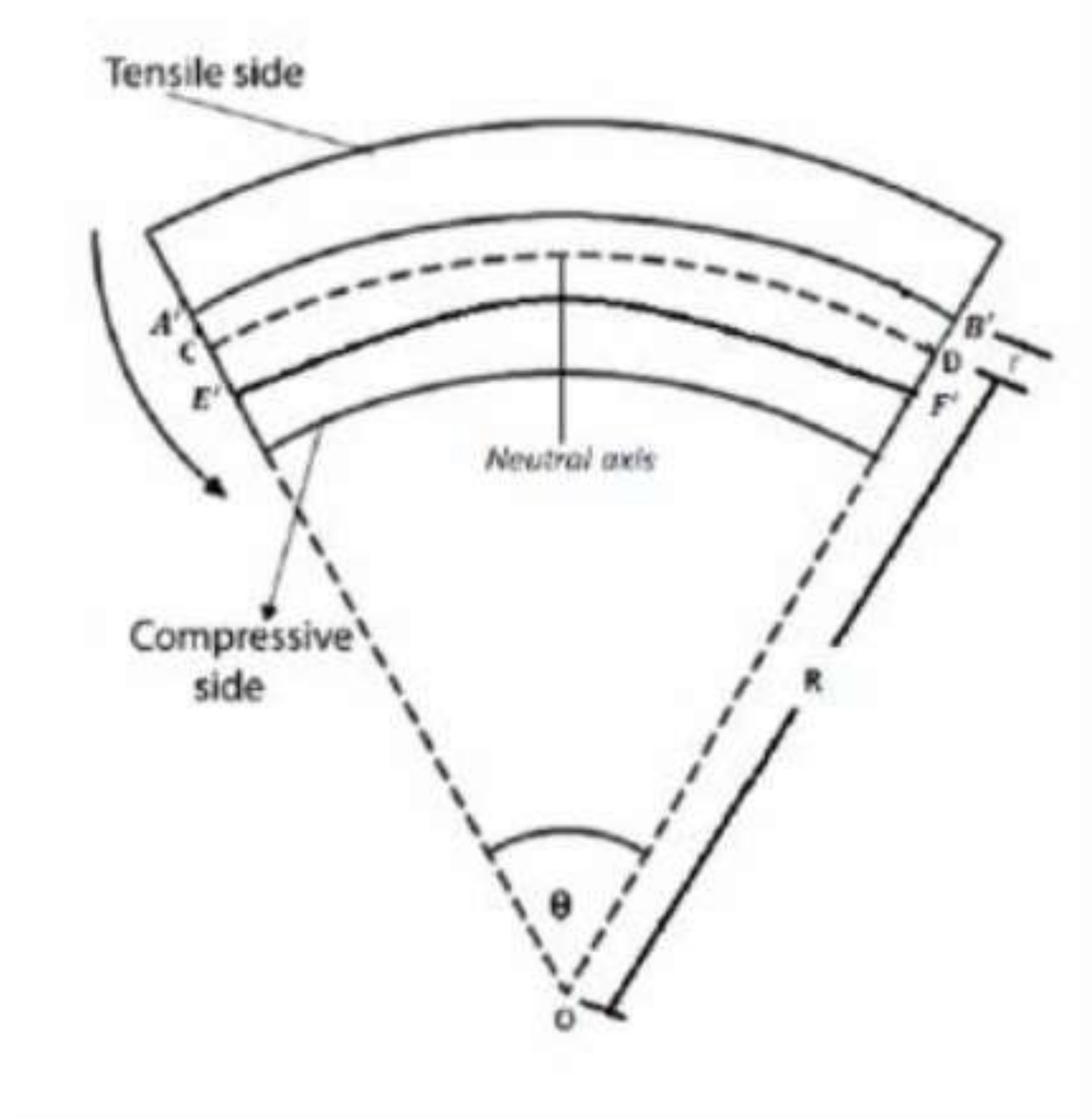


Figure 3.7: Sectional View of the Beam

From the figure $CD = R\theta$ and $A'B' = (R + r)\theta$. The change in length of the layer AB is given by

$$A'B' - AB = A'B' - CD = R\theta - (R + r)\theta$$

Thus the change in length is give by

$$Chane\ in\ Lenth = R\theta - R\theta + r\theta = r\theta$$

The original length $AB = CD = R\theta$, Thus the linear Strain is given by

$$Lie\ Stain = \frac{r\theta}{R\theta} = \frac{r}{R} \tag{3.23}$$

Let Y be the Young's Modulus of the material. According to Hooke's Law,

$$Lontudial\ Stress = Y \times Liea\ Stan \tag{3.24}$$

the longitudinal strain is also given by $\frac{F}{a}$. Thus 3.24 could be written as

$$\frac{F}{a} = Y \times Lina\ Strai$$

$$F = \frac{Ya}{R} \tag{3.25}$$

Moment of Force of a layer with respect to neutral layer is given by

$$Moment\ of\ Foce = F \times \perp Disace$$

$$Moment\ of\ Foce = F\ r = \frac{Ya^{\ 2}}{R} \tag{3.26}$$

The Bending moment is the moment of force acting on the entire beam and is given by

$$Bendin\ Moment = \sum \frac{Ya^{\ 2}}{R} = \frac{Y}{R} \sum a^{\ 2} \tag{3.27}$$

Here $\sum a^{\ 2} = I_g$ is called Geometrical Moment of Inertia of inertia.

$$Bendin\ Moment = \frac{Y}{R} I_g \tag{3.28}$$

Thus the expression for Bending Moment.

3.13.3 Geometrical MI of Rectangular and Circular Beams

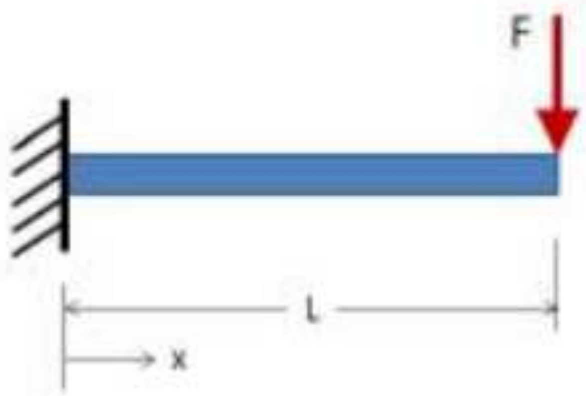
The Geometrical moment of inertia I_g depends of the geometrical shape of beam's cross-section.

1. For a beam of rectangular cross-section of breadth b and thickness d, $I_g = \frac{bd^3}{12}$.
2. For a beam of circular cross-section of radius x, $I_g = \frac{\pi x^4}{4}$.

3.14 Cantilever and I Section Girders

3.14.1 Cantilever

Definition: The cantilever is a beam fixed at one end and free at the opposite end.

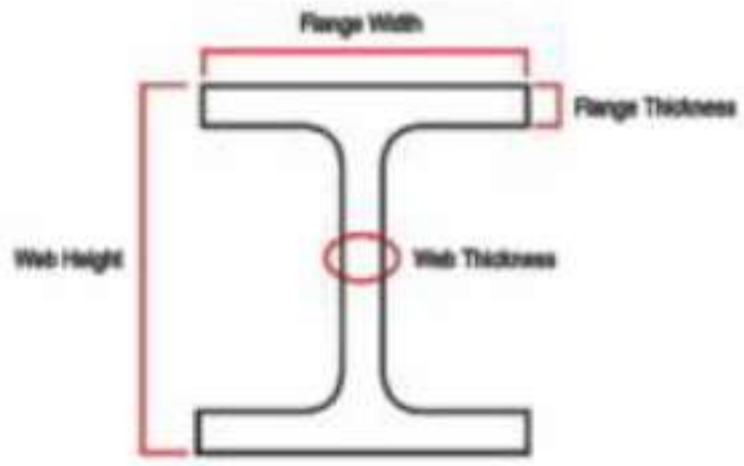


Structure : Typically it is firmly attached to a flat vertical surface like wall and extends from it outwards. It distributes the load back to support that force against a flash and shear stress.

Applications : Cantilever construction allows overhanging structures without additional supports and bracing. A cantilever beam allows the creation of a bay window, some bridges, balconies, truss, or slab.

3.14.2 I Section Girders

Definition I-shaped girders are made like the I-beam section and are composed of couple of load bearing flanges and web.



Structure and Purpose The top and bottom flanges of the "I" shape provide horizontal support and resistance against bending, while the vertical web connects the two flanges and provides resistance against shearing forces. The "I" shape of steel girders and rails provides structural efficiency, material efficiency, and cost-effectiveness.

Uses They are used in building of pulls, fly overs etc.They are also used in construction of public shades and public places.

3.15 Types of Elastic Materials

- Linear Elastic Materials : On loading exhibits a linear graph between of stress vs strain passing through the origin. Eg. Metals.
- Non-Linear Elastic Material : On loading Exhibits non linear graph of stress vs strain passing through the origin. Eg. Large strain rubber or Plastic.
- Cauchy-elastic material : In this material the stress at each point is determined only by the current state of deformation with respect to an arbitrary reference configuration.
- Hyper-elastic materials : They are conservative models that are derived from a strain energy density function.
- Hypo-elastic Materials : Hypo-elasticity implies that stress is not derivable from an energy potential.
- Elastomers : Rubbery material composed of long chain-like molecules (polymers) that are capable of

recovering their original shape after being stretched to great extents. eg. Polyurethane.

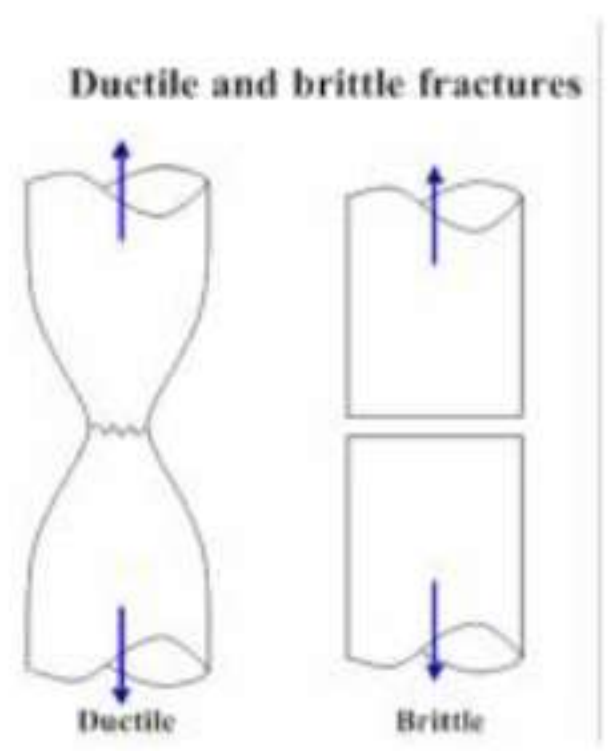
- Some Examples of Elastic materials are Thermo-Plastic Elastomer, Silicone Rubber
- Elastin, Poly Amide (Nylon), Natural Gum, Wool, Lycra.

3.16 Elastic Fatigue and Fracture

Consider an Elastic material which is subjected to number of cycles of loading and unloading. Thus the material experiences repeated strain and relief. During the course the material experiences " Elastic tiredness" or "Elastic Fatigue". A material experiencing fatigue develops cracks which grow over cycles and the materials undergoes fracture and hence called *fatigue failure*. Thus the material(part) fails during the operation. Hence fatigue is an important concern in the design of some mechanical instruments and devices like Engine Pistons, Suspension springs etc.

Fracture is classified into two types.

- Brittle fracture
- Ductile fracture



3.16.1 Brittle Fracture

Brittle fracture means fracture of material without or with very small plastic deformation before fracture. Eg. Rock, concrete, and cast iron. Such materials are called brittle materials.

3.16.2 Ductile Fracture

Ductile fracture is a type of failure seen in readily deformable (malleable) materials and is characterized by extensive plastic deformation or necking that occurs before the material finally cracks or breaks apart. Eg. Aluminium , Copper. Such materials are called ductile materials.

3.17 Stress Concentration and Concentration Factor

3.17.1 Stress Concentration

A stress concentration is a point in a part where the stress is significantly greater than its surrounding area. Stress concentrations occur as a result of irregularities in the geometry or within the material of a component structure types of Elastic Materials that cause an interruption of the stress flow. These interruptions typically arise from discontinuities such as holes, grooves, notches and fillets. Stress concentrations may also be caused by accidental damage such as nicks and scratches.

3.17.2 Concentration Factor

A stress concentration factor (K_t) is a dimensionless factor that is used to quantify how concentrated the stress is in a mechanical part. It is defined as the ratio of the highest stress in the part compared to a reference stress (Nominal Stress).

$$K_t = \frac{S_{max}}{S_{ref}} \tag{3.29}$$

3.18 Factors Affecting Fatigue

3.18.1 Surface Effect

There are always uneven machining marks on the machinated surface. These marks are equivalent to tiny gaps, which cause stress concentration on the surface of the material, thus reducing the fatigue strength of the material.

3.18.2 Design Effect

Any notch or geometrical discontinuity can act as a stress raiser and fatigue crack initiation site; these design features include grooves, holes, keyways, threads, and so on.The sharper the discontinuity (i.e., the smaller the radius of curvature),the more severe the stress concentration.

3.18.3 Environmental Effects

Two types of environment-assisted fatigue are thermal fatigue and corrosion fatigue.

Thermal Fatigue

Thermal fatigue is normally induced at elevated temperatures by fluctuating thermal stresses.

Corrosion Fatigue

Failure that occurs by the simultaneous action of a cyclic stress and chemical attack is termed corrosion fatigue.

Corrosive environments have a deleterious influence and produce shorter fatigue lives.

3.19 Model Questions

1. State Hooke's Law and Explain Stress Vs Strain curve with the help of a neat sketch.

2. Discuss Strain Hardening and Softening.

3. Define Elastic Modulus and Explain the three types of Elastic Moduli.

4. Explain Longitudinal and Lateral Strain coefficients and hence define Poisson's ration.

5. Derive the relation between y, n and σ.

6. Discuss the limiting values of Poisson's Ratio.

7. Define Beam and explain the classification of beams.

8. Define Bending Moment, Restoring Moment, Neutral Surface, Neutral Axis.

9. Derive an expression for the Bending Moment of a beam.

10. Describe Cantilever and I section Girder with the held of neat sketches.

11. Discuss the types of elastic material.

12. Elucidate the stress concentration and concentration factor.

13. Discuss the Surface, Design and Environment factors affecting Fatigue.

3.20 Numerical Problems

1. Consider a steel wire of radius 0.13 mm and length 2m. If the wire is rigidly fixed at one and loaded at the other with a mass of 1.5 kg the extension observed is 2 mm. Calculate the Young's Modulus of the material of the wire. Ans : 200 GPa

2. Calculate the force required to produce an extension of 1 mm in steel wire of length 1 m and diameter 1 mm. Given Y = 100 GPa. Ans: F = 157.08 N.

3. A solid sphere of radius 10.3 meter is subjected to a normal pressures of 10^6 Nm^{-2} acting all over the surface. Determine the change in its volume.

4. A bar is subjected to a tensile load of 55 kN. diameter=31 mm; Gauge length=300mm; extension=.115mm; change in diameter= 0.00367mm. Find: Poisson's ratio, Young's modulus, Bulk modulus, and modulus of rigidity. Ans: Poisson's Ratio σ= 0.308, Y = 190 GPa, K = 165 GPa, η = 72.7 GPa.

5. a block of gelatin is 60mm by 60mm by 20mm when unstressed. A force of 0.245 N is applied tangentially to the upper surface causing a 5mm displacement relative to the lower surface. The block is placed such that 60x60 comes on the lower and upper surface. Find the shearing stress, shearing strain and shearing modulus.Ans: Shearing stress 68.1 Pa, Shearing Strain=0.25, Shearing Modulus=274.4 Pa.

6. A steel plate is bent into a circular path of radius 10 metres. If the plate section be 120 mm wide and 20 mm thick, then calculate the Bending Moment and maximum bending stress given Y=200 GPa. Hint - $\frac{M}{I_g} = \frac{Y}{R} = \frac{\S_b}{y}$

7. Calculate the maximum stress due to Bending in a steel strip of 30 mm thick and 60 mm wide is bent around a circular drum of 3.6 m diameter [Take Young's modulus = 200kN/m2]. Hint - $\frac{M}{I_g} = \frac{Y}{R} = \frac{\S_b}{y}$

8. A steel plate is bent into a circular path of radius 10 m. If the plate section be 120 mm wide and 20 mm thick, then calculate the Bending Moment and maximum bending stress given Y=200 GPa.

Part III

Module-3 - Thermo-electric Materials

Chapter 4

Thermo-Electric Materials

4.1 Thermo EMF and Thermo Current

When two dissimilar metals are joined at their ends to form two junctions and if these two junctions are maintained at two different temperatures a current is found to flow in the closed circuit and hence emf is produced. The emf is called as Thermo Emf and current is known as Thermo Current. Thermo emf is generated due to the thermal gradient.

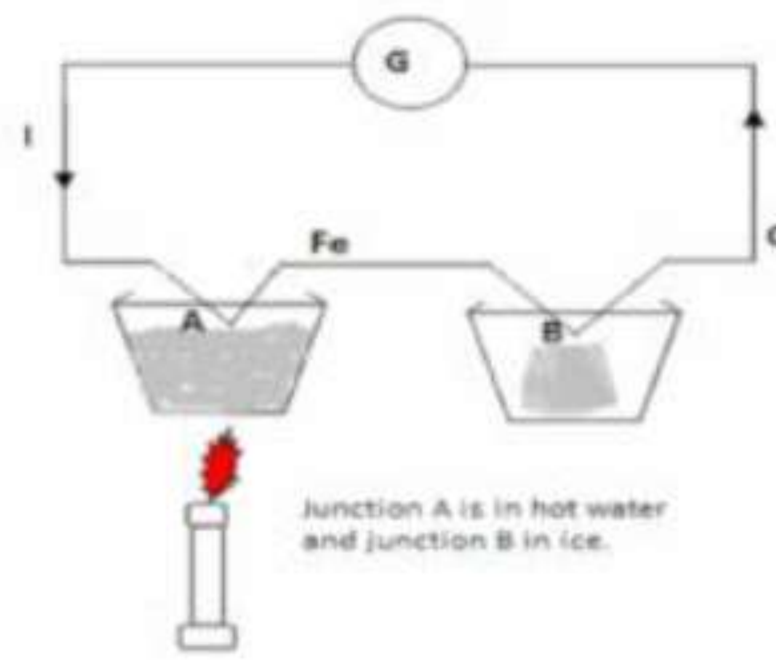


Figure 4.1: Thermoelectricity

Consider two junctions formed between two metals Fe and Cu as in the figure 4.1. One junction is kept in hot water and the other in ice. Thus Thermo current flows through the circuit indicated by the deflection in the galvanometer. The flow of Thermo current indicates the existence of Thermo Emf. The pair of metals forming the circuit is called a Thermocouple .

4.2 Seebeck Effect and Seebeck Coefficient

4.2.1 Seebeck Effect

Seebeck Effect was discovered in the year 1821 by a German Physicist Thomas Johann Seebeck.

The **Seebeck effect** is a phenomenon in which a temperature difference between two dissimilar electrical conductors or semiconductors produces a voltage difference between the two substances (Fig. 4.6). The voltage (Thermo electric emf) created is of the order of several micro volts per kelvin difference. If the circuit forms a closed loop then Thermo-current flows through the circuit.

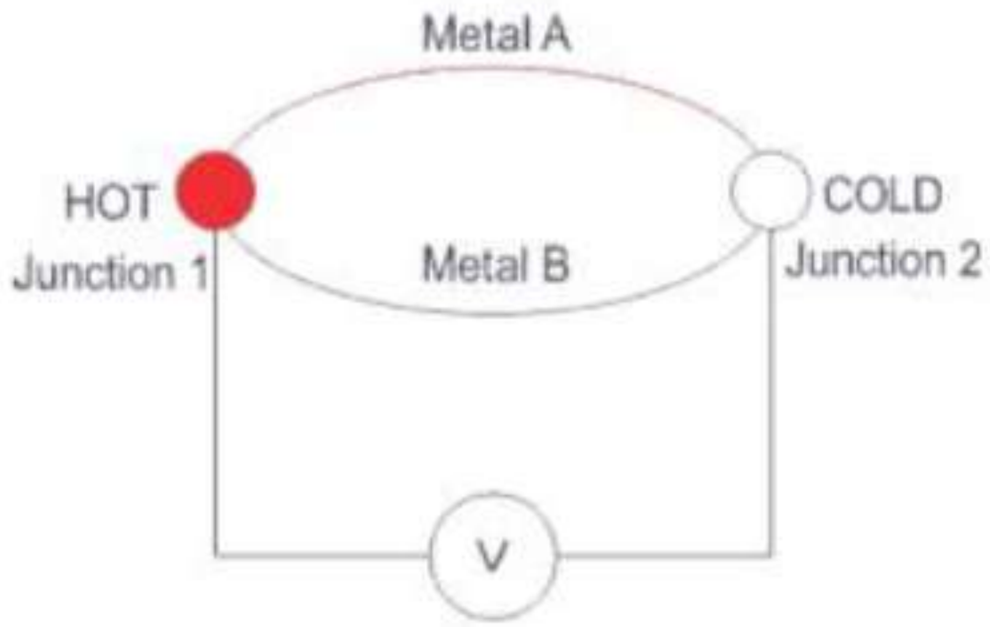


Figure 4.2: Seebeck Effect

The Thermo EMF developed is directly proportional to the difference is the temperatures of cold and hot junctions.

$$V = \alpha(T_2 - T_1) \quad (4.1)$$

$\alpha = \alpha_B - \alpha_A$, here α_A and α_B are Seebeck Coefficients of Metal A and Metal B and T_1 and T_2 are the temperatures of cold and hot Junctions.

The magnitude and direction of thermometric current depends on the types of metals used and the temperature between the hot and cold ends.

4.2.2 Seebeck Coefficient

The Seebeck coefficient is a property that measures the magnitude of voltage generated per unit temperature difference for a given material. It is defined as the ratio of open circuit potential difference generated (ΔV) to the difference in temperature between the junctions (ΔT).

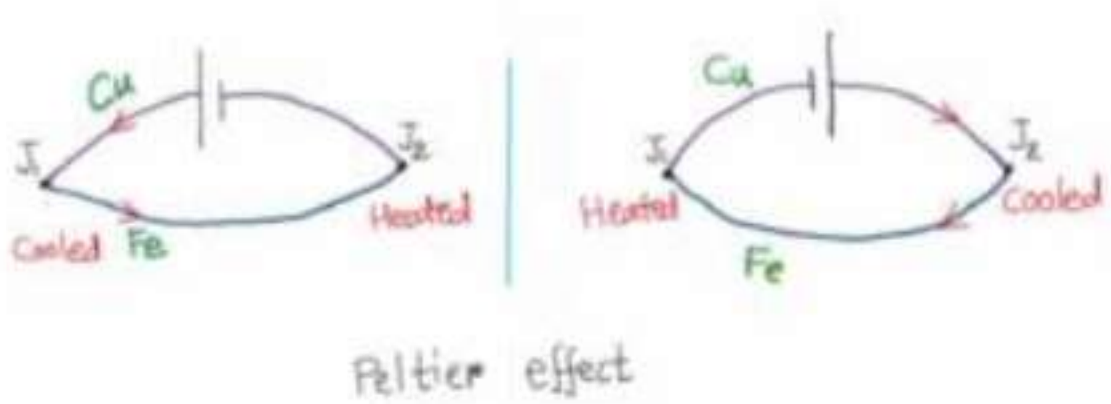
$$\alpha = -\frac{\Delta V}{\Delta T} \quad (4.2)$$

α defines the efficiency of thermometric material.

4.3 Peltier Effect and Peltier Coefficient

4.3.1 Peltier Effect

It was discovered by **Jean-Charles-Athanase Peltier** in the year 1834 Peltier. If electric current passed in a circuit consisting of two dissimilar metals, heat is evolved at one junction and absorbed at the other junction. This is known as Peltier effect. It is the inverse of the Seebeck effect. There is heat absorption or generation at the junctions depending on the direction of current flow. The Peltier effect depicts the transformation of electrical energy into heat energy. The heating and cooling effect depends on the direction of flow of current and is as shown in the figure 4.3.1



4.3.2 Peltier Coefficient

The Peltier coefficient is defined as the amount of heat energy absorbed or evolved at the junction of two dissimilar metals when one ampere of current flows through it for one second. It is denoted by π and expressed in volts. It is a property that depends on both the materials of the junction.

$$\text{Heat absorbed in one second} = \pi_{ab} I \tag{4.3}$$

$$\text{Heat absorbed in 't' Seconds} = H = \pi_{ab} It = \pi_{ab} q \tag{4.4}$$

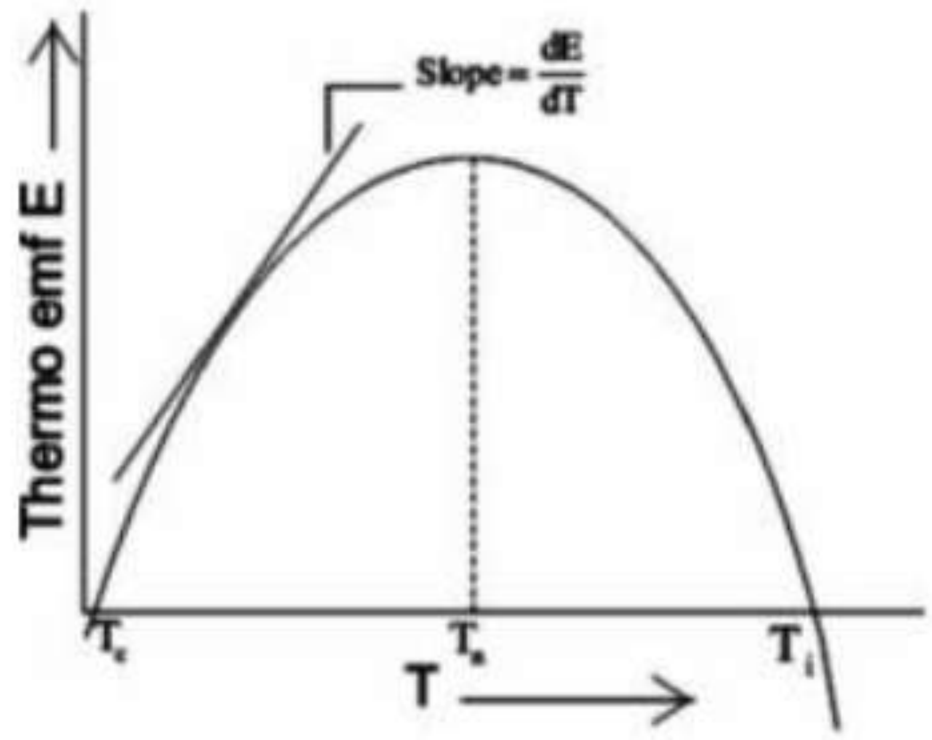
The junction emf π_{ab} , is known as Peltier coefficient and is given by

$$\pi_{ab} = \frac{H}{It} \tag{4.5}$$

the sign of π_{ab} depends on the direction of flow of current.

4.4 Variation of Thermo EMF with Temperature

Let us take a copper-iron Thermo-couple, with a galvanometer G in the circuit. Let the temperature of the cold junction be fixed at $T_c = 0$ °C.



Now the temperature of hot junction is gradually increased and the corresponding Thermo EMF is noted. A graph is plotted between the temperature of the hot junction and the Thermo EMF. The graph is shown in the figure 4.4. From the graph it is evident that

- When both the junctions are at the same temperature (0°C), then Thermo EMF is also zero.
- As the temperature of the hot junction increases, Thermo EMF also increases, till it becomes maximum.
- The temperature at which Thermo EMF becomes maximum is called ***Neutral temperature***. It is represented by T_n.
- When the temperature of the hot junction is increased beyond neutral temperature the Thermo EMF starts decreasing instead of increasing.
- At another particular temperature of the hot junction, the Thermo EMF becomes zero on heating slightly further, the direction of Thermo EMF is reversed called ***Inversion Temperature*** (T_i).

4.5 Relation between T_i and T_n

The variation of Thermo EMF as a function of temperature is given according to the equation

$$e = at + \frac{1}{2} bt^2 \tag{4.6}$$

Here a and b are Seebeck constants, $t = T_i - T_n$, and the equation 4.6 is called Seebeck equation.

Differentiating equation 4.6 with respect to temperature 't' we get

$$\frac{\partial e}{\partial t} = a + bt \tag{4.7}$$

at $T = T_n$, e is maximum and hence $\frac{\partial}{\partial} = 0$. Therefore from equation 4.7 Thus

$$0 = a + b \ _n$$

and hence

$$T_n = -\frac{a}{b} \tag{4.8}$$

At $T = Ti$, $e = 0$. Thus from equation 4.6

$$0 = a \;_i + \frac{1}{2} b \;_i^2$$

$$T_i \left(a + \frac{1}{2} b \;_i \right) = 0$$

Since $T_i \neq 0$

$$\left(a + \frac{1}{2} b \;_i \right) = 0$$

$$T_i = -2\frac{a}{b} \tag{4.9}$$

Thus from equations 4.8 and 4.9

$$T_i \;= 2T_n \tag{4.10}$$

Note : Relation between T_i, T_c and T_n

$$T_n - T_c = T_i - T_n \tag{4.11}$$

For $T_c = 0$ in equation 4.11 we get $Ti = 2Tn$.

4.6 Thermo-electric Power

The rate of change of emf with temperature is called Thermo-electric power and is denoted by P.

$$P = \frac{d}{d} \tag{4.12}$$

Relation between Peltier coefficient and Thermo-electric power is given by $\pi = TP$, here T is the temperature of the junction and P is the thermometric power at that temperature.

4.7 Figure of Merit

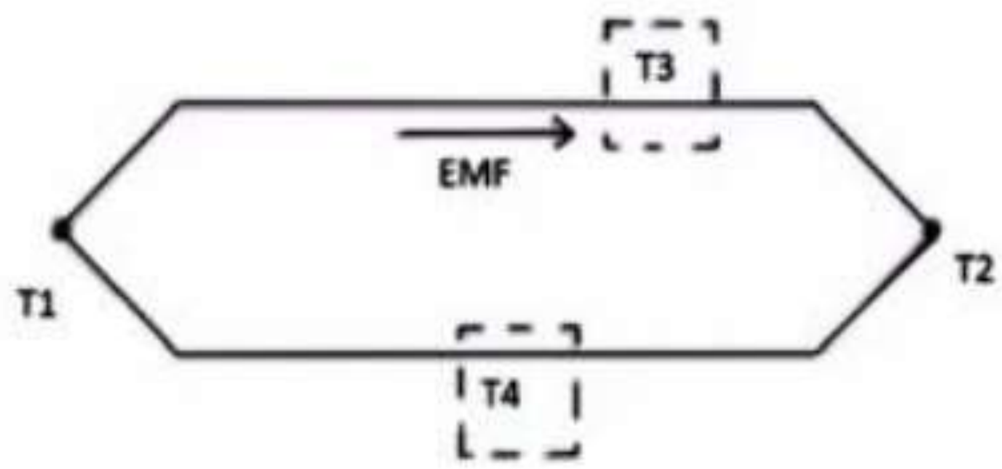
The usefulness of a material in Thermo-electric systems is determined by the device efficiency. This is determined by the material's electrical conductivity (σ), thermal conductivity (κ), and Seebeck coefficient (α), which change with temperature (T). The maximum efficiency of the energy conversion process (for both power generation and cooling) at a given temperature point in the material is determined by the figure of merit z of Thermo-electric materials.

$$z = \frac{\alpha^2\sigma}{\kappa}T \tag{4.13}$$

4.8 Laws of Thermo-electricity

4.8.1 Law of homogeneous circuits

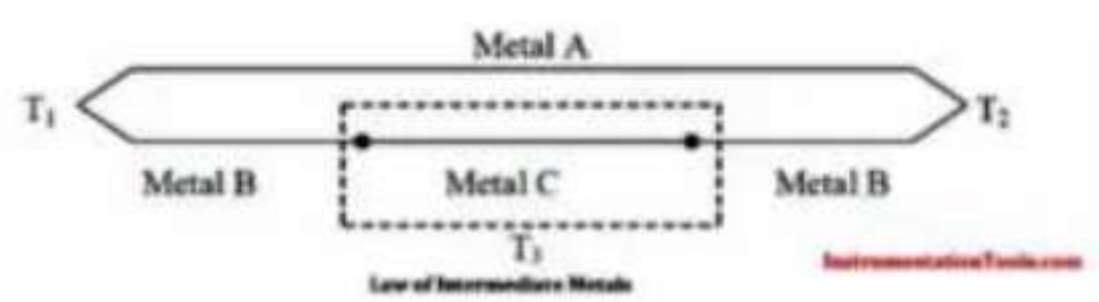
Statement: A Thermo-electric current cannot be sustained in a circuit of single homogeneous material by the application of heat alone.The thermal emf produced by two thermocouples at T_1 and T_2 is independent of and unaffected by any temperature variation down the wires.



Practical significance: Two different materials are required for any thermocouple circuit to produce thermo emf.

4.8.2 Law of intermediate metals

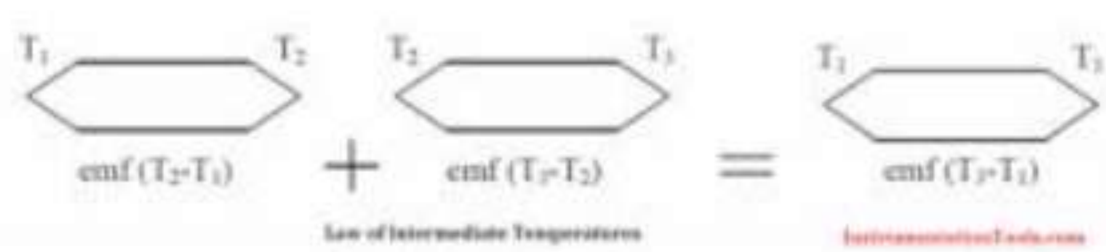
Statement: A third metal may be inserted into a thermocouple system without effecting the emf generated, if and only if, the junctions with the third metal are kept at the same temperature.



Practical Significance It allows the use of extension wires of metal for measuring instruments and soldering different from the metal used to form Thermocouple.

4.8.3 Law of intermediate temperature

Statement: the sum of the EMFs developed by a thermocouple with its junctions at temperatures T_1 and T_2, and with its junctions at temperatures T_2 and T_3, will be the same as the EMF developed if the thermocouple junctions are at temperatures T_1 and T_3.

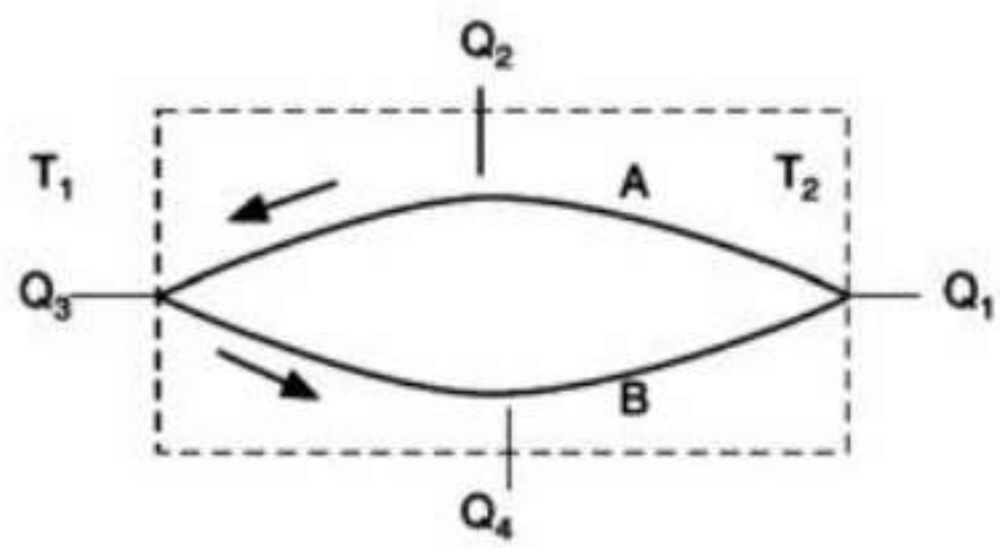


Practical significance: Referring to the figure, This law is useful in practice because it helps in giving a suitable correction in case of a reference junction temperature other than 0 °C is employed.

Note: For example , if a thermocouple is calibrated with reference junction a 0 °C and is with a junction temperature of 20 °C, then correction required for the observation would be the emf produced by the thermocouple between 0 °C and 20 °C

4.9 Expression for Thermo EMF in terms of Temperature of Cold (T_1) and Hot (T_2) Junctions

Consider circuit of two dissimilar metals A and B with two junctions. Let T_1K and T_2K be the temperatures of cold and hot junctions. The electric current flows in the circuit due to Seebeck effect. Due to the flow of electric current through hot and cold junctions the Peltier effect will be in action which results in the absorption of heat by Hot junction and loss of heat at the cold junction. Let π_1 and π_2 be the Peltier coefficients at cold and hot junctions.



Thus $Q = \pi_1 q$ is the heat evolved at cold junction $Q^{'} = \pi_2 q$ is the heat absorbed at hot junction. The total energy (PD) used in driving the current through the circuit is given by $(\pi_2 - \pi_1)q$. Since the Peltier coefficient represents voltage the Thermo EMF is given by

$$e = \pi_2 - \pi_1 \tag{4.14}$$

Considering the process as similar to the Carnot's engine, hot junction representing the source and cold junction representing the sink and by the definition of Peltier coefficients we can write

$$\frac{\pi_1 q}{T_1} = \frac{\pi_2 q}{T_2} \tag{4.15}$$

$$\frac{\pi_1}{T_1} = \frac{\pi_2}{T_2}$$

$$\frac{\pi_1}{\pi_2} = \frac{T_1}{T_2}$$

$$\frac{\pi_2}{\pi_1} = \frac{T_2}{T_1}$$

$$\frac{\pi_2}{\pi_1} - 1 = \frac{T_2}{T_1} - 1$$

$$\frac{\pi_2 - \pi_1}{\pi_1} = \frac{T_2 - T_1}{T_1}$$

$$\pi_2 - \pi_1 = \frac{\pi_1}{T_1}(T_2 - T_1)$$

Thus the expression for Thermo EMF in terms of T_1 and T_2 is given by

$$e = \frac{\pi_1}{T_1}(T_2 - T_1) \tag{4.16}$$

If the temperature of the cold junction T_1 is constant then the Peltier coefficient π_1 is also constant and hence

$$e \propto (T_2 - T_1) \tag{4.17}$$

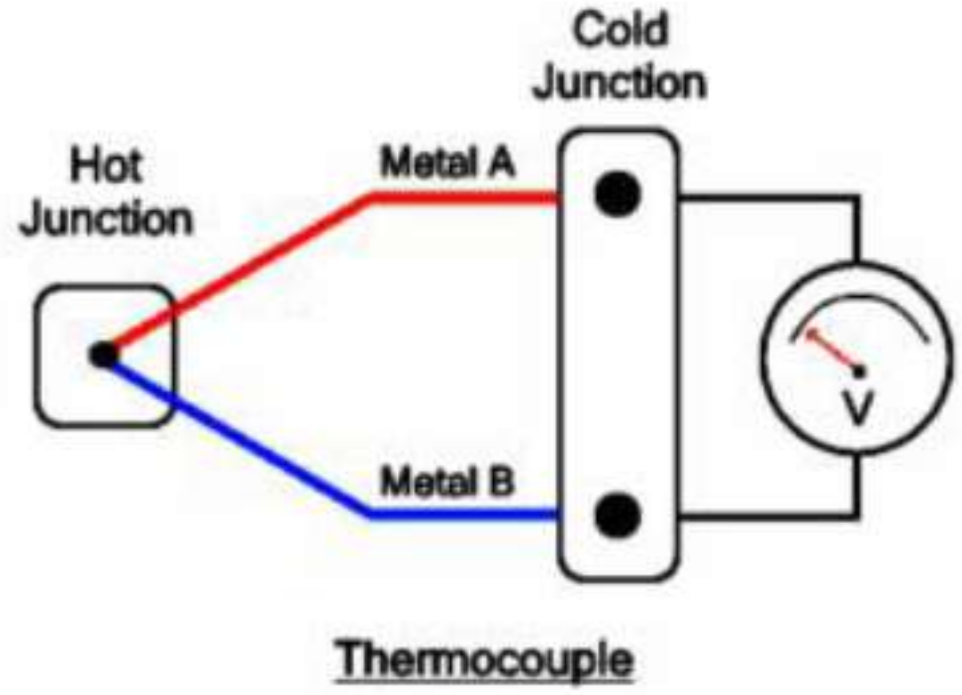
4.10 Thermo-couple and Thermo-pile

4.10.1 Thermocouple

Principle: It is based on the principle of Seebeck effect.

Description: A thermocouple is a transducer that converts thermal energy into electrical energy. It is used to measure unknown temperature.

Construction: Thermo-couple is constructed by joining wires made from dissimilar metals to form a junction. Voltage is produced when the temperature at the junction changes. A is used to measure Thermo EMF.



Working - Measurement of Temperature : First the thermocouple is calibrated by studying the variation of Thermo EMF as a function of the temperature of HOT junction keeping the temperature of cold junction constant, say 0°C. A graph of Thermo EMF Vs Temperature of Hot junction is plotted This graph is known as the calibration graph.

Next, the hot junction is placed in a bath of unknown temperature and the EMF developed in the thermocouple is noted. The temperature corresponding to this EMF is read from the calibration graph.

Advantages

1. It is an active transducer i.e., it operates without any external power source.
2. Measurement of wide ranges of temperature from -200°C to 2800°C.
3. The response time is fast, which can measure fast-changing temperatures.
4. The cost of thermocouples is low compared to thermistors.
5. Able to measure temperatures at desired points.

Disadvantages

1. The output voltage produced is low.
2. The stray magnetic field can introduce errors in output voltage. Accuracy is low

4.10.2 Thermo-pile

Description: A thermopile is an electronic device that converts thermal energy into electrical energy. It is composed of several thermocouples connected usually in series or less commonly in parallel.

Construction The structure of the Thermopile is shown in figure 4.3. The output voltage of a single Thermo-electric cell is extremely small. So a number of these cells is connected in series/parallel to get a larger signal output. The arrangement of this thermocouple stack is called "thermopile".

A thermopile includes a series of thermocouples where each includes two special materials with large thermoelectric power and reverse polarities which are interconnected in series.

Working: These thermocouples are arranged throughout the cold and hot areas and the hot junctions are isolated thermally from the cold junctions. In reply to the temperature variation across the material, the output voltage of the thermopile is called a Seebeck coefficient or thermoelectric coefficient. So it is measured per kelvin (V/K) otherwise m/K in volts.

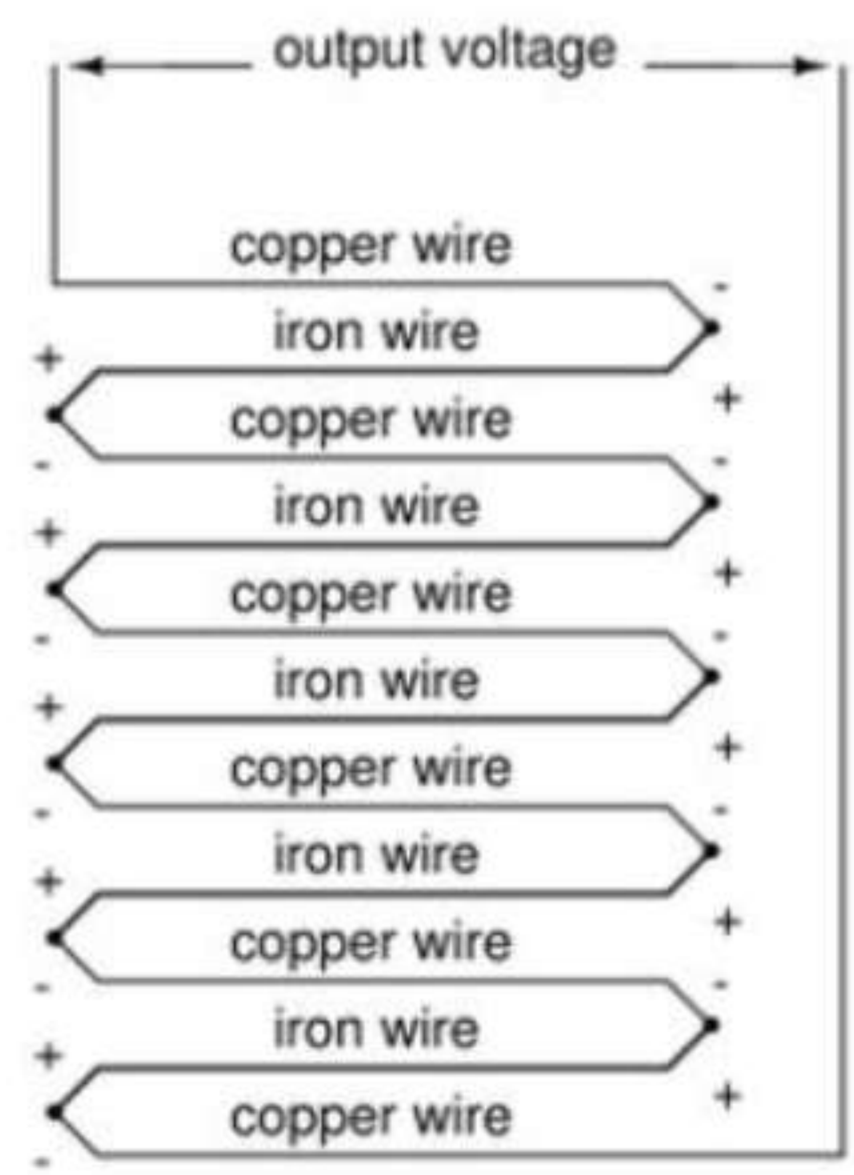


Figure 4.3: Thermopile

Advantages

1. It doesn't need an external power supply.
2. It gives a stable response to radiation which is gone from temperature-measuring bodies.
3. It has stable response characteristics.
4. Thermopile is a non-contact temperature-detecting device that uses IR radiation to transfer heat.
5. These are available in small sizes.
6. It is less costly.
7. It generates larger output voltage because of the usage of several thermocouple devices.

Disadvantages

1. The thermopile that are not in use should be stored within conductive material to defend them from static discharges and static fields.
2. These can be damaged due to stress and reverse the polarity of the supply.
3. These should not be directly exposed to moisture or sunlight because this may harm or will have corrosion on the device's performance.
4. This device should not be operated with dirty or oily fingers because this dust will affect the device's performance. For superior performance, we need to clean with cotton swabs or alcohol.

5. For precise temperature measurement, an object should fill the field of view completely of the thermopile device.

4.11 Thermo-electric Generators (TEG), Thermo-electric Coolers (TEC)

4.11.1 Thermoelectric Generators (TEG)

Principle: Thermoelectric generators are the devices that convert the temperature difference that is generated between the two sections into the electrical form of energy (DC Voltage). When a load is properly connected, electrical current flows.

Construction: The simplest thermoelectric generator consists (figure 4.4) of a thermocouple, comprising a p-type and n-type thermo-element connected electrically in series and thermally in parallel (Fig).The P-type and N-type semiconductors are interconnected through a metal. Load is connected to free end of P and N type semiconductors.

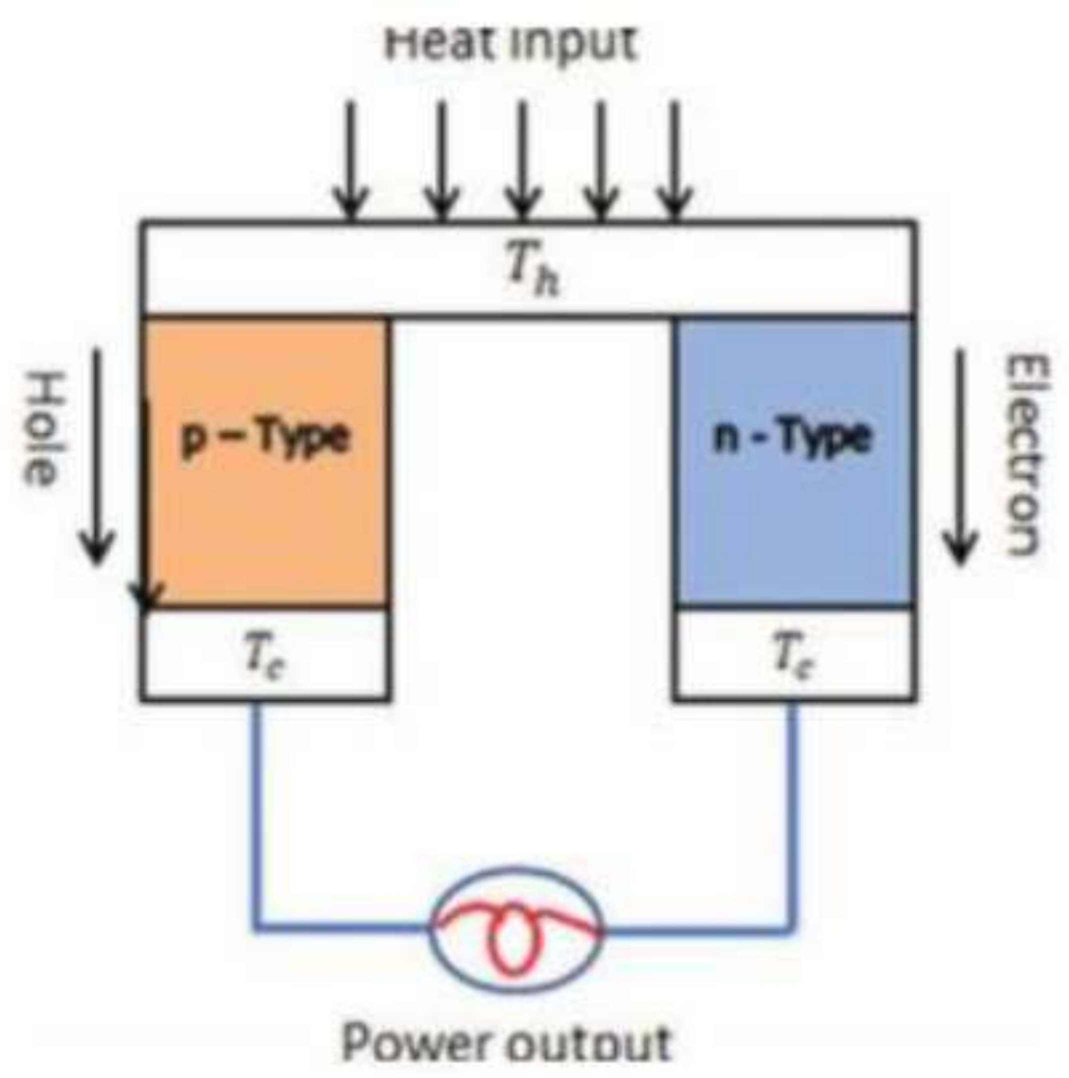


Figure 4.4: Thermoelectric Generator

Working: Heat is pumped into one side of the couple and rejected from the opposite side. The electrons present at the hot end would be at a high energy level as compared to electrons present at the cool end side. This means that when temperature gradient is produced hot electrons will tend to move towards the cool end due and DC voltage is produced. An electrical current is produced, proportional to the temperature gradient between the hot an cold junctions. Some examples are semiconductors based on bismuth telluride, lead telluride and silicon-germanium alloys are found to be the best.

Applications:

1. TEGs enhance the fuel performance of cars by converting heat liberated into electricity.
2. Seebeck Power Generation is utilized to provide power for the spacecraft.
3. Thermoelectric generators to implemented provide power for the remote stations such as weather systems, relay networks, and others

Typical applications for this technology include providing power for remote telecommunication, navigation, and petroleum installations.

4.11.2 Thermo-electric Coolers

Thermoelectric coolers are solid state heat pump used in applications where temperature stabilization, temperature cycling, or cooling below ambient are required.

Principle:

The principle used in this is Peltier effect. i.e: 'when electric current passed in a circuit consisting of two dissimilar metals, heat is evolved at one junction and absorbed at the other junction.'

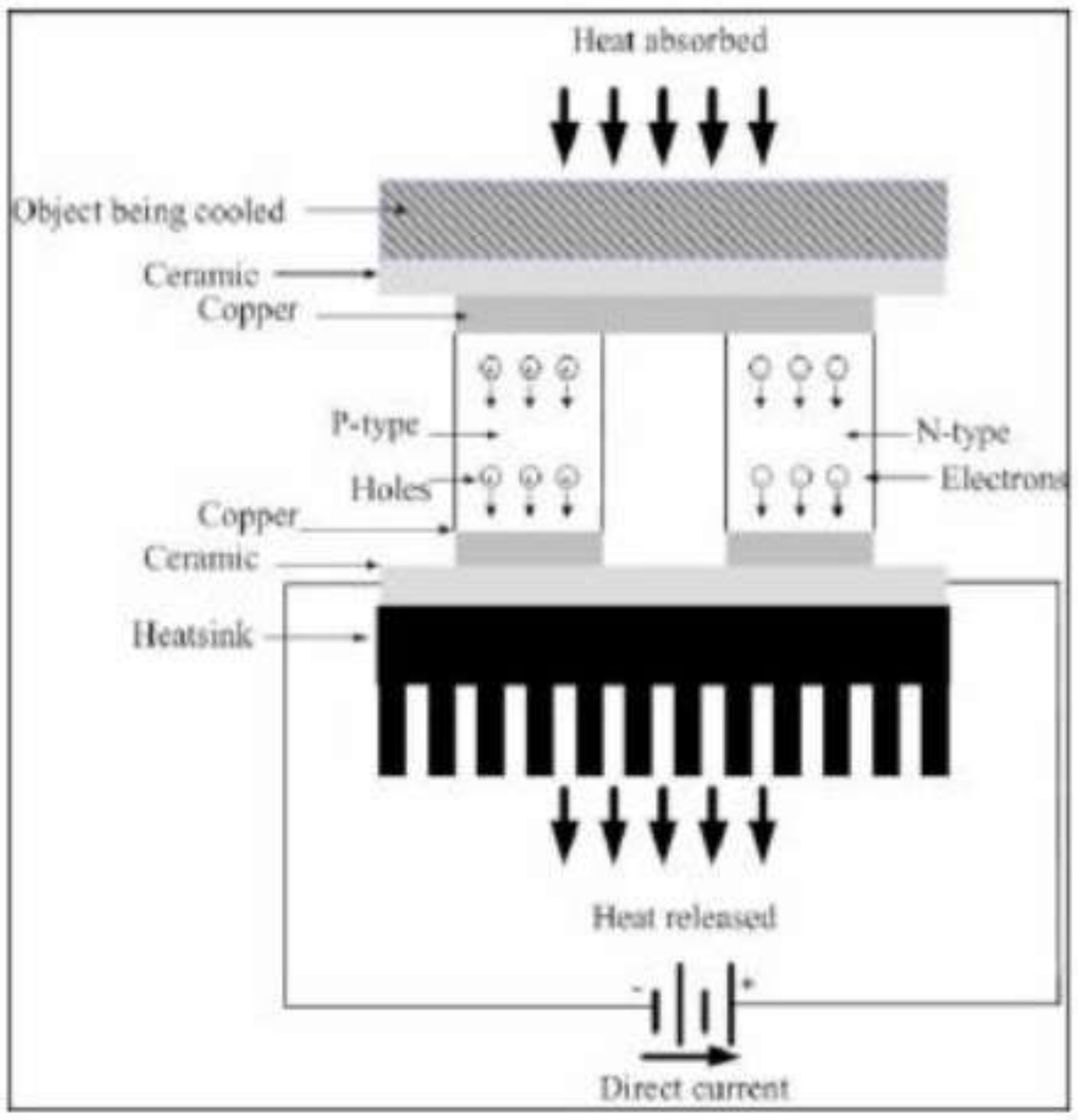


Figure 4.5: Thermoelectric Coolers

4.11.3 Construction

A thermoelectric cooling arrangement is as shown in figure 4.5. It consists of a thermoelectric module, a heat sink and the object to be cooled. A typical thermoelectric module consists of an array of bismuth telluride semiconductor pellets that have been "doped" so that one type of charge carrier-either positive or negative carries the majority of current. The pairs of P/N pellets are configured so that they are connected electrically in series, but thermally in parallel. Metalized ceramic substrates provide the platform for the pellets and the small conductive tabs that connect them.The ceramic material on both sides of the thermoelectric adds rigidity and the necessary electrical insulation.The pellets, tabs and substrates thus form a layered configuration.Thermoelectric modules can function singularly,in groups, or as stacked multi-stage modules.

4.11.4 Working:

Working: When DC voltage is applied to the module, the positive and negative charge carriers in the pellet array absorb heat energy from one substrate surface and release it to the substrate at the opposite side.The surface where heat energy is absorbed becomes cold the opposite surface where heat energy is released, becomes hot. These devices cannot only pump appreciable amount of heat, but with their series electrical connection, are suitable to be used as DC power supplies.

4.12 Thermo-electric Materials

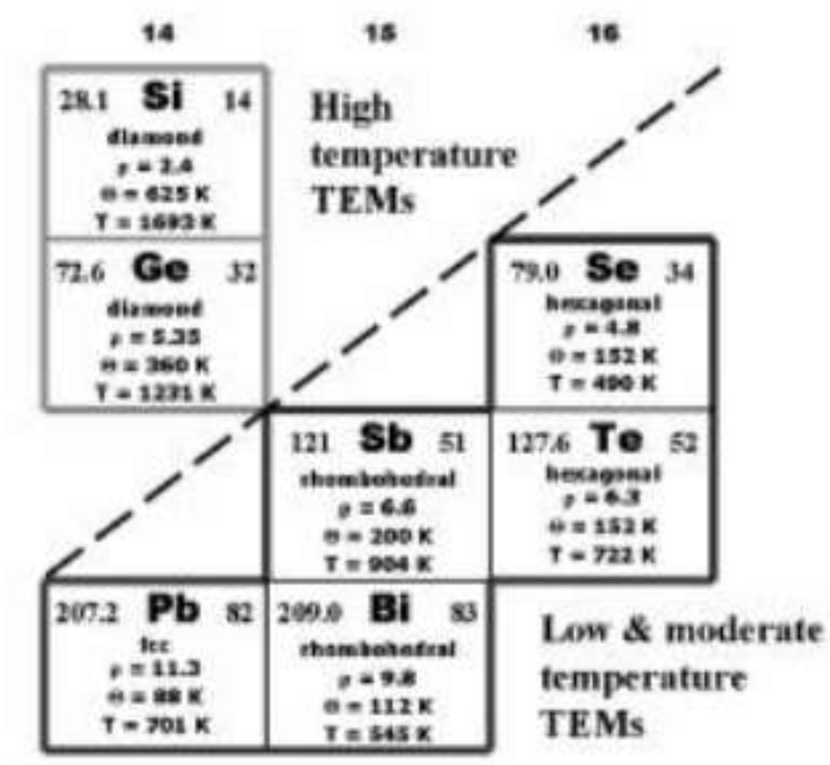
Thermoelectric (TE) materials have the capability of converting heat into electricity, which can improve fuel efficiency as well as provide a robust alternative energy supply in multiple applications by collecting wasted heat, and therefore assist in finding new energy solutions.

Classification: The thermoelectric materials can be classified into the following three categories according to their operating temperature. They are low, mid and high temperature thermometric materials.

4.12.1 Low and Near Room Temperature Thermoelectric Materials (<300K and 300K to 500K)

Bismuth Telluride/Antimony Telluride

Bismuth telluride/Antimony Telluride (B_2Te_3/Sb_2Te_3) is a well-established and effective thermoelectric semiconductor materials particularly suited for thermoelectric applications below room temperature. Figure of Merit (ZT) Bi-Sb compounds varies between 1.1 to 1.8 for temperatures between 300 to 346K.



Iron Antimonide ($FeSb_2$)

The $FeSb_2$ is low temperature thermoelectric materials considered as a potential candidate for Peltier cooling applications because of its colossal value of Seebeck coefficient ($45,000\mu K^{-1}$) at around 10 K. The figure of merit ZT is around 0.71 at 55K.

4.12.2 Mid Temperature Thermoelectric Materials (500 to 800K)

Skutterudites

The word "skutterudite" is derived from a town in Norway where minerals with this crystalline structure were first discovered. Binary skutterudites compounds crystallize in a BCC. Yb filled $CoSb_{12}$ skutterudites have ZT between 1.1 to 1.35 around 800K.

Tetrahedrites

Tetrahedrites are natural earth-abundant minerals consisting of environmentally-friendly elements of copper and sulphur. The thermoelectric figure of merit zT of around unity at 723 K for many doped and natural tetrahedrite materials

4.12.3 High Temperature Thermoelectric Materials (>800K)

Lathanum Telluride (Rare Earth Telluride)

Lanthanum telluride $La_{3-x}Te_4$ was first considered for TEGs application during the late 1980s. It is an n-type material with high TE conversion efficiency at temperatures above 1000 K (ZT of 1.2 at 1275 K).

SiGe

$S_{1-x}Ge_x$ alloy is a solid solution semiconductor with a cubic diamond-type structure. This alloy is the preferred TEM for TE power generation at elevated temperatures. This is because bulk SiGe devices (ZT=0.7 to 1.0 at about

1200K) can operate at temperatures up to 1300 K without significant degradation.

Compound	Structure	n_L	n_V	n_E	κ ($Wm^{-1}K^{-1}$)	ZT	T^* (K)
BiSb	R3m	2	2	0	4.3	0.40	80
Sb_2Te_3	R3m	2	5	0	2.4	0.26	350
Bi_2Te_3	R3m	2	5	0	1.9	0.66	300
ZnSb	Pbca	3	16	1	1.4	0.70	570
Zn_4Sb_3	R3c	2	66	0	0.9	1.30	670
PbTe	Fm3m	1	4	0	1.8 – 2.4	0.8 – 1.0	600-800
SiGe	diamond	1	8	0	4-10	0.7-1.0	1200

4.13 Applications

4.13.1 Exhaust of Automobiles - ATEG

Principle An automotive thermoelectric generator (ATEG) is a device that converts some of the waste heat of an internal combustion engine (IC) into electricity using the Seebeck Effect. ATEGs can convert waste heat from an engine's coolant or exhaust into electricity.

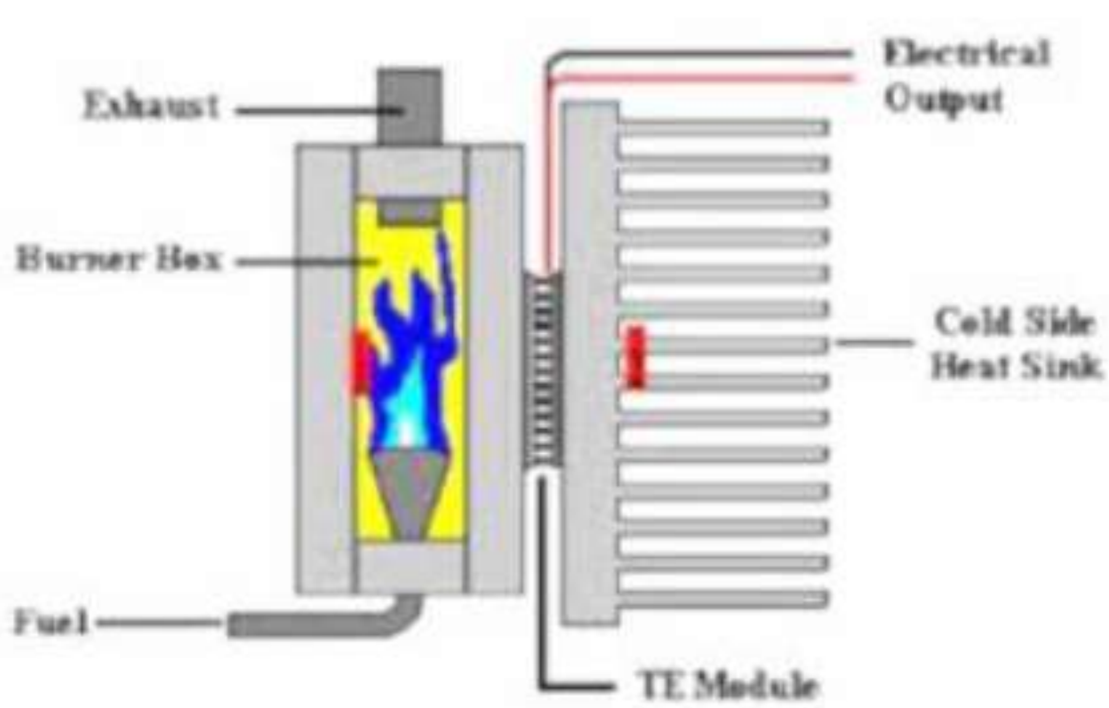


Figure 4.6: Automobile Thermoelectric Exhaust Generator

Construction A typical ATEG consists of four main elements: A hot-side heat exchanger, a cold-side heat exchanger, thermoelectric materials, and a compression assembly system. In ATEGs, thermoelectric materials are packed between the hot-side and the cold-side heat exchangers. The thermoelectric materials are made up of p-type and n-type semiconductors, while the heat exchangers are metal plates with high thermal conductivity.

Working When hot exhaust from the engine passes through an exhaust ATEG, the charge carriers of the semiconductors within the generator diffuse from the hot-side heat exchanger to the cold-side exchanger. The build-up of charge carriers results in a net charge, producing an electrostatic potential while the heat transfer drives a current. The temperature difference of several hundred degrees (700°C)is capable of generating 500 to 750W of electricity.

4.13.2 Thermoelectric Refrigeration

Thermoelectric Refrigeration Thermoelectric Refrigeration System works on the principle of Peltier effect according to which heat energy is evolved at one junction and absorbed at the other one when direct current is passed through a junction of two dissimilar metals like antimony and bismuth. It consists of a number of thermoelectric module assemblies in series joined by copper strips, as shown in figure 4.7. Each thermoelectric module is built up of a large number of thermocouples

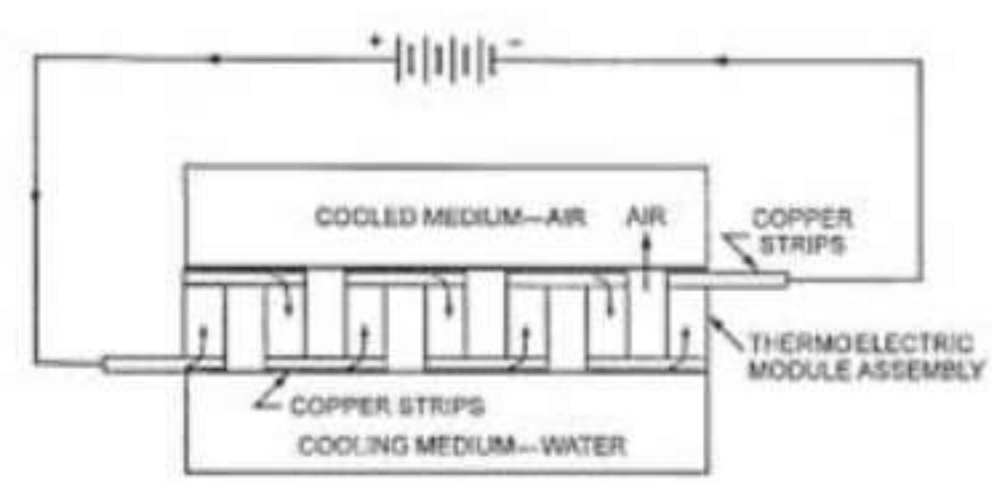


Figure 4.7: Thermoelectric Refrigerator

Working When the direct current is passed through the thermoelectric module assemblies in the direction shown in figure 4.7, junctions at the top of the assembly are cooled and those at the bottom are heated up. Thus top junctions abstract heat from the surroundings and produce refrigerating effect and bottom junctions require cooling by water. Module assembly, therefore, abstracts heat from the medium at top and rejects the same to the medium at bottom.

Advantages The main advantages of this thermoelectric refrigeration system are absence of moving parts and ease of automatic control by controlling the magnitude of current.

Disadvantages The only drawback of this system of refrigeration is its very high initial cost.

4.13.3 Space Program - Radioisotope Thermoelectric Generator(RTG)

Radioisotope Thermoelectric Generators (RTGs) are lightweight, compact spacecraft power systems that are extraordinarily reliable.RTGs provide electrical power using heat from the natural radioactive decay of plutonium-238, in the form of plutonium dioxide. The large difference in

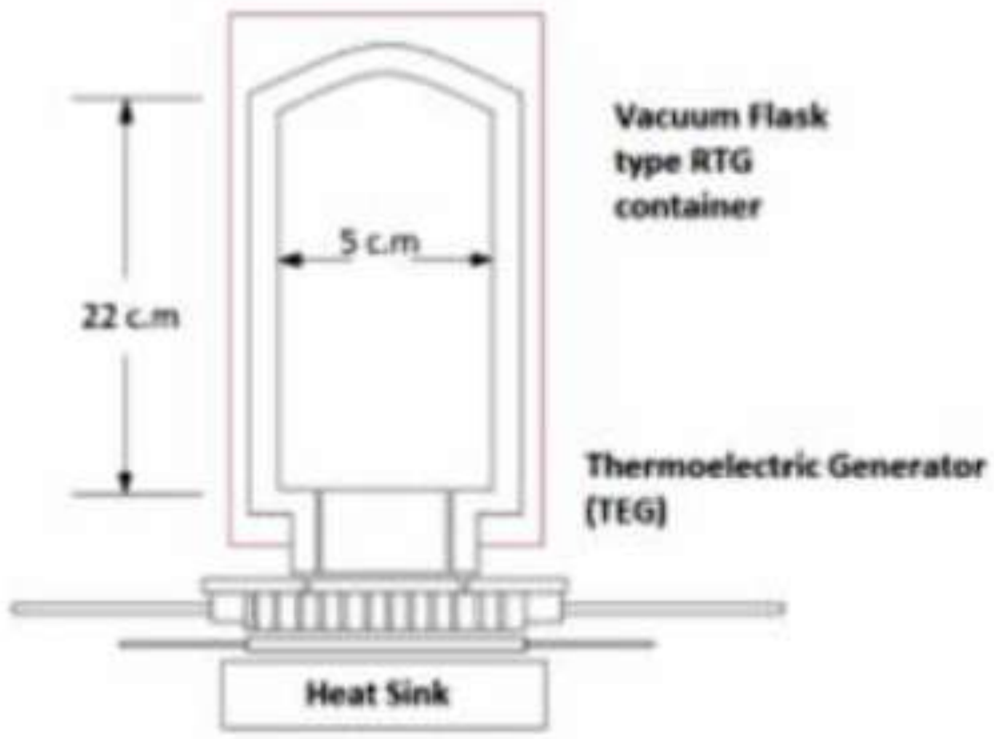


Figure 4.8: Radioisotope Thermoelectric Generator

temperature between this hot fuel and the cold environment of space is applied across special solid-state metallic junctions called thermocouples, which generates an electrical current using no moving parts.

The design of an RTG is simple by the standards of nuclear technology: the main component is a sturdy container of a radioactive material (the fuel). Thermocouples are placed in the walls of the container, with the outer end of each thermocouple connected to a heat sink. Radioactive decay of the fuel produces heat. It is the temperature difference between the fuel and the heat sink that allows the thermocouples to generate electricity.

4.14 Model Questions

1. State and Explain Seebeck Effect and hence define Seebeck coefficient.
2. State and Explain Peltier Effect and hence define peltier coefficient.
3. Discuss the variation of Thermo EMF with respect to temperature.
4. Define neutral temperature and inversion temperature. and hence Deduce the relation between them.
5. Elucidate the laws of Thermo-electricity.
6. Derive an expression for Thermo EMF in terms of temperatures of hot and cold junctions.
7. Explain the principle, construction and working of thermocouple with a neat sketch.
8. Mention the advantages and disadvantages of thermocouples.
9. Mention the need for thermopile and hence describe its construction and working.
10. Enumerate the advantages and disadvantages of thermopile.
11. Discuss the principle construction and working of Thermo Electric Generator.
12. Explain the principle, construction, working and application of thermoelectric coolers.
13. Discuss Low temperature, Moderate temperature and High Temperatures with suitable examples and mentions their ZT.
14. Describe the application of thermoelectrics in Automobile Thermo Electric Generators.
15. Explain thermoelectric refrigeration.
16. Describe Radioisotope Thermometric Generator and its applications.

4.15 Numerical Problems

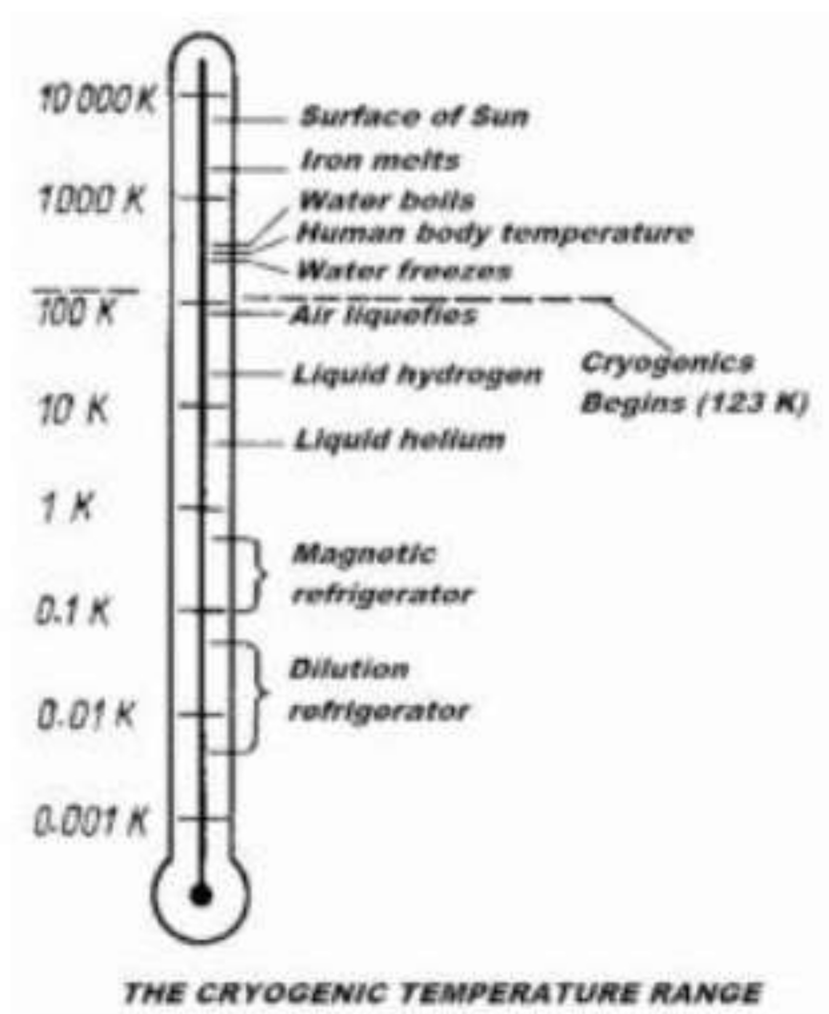
1. Calculate the neutral temperature for an iron - silver thermo couple. The values of a and b are 16.65 and –0.096 for iron and 2.86 and 0.017 for silver respectively.[Ans: 61°C]
2. The neutral temperature of a thermocouple is 300 °C. When its junctions are kept at temperatures 0°C and 100 °C, the e.m.f. generated is 1300 mV. Calculate the values of the coefficients a and b. [Ans: 15.6 × 10–6]
3. For Fe-Cu thermocouple it is observed that the thermo e.m.f is zero when one of the junction is at 20 °C and the other one is at some higher temperature. If the neutral temperature is 285°C,calculate the higher temperature. Hence find out the temperature of inversion, if the cold junction temperature is at –20°C. [Ans: 650°C]
4. The e.m.f of Fe-Pb thermocouple when one junction is at 0°C and the other at 100 °C is 1185 mV. When the second junction is at 300°C the e.m.f. is 675 mV. Similar readings with Ag-Pb thermocouple are 371 mV and 1623 mV respectively. Calculate the neutral temperature for Fe-Ag thermocouple. [Ans: 122°C]
5. The thermoelectr°C and zero at 400 °C. That for copper is 6 mV / °C at 500 °C and zero at –50 °C. Find the e.m.f. for steel-copper thermocouple with one junction at its neutral temperature and other at 0 °C.[Ans : 2.7 mV]
6. The thermoelectric power of iron is 1734 – 4.87 t and that of copper is 136–0.95t, where t is the temperature in °C. Show that the e.m.f. of thermocouple of iron-copper, the junctions of which are at 0 °C and 100 °C is 0.14 V.

Part IV

Module 4 - Cryogenics

4.16 Introduction to Production of Low Temperature

Cryogenics is defined as a branch of physics that deals with the production of very low temperatures and their effect on matter. It is a formulation which addresses both aspects of attaining low temperatures which do not naturally occur on Earth and its application. It is also defined as the science and technology of temperatures below 120 K.



The limit temperature of 120 K comprehensively includes the normal boiling points of the main atmospheric gases, as well as of methane. Methane constitutes the principal component of natural gas. Presently, the liquid natural gas (LNG) represents one of the largest and fast-growing industrial domains of application of cryogenics, together with the liquefaction and separation of air gases.

4.17 Joule-Thomson Effect

Statement: Joule-Thomson effect is the change in temperature that accompanies expansion of a gas without production of work or transfer of heat.

Explanation: If a gas initially at constant high pressure is allowed to suffer throttle expansion through the porous plug of silk, wool or cotton wool having a number of fine pores, to a region of constant low pressure adiabatically, a change in temperature of gas (either cooling or heating) is observed. This effect is called as Joule–Thomson or Joule-Kelvin effect.

Theory: Consider a gas which expands through fine pores. Let the initial state and final states of the gas be (P_1, V_1, T_1) and (P_2, V_2, T_2). The Change in temperature $T_1 - T_2 = \partial T$ is due to Joule-Thomson Effect. The external work done by the gas is given by

$$W_1 = P_2V_2 - P_1V_1 \tag{4.18}$$

The internal work done by the gas to overcome the inter-molecular gases is given by

$$W_2 = -\frac{a}{V_2} + \frac{a}{V_1} \tag{4.19}$$

The total work done is by the gas is given by

$$W = W_1 + W_2 = P_2V_2 - P_1V_1 - \frac{a}{V_2} + \frac{a}{V_1} \tag{4.20}$$

Van der Waals state equation for the gas is given by

$$\left(P + \frac{a}{V^2}\right)(V - b) = R \tag{4.21}$$

Neglecting $\frac{ab}{V^2}$ the equation 4.21 could be reduced to

$$P = R + Pb - \frac{a}{V} \tag{4.22}$$

Thus for the initial and final states of the gas we can write

$$P_1 V_1 = R\ {}_1 + P_1 b - \frac{a}{V_1} \tag{4.23}$$

$$P_2 V_2 = R\ {}_2 + P_2 b - \frac{a}{V_2} \tag{4.24}$$

Substituting equations 4.23 and 4.24 in equation 4.20 and on simplification we get

$$W = R(T_2 - T_1) - b(P_1 - P_2) + \frac{2a}{V_1} - \frac{2a}{V_2} \tag{4.25}$$

for very small values of a and b in Van der Walls equation it reduces to $P = R$ and thus $V = \frac{R}{P}$. Thus substituting for V_1 and V_2 in equation 4.25 we get

$$W = R(T_2 - T_1) - b(P_1 - P_2) + \frac{2aP_1}{R} - \frac{2aP_2}{R}$$

$$W = R(T_2 - T_1) - b(P_1 - P_2) + \frac{2a}{R}(P_1 - P_2)$$

Since $\partial T = T_1 - T_2$ and re-organizing the terms we get

$$W = -R\partial T - b(P_1 - P_2) + \frac{2a}{R}(P_1 - P_2) \tag{4.26}$$

$$W = -R\partial T + (P_1 - P_2)\left(\frac{2a}{R} - b\right)$$

$$W = (P_1 - P_2)\left(\frac{2a}{R} - b\right) - R\partial \tag{4.27}$$

Since the gas is thermally insulated the expansion leads to reduction in temperature ∂T and the ***Heat lost by the gas*** is given by

$$W = C_v \partial \tag{4.28}$$

Thus the equation 4.27 becomes

$$C_v\partial = (P_1 - P_2)\left(\frac{2a}{R} - b\right) - R\partial$$

$$(C_v + R)\partial = (P_1 - P_2)\left(\frac{2a}{R} - b\right)$$

The change in pressure $P_1 - P_2 = \partial P$ and $C_p - C_v = R$ the equation 4.17 could be written as

$$\partial = \partial P \; \frac{1}{C_p}\left(\frac{2a}{R} - b\right)$$

Thus The Joule-Thomson Co-efficient is given by μ_{JT}

$$\mu_{JT} = \frac{\partial}{\partial P} = \frac{1}{C_p}\left(\frac{2a}{R} - b\right) \qquad (4.29)$$

The equation 4.29 gives the change in temperature of Van der Walls gas when subjected to throttling process.

The Three Cases:

1. if $\frac{2a}{R} > b$ then ∂ is positive and there will be cooling effect.
2. if $\frac{2a}{R} < b$ then ∂ is negative and there will be heating effect.
3. if $\frac{2a}{R} = b$ then $\partial = 0$ indicating neither cooling and nor heating.

NOTE: For a gas temperature that is above the inversion temperature, the μ_{JT} would be negative. The ∂P shall be always negative in this case, which means that the ∂P must be positive. Consequently, the warming of the gas will take place.

4.18 Inversion Temperature

The temperature at which the Joule –Thomson effect changes sign is called the temperature of Inversion. T_i. At this temperature $\frac{2a}{R_i} = b$ and hence $T_i = \frac{2a}{Rb}$. Above T_i Joule-Thomson effect produces heating and below T_i it produces cooling effect.

4.19 Porous Plug Experiment

Construction : Joule in collaboration with Thomson [Lord Kelvin] devised a very sensitive technique known as Porous Plug experiment. The experiment set up of porous plug experiment to study the Joule-Thomson effect is shown in fig. 4.9. It consists of the following main parts.

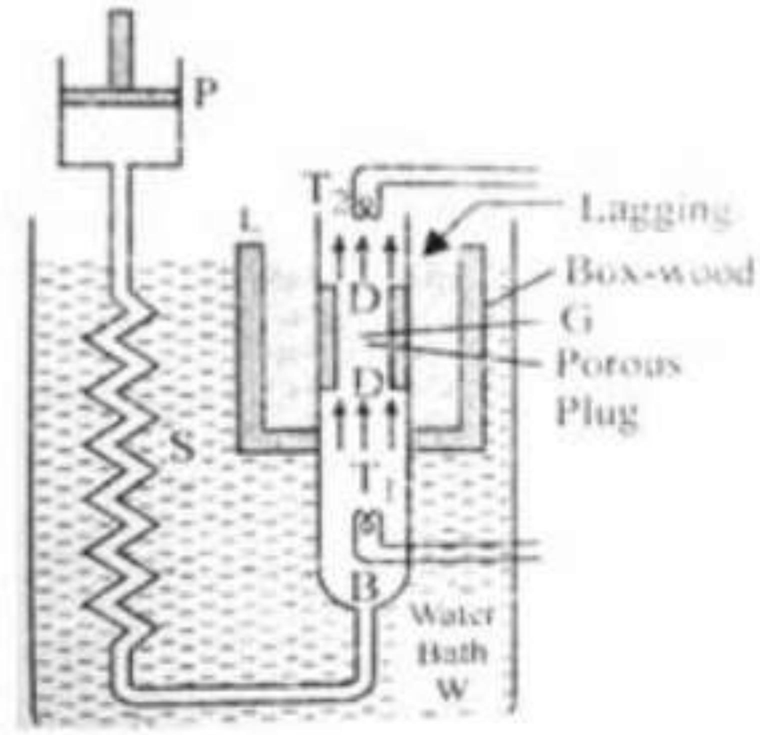


Figure 4.9: Porous-Plug Experiment

Construction:

1. A Porous plug has two perforated - brass discs $D\&D$ and the space between is filled with cotton wool or silk fibers.
2. The porous plug G is fitted in a cylindrical box-wood which is surrounded by a vessel L containing cotton wool to avoid heat exchange with surrounding.
3. $T_1\&T_2$ are two sensitive platinum resistance thermometers used to measure the temperatures of the incoming and outgoing gas.
4. The gas is compressed to a high pressure with the help of piston P and it is passed through a spiral tube immersed in water bath maintained at a constant temperature.

Experimental Procedure:

1. The experimental gas is compressed by Pump P and is passed slowly and uniformly through the porous plug keeping the high pressure constant which read by a pressure gauge.
2. During the passage of the gas through the porous plug it is throttled.
3. The gas emerging from the pores of the porous plug expands to atmospheric pressure.
4. Since the expansion of the gas occurs under thermal isolation it is adiabatic.
5. The initial and final temperatures are noted by platinum resistance thermometers $T_1\&T_2$.

Experimental Results: A simple arrangement of porous plug experiment is shown in fig. 4.10. The behavior of large number of gases was studied at various inlet temperatures of the gas and the results are as follows:

- At sufficiently low temperatures, all gases show a cooling effect.
- At ordinary temperatures, all gases except Hydrogen and Helium show cooling effect. The Hydrogen and Helium show heating effect.
- The fall in temperature is directly proportional to the difference in pressure on the two sides of porous plug.
- The fall in temperature for a given difference of pressure decreases with rise in the initial temperature of the gas.
- It was found that the cooling effect decreased with the increase of initial temperature and becomes zero at a certain temperature and at a temperature higher than the temperature instead of cooling heating was observed. This particular temperature at which the Joule – Thomson effect changes sign is called temperature of inversion.

4.20 Thermodynamic Analysis of Joule Thomson Effect

The arrangement of the porous plug experiment is shown in Fig. The gas passes through the porous plug from the high pressure side to the low pressure side. Consider one mole of the gas. Let (P_1, V_1) and (P_2, V_2) represent the pressure and volume of the two sides of the porous plug. Let d be the distance through which each piston moves to the right.

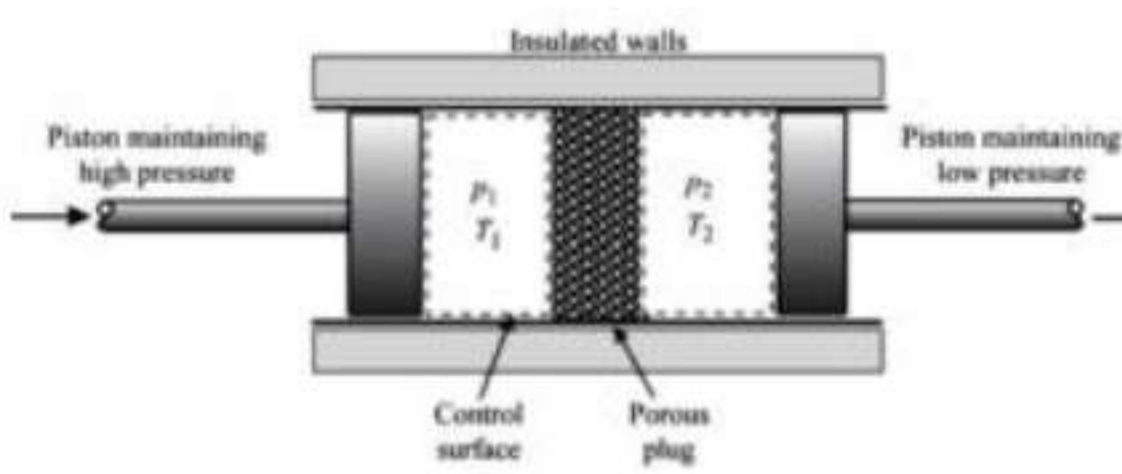


Figure 4.10: Porous-Plug Experiment

$$\text{Work done by piston 1 on the gas} = P_1A_1d = P_1V_1 \quad (4.30)$$

$$\text{Work done by the gas on piston 2} = P_2A_2d = P_2V_2 \quad (4.31)$$

$$\text{Net external work done} = (P_2V_2 - P_1V_1) \quad (4.32)$$

Let w be the work done by the gas in separating the molecules against their inter-molecular attraction.

$$\text{Total amount of work done by the gas} = (P_2V_2 - P_1V_1) + w \quad (4.33)$$

Three cases could be considered based on the initial temperature of the gas.

1. Below the Boyle temperature: If $P_2V_2 > P_1V_1$, then $P_2V_2 - P_1V_1$ is +ve. w must be either positive or zero. Thus, a net +ve work is done by the gas and there must be a cooling effect.

2. At the Boyle temperature: If $P_2V_2 = P_1V_1$, then $P_2V_2 - P_1V_1$ is 0. The total work done by the gas is w. Therefore, cooling effect at this temperature is only due to the work done by the gas in overcoming inter-molecular attraction.

3. Above the Boyle temperature: If $P_2V_2 < P_1V_1$. Then $P_2V_2 - P_1V_1$ is -ve.Thus the net workdone by the gas and the observed effect will depend upon whether $(P_2V_2 - P_1V_1)$ is greater than or less than w.
   >
   If $w > (P_2V\_P_1V_1)$, cooling will be observed.
   If $w < (P_2V_2 - P_1V_1)$, heating will be observed.

Thus, the cooling or heating of a gas depends on

1. The deviation from Boyle's law

2. Work done in overcoming inter-molecular attraction.

4.21 Liquefaction of Gasses by Cascade Process

Definition : Liquefaction of gases means the process into which the gas substances are converted from gases to a liquid state.

4.21.1 Principles of Liquefaction of Gases

Principle 1 : When a gas is compressed by a sufficient amount of pressure below its critical temperature results in the reduction of temperature.

Principle 2 : If a gas is allowed to evaporate under reduced pressure results in the reduction of temperature.

Principle 3 : On the basis of the Joule Thomson effect (Porous plug experiment).

4.22 Liquefaction of Oxygen by Cascade Process - Pictet Process

Principle When a liquid is allowed to evaporate under reduced pressure, it produces high cooling.

Cascade system or Process: When a single stage is not enough to produce the desired result, therefore the process takes place in a number of stages in a sequence.

Construction: The apparatus modified by H K Onnes consists of compression pumps P_1, P_2 and P_3. T_1, T_2 and T_3 are the three tubes which are surrounded by outer jackets A, B and C. 'D' is a Dewar's flask which collects liquid oxygen

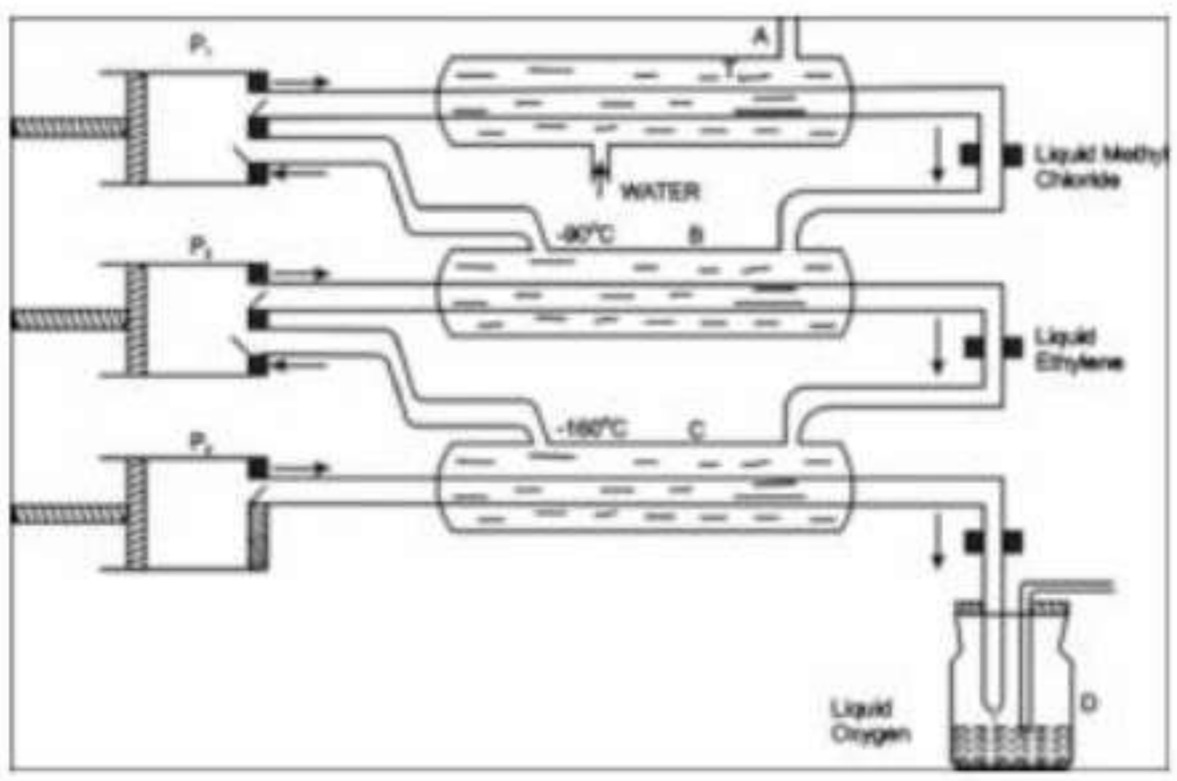


Figure 4.11: Cascade Liquefaction

Working:

1. The methyl chloride gas of critical temperature 145°C is compressed by the pump P_1 through the tube T_1. It is cooled by the cold water circulating in the jacket A.

2. Here the methyl chloride reaches the temperature lower than its critical temperature. Then it is liquefied under high pressure. The liquid methyl chloride is collected in the jacket B and evaporates under reduced pressure lowering the temperature to -90°C.

3. The ethylene gas of critical temperature 10°C is compressed by the pump P_2 through the tube T_2. It is cooled to -90°C by liquid methyl chloride.

4. Then it is liquefied under high pressure. The liquid ethylene is collected in the jacket C and evaporates under reduced pressure lowering the temperature to -160°C.

5. The oxygen gas of critical temperature - 119°C is compressed to 50 atmospheric pressure by the pump P_3 and passed through the tube T_3. It is cooled to - 160°C by liquid ethylene in R. Then it is liquefied and the liquid oxygen is collected in the Dewar flask D.

Limitations: Cascade method cannot liquefy the gases that have very low critical temperatures like Hydrogen and Helium.

4.23 Linde's Air Liquefier

Principle: Linde's process is known as Adiabatic expansion of compressed gas. The process is based upon the combined effect of Joule – Thomson Effect and regenerative cooling. The compressed gas is made to expand adiabatically and repeatedly.

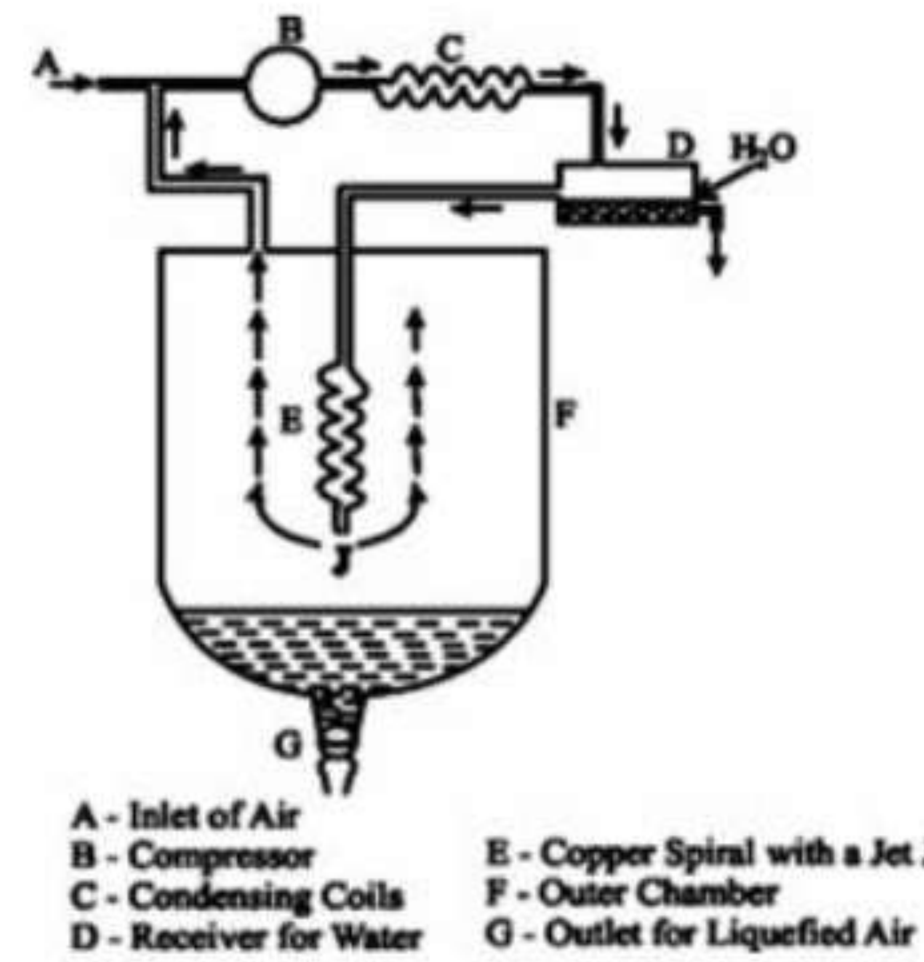


Figure 4.12: Linde's Air Liquefaction

Construction: The apparatus used is shown in figure 4.12 and the parts are labeled.

Process:

- The air is first compressed to about 200 atmospheres in the compressor and then passed through the condensing coils.
- As a result, the air is cooled and the water vapours present condense to form the water which is removed from receiver.
- The dry air is then passed through the spiral copper tube which terminates into a jet .
- The air expands through the jet into the chamber where pressure is about 50 atmospheres.
- Thus, as a result of Joule-Thomson effect, the air is cooled.
- As this cooled air moves up, it further cools the incoming air.
- The cooled air is sent to the compressor again and the process is repeated a number of times till ultimately the air is cooled to such an extent that it liquefies.
- The liquefied air gets collected at the bottom of the outer chamber and can be drawn off. Any uncondensed air is recirculated.

4.24 Liquefaction of Helium and its Properties

4.24.1 Liquefaction of Helium

Introduction: Helium is the only substance which remains as fluid at temperatures below -269 °C (4K) and its inversion point is much lower than that of Hydrogen namely around -233 °C(40K). The boiling point of Helium is just around -268.9 °C which is quite close to absolute zero on the Kelvin scale

Principle: Claude's method works on two principles. i.e: Joule Thomson effect and mechanical expansion.

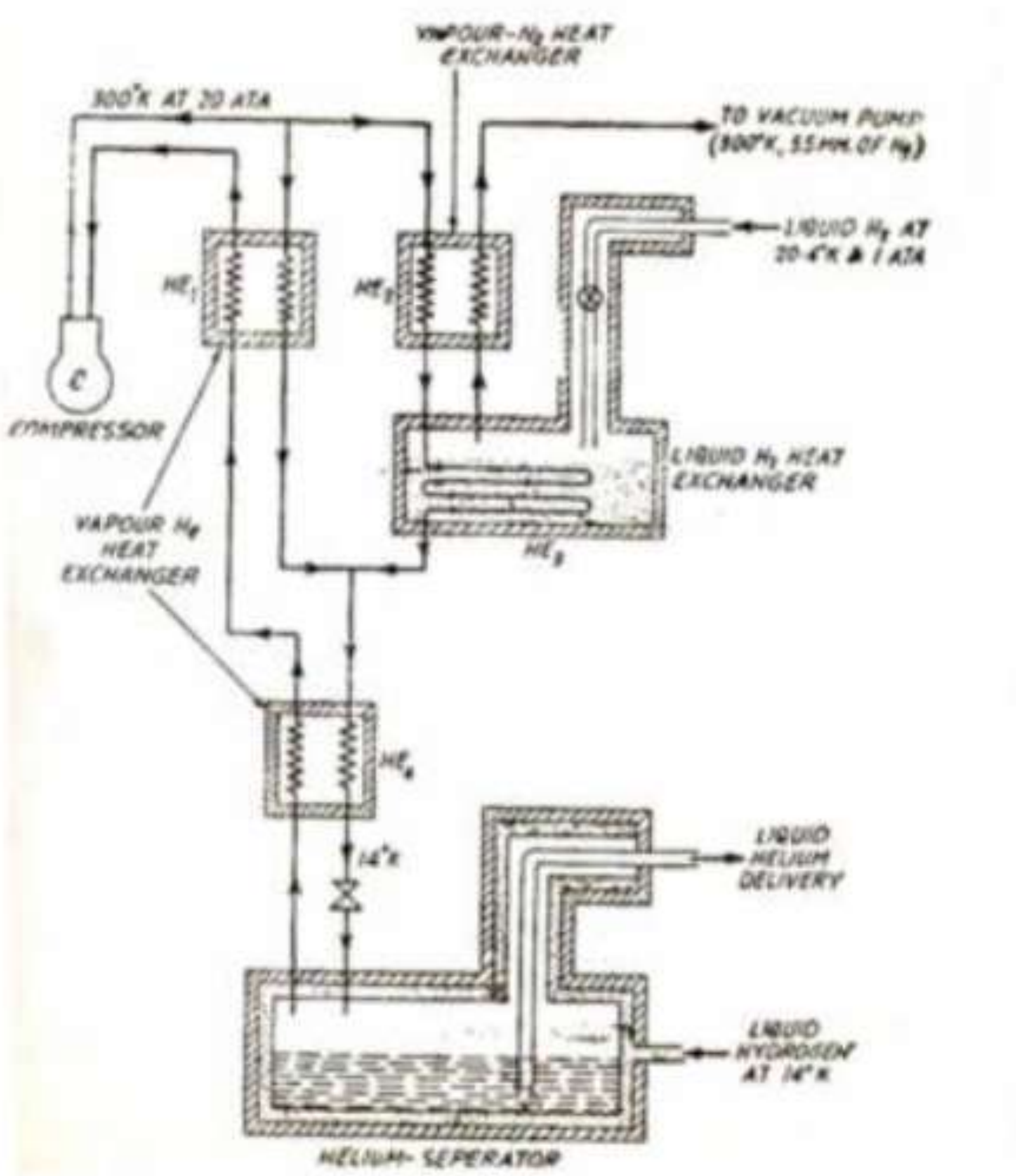


Figure 4.13: Liquefaction of Helium

Working :

- Initially Helium is compressed to a pressure of 20 atmospheres which raises its temperature to the region of 300 degrees Kelvin.
- This compressed high temperature Helium is then split into two streams along different paths.
- The first part of the stream is cooled in the heat exchanger labeled HE_1 with the help of Helium vapors
- The second part fo the stream passes through the heat exchanger HE_2 to be cooled with Nitrogen vapors cooled by liquid Hydrogen.
- The second part is then passed through liquid Hydrogen heat exchanger HE_3
- Further the first and second streams are combined and then passed through HE_4 and again cooled by Helium vapors.
- Finally the throttle valve is used to initiate the Joule Thomson effect and Helium is collected in the liquid state in the Helium separator.

4.24.2 Properties and Uses

Properties:

- There are two main isotopes of Helium used for liquefaction namely Helium-4 and Helium-3.
- There is a slight difference between the properties of the two in terms of their boiling point and critical temperature.
- The boiling point of Helium-3 is even one degree lower than Helium-4.
- Liquid Helium exhibits super-fluidity.

Uses:

- Lord Kammerlingh Onnes used liquid helium to study superconductivity of mercury.
- Liquid Helium is used extensively for use in superconducting magnets which need to be cooled to extremely low temperatures. The Superconducting Magnets are in turn used in Magnetic Resonance Imaging and Nuclear Magnetic Resonance.

4.25 Platinum Resistance Thermometer - PRT

Introduction The Platinum Resistance Thermometer uses platinum for determining the temperature. It works on the principle of positive temperature coefficient of resistance. Hence, the resistance of platinum increases with increase in temperature. The platinum is a chemically inert metal and can easily be drawn into fine wires. Because of these properties of platinum, it is used as a sensing element in thermometer.

Construction: The PRT consists of pure platinum wire wound on hollow pipe made up of insulating mica or ceramic, which is placed in porcelain sheath. Free ends of platinum wire are attached to long leads of low resistance copper wires (Fig.1). To measure change in resistance, Wheatstone bridge is used. Two long extension leads form one arm of Wheatstone bridge connect to the copper leads of encapsulated platinum wire(fig.2).

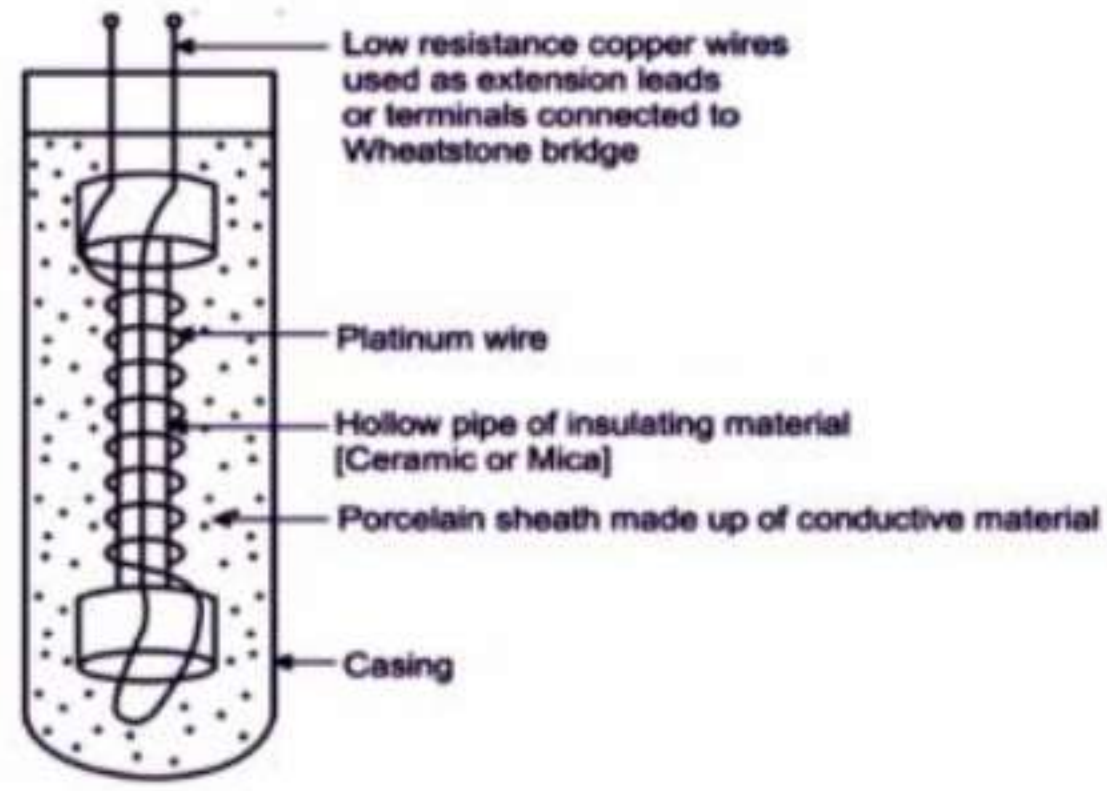


Figure 4.14: Platinum Resistance Thermometer

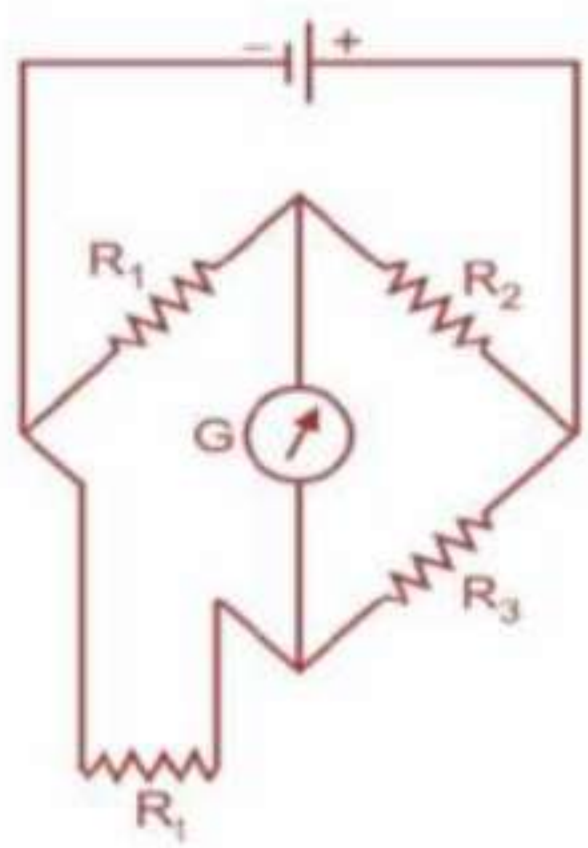


Figure 4.15: PRT - Wheatstone's Bridge

Advantages:

- Linear relationship of resistance with temperature
- The meter gives the precise reading of temperature.
- The thermometer has a wide range from -200°C to 1200°C.
- The thermometer is quite sensitive and posses high accuracy (0.01 °C).
- Platinum possess stable value of resistance at the given temperature.
- Platinum is chemically inert

Disadvantages:

- The thermometer gives the slow response.
- The melting point of the thermometer is 1800°C.
- But, if platinum measures the temperature higher than 1200°C it start evaporating.

4.26 Applications of Cryogenics

4.26.1 Aerospace

- For critical components in the aerospace industry the cryogenic hardening can be used to strengthen metal parts in communications systems, guidance systems, landing gear, and more.
- Using cryogenic hardening to prep critical components may help the rover survive these bitter cold conditions of lunar night.
- Cryogenics is used to design the multi-layer insulation blankets to insulate the spacecraft.
- Cryogenic engine makes use of Liquid Oxygen (LOX) and Liquid Hydrogen (LH2) as propellants which liquefy at -183 deg C and -253 deg C respectively.

4.26.2 Tribology - Cryogenic Treatment of Metals

- Cryogenic processing (Cryogenic Treatment) typically entails a slow cooling to cryogenic temperatures, a dwell period at this low temperature followed by a slow return to room temperature and finally a first stage tempering.
- The cryogenic hardening for steel induces a phase transformation from austenite to martensite, among other changes. This improves qualities such as hardness, electrical conductivity, wear resistance,lowering friction and susceptibility to corrosion
- Cryogenic treatment works in creases life of the tools like n Reamers, Tool bits, Tool punches, Carbide Drills, Carbide Cutters, Milling Cutters, Files, Knives, Reciprocating Blades, Dies and cutting tools.
- Stress relieved ferrous and non ferrous castings and forgings for enhanced dimensional stability and surface finish.

The following are the types of cryogenic treatment

Shallow Treatment: The objects are cooled down to temperature of approximately -85 °C.

Flooding: First the object is taken to -85°C, then the chamber is flooded with liquid nitrogen to reduce the temperature further.

Deep Cryogenic Treatment (DCT): Subjects the objects to the temperature of approximately -185 °C

4.26.3 Food Processing

- Cryogenic quick freezer is widely used in the food cold chain resulting from the better quality of frozen food.
- Due to the rapid freezing rate, water in food will become total or partial vitrification with cryogenic technology.
- Liquid nitrogen (LN2) is used in the freezing process of food.
- Cryogenics is also used transportation of large masses of frozen food.
- Cryogenic food freezing is also helpful for large scale food processing industries.

4.27 Model Questions

1. Define Cryogenics and explain the selection of 120K as the reference temperature in cryogenics.
2. State and Explain Joule-Thomson Effect.
3. Define Joule-Thomson effect and hence discuss the theory.
4. Derive the expression for Joule-Thomson Coefficient.
5. Explain Joule-Thomson effect and interpret the cases in Joule-Thomson effect.
6. Describe Porus Plug Experiment with the help of neat sketches.
7. Discuss the Thermodynamical Analysis of Joule-Thomson Effect.
8. Explain the three principles of Liquefaction of Gasses.
9. Discuss the Liquefaction of Oxygen by Cascade process with the help of a neat sketch.
10. Explain the construction and working of Linde's Air Liquefier with the help of a neat diagram.
11. Describe the Liquefaction of Helium using Claude's Method.
12. Explain the properties of Liquid Helium and its uses.
13. With the help of neat sketches explain the construction and working of Platinum Resistance Thermometers.
14. Discuss the applications of Cryogenics in aerospace, Tribology and Food processing.
15. Explain the application of Cryogenics in Aerospace.
16. Explain the applications of Cryogenics in Cryogenic Treatment of metals and advantages.
17. Mention the applications of Cryogenics in food processing.

4.28 Numerical Problems

1. In Joule Thomson experiment Temperature changes from 100 °C to 150 °C for pressure change of 20 MPa to 170 MPa. Calculate Joule Thomson Coefficient. Ans: $\frac{\partial}{\partial P} = 0.333\mu KPa^{-1}$

Part V

Module - 5 - Material Characterization and Instrumentation Technique

Chapter 5

Introduction to materials: Nanomaterials and Nanocomposites

5.1 Nanomaterials

Nanomaterials can be defined as materials possessing, at minimum, one external dimension measuring 1-100nm. The definition given by the European Commission states that the particle size of at least half of the particles in the number size distribution must measure 100nm or below.

Nanomaterials can occur naturally, be created as the by-products of combustion reactions, or be produced purposefully through engineering to perform a specialized function. These materials can have different physical and chemical properties to their bulk-form counterparts.

5.1.1 Classification of Nanomaterials

Based on the number of nano dimensions nano materials are classified in to the following types.

Zero-dimensional(0D) nanomaterials: In case of OD nanomaterials all three dimensions (x, y, z) are at nanoscale, i.e., no dimensions are greater than 100 nm. It includes nanospheres and nanoclusters.

One-dimensional (1D) nanomaterials: Here, two dimensions (x, y) are at nanoscale and the other is outside the nanoscale. This leads to needle shaped nanomaterials. It includes nanofibres, nanotubes, nanorods, and nanowires.

Two-dimensional (2D) nanomaterials: Here, one dimension (x) is at nanoscale and the other two are outside the nanoscale. The 2D nanomaterials exhibit platelike shapes. It includes nanofilms, nanolayers and nanocoatings with nanometre thickness.

Three-dimensional(3D) nanomaterials: These are the nanomaterials that are not confined to the nanoscale in any dimension. These materials have three arbitrary dimensions above 100 nm. The bulk (3D) nanomaterials are composed of a multiple arrangement of nanosize crystals in different orientations. It includes dispersions of nanoparticles, bundles of nanowires and nanotubes as well as multinanolayers (polycrystals) in which the 0D, 1D and 2D structural elements are in close contact with each other and form interfaces.

The steel plate is bent into a circular path of radius 10 metres. If the plate section be 120 mm wide and 20 mm thick, then calculate the Bending Moment and maximum bending stress. Given Young's modulus = 200 GPa.

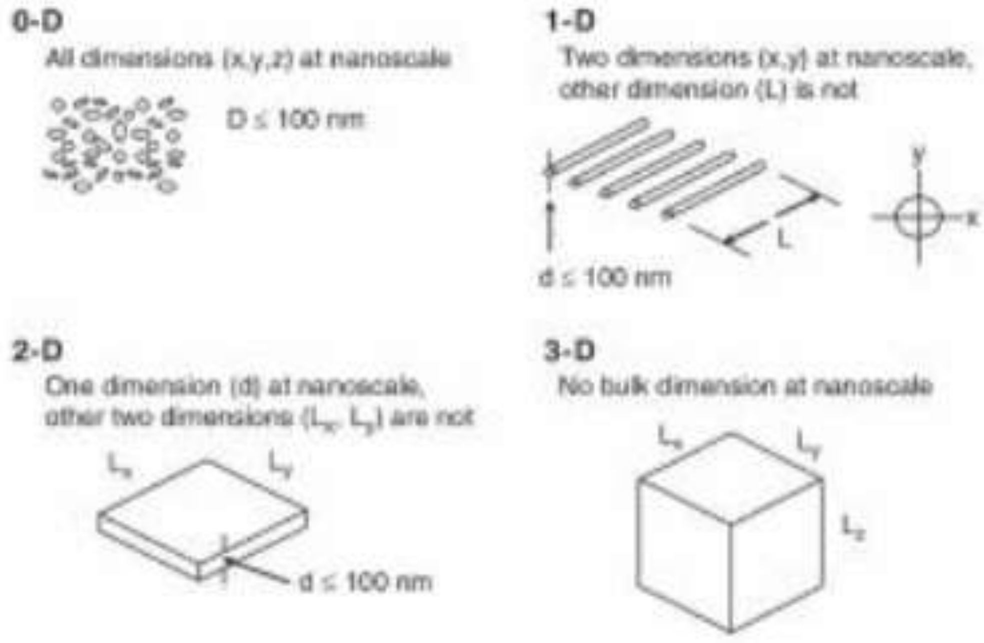


Figure 5.1: Classification of Nanomaterials

5.2 Nanocomposites

5.2.1 Introduction and Properties :

Nanocomposites are materials that incorporate nanosized particles into a matrix of standard material. The result of the addition of nanoparticles is to improve the properties of the materials that can include mechanical strength, toughness and electrical or thermal conductivity. Nanocomposites can improve properties like,

- Mechanical properties including strength, modulus and dimensional stability.
- Electrical conductivity
- Thermal stability
- Chemical resistance

- Optical clarity
- Surface to volume ratio (SA:V)

5.2.2 Classification of Nanocomposites based on the Matrix

1. Polymer Matrix Nanocomposites
   - Polymer - Polymer Nanocomposite
   - Polymer - Metall Nanocomposite
   - Polymer - Ceramic Nanocomposite
   - Polymer - Clay Nanocomposite
2. Non-Polymer Matrix Nanocomposites
   - Metal Matrix and Metal Filler Nanocomposite
   - Metal Matrix and Ceramic Filler Nanocomposite
   - Ceramic Matrix and Ceramic Filler Nanocomposite

5.2.3 Types of Nanocomposites

The nanocomposites are of several types out of which the following are some of the types

1. Thermoplastic Nanocomposites : These re the composites in which nanofillers are incorporated into a thermo-plasitic resins.
2. Thermoset Nanocomposites : The incorporation of nanofillers in to the thermosets are called Thermoset Nanocomposites which find application in mechanical and tribological applications.
3. These nanomaterials are mixed with elastomers. They main aim is to enhance the mechanical, the electrical, as well as the wear performance. They include inorganic or organic fillers of metallic, ceramic, or polymeric structure.
4. Carbon nanotube nanocomposites: The carbon nanotube nanocompostes are fabricated by dispersig carbonnano tubes in metal matrix or polymer matrix.
5. Graphene and Graphene sheet based nano composites : Graphene-based nanocomposites, derived from the decoration of graphene sheets with metal/metal oxide nanoparticles, are emerging as a new class of exciting materials that hold promise for many applications.
6. POSS (Poly Oligomeric Silsequioxane) nanocomposite : Polyhedral oligomeric silsesquioxanes (POSS) are a class of nanostructured silica-based compounds that can be considered as an organic and inorganic hybrid material.
7. Zeolites and Composites :Zeolites are microporous, three-dimensional crystalline solids of aluminium silicate. Zeolites have small openings of fixed size in them which allow small molecules to pass through them easily but larger molecules cannot pass through them. Geopolymer-zeolite composites may merge interesting properties of both aluminosilicate materials.

5.2.4 Applications of Nanocomposites

Applications of Nanocomposites are

- Thin-film capacitors for computer chips.
- Solid polymer electrolytes for batteries.
- Automotive engine parts and fuel tanks.
- Impellers and blades.
- Oxygen and gas barriers.
- Food packaging.

Chapter 6

Material Characterization and Instrumentation Techniques

6.1 X-ray Diffraction and Bragg's Law

6.1.1 X-ray Diffraction

The wavelength of X-Ray is comparable with the dimension of atoms. Thus crystals diffract X-rays and the diffraction patter depends on the crystal structure.

6.1.2 Bragg Diffraction/Reflection

Definition

The condition for the constructive interference for the diffraction of X-rays from the crystal surface is given Bragg's Law. The diffraction satisfies the laws of reflection and hence called Bragg Reflection.

Mathematical Form

Consider an X-ray beam incident on a crystal at glancing angle "θ" with a lattice spacing "d". The condition for constructive interference is given by Braggs' Law.

$$2dsn\theta = n \tag{6.1}$$

Here 'n' is the order of diffraction.

6.2 X-ray Diffractometer-XRD

introduction

XRD technique is used to examine the crystal structure of powder samples, so it is also called as X-ray powder diffractometry.

Principle

X-ray diffractometer is based on the principle of Braggs' Law The constructive interference of monochromatic X-rays reflected by a crystalline sample is studied.

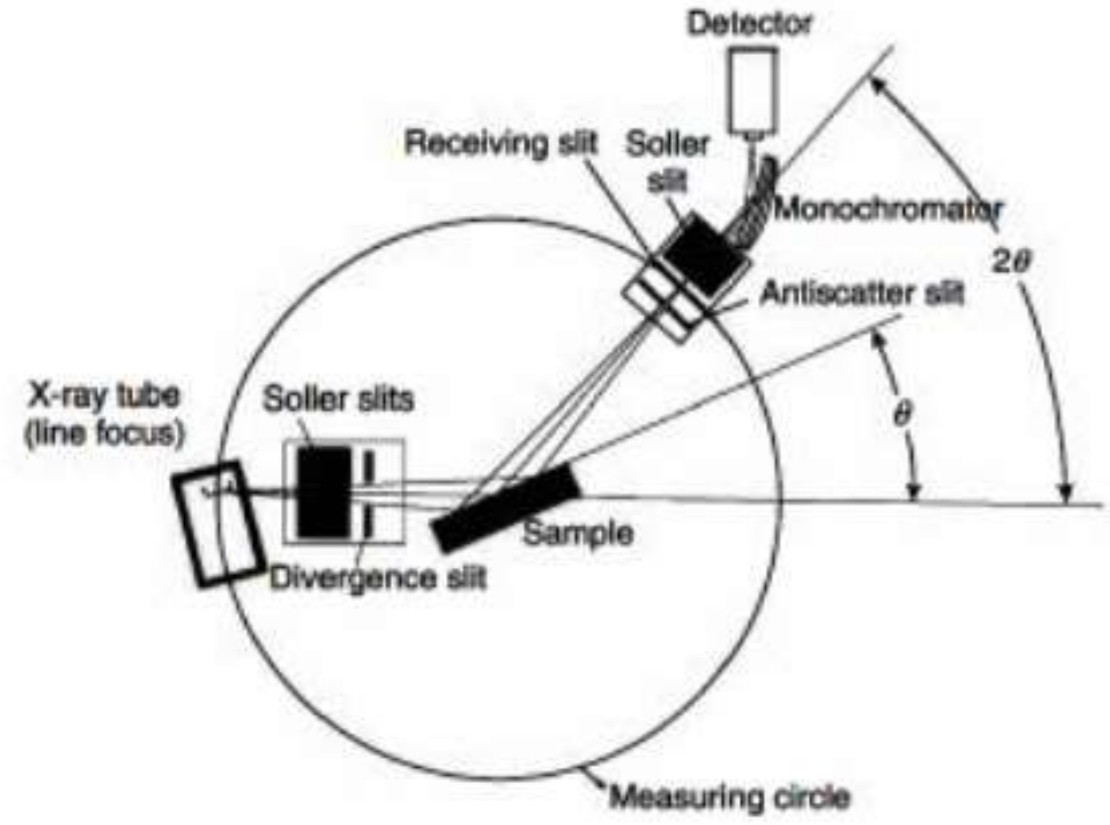


Figure 6.1: XRD

Construction

- The main arrangements consist of X-Ray source, Specimen and Detector.
- The X-ray radiation generated by an X-ray tube is collimated by passing through **soller slits**.
- Soller Slits are the arrangement of closely spaced thin metal plates in parallel to prevent beam divergence.
- A divergent X-ray beam from the soller slits strikes the specimen.
- X-rays are diffracted by the specimen on the **mount(turntable)** and form a convergent beam at the receiving soller slits before they enter a detector.
- The diffracted X-ray beam is passed through the **monochromator** before being received by the detector.
- The monochromatic filter is used suppress wavelengths other than K_α radiation.

- The detector and the turn-table are so coupled, if the crystal is rotated through an angle θ then the detector turns through an angle 2θ.

working

- The basic function of a diffractometer is to detect X-ray diffraction from materials and to record the diffraction intensity in a range of the diffraction angle (2θ).
- Diverging X-ray beam is collimated and incident at a fixed angle on the sample mounted on the turntable.
- The detector scans for the diffracted X-rays.
- The reflected X-rays are collimated and manochromatic X-rays from the monochromator are collected by the detector.
- The intensity of the reflected X-ray beam as a function of 2θ is recorded in a computer.
- A plot of Intensity as a function of 2θ. thin-film XRD requires a parallel incident beam, not a divergent beam as in regular diffractometry.
- The small glancing angle of the incident beam ensures that sufficient diffraction signals come from a thin film or a coating layer instead of the substrate.

Applications

1. XRD is a non-destructive technique
2. To identify crystalline phases and orientation .
3. To determine structural properties: Lattice parameters (10^{-4},), strain, grain size, expitaxy, phase composition, preferred orientation (Laue) order-disorder transformation, thermal expansion.
4. To measure thickness of thin films and multi-layers
5. To determine atomic arrangement.

Uses

1. Obtain XRD pattern
2. Measure d-spacing
3. Obtain integrated intensities

6.3 Determination of Crystallite Size - Scherrer Equation

The Scherrer equation, in X-ray diffraction and crystallography, is a formula that relates the size of sub-micrometre crystallites in a solid to the broadening of a peak in a diffraction pattern. It is often referred to, incorrectly, as a formula for particle size measurement or analysis.

The Scherrer equation is given by

$$B\,(2\theta) = \frac{K\lambda}{L\cos\theta} \tag{6.2}$$

Here

- Peak width (B) or Full Width at Half Maximum (FWHM) is inversely proportional to **crystallite size (L)**.
- λ is the wavelength of X-ray used.
- θ is the glancing angle.
- K Schrerrer Constant : The constant of proportionality.
- K (the Scherrer constant)depends on the how the width is determined, the shape of thecrystal, and the size distribution.
- The most common values for K are: 0.94 for FWHM (fullwidth at half maximum) of spherical crystals with cubic symmetry.

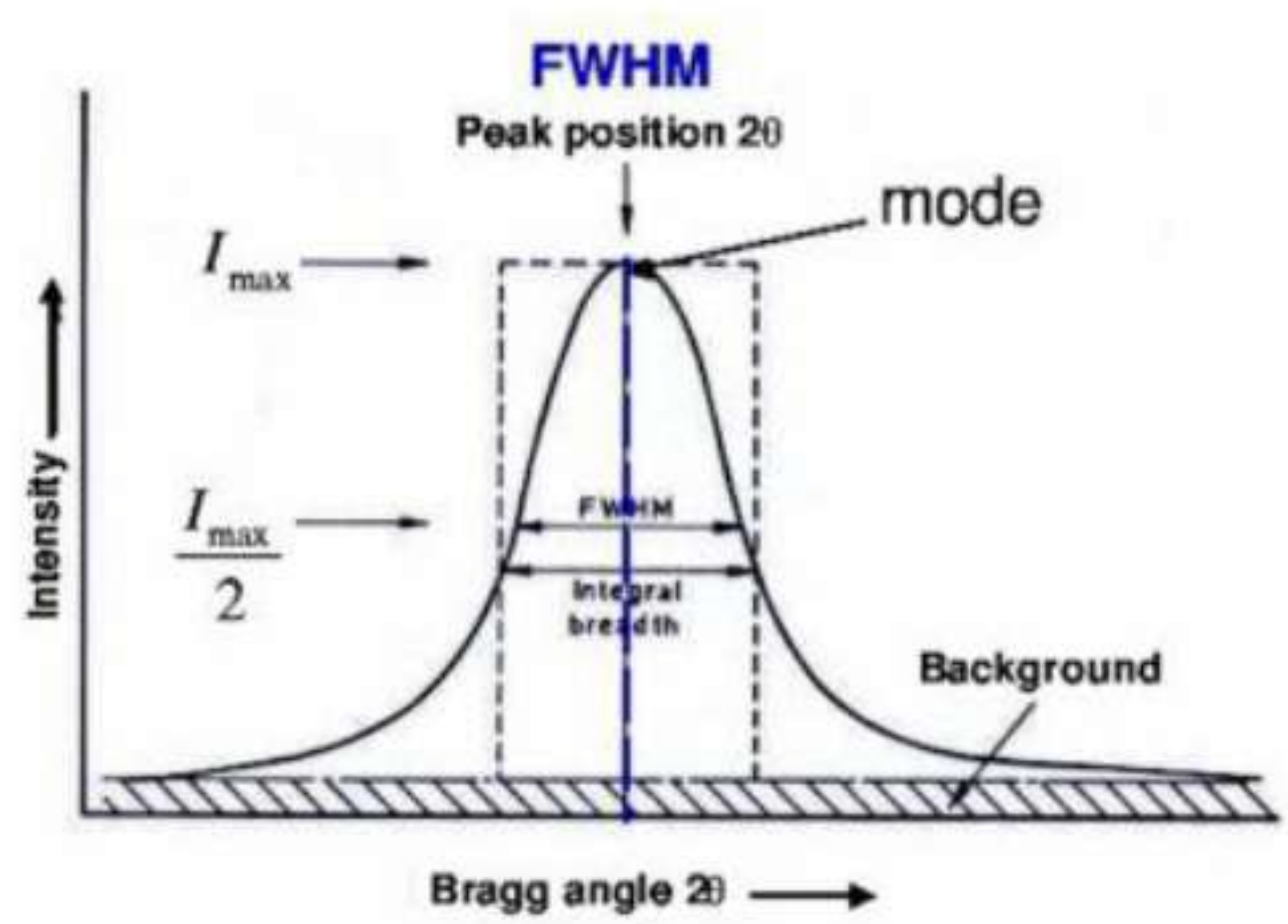


Figure 6.2: Full Width at Half Maximum (FWHM)

6.4 Atomic Force Microscoph (AFM)

Introduction

- The atomic force microscopy (AFM) is a subcategory of scanning probe microscopy (SPM).
- It uses raster-scanning tip to measure surface properties such as the local height, friction, electronic or magnetic properties, and construct a map of this data to form an image.

Principle

- The AFM measures the forces acting between a fine tip and a sample.
- The tip is attached to the free end of a spring cantilever and is brought very close to a surface.
- The inter-atomic potential developed between the tip and the sample at inter-atomic separation results in attractive or repulsive forces.
- As the tip scans the surface of the sample, the force between the tip and the sample varies which is sensed by the tip.
- The amount of force between the probe and the sample is dependent on the spring constant of the cantilever and the distance between the probe and the sample surface. This force can be characterized with Hooke's Law.

$$F = kx \tag{6.3}$$

here F = Force, k = spring constant, x = cantilever deflection

- If the spring constant of cantilever (typically, 0.1 - 1 N/m) is less than surface, cantilever bends and the deflection is monitored.

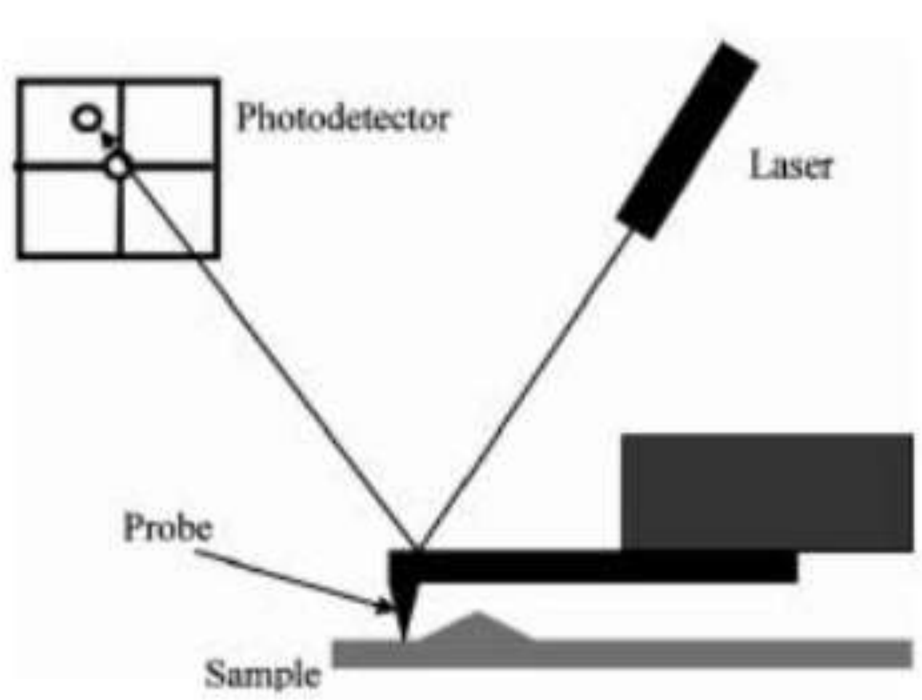


Figure 6.3: Schematic - Atomic Force Microscope

Working

- As the tip travels across the sample surface the interaction with the asperities (Surface Profile) causes up and down motion.
- These fluctuations are sourced by the interactions (electrostatic, magnetic, capillary, Van der Waals forces) between the tip and the sample.
- The displacement of the Laser beam reflected from the backside of the tip traces the surface profile.
- The laser and the photo-detector arrangement detects the position of the cantilever
- The vertical deflection measures the interaction forces while the horizontal deflection measures the lateral forces.
- Thus a three-dimensional image of the surface topography of the sample under a constant applied force (as low as nano-Newton range) is obtained.

Uses

- An AFM is used to achieve 3D topographical image of the surface on a nanoscale.
- It is a versatile and powerful tool for imaging and measuring small-scale objects such as nanoparticles, single molecules, semiconductor devices and etc., at atomic resolutions of $10^{-10}m$ or one tenth of nanometre.

6.5 X-ray Photoelectron Spectroscopy (XPS)

Introduction

- In the X-ray photo-electron spectroscopy (XPS) the sample under investigation is bombarded with photons or charges for exciting the emission of photons and charges.
- Highest available vacuum conditions are required for XPS experiments.
- Contamination is an important concern in XPS.
- it is a surface sensitive technique with a sampling depth of a few nano-meter.

Principle

- Surface analysis by XPS involved irradiation of the sample by low-energy (and mono-energetic) X-rays and the subsequent analysis of the energy of emitted electrons. Typically used x-rays are K lines of Mg (1.2536 keV) and Al (1.4866 keV).
- The penetration depth of these photons in solids is limited to a few microns. Thus, interactions take place between the incident photons are the surface atoms leading to the photoelectric emission of electrons.
- The kinetic energy (K.E.) of the emitted electrons is expressed as $KE = h\nu - (B.E. + \phi_s)$. Here $h\nu$ is the energy of the photon, B.E. is the binding energy of the atomic orbital from the electron is realeased, ϕ_s is the work-function of the spectrometer.

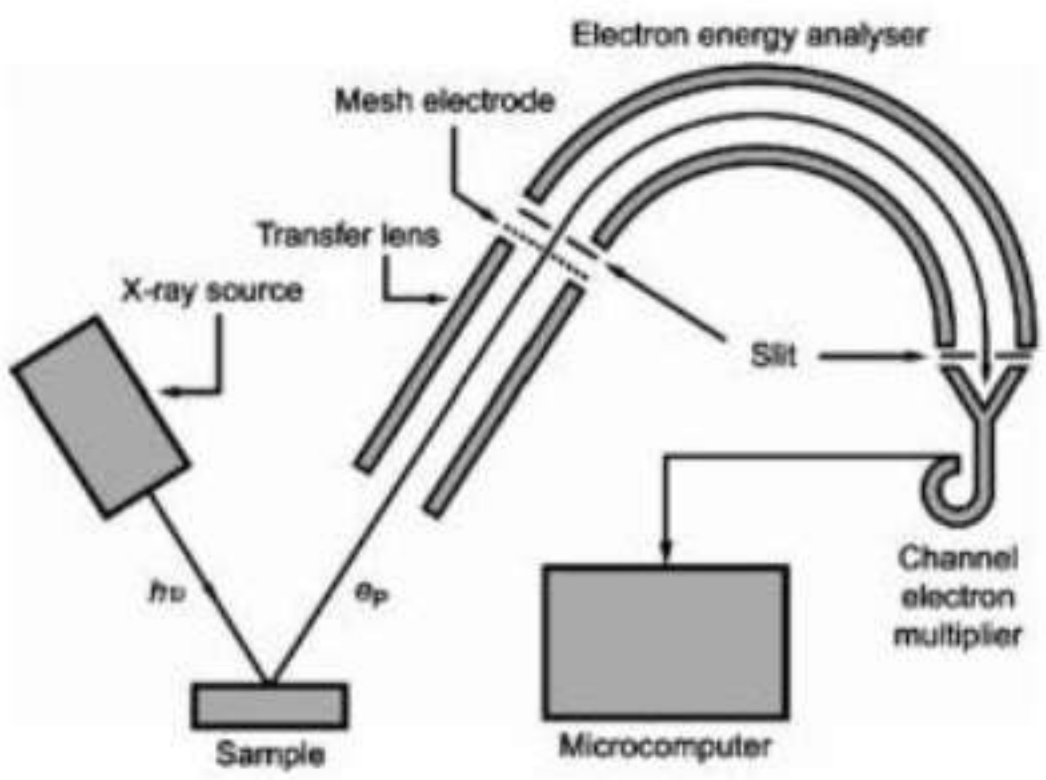


Figure 6.4: Schematic - Xray Photoelectron Spectroscopy

Construction and Working

1. A typical X-ray photoelectron spectrometer consists of an X-ray source, an electron energy analyser, and a photo-electron detector.
2. X-ray source emits monochromatized x-ray with a narrow line width of energy $h\nu$ falls on the sample and photo electrons are emitted from the sample.
3. The ejected photo-electrons are transferred to an electron energy analyser(An electrostatic hemispherical analyzer (HSA)) through the electron lens and separated according to their kinetic energy.
4. The separated electrons are subjected to channel electron multiplier which is connected to Microcomputer.
5. The transfer lens system retards the photo-electrons there by enhancing the energy resolution of the analyzer (≤ 0.1 eV).

Uses of XPS

1. Elemental composition of the surface (top 1–10 nm usually)
2. Empirical formula of pure materials
3. Elements that contaminate a surface
4. Chemical or electronic state of each element in the surface
5. Uniformity of elemental composition across the top surface (or line profiling or mapping)
6. Uniformity of elemental composition as a function of ion beam etching (or depth profiling)

6.6 Scanning Electron Microscope (SEM)

Introduction

Scanning Electron Microscope (SEM) is a type of electron microscope that scans surfaces of microorganisms that uses a beam of electrons moving at low energy to focus and scan specimens.

Principle

- The Scanning electron microscope works on the principle of applying kinetic energy to produce signals on the interaction of the electrons with the specimen.
- These electrons are secondary, backs-cattered, diffracted backs-cattered electrons and auger electrons are used to view crystallized elements and photons.

Working

1. A SEM consists of an field emission electron gun and a series of electromagnetic lenses and apertures. The electron beam emitted from an electron gun is condensed to a fine probe for surface scanning.
2. An electron gun—emits electrons that get accelerated by an applied voltage. Magnetic lenses converge the stream of electrons into a focused beam, which then hits the sample surface in a fine, precise spot.
3. The electron beam then scans the surface of the specimen in a rectangular raster.
4. The user can increase the magnification by reducing the size of the scanned area on the specimen.

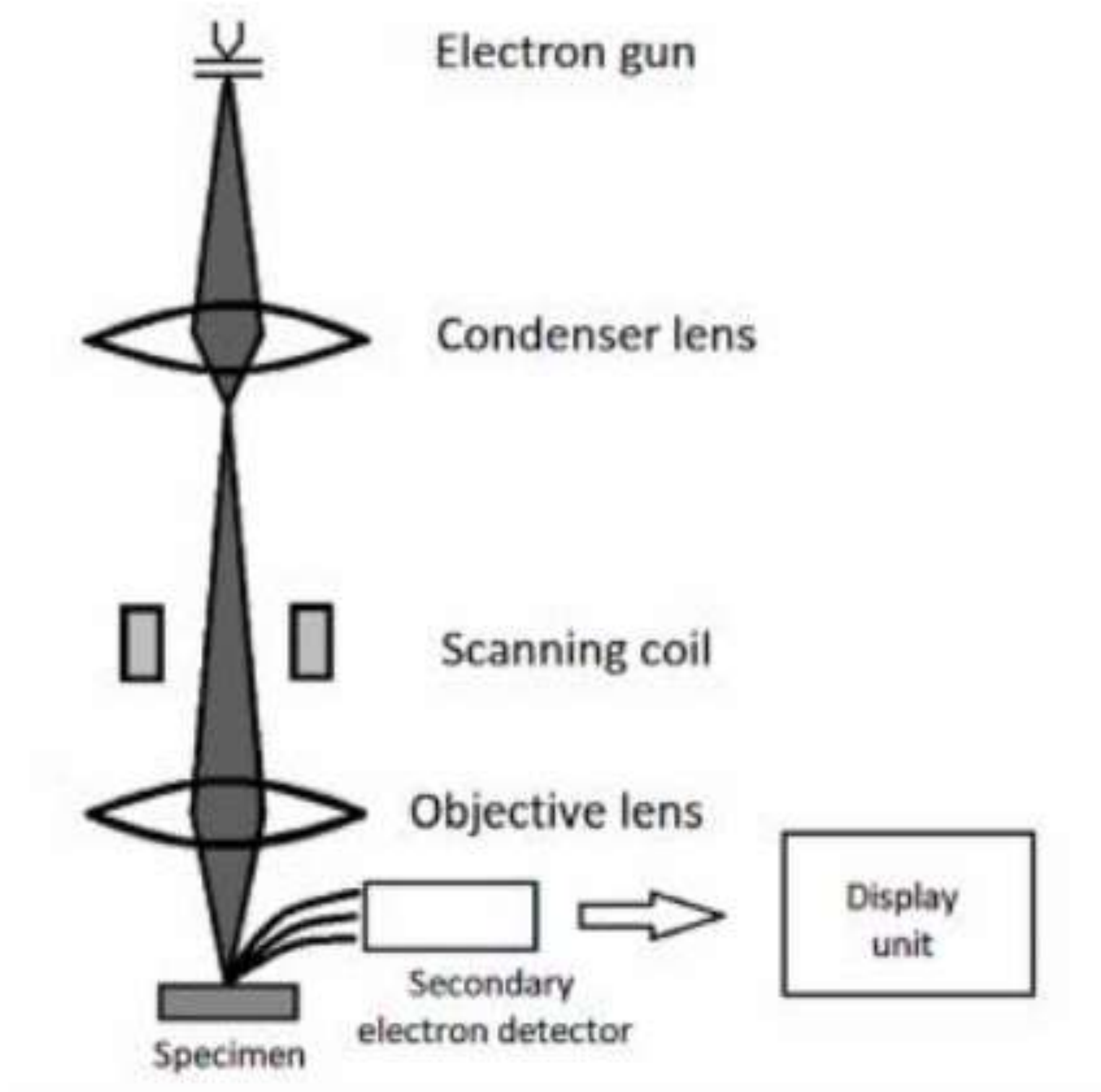


Figure 6.5: Schematic -Scanning Electron Microscope

5. Detectors collect the backscattered and secondary electrons (SE). The corresponding signals are measured and the values are mapped as variations in brightness on the image display.

6. The secondary electrons are more frequently used as read-out signal. They highlight the topography of the sample surface: bright areas represent edges while dark regions represent recesses.

Applications of SEM

1. For investigation of virus structure.

2. 3D tissue imaging

3. Insect, spore other microorganism or cellular component visualize.

4. Geologist often use same to learn more about crystalline structure.

5. Industries including Microelectronics, medical devices, food processing, all use SEM as a way to examine the surface composition of component and products.

6.7 Transmission Electron Microscope

Introduction

Transmission electron microscopes (TEM) are microscopes that use a particle beam of electrons to visualize specimens and generate a highly-magnified image. TEMs can magnify objects up to 2 million times.

Principle

It is based on the principle of diffraction of electrons while transmitting through the thin sample specimen.

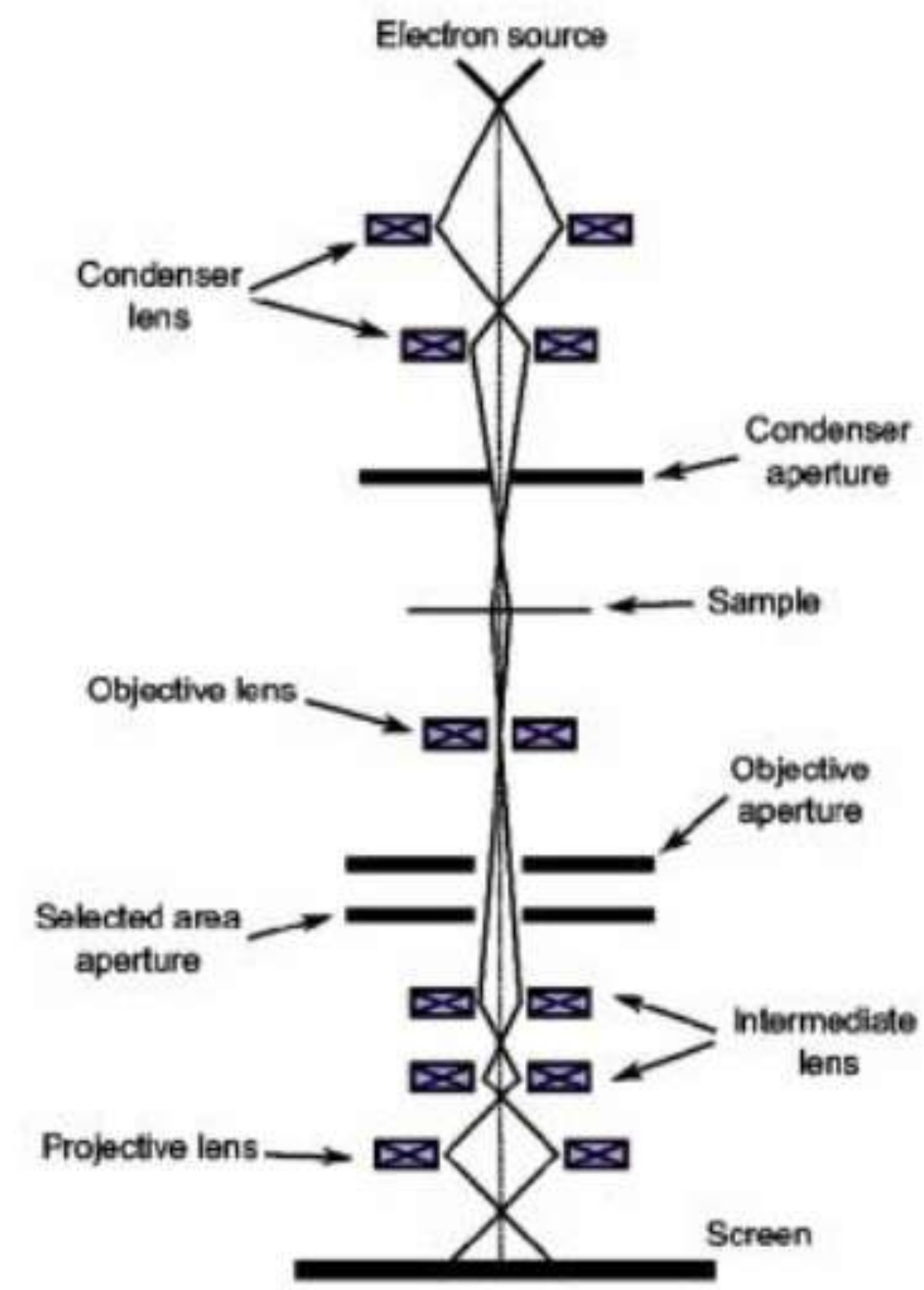


Figure 6.6: Schematic -Transmission Electron Microscope

Construction and Working

- An extremely thin sample is required for scanning in TEM from which electron beam is passed through rendering its interaction with the sample as a result of which image is produced.

- This image can be magnified and focused on the device used for imaging like fluorescent screen, photographic film, CCD Camera.

- A heated tungsten filament in the electron gun produces electrons that get focus on the specimen by the condenser lenses.

- Magnetic lenses are used to focus the beam of electrons on the specimen. The vacuum in the column tube allows electrons to produce a clear image without collision with any air molecules which may deflect them.

- On reaching the specimen, the specimen scatters the electrons focusing them on the magnetic lenses forming a large clear image, and if it passes through a fluorescent screen it forms a polychromatic image.

- The denser the specimen, the more the electrons are scattered forming a darker image because fewer electron reaches the screen for visualization while thinner, more transparent specimens appear brighter.

Applications of TEM TEM is used

1. In a wide variety of fields From Biology, Microbiology, Nanotechnology, forensic studies.
2. To visualize and study the cell structures of bacteria, viruses, and fungi and also shapes and sizes of microbial cell organelles.
3. To study and differentiate between plant and animal cells.
4. It is also used in nanotechnology to study nanoparticles such as ZnO nanoparticles.
5. It is used to detect and identify fractures, damaged microparticles which further enables repair mechanisms of the particles.

Difference Between TEM and SEM

The differences between the Transmission Electron Microscope and Scanning Electron Microscope is as given below.

TEM	SEM
Electron beam passes through thin sample.	Electron beam scans over surface of sample.
Specially prepared thin samples, which are supported on TEM grids.	Samples can be any thickness and is mounted on an aluminium stub.
Specimen stage halfway down column.	Specimen stage in the chamber at the bottom of the column.
Image shown on fluorescent screen.	Image shown on TV monitor.
Image is a two-dimensional projection of the sample.	Image is of the surface of the sample.

Figure 6.7: SEM and TEM Differences

Numerical Problems

1. X-Rays are diffracted in the first order from a crystal with d spacing $2.8 \times 10^{-10} m$ at a glancing angle 60 °. Calculate the wavelength of X-rays.
2. Determine the Crystallite size given the Wavelength of X-Rays 1.541 , the Peak Width 0.3 °and peak position 23.3 °for a cubic crystal given K = 0.94. Ans 28.25 nm
3. Determine the Crystallite size given the Wavelength of X-Rays 1.00 , the Peak Width 0.5 °and peak position 25 °for a cubic crystal given K = 0.94. Ans 11.04 nm

Model Questions

1. Write a Note on Nanomaterials.
2. Explain the types of nanocomposites. Mention the applications of nanocomposites.
3. State and derive Braggs'law of X-ray Diffraction.
4. With a neat sketch explain the construction and working of X-ray Diffractometer.
5. Mention Scherrers Equation. Explain the calculation of crystallite size using Scherrer's equation.
6. Explain the construction and working of X-ray Photoelectron Spectrometer with the help of a neat sketch.
7. Describe the construction and working of Scanning Electron Microscope using necessary diagrams.
8. Explain the principle, Construction and working of Transmission Electron Microscope with neat sketch.
9. Discuss the Scanning Tunneling Electron Microscope.
10. Distinguish between TEM and SEM.
11. Mention the applications of TEM and SEM
12. State the applications of XRD and XPS.
13. Demonstrate STEM is based on the combined principles of TEM and SEM.